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On Continuity of Soft Mappings

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Abstract: In this paper, we give some new characterizations of soft continuity, soft openness and soft closedness of soft mappings. We study restriction of a soft mapping and generalize the pasting lemma to the soft topological spaces. We also investigate the behavior of soft separation axioms under the soft continuous, open and closed mappings.

Keywords: Soft set, soft continuity, soft mapping, soft separation axiom

1 Introduction

In 1999, Molodtsov [20] introduced the concept of soft sets, which is a completely new mathematical tool to deal with uncertainties while modeling problems in engineering physics, computer science, economics, social sciences and medical sciences. Soft set theory has rich potential for practical applications in several domains, a few of which are indicated by Molodtsov in [20] and [21]. Maji et al. [16,17] described an application of soft set theory to a decision-making problem and gave the operations of soft sets and their properties. Chen et al. [3] improved the work of Maji et al. [17]. Pei and Miao [22] investigated the relationships between soft sets and information systems. They showed that soft sets are a class of special information systems.

Theoretical studies of soft set theory has also been studied by some authors. Aktaş and Çağman [1] have introduced the notion of soft groups. Jun [11] applied soft sets to the theory of BCK/BCI-algebras and introduced the concept of soft BCK/BCI-algebras. Jun and Park [12] and Jun et al. [13,14] reported the applications of soft sets in ideal theory of BCK/BCI-algebras and d-algebras. Feng et al. [7] defined soft semirings and several related notions to establish a connection between soft sets and semirings. Sun et al. [24] presented the definition of soft modules and construct some basic properties using modules and Molodtsov's definition of soft sets. On the other hand, topological structures of soft sets introduced by Shabir and Naz [23]. Shabir and Naz also studied separation axioms in soft topological spaces. Min [19] presented some new results deal with soft separation axioms. Zorlutuna et al. [27] studied some concepts in soft topological spaces such as interior point, interior, neighborhood, continuity, and compactness. Aygünoğlu and Aygün [2] introduced soft product topology and generalized Alexander subbase theorem and Tychonoff theorem to the soft topological spaces. Some other studies on soft topological spaces can be listed as [6,9,25,26]. More recently, Chen [4] defined soft semi-open sets and studied related properties in soft topological spaces and Georgiou [8] presented new definitions, characterizations, and results concerning separation axioms, convergence and defined soft θ-topology, and soft θ-continuity.

The purpose of this paper is to study some new properties of soft continuous mappings. We first give, as a preliminaries, some well-known results in soft set theory such as set theoretic operation and the properties of image and preimage of soft sets under soft mappings. We generalize the pasting lemma in line with soft set theory. We give some new characterizations of soft continuous, soft open and soft closed mappings and also soft homeomorphisms. Lastly, we observe the behavior some soft separation axioms under the soft continuous, open and closed mappings.

2 Preliminaries

Throughout this paper, X refers to an initial universe, E is the set of all parameters for X.

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Definition 1.[16] *A pair (F,A) is called a soft set over the universe X, where F is a mapping given by $F : A \to P(X)$ and $A \subseteq E$.*

According to [18], any soft set (F,A) can be extended to a soft set of type (F,E), where $F(e) \neq \varnothing$ for all $e \in A$ and $F(e) = \varnothing$ for all $e \in E \backslash A$. From now on, $S(X,E)$ denotes the family of all soft sets over X.

Definition 2.[5] *Let (F,A), $(G,B) \in S(X,E)$.*

(1) (F,A) is a soft subset of (G,B), denoted by $(F,A)\widetilde{\subseteq}(G,B)$, if $F(e) \subseteq G(e)$ for each $e \in E$.

(2) (F,A) and (G,B) are said to be soft equal, denoted by $(F,A) = (G,B)$ if $(F,A)\widetilde{\subseteq}(G,B)$ and $(G,B)\widetilde{\subseteq}(F,A)$.

(3) Union of (F,A) and (G,B) is a soft set (H,C), where $C = A \cup B$ and $H(e) = F(e) \cup G(e)$ for each $e \in E$. This relationship is written as $(F,A)\widetilde{\cup}(G,B) = (H,C)$.

(4) Intersection of (F,A) and (G,B) is a soft set (H,C), where $C = A \cap B$ and $H(e) = F(e) \cap G(e)$ for each $e \in E$. This relationship is written as $(F,A)\widetilde{\cap}(G,B) = (H,C)$.

*Remark.*In above Definition 2(4), intersection for soft sets is a partial operation and may cause difficulties, when $A \cap B$ is an empty set [10]. However, since we study on the collection of all soft sets defined over X with a fixed parameters set E, there is no such problem in here.

Definition 3.[10] *The complement of a soft set (F,A), denoted by $(F,A)^c$, is defined by $(F,A)^c = (F^c,A)$. $F^c : A \to P(X)$ is a mapping given by $F^c(\alpha) = X - F(\alpha)$, $\forall \alpha \in E$. F^c is called the soft complement function of F. Clearly, $(F^c)^c$ is the same as F and $((F,A)^c)^c = (F,A)$.*

Definition 4.[5] *Let $(F,E) \in S(X,E)$. (F,E) is said to be a null soft set, denoted by Φ, if $\forall e \in E$, $F(e) = \varnothing$.*

Definition 5.[5] *Let $(F,E) \in S(X,E)$. (F,E) is said to be an absolute soft set, denoted by $\widetilde{X}$, if $\forall e \in E$, $F(e) = X$.*

Clearly, $(\widetilde{X})^c = \Phi$ and $\Phi^c = \widetilde{X}$.

Definition 6.[27] *Let I be an arbitrary index set and $\{(F_i,E)\}_{i\in I} \subseteq S(X,E)$.*

(1) The union of these soft sets is the soft set (G,E), where $G(e) = \cup_{i\in I}F_i(e)$ for each $e \in E$. We write $\widetilde{\cup}_{i\in I}(F_i,E) = (G,E)$.

(2) The intersection of these soft sets is the soft set (H,E), where $H(e) = \cap_{i\in I}F_i(e)$ for all $e \in E$. We write $\widetilde{\cap}_{i\in I}(F_i,E) = (H,E)$.

Proposition 1.(see [2]) *Let I be an arbitrary index set, $\{(F_i,E)\}_{i\in I} \subseteq S(X,E)$ and $(F,E) \in S(X,E)$. Then $(F,E)\widetilde{\cap}(\widetilde{\cup}_{i\in I}(F_i,E)) = \widetilde{\cup}_{i\in I}[(F,E)\widetilde{\cap}(F_i,E)]$.*

Proposition 2.*Let (F,E), $(G,E) \in S(X,E)$, then*

(1) $((F,E)\widetilde{\cup}(G,E))^c = (F,E)^c\widetilde{\cap}(G,E)^c$ [23].

(2) $((F,E)\widetilde{\cap}(G,E))^c = (F,E)^c\widetilde{\cup}(G,E)^c$ [23].

(3) $(F,E)\widetilde{\cap}\widetilde{X} = (F,E)$ [16].

(4) $(F,E)\widetilde{\subseteq}(G,E)$ iff $(G,E)^c\widetilde{\subseteq}(F,E)^c$ [5].

Definition 7.[15] *Let $S(X,E)$ and $S(Y,K)$ be families of soft sets. Let $u : X \to Y$ and $p : E \to K$ be mappings. Then a mapping $f_{pu} : S(X,E) \to S(Y,K)$ is defined as:*

(1) Let (F,A) be a soft set in $S(X,E)$. The image of (F,A) under f_{pu}, written as $f_{pu}(F,A) = (f_{pu}(F), p(A))$, is a soft set in $S(Y,K)$ such that

$$f_{pu}(F)(k) = \begin{cases} \bigcup\limits_{e\in p^{-1}(k)\cap A} u(F(e)), & p^{-1}(k) \cap A \neq \varnothing \\ \varnothing, & \text{otherwise} \end{cases}$$

for all $k \in K$.

(2) Let (G,B) be a soft set in $S(Y,K)$. The inverse image of (G,B) under f_{pu}, written as $f^{-1}_{pu}(G,B) = (f^{-1}_{pu}(G), p^{-1}(B))$, is a soft set in $S(X,E)$ such that

$$f^{-1}_{pu}(G)(e) = \begin{cases} u^{-1}(G(p(e))), & p(e) \in B \\ \varnothing, & \text{otherwise} \end{cases}$$

for all $e \in E$.

The soft mapping f_{pu} is called surjective if p and u are surjective. The soft mapping f_{pu} is called injective if p and u are injective.

Theorem 1.[15] *Let $\{(F_i,E)\}_{i\in I} \subseteq S(X,E)$ and $\{(G_i,K)\}_{i\in I} \subseteq S(Y,K)$. Then for a soft mapping $f_{pu} : S(X,E) \to S(Y,K)$, the following are true.*

(1) If $(F_1,E)\widetilde{\subseteq}(F_2,E)$, then $f_{pu}(F_1,E)\widetilde{\subseteq} f_{pu}(F_2,E)$.

(2) If $(G_1,K)\widetilde{\subseteq}(G_2,K)$, then $f^{-1}_{pu}(G_1,K)\widetilde{\subseteq} f^{-1}_{pu}(G_2,K)$.

(3) $f_{pu}((F_1,E)\widetilde{\cup}(F_2,E)) = f_{pu}(F_1,E)\widetilde{\cup} f_{pu}(F_2,E)$. In general $f_{pu}(\widetilde{\cup}_i(F_i,E)) = \widetilde{\cup}_i f_{pu}(F_i,E)$.

(4) $f^{-1}_{pu}((G_1,K)\widetilde{\cap}(G_2,K)) = f^{-1}_{pu}(G_1,K)\widetilde{\cap} f^{-1}_{pu}(G_2,K)$.

(5) $f^{-1}_{pu}((G_1,K)\widetilde{\cup}(G_2,K)) = f^{-1}_{pu}(G_1,K)\widetilde{\cup} f^{-1}_{pu}(G_2,K)$.

Theorem 2.[27] *For a soft mapping $f_{pu} : S(X,E) \to S(Y,K)$, the following are true.*

(1) $f^{-1}_{pu}((G,K)^c) = (f^{-1}_{pu}(G,K))^c$ for every $(G,K) \in S(Y,K)$

(2) $f_{pu}(f^{-1}_{pu}(G,K))\widetilde{\subseteq}(G,K)$ for every $(G,K) \in S(Y,K)$. If f_{pu} is surjective, the equality holds.

(3) $(F,E)\widetilde{\subseteq} f^{-1}_{pu}(f_{pu}(F,E))$ for every $(F,E) \in S(X,E)$. If f_{pu} is injective, the equality holds.

Definition 8.[23] *Let $\tau \subseteq S(X,E)$. Then τ is called a soft topology on X if*

(1) Φ, $\widetilde{X}$ belong to τ,

(2) the union of any number of soft sets in τ belongs to τ,

(3) the intersection of any two soft sets in τ belongs to τ.

The triplet (X,τ,E) is called soft topological space over X. The members of τ are called soft open sets in X and complements of their are called soft closed sets in X. The family of all soft closed sets in X is denoted by τ'.

Definition 9.[23] *Let (F,E) be a soft set over X and Z be a non-empty subset of X. Then the sub soft set of (F,E) over Z denoted by $({}^ZF,E)$, is defined as follows ${}^ZF(\alpha) = Z \cap F(\alpha)$, for all $\alpha \in E$. In other words $({}^ZF,E) = \widetilde{Z}\widetilde{\cap}(F,E)$.*

Definition 10.[23] *Let (X,τ,E) be a soft topological space over X and Z be a non-empty subset of X. Then*

$$\tau_Z = \{({}^ZF,E)|(F,E) \in \tau\}$$

is said to be the soft relative topology on Z and (Z,τ_Z,E) is called a soft subspace of (X,τ,E).

Theorem 3.[23] *Let (Z,τ_Z,E) be a soft subspace of a soft topological space (X,τ,E) and (F,E) be a soft open set in Z. If $\widetilde{Z} \in \tau$, then $(F,E) \in \tau$.*

Theorem 4.*Let (Z,τ_Z,E) be a soft subspace of a soft topological space (X,τ,E) and (F,E) be a soft closed set in Z. If $\widetilde{Z} \in \tau'$, then $(F,E) \in \tau'$.*

*Proof.*It can be proved directly.

Theorem 5.[27] *A soft mapping $f_{pu} : (X,\tau,E) \to (Y,\upsilon,K)$ is soft continuous iff $f_{pu}^{-1}(G,K) \in \tau$ for every $(G,K) \in \upsilon$.*

3 Some properties of soft mappings

In this section, firstly we study on some constructings soft continuous mappings from one soft topological space to another. Secondly we give some new characterizations of soft continuous, soft open and soft closed mappings.

Definition 11.*Let $f_{pu} : S(X,E) \to S(Y,K)$ be a soft mapping and $Z \subseteq X$. Then the restriction of f_{pu} to $S(Z,E)$ is the soft mapping $f_{pu}|_{S(Z,E)}$ from $S(Z,E)$ to $S(Y,K)$ which defined by the functions $p : E \to K$ and $u|_Z : Z \to Y$ where $u|_Z$ is the restriction of u to Z.*

Proposition 3.*Let $f_{pu} : S(X,E) \to S(Y,K)$ be a soft mapping and $Z \subseteq X$. Then for all $(G,K) \in S(Y,K)$,*

$$(f_{pu}|_{S(Z,E)})^{-1}(G,K) = f_{pu}^{-1}(G,K)\widetilde{\cap}\widetilde{Z}$$

*Proof.*This follows from the equality $(u|_Z)^{-1}(Y') = u^{-1}(Y') \cap Z$ for all $Y' \subseteq Y$.

Theorem 6.*If $f_{pu} : (X,\tau,E) \to (Y,\upsilon,K)$ is soft continuous, then $f_{pu}|_{S(Z,E)} : (Z,\tau_Z,E) \to (Y,\upsilon,K)$ is soft continuous for every $Z \subseteq X$.*

*Proof.*This follows from Proposition 3 and definition of soft relative topology.

Theorem 7.*Let (X,τ,E) and (Y,υ,K) be any soft topological spaces.*

(1) Let $\{Z_i\}_{i\in I}$ be a family of subsets of X with $\widetilde{Z}_i$' s are soft open sets in X and $\widetilde{X} = \widetilde{\cup}_{i\in I}\widetilde{Z}_i$. Then the soft mapping $f_{pu} : (X,\tau,E) \to (Y,\upsilon,K)$ is soft continuous if and only if $f_{pu}|_{S(Z_i,E)} : (Z_i,\tau_{Z_i},E) \to (Y,\upsilon,K)$ is soft continuous for every $i \in I$.

(2) Let $\widetilde{Z}_1,\widetilde{Z}_2,...,\widetilde{Z}_n$ be soft closed sets in X and $\widetilde{X} = \cup_{i=1}^n\widetilde{Z}_i$. Then the soft mapping $f_{pu} : (X,\tau,E) \to (Y,\upsilon,K)$ is soft continuous if and only if $f_{pu}|_{S(Z_i,E)} : (Z_i,\tau_{Z_i},E) \to (Y,\upsilon,K)$ is soft continuous for every $i = 1,2,...,n$.

Proof.(1) ($\Rightarrow$) This is Theorem 6.

($\Leftarrow$) Let (G,K) be a soft open set in Y. Since for all $i \in I$, $f_{pu}|_{S(Z_i,E)}$ is soft continuous, $(f_{pu}|_{S(Z_i,E)})^{-1}(G,K)$ is soft open set in Z_i. Again since $\widetilde{Z}_i$ is soft open in X, by Theorem 3, $(f_{pu}|_{S(Z_i,E)})^{-1}(G,K)$ is soft open set in X. Therefore,
$f_{pu}^{-1}(G,K) = f_{pu}^{-1}(G,K)\widetilde{\cap}\widetilde{X} = f_{pu}^{-1}(G,K)\widetilde{\cap}(\widetilde{\cup}_{i\in I}\widetilde{Z}_i) = \widetilde{\cup}_{i\in I}(f_{pu}^{-1}(G,K)\widetilde{\cap}\widetilde{Z}_i) = \widetilde{\cup}_{i\in I}(f_{pu}|_{S(Z_i,E)})^{-1}(G,K)$ is soft open in X. This completes the proof.

(2) It can be proved in similar way.

Now we will generalize the pasting lemma to soft mappings, which is one of the most important theorems in classical topological spaces. Because in order to have a continuous function on whole space X, one needs to combine functions which are defined on subsets of X and agree on the overlapping part of their domains.

Theorem 8.*(The pasting lemma) Let $\widetilde{X} = \widetilde{Z}\widetilde{\cup}\widetilde{W}$, where $\widetilde{Z}$ and $\widetilde{W}$ are soft open in X. Let $f_{p_1u_1} : (Z,\tau_Z,E) \to (Y,\upsilon,K)$ and $f_{p_2u_2} : (W,\tau_W,E) \to (Y,\upsilon,K)$ be soft continuous mappings where $p_1 = p_2 : E \to K$, $u_1 : Z \to Y$ and $u_2 : W \to Y$ are functions. If $u_1(x) = u_2(x)$ for every $x \in Z \cap W$, then $f_{p_1u_1}$ and $f_{p_2u_2}$ combine to give a soft continuous mapping $f_{pu} : (X,\tau,E) \to (Y,\upsilon,K)$ defined by the functions $p = p_1 = p_2$ and $u(x) = u_1(x)$ if $x \in Z$, and $u(x) = u_2(x)$ if $x \in W$.*

*Proof.*Let (G,K) be a soft open set in Y. It is easily seen that $f_{pu}^{-1}(G,K) = f_{p_1u_1}^{-1}(G,K)\widetilde{\cup}f_{p_2u_2}^{-1}(G,K)$. Since $f_{p_1u_1}$ is soft continuous, $f_{p_1u_1}^{-1}(G,K)$ is soft open in Z and therefore soft open in X. Similarly, $f_{p_2u_2}^{-1}(G,K)$ soft open in W and therefore soft open in X. Their union $f_{pu}^{-1}(G,K)$ is thus soft open in X.

In the upper Theorem, if $\widetilde{Z}$ and $\widetilde{W}$ are taken as soft closed in X, then we get also same result.

From now on, we give a set of new characterizations of soft continuous, soft open and soft closed mappings.

Definition 12.*Let (X,τ,E) be a soft topological space and let $(F,E) \in S(X,E)$.*

(1) The soft closure of (F,E) is the soft set $\overline{(F,E)} = \widetilde{\cap}\{(G,E) : (G,E) \in \tau'$ and $(F,E)\widetilde{\subseteq}(G,E)\}$ [23].

(2) The soft interior of (F,E) is the soft set $(F,E)^\circ = \widetilde{\cup}\{(G,E) : (G,E) \in \tau$ and $(G,E)\widetilde{\subseteq}(F,E)\}$ [27].

(3) The soft boundary of (F,E) is the soft set $\partial(F,E) = \overline{(F,E)}\widetilde{\cap}\,\overline{(F,E)^c}$ [9].

Theorem 9. *Let (X,τ,E) be a soft topological space and let $(F,E),(G,E)\in S(X,E)$. Then*

(1) (F,E) is soft closed iff $(F,E)=\overline{(F,E)}$ [23].
(2) If $(F,E)\tilde{\subseteq}(G,E)$, then $\overline{(F,E)}\tilde{\subseteq}\overline{(G,E)}$ [23].
(3) (F,E) is soft open iff $(F,E)=(F,E)^{\circ}$ [27].
(4) If $(F,E)\tilde{\subseteq}(G,E)$, then $(F,E)^{\circ}\tilde{\subseteq}(G,E)^{\circ}$ [27].
(5) $\left(\overline{(F,E)}\right)^{c}=((F,E)^{c})^{\circ}$ [27].
(6) $((F,E)^{\circ})^{c}=\overline{((F,E)^{c})}$ [27].
(7) (F,E) is soft closed iff $\partial(F,E)\tilde{\subseteq}(F,E)$ [9].
(8) $(\partial(F,E))^{c}=(F,E)^{\circ}\tilde{\cup}((F,E)^{c})^{\circ}$ [9].

Corollary 1. $\overline{(F,E)}=\partial(F,E)\tilde{\cup}(F,E)$

Theorem 10. *Let $f_{pu}:(X,\tau,E)\rightarrow(Y,\upsilon,K)$ be a soft mapping. Then the following statements are equivalent.*

(1) f_{pu} is soft continuous;
(2) $f_{pu}^{-1}(G,K)\in\tau'$, $\forall(G,K)\in\upsilon'$;
(3) $\overline{f_{pu}^{-1}(G,K)}\tilde{\subseteq}f_{pu}^{-1}\left(\overline{(G,K)}\right)$, $\forall(G,K)\in S(Y,K)$;
(4) $\partial(f_{pu}^{-1}(G,K))\tilde{\subseteq}f_{pu}^{-1}(\partial(G,K))$, $\forall(G,K)\in S(Y,K)$;
(5) $f_{pu}(\partial(F,E))\tilde{\subseteq}\partial(f_{pu}(F,E))$, $\forall(F,E)\in S(X,E)$;
(6) $f_{pu}\left(\overline{(F,E)}\right)\tilde{\subseteq}\overline{f_{pu}(F,E)}$, $\forall(F,E)\in S(X,E)$;
(7) $f_{pu}^{-1}((G,K)^{o})\tilde{\subseteq}\left(f_{pu}^{-1}(G,K)\right)^{o}$, $\forall(G,K)\in S(Y,K)$.

Proof. (1)⇔(2) Theorem 6.3 in [27].

(2)⇒(3) Let (G,K) be a soft set set over Y. Then $(G,K)\tilde{\subseteq}\overline{(G,K)}$. Therefore, we have $f_{pu}^{-1}(G,K)\tilde{\subseteq}f_{pu}^{-1}\left(\overline{(G,K)}\right)$ and so, by using (2), we obtain that $\overline{f_{pu}^{-1}(G,K)}\tilde{\subseteq}\overline{f_{pu}^{-1}\left(\overline{(G,K)}\right)}\tilde{\subseteq}f_{pu}^{-1}\left(\overline{(G,K)}\right)$. This shows that $\overline{f_{pu}^{-1}(G,K)}\tilde{\subseteq}f_{pu}^{-1}\overline{(G,K)}$.

(3)⇒(4) Let (G,K) be a soft set over Y. By (3), Theorem 1(4) and Theorem 2(1), $\partial\left(f_{pu}^{-1}(G,K)\right)=\overline{f_{pu}^{-1}(G,K)}\tilde{\cap}\overline{\left(f_{pu}^{-1}(G,K)\right)^{c}}\tilde{\subseteq}f_{pu}^{-1}\left(\overline{(G,K)}\right)\tilde{\cap}f_{pu}^{-1}\left(\overline{(G,K)^{c}}\right)=f_{pu}^{-1}\left(\overline{(G,K)}\tilde{\cap}\overline{(G,K)^{c}}\right)=f_{pu}^{-1}(\partial(G,K))$ and hence we have $\partial\left(f_{pu}^{-1}(G,K)\right)\tilde{\subseteq}f_{pu}^{-1}(\partial(G,K))$.

(4)⇒(5) Let (F,E) be a soft set over X. Then for $f_{pu}(F,E)\in S(Y,K)$, by (4) $\partial\left(f_{pu}^{-1}(f_{pu}(F,E))\right)\tilde{\subseteq}f_{pu}^{-1}(\partial(f_{pu}(F,E)))$ and so $\partial(F,E)\tilde{\subseteq}f_{pu}^{-1}(\partial(f_{pu}(F,E)))$. Therefore, we have $f_{pu}(\partial(F,E))\tilde{\subseteq}\partial(f_{pu}(F,E))$.

(5)⇒(4) Let (G,K) be a soft set over Y. Then for $f_{pu}^{-1}(G,K)\in S(X,E)$, by (5) $f_{pu}\left(\partial(f_{pu}^{-1}(G,K))\right)\tilde{\subseteq}\partial(f_{pu}\left(f_{pu}^{-1}(G,K)\right))$ and so $f_{pu}\left(\partial(f_{pu}^{-1}(G,K))\right)\tilde{\subseteq}\partial(G,K)$.
Therefore, we have $\partial(f_{pu}^{-1}(G,K))\tilde{\subseteq}f_{pu}^{-1}(\partial(G,K))$.

(4)⇒(2) Let (G,K) be a soft closed set in Y. Then $\partial(G,K)\tilde{\subseteq}(G,K)$ and $f_{pu}^{-1}(\partial(G,K))\tilde{\subseteq}f_{pu}^{-1}(G,K)$. By (4), we have $\partial(f_{pu}^{-1}(G,K))\tilde{\subseteq}f_{pu}^{-1}(G,K)$. This shows that $f_{pu}^{-1}(G,K)$ is soft closed set in X.

(2)⇒(6) Let (F,E) be a soft set over X. Since $(F,E)\tilde{\subseteq}f_{pu}^{-1}(f_{pu}(F,E))\tilde{\subseteq}f_{pu}^{-1}\left(\overline{f_{pu}(F,E)}\right)\in\tau'$, we have $\overline{(F,E)}\tilde{\subseteq}f_{pu}^{-1}\left(\overline{f_{pu}(F,E)}\right)$. By Theorem 1 and Theorem 2, we get $f_{pu}\left(\overline{(F,E)}\right)\tilde{\subseteq}\overline{f_{pu}(F,E)}$.

(6)⇒(7) Let (G,K) be a soft set over Y. Then $f_{pu}^{-1}((G,K)^{c})$ is a soft set over X. From (6), Theorem 2(2) and Theorem 9(5), $f_{pu}\left(\overline{\left(f_{pu}^{-1}((G,K)^{c})\right)}\right)\tilde{\subseteq}\overline{f_{pu}\left(f_{pu}^{-1}((G,K)^{c})\right)}\tilde{\subseteq}\overline{(G,K)^{c}}=((G,K)^{o})^{c}$. Therefore, we have $\overline{f_{pu}^{-1}((G,K)^{c})}\tilde{\subseteq}f_{pu}^{-1}\left(((G,K)^{o})^{c}\right)=\left(f_{pu}^{-1}((G,K)^{o})\right)^{c}$. Since $\overline{f_{pu}^{-1}((G,K)^{c})}=\overline{\left(f_{pu}^{-1}(G,K)\right)^{c}}=\left(\left(f_{pu}^{-1}(G,K)\right)^{\circ}\right)^{c}$, by Proposition 2(4) we obtain that $f_{pu}^{-1}((G,K)^{o})\tilde{\subseteq}\left(f_{pu}^{-1}(G,K)\right)^{o}$.

(7)⇔(3) These follow from Theorem 2(1) and Theorem 9(5).

Definition 13. *Let (X,τ,E) and (Y,υ,K) be soft topological spaces. A soft mapping $f_{pu}:(X,\tau,E)\rightarrow(Y,\upsilon,K)$ is called*

(1) soft open if $f_{pu}(F,E)\in\upsilon$ for each $(F,E)\in\tau$ [2].
(2) soft closed if $f_{pu}(F,E)\in\upsilon'$ for each $(F,E)\in\tau'$.

Theorem 11. *Let $f_{pu}:(X,\tau,E)\rightarrow(Y,\upsilon,K)$ be a soft mapping. Then the following statements are equivalent.*

(1) f_{pu} is soft open;
(2) $f_{pu}((F,E)^{o})\tilde{\subseteq}(f_{pu}(F,E))^{o}$, $\forall(F,E)\in S(X,E)$;
(3) $\left(f_{pu}^{-1}(G,K)\right)^{o}\tilde{\subseteq}f_{pu}^{-1}((G,K)^{o})$, $\forall(G,K)\in S(Y,K)$;
(4) $f_{pu}^{-1}(\partial(G,K))\tilde{\subseteq}\partial(f_{pu}^{-1}(G,K))$, $\forall(G,K)\in S(Y,K)$;
(5) $f_{pu}^{-1}\left(\overline{(G,K)}\right)\tilde{\subseteq}\overline{f_{pu}^{-1}(G,K)}$, $\forall(G,K)\in S(Y,K)$.

Proof. (1)⇒(2) Let (F,E) be a soft set over X. Then $(F,E)^{o}\tilde{\subseteq}(F,E)$. By using (1), we have $f_{pu}((F,E)^{o})\tilde{\subseteq}(f_{pu}(F,E))^{o}$.

(2)⇒(3) Let (G,K) be a soft set over Y. Then $f_{pu}^{-1}(G,K)$ is a soft set over X. By (2), $f_{pu}\left(\left(f_{pu}^{-1}(G,K)\right)^{o}\right)\tilde{\subseteq}\left(f_{pu}\left(f_{pu}^{-1}(G,K)\right)\right)^{o}\tilde{\subseteq}(G,K)^{o}$. Therefore, we have $\left(f_{pu}^{-1}(G,K)\right)^{o}\tilde{\subseteq}f_{pu}^{-1}((G,K)^{o})$.

(3)⇒(4) Let (G,K) be a soft set over Y. Then by using (3) and Theorem 9(8), $\left(\partial(f_{pu}^{-1}(G,K))\right)^{c}=\left(f_{pu}^{-1}(G,K)\right)^{o}\tilde{\cup}\left(\left(f_{pu}^{-1}(G,K)\right)^{c}\right)^{\circ}\tilde{\supseteq}f_{pu}^{-1}((G,K)^{o})\tilde{\cup}f_{pu}^{-1}\left(((G,K)^{c})^{\circ}\right)=f_{pu}^{-1}((G,K)^{\circ}\tilde{\cup}((G,K)^{c})^{o})=f_{pu}^{-1}((\partial(G,K))^{c})=\left(f_{pu}^{-1}(\partial(G,K))\right)^{c}$ and so we have $f_{pu}^{-1}(\partial(G,K))\tilde{\subseteq}\partial(f_{pu}^{-1}(G,K))$.

(4)⇒(5) Let (G,K) be a soft set over Y. By (4) and Corollary 1, $f_{pu}^{-1}\left(\overline{(G,K)}\right)=f_{pu}^{-1}((G,K)\tilde{\cup}\partial(G,K))=f_{pu}^{-1}(G,K)\tilde{\cup}f_{pu}^{-1}(\partial(G,K))\tilde{\subseteq}f_{pu}^{-1}(G,K)\tilde{\cup}\partial(f_{pu}^{-1}(G,K))=\overline{f_{pu}^{-1}(G,K)}$.

(5)⇔(3) These follow from Theorem 2(1) and Theorem 9(5).

(3)⇒(1) Let (F,E) be a soft open set in X. Then for $f_{pu}(F,E) \in S(Y,K)$, by (3) $(f_{pu}^{-1}(f_{pu}(F,E)))^{\circ} \widetilde{\subseteq} f_{pu}^{-1}((f_{pu}(F,E))^{\circ})$. Again since $(F,E) = (F,E)^{\circ}$, $(F,E) \widetilde{\subseteq} (f_{pu}^{-1}(f_{pu}(F,E)))^{\circ} \widetilde{\subseteq} f_{pu}^{-1}((f_{pu}(F,E))^{\circ})$ and so $f_{pu}(F,E) \widetilde{\subseteq} (f_{pu}(F,E))^{\circ}$. This shows that f_{pu} is soft open.

Theorem 12. *Let $f_{pu} : (X,\tau,E) \to (Y,\upsilon,K)$ be a soft bijection. Then f_{pu} is soft continuous if and only if $(f_{pu}(F,E))^{\circ} \widetilde{\subseteq} f_{pu}((F,E)^{\circ})$ for every $(F,E) \in S(X,E)$.*

Proof.(⇒) Let $(F,E) \in S(X,E)$. Then for $f_{pu}(F,E) \in S(Y,K)$, $(f_{pu}(F,E))^{\circ} \widetilde{\subseteq} f_{pu}(F,E)$ and so $f_{pu}^{-1}((f_{pu}(F,E))^{\circ}) \widetilde{\subseteq} f_{pu}^{-1}(f_{pu}(F,E))$. Since f_{pu} is injective and soft continuous, $f_{pu}^{-1}((f_{pu}(F,E))^{\circ}) \widetilde{\subseteq} (F,E)^{\circ}$. Again since f_{pu} is surjective, $(f_{pu}(F,E))^{\circ} \widetilde{\subseteq} f_{pu}((F,E)^{\circ})$ as claimed.

(⇐) Let (G,K) be a soft open set in Y. Then since f_{pu} is surjective, $(G,K) = (G,K)^{\circ} = \left(f_{pu}\left(f_{pu}^{-1}(G,K)\right)\right)^{\circ}$. By using hypothesis, $(G,K) \widetilde{\subseteq} f_{pu}\left(\left(f_{pu}^{-1}(G,K)\right)^{o}\right)$. Since f_{pu} is injective, $f_{pu}^{-1}(G,K) \widetilde{\subseteq} \left(f_{pu}^{-1}(G,K)\right)^{\circ}$. This shows that $f_{pu}^{-1}(G,K)$ is soft open set in X.

Theorem 13. *A soft mapping $f_{pu} : (X,\tau,E) \to (Y,\upsilon,K)$ is soft closed if and only if $\overline{f_{pu}(F,E)} \widetilde{\subseteq} f_{pu}\left(\overline{(F,E)}\right)$ for every $(F,E) \in S(X,E)$.*

Proof. It can be proved directly.

Theorem 14. *Let $f_{pu} : (X,\tau,E) \to (Y,\upsilon,K)$ be a soft bijection. Then f_{pu} is soft closed if and only if $f_{pu}^{-1}\left(\overline{(G,K)}\right) \widetilde{\subseteq} \overline{f_{pu}^{-1}(G,K)}$ for every $(G,K) \in S(Y,K)$.*

Proof. It is similar to that of Theorem 12.

A soft mapping $f_{pu} : (X,\tau,E) \to (Y,\upsilon,K)$ is called soft homeomorphism if f_{pu} is soft continuous, soft open, surjective and injective [26]. Then we have the following Theorem.

Theorem 15. *Let $f_{pu} : (X,\tau,E) \to (Y,\upsilon,K)$ be a soft mapping. Then the following statements are equivalent.*

(1) f_{pu} is soft homeomorphism;

(2) $f_{pu}((F,E)^{o}) = (f_{pu}(F,E))^{o}, \forall (F,E) \in S(X,E)$;

(3) $\left(f_{pu}^{-1}(G,K)\right)^{o} = f_{pu}^{-1}((G,K)^{o}), \forall (G,K) \in S(Y,K)$;

(4) $f_{pu}^{-1}(\partial(G,K)) = \partial(f_{pu}^{-1}(G,K)), \forall (G,K) \in S(Y,K)$;

(5) $f_{pu}^{-1}\left(\overline{(G,K)}\right) = \overline{f_{pu}^{-1}(G,K)}, \forall (G,K) \in S(Y,K)$;

(6) $f_{pu}\left(\overline{(F,E)}\right) = \overline{f_{pu}(F,E)}, \forall (F,E) \in S(X,E)$.

4 On soft separation axioms

In this section, we investigate the behavior some separation axioms under the soft continuous, open and closed mappings. Moreover, we give some new characterizations of these.

Definition 14. *[23] Let (F,E) be a soft set over X and $x \in X$. We say that $x \in (F,E)$, read as x belongs to the soft set (F,E), whenever $x \in F(e)$ for all $e \in E$.*

Note that for any $x \in X$, $x \notin (F,E)$, if $x \notin F(e)$ for some $e \in E$.

Definition 15. *[23] Let (X,τ,E) be a soft topological space over X and $x, y \in X$ such that $x \neq y$.*

(1) If there exist soft open sets (F,E) and (G,E) such that $x \in (F,E)$ and $y \notin (F,E)$ or $y \in (G,E)$ and $x \notin (G,E)$, then (X,τ,E) is called a soft T_0-space.

(2) If there exist soft open sets (F,E) and (G,E) such that $x \in (F,E)$ and $y \notin (F,E)$ and $y \in (G,E)$ and $x \notin (G,E)$, then (X,τ,E) is called a soft T_1-space.

(3) If there exist soft open sets (F,E) and (G,E) such that $x \in (F,E)$, $y \in (G,E)$ and $(F,E) \widetilde{\cap} (G,E) = \Phi$, then (X,τ,E) is called a soft T_2-space.

Theorem 16. *If f_{pu}: $(X,\tau,E) \to (Y,\upsilon,K)$ is soft continuous injection and (Y,υ,K) is soft T_0, then (X,τ,E) is soft T_0-space.*

Proof. Suppose that (Y,υ,K) is soft T_0. For any distinct points x_1 and x_2 in X, there exists soft open sets (F,K), (G,K) in Y such that $u(x_1) \in (F,K)$, $u(x_2) \notin (F,K)$ or $u(x_1) \notin (G,K)$, $u(x_2) \in (G,K)$. Since f_{pu} is soft continuous, $f_{pu}^{-1}(F,K)$ and $f_{pu}^{-1}(G,K)$ are soft open sets in X. Moreover, it is easily seen that $x_1 \in f_{pu}^{-1}(F,K)$, $x_2 \notin f_{pu}^{-1}(F,K)$ or $x_1 \notin f_{pu}^{-1}(G,K)$, $x_2 \in f_{pu}^{-1}(G,K)$. This shows that (X,τ,E) is soft T_0.

Theorem 17. *If f_{pu}: $(X,\tau,E) \to (Y,\upsilon,K)$ is soft continuous injection and (Y,υ,K) is soft T_1, then (X,τ,E) is soft T_1-space.*

Proof. Similar to that of Theorem 16.

Theorem 18. *If f_{pu}: $(X,\tau,E) \to (Y,\upsilon,K)$ is soft continuous injection and (Y,υ,K) is soft T_2, then (X,τ,E) is soft T_2-space.*

Proof. For any pair of distinct points x_1 and x_2 in X, there exist disjoint soft open sets (F,K) and (G,K) in Y such that $u(x_1) \in (F,K)$ and $u(x_2) \in (G,K)$. Since f_{pu} is soft continuous, $f_{pu}^{-1}(F,K)$ and $f_{pu}^{-1}(G,K)$ are soft open in X containing x_1 and x_2 respectively. Moreover, it is clear that $f_{pu}^{-1}(F,K) \widetilde{\cap} f_{pu}^{-1}(G,K) = \Phi$. This shows that (X,τ,E) is soft T_2.

Theorem 19. *If f_{pu} is soft open function from a soft T_0-space (X,τ,E) onto a soft topological space (Y,υ,K), then (Y,υ,K) is soft T_0-space.*

Proof. Let y_1 and y_2 be distinct points of Y. Since u is surjective, there exist distinct points x_1 and x_2 in X such that $u(x_1) = y_1$ and $u(x_2) = y_2$. Again since (X,τ,E) is soft T_0-space, there exist soft open sets (F,E) and (G,E) in X such that $x_1 \in (F,E)$, $x_2 \notin (F,E)$ or $x_1 \notin (G,E)$,

$x_2 \in (G,E)$. Then $f_{pu}(F,E)$ and $f_{pu}(G,E)$ are soft open sets in Y. Because f_{pu} is soft open. Moreover, it is easily seen that $y_1 \in f_{pu}(F,E)$, $y_2 \notin f_{pu}(F,E)$ or $y_1 \notin f_{pu}(G,E)$, $y_2 \in f_{pu}(G,E)$. This shows that (Y,υ,K) is soft T_0-space.

Theorem 20.*If f_{pu} is soft open function from a soft T_1-space (X,τ,E) onto a soft topological space (Y,υ,K), then (Y,υ,K) is soft T_1-space.*

*Proof.*Similar to that of Theorem 19.

Theorem 21.*[26] If f_{pu} is injective soft open function from a soft T_2- space (X,τ,E) onto a soft topological space (Y,υ,K), then (Y,υ,K) is soft T_2- space.*

Definition 16.*[23] Let (X,τ,E) be a soft topological space over X, (G,E) be a soft closed set in X and $x \in X$ such that $x \notin (G,E)$. If there exist soft open sets (F_1,E) and (F_2,E) such that $x \in (F_1,E)$, $(G,E)\tilde{\subseteq}(F_2,E)$ and $(F_1,E)\tilde{\cap}(F_2,E) = \Phi$, then (X,τ,E) is called a soft regular space.*

(X,τ,E) is said to be a soft T_3-space if it is soft regular and soft T_1-space.

Theorem 22.*If f_{pu} is soft continuous and soft open bijection from a soft regular space (X,τ,E) to a soft topological space (Y,υ,K), then (Y,υ,K) is soft regular.*

*Proof.*Let $y \in Y$ and $y \notin (G,K) \in \upsilon'$. Since u is surjective, there exists $x \in X$ such that $u(x) = y$. Again since f_{pu} is soft continuous, $f_{pu}^{-1}(G,K) \in \tau'$ and $x \notin f_{pu}^{-1}(G,K)$. By soft regularity of (X,τ,E), there exist disjoint soft open sets (F_1,E) and (F_2,E) such that $x \in (F_1,E)$, $f_{pu}^{-1}(G,K)\tilde{\subseteq}(F_2,E)$. Thus, we obtain disjoint soft open sets $f_{pu}(F_1,E)$ and $f_{pu}(F_2,E)$ such that $y \in f_{pu}(F_1,E)$ and $(G,K)\tilde{\subseteq}f_{pu}(F_2,E)$. Because f_{pu} is bijective and soft open. Thus, (Y,υ,K) is soft regular.

Corollary 2.*If f_{pu} is soft continuous and soft open bijection from a soft T_3- space (X,τ,E) to a soft topological space (Y,υ,K), then (Y,υ,K) is T_3- space.*

Definition 17.*[23] Let (X,τ,E) be a soft topological space over X. (F,E) and (G,E) soft closed sets in X such that $(F,E)\tilde{\cap}(G,E) = \Phi$. If there exist soft open sets (F_1,E) and (F_2,E) such that $(F,E)\tilde{\subseteq}(F_1,E)$, $(G,E)\tilde{\subseteq}(F_2,E)$ and $(F_1,E)\tilde{\cap}(F_2,E) = \Phi$, then (X,τ,E) is called a soft normal space.*

(X,τ,E) is said to be a soft T_4-space if it is soft normal and soft T_1-space.

Theorem 23.*If f_{pu} is soft continuous and soft open bijection from a soft normal space (X,τ,E) to a soft topological space (Y,υ,K), then (Y,υ,K) is soft normal.*

*Proof.*Similar to that of Theorem 22.

Corollary 3.*If f_{pu} is soft continuous and soft open bijection from a soft T_4-space (X,τ,E) to a soft topological space (Y,υ,K), then (Y,υ,K) is soft T_4-space.*

Theorem 24.*(X,τ,E) is soft regular space if and only if for every $x \in X$ and every soft open set (F,E) with $x \in (F,E)$, there exists a soft open set (H,E) such that $x \in (H,E)\tilde{\subseteq}\overline{(H,E)}\tilde{\subseteq}(F,E)$.*

*Proof.*Suppose (X,τ,E) is soft regular, (F,E) is soft open in X and $x \in (F,E)$. Then $x \notin (F,E)^c$ and $(F,E)^c$ is a soft closed set. Hence disjoint soft open sets (H,E) and (G,E) can be found with $x \in (H,E)$ and $(F,E)^c\tilde{\subseteq}(G,E)$. Then $(G,E)^c$ is soft closed set contained in (F,E) and containing (H,E). This implies that $x \in (H,E)\tilde{\subseteq}\overline{(H,E)}\tilde{\subseteq}(F,E)$.

To prove the converse, suppose the point x and the soft closed set (G,E) not containing x are given. Then $(G,E)^c$ is a soft open set in X. By hypothesis, there is a soft open set (H,E) such that $x \in (H,E)\tilde{\subseteq}\overline{(H,E)}\tilde{\subseteq}(G,E)^c$. The soft open sets (H,E) and $\left(\overline{(H,E)}\right)^c$ are disjoint soft open sets which contain x and (G,E), respectively.

Theorem 25.*(X,τ,E) is soft normal space if and only if for every soft closed set (G,E) and every soft open set (F,E) with $(G,E)\tilde{\subseteq}(F,E)$, there exists a soft open set (H,E) such that $(G,E)\tilde{\subseteq}(H,E)\tilde{\subseteq}\overline{(H,E)}\tilde{\subseteq}(F,E)$.*

*Proof.*This proof uses exactly the same argument; one just replaces the point x by the soft set (G,E) throughout.

5 Conclusion

In the present study, we have continued to study the properties of soft continuous, soft open and soft closed mappings between soft topological spaces. We obtain new characterizations of these mappings and investigate preservation properties. We expect that results in this paper will be basis for further applications of soft mappings in soft sets theory.

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