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Soft Fuzzy Syntopogenous Spaces

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Abstract: In this paper , we introduce the conceptes of semi-topogenous (resp. topogenous)soft fuzzy order and the syntopogenous soft fuzzy structure and study many of their properties and show that there is a one-one corresponds between perfect topogenous soft fuzzy structures and soft fuzzy topological structures

Keywords: soft set - soft fuzzy set - soft fuzzy topological space - soft fuzzy topogenous order - soft fuzzy syntopogenous structure.

1 Introduction

The soft set theory of Molodstov [6] offers a general mathematical tool for dealing with uncertainly and vaguness of objects . In the last three years many structures using the soft set theory are progressing rapidly [1,2,4,11,13] . Maji et al. [8,9] proposed the concept of soft fuzzy set and developed some of their properties . In recent years, the researchers have contributed a lot towards the fuzzyfication of the soft set theory . In 1963 Csaszar [3] introduced the syntopogenous structures which are a unified theory of topologies, proximities and uniformities, and in 1983 Katsasars and Petalas [4,5] used the ideas of Csaszar and the concept of fuzzy set to introduce the fuzzy syntopogenous structure which is a generalization of the fuzzy topology, fuzzy proximity and fuzzy uniformity structures . In this paper , we study the semi -topogenous (resp. topogenous) soft fuzzy orders which is a generalization of the ordinary semi -topogenous (resp. topogenous) fuzzy orders and also study of its properties continuous functions , images and inverse images of semi -topogenous (resp. topoenous) soft fuzzy order under the continuous functions are also studied . We show that any topogenous (resp. perfect, biperfect) soft fuzzy order on U is a parameterized collection of topogenous (resp. perfect , biperfect) fuzzy orders on U. Also, we show that there is a one to one correspondence between soft fuzzy topological structures and perfect topogenous soft fuzzy structures.

2 Preliminaries

In this section , we recall the basic definitions and results of soft set and soft fuzzy set theory are which may be found in [1,2,7,8,9]

Definition 1. [7,8] Let U be a universal set, E be a set of parameters and let $A \subset E$. A pair (F_A, E) is said to be a soft fuzzy subset of U with support A, if F_A is a mapping $F_A : E \to I^U$ for which $F_A(e) \neq \underline{0}$ only for every $e \in A$ In other words, A soft fuzzy set is a parameterized collection of fuzzy sets. The collection of all soft fuzzy sets over (U,E) is denoted by $SFS(U,E)$. The soft set $\tilde{\phi} = (\phi_E, E)$ defined by

$$\phi_E : E \to I^U, \ \phi_E(e) = \underline{0} \ \forall\, e \in E$$

is called the null soft fuzzy set on U also, the universal soft fuzzy set denoted $\tilde{U} = (F_E, E)$ is defined by $F_E(e) = \underline{1}_U \forall\, e \in E$.

Definition 2. [2,8,9] Let F_A, G_B be two soft fuzzy sets. F_A is said to be a soft fuzzy subset of G_B denoted by $F_A \tilde{\leq} G_B$ if $F_A(e) \leq G_B(e)\ \forall\, e \in E$. Also, F_A and G_B are called equals denoted by $F_A \cong G_B$ if $F_A \tilde{\leq} G_B$ and $G_B \tilde{\leq} F_A$.
Union, intersection and difference between soft fuzzy sets are given as follows.

Definition 3. [7,8] Let $F_A, G_B \in SFS(U,E)$
(1) The union F_A and G_B denoted by $F_A \tilde{\vee} G_B$ is the soft fuzzy set H_c denoted by $H_C(e) = F_A(e) \vee G_B(e)\ \forall\, e \in E$, where $C = A \cup B$.
(2) The intersection of F_A and G_B denoted by $F_A \tilde{\wedge} G_B$ is the soft fuzzy set H_C denoted by $H_C(e) = F_A(e) \wedge G_B(e)\ \forall\, e \in E$ where $C = A \cap B$.

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(3) The difference $F_A - G_B$ is the soft fuzzy set H_C defined by

$$H_C(e) = F_A(e) \wedge (\underline{1}_U - G_B(e)) \ \forall\, e \in E.$$

(4) The complement of a soft fuzzy set F_A denoted by F_A^c and defined by $F_A^c = \tilde{U}_E - F_A$, i.e $F_A^c(e) = \underline{1}_U - F_A(e)\ \forall e \in E$.
Note that the support of F_A^c equals $A^c \cup \{e : e \in A, F_A(e) \neq \underline{1}_U\}$.

Theorem 1. [8,7] $(SFS(U,E), \tilde{\vee}, \tilde{\wedge}, c)$ is a deMorgan algebra.

Definition 4. [2,6,7] A soft fuzzy point is a soft fuzzy set with singlton support $\{e_0\}$ and fuzzy point image $\{x_t\}$. i.e. $F_{\{e_0\}}$ where

$$F_{\{e_0\}}(e) = \begin{cases} \{x_t\} & je = e_0 \\ \underline{0} & e \neq e_0 \end{cases}$$

for every $t \in (0,1], e \in E, x \in U$. $F_{\{e_0\}}$ is sometimes denoted by $(x_t)_{e_0}$. Also, the soft fuzzy point $(x_t)_{e_0}$ is called belongs to a soft fuzzy set F_A denoted by $(x_t)_{e_0} \tilde{\in} F_A$ if $x_t \in F_A(e_0)$ i.e $(F_A(e_0))(x) \geq t$.

Definition 5.[1,2,6] Let $SFS(U,E)$ and $SFS(V,K)$ be the collections of all soft fuzzy sets over (U,E) and (V,K), respectively.
A soft mapping (φ,ψ) from (U,E) to (V,K) is an ordered pair of mappings $\varphi : U \to V$ and $\psi : E \to K$. The image of any soft fuzzy set F_A over (U,E) under (φ,ψ) denoted by $(\varphi,\psi)(F_A)$ is the soft fuzzy set over (V,K), defined by:

$$(\varphi,\psi)(F_A)(k) = \begin{cases} \tilde{\vee}_{e \in A \cap \psi^{-1}(k)} \varphi(F(e)) & \text{if } A \cap \psi^{-1}(k) \neq \phi \\ \underline{0} & \text{otherwise} \end{cases},$$

where $\tilde{\varphi} : I^U \to I^V$ is the fuzzy mapping induced by $\varphi : U \to V$ as usual.
The preimage of a soft fuzzy set G_B over (V,K) under (φ,ψ), denoted by $(\varphi,\psi)^{-1}(G_B)$ is the fuzzy soft set over (U,E), defined by

$$(\varphi,\psi)^{-1}(G_B)(e) = \begin{cases} \varphi^{-1}(G_B(\psi(e))) & \forall\, e \in \psi^{-1}(B) \\ \underline{0} & \text{otherwise} \end{cases}$$

Definition 6. [2,10] A soft fuzzy topology τ on (U,E) is a family of soft fuzzy sets over (U,E) satisfies :
(1) $\tilde{\phi}, \tilde{U} \in \tau$
(2) $F_A, G_B \in \tau \Rightarrow F_A \tilde{\wedge} G_B \in \tau$
(3) $F^{\alpha}_{A_\alpha} \in \tau, \alpha \in \Gamma \Rightarrow \tilde{\vee}_{\alpha \in \Gamma} F^{\alpha}_{A_\alpha} \in \tau$
The triple (U,E,τ) is called a soft fuzzy topological space, members of τ are called open soft fuzzy sets and their complements are called closed soft fuzzy sets.

theorem 2. A soft fuzzy topological space is a collection of parameterized fuzzy topological spaces . And also any parameterized collection of fuzzy topological spaces is a soft fuzzy topological space .

proof: Straightforward

Definition 7. [3,5,6] A semi-topogenous order on a non-empty set X is a binary relation R on $P(X)$ satisfies the following conditions:
(i) $\phi R \phi . XRX$
(ii) $ARB \Rightarrow A \leq B$
(iii)$A_1 \leq\ ARB \leq B_1 \Rightarrow A_1 R B_1$
A semi-topogenous order R on $P(X)$ is called
(1) topogenous (or top. for short)if it satisfies
$A_i R B_i \forall i \in \{1,2,\ldots,n\} \Rightarrow (\cup_i^n A_i) R (\cup_{i=1}^n B_i)$ and $(\cap_{i=1}^n A_i) R (\cap_{i=1}^n B_i)$
(2) perfect semi-topogenous if

$$A_i R B_i \forall i \in \Delta \Rightarrow (\cup_{i\in\Delta} A_i) R (\cup_{i\in\Delta} B_i),\ for\ any\ index\ set\ \Delta$$

.
(3) biperfect topogenous if

$$A_i R B_i \forall i \in \Delta \Rightarrow (\cup_i A_i) R (\cup_i B_i)\ and\ (\cap_i A_i) R (\cap_i B_i),$$

for any index set Δ.

Definition 8. [3,6] The complement of a semi-topogenous (resp. top., perfect semi-top., biperfect top.) order R on $P(X)$ denoted by R^c and is defined by

$$AR^cB \Leftrightarrow B^c R A^c$$

where A^c is the complement of A, and B^c is the complement of B .

Proposition 3. [3] The complement of a semi-topogenous order on X is also a semi-topogenous order on X

Definition 9. [3,6,12] A syntopogenous structure on a set $X \neq \phi$ is a non-empty family S of topogenous orders on X satisfies the following conditions
(S1) $\forall R_1, R_2 \in S \exists R_3 \in S$ s.t $R_1 \leq R_3, R_2 \leq R_3$
(S2) $\forall R \in S \exists R^* \in S$ s.t $R \leq R^* \circ R^*$.
The pair (X,S) is called a syntopogenous space. In case S consists of a single topogenous order, it is called a topogenous structure. If all topogenous orders on a syntopogenous structure S are perfect (resp. biperfect), it is called perfect (resp. biperfect) syntopogenous structure.

Definition 10. [3,5] A syntopogenous structure S_1 on a set X is called finer than another one S_2 on the same set X if for each $R \in S_2$ there exists a member of S_1 finer than R.

3 Soft fuzzy topogenous orders

In this section the soft fuzzy topogenous orders are introduced as a generalization of the ordinary fuzzy topogenous orders and many of their properties are given.

Definition 11. A relation R on $SFS(U,E)$ is said to be a semi-topogenous soft fuzzy order (s.t.sfo. for short) if it satisfies the following condition; for any F_A, G_B, H_C and $K_D \in SFS(U,E)$
(1) $\tilde{\phi} R \tilde{\phi}, \tilde{U} R \tilde{U}$
(2) $F_A R G_B \Rightarrow F_A \tilde{\leq} G_B$

(3) $H_C \tilde{\leq} F_A R G_B \tilde{\leq} K_D \Rightarrow H_C R K_D$
Also , a semi-topogenous soft fuzzy order R is called a topogenous soft fuzzy order (t.sfo. for short) if it satisfies the condition:
(i) $F_A R G_B$ and $H_C R K_D \Rightarrow (F_A \tilde{\vee} H_C) R (G_B \tilde{\vee} K_D)$ and $(F_A \tilde{\wedge} H_C) R (G_B \tilde{\wedge} K_D)$, semi-topogenous soft fuzzy order R is called
(ii) Perfect if $(F_A)_i R (G_B)_i, i \in J \Rightarrow (\tilde{\vee}_{i\in J}(F_A)_i) R (\tilde{\vee}_{i\in J}(G_B)_i)$
(iii) biperfect if $(F_A)_i R (G_B)_i, i \in J \Rightarrow (\tilde{\vee}_{i\in J}(F_A)_i) R (\tilde{\vee}_{i\in J}(G_B)_i)$ and $(\tilde{\wedge}_{i\in J}(F_A)_i) R (\tilde{\wedge}_{i\in J}(G_B)_i)$

Definition 12. Let R_1 and R_2 be two semi-topogenous soft fuzzy orders on (U,E), then we say R_2 is finer than R_1 or R_1 is coarser than R_2, denoted by $R_1 \tilde{\sqsubseteq} R_2$ if

$$F_A R_1 G_B \Rightarrow F_A R_2 G_B, \forall\, F_A, G_B \in SFS(U,E).$$

In the following theorem a generation of a semi-topogenous soft fuzzy order is given using a collection of soft fuzzy sets.

Theorem 4. Let $\mathscr{D}$ be an arbitrary family of soft fuzzy sets on (U,E), such that $\tilde{\Phi}, \tilde{U} \in \mathscr{D}$, and let $R_{\mathscr{D}}$ be the binary relation on $SFS(U,E)$ defined by,

$$F_A R_{\mathscr{D}} G_B \text{ if } \exists H_C \in \mathscr{D} \ni \mathscr{F}_{\mathscr{A}} \tilde{\leq} \mathscr{H}_{\mathscr{C}} \tilde{\leq} \mathscr{G}_{\mathscr{B}}, \quad \forall\, F_A, G_B \in SFS(U,E)$$

then:
(I) the binary relation $R_{\mathscr{D}}$ is a semi-topogenous soft fuzzy order on $SFS(U,E)$ which is called generated by $\mathscr{D}$.
(II) The relation $R_{\mathscr{D}}$ is a topogenous soft fuzzy order on $SFS(U,E)$ if $\mathscr{D}$ is closed under finite union and finite intersection i . e .

$$F_A, G_B \in \mathscr{D} \Rightarrow \mathscr{F}_{\mathscr{A}} \tilde{\vee} \mathscr{G}_{\mathscr{B}} \in \mathscr{D} \text{ and } \mathscr{F}_{\mathscr{A}} \tilde{\wedge} \mathscr{G}_{\mathscr{B}} \in \mathscr{D}$$

(III) The relation $R_{\mathscr{D}}$ is a perfect semi-topogenous soft fuzzy order if $\mathscr{D}$ is closed under arbitrary union i . e .

$$(F_A)_i \in \mathscr{D}, \rangle \in \mathscr{J} \Rightarrow \tilde{\vee}_{\rangle \in \mathscr{J}}(\mathscr{F}_{\mathscr{A}})_{\rangle} \in \mathscr{D}.$$

(iv) The relation $R_{\mathscr{D}}$ is a biperfect topogenous soft fuzzy order if $\mathscr{D}$ is closed under arbitrary union and arbitrary intersection , i.e

$$(F_A)_i \in \mathscr{D}, \rangle \in \mathscr{J} \Rightarrow \tilde{\vee}_{\rangle \in \mathscr{J}}(\mathscr{F}_{\mathscr{A}})_{\rangle}, \tilde{\wedge}_{\rangle \in \mathscr{J}}(\mathscr{F}_{\mathscr{A}})_{\rangle} \in \mathscr{D}.$$

Proof (I) (1) Since $\tilde{\Phi}, \tilde{U} \in \mathscr{D}$, then $\tilde{\Phi} R_{\mathscr{D}} \tilde{\Phi}$ and $\tilde{U} R_{\mathscr{D}} \tilde{U}$
(2) $F_A R_{\mathscr{D}} G_B \Rightarrow \exists H_C \in \mathscr{D}$ s.t. $F_A \tilde{\leq} H_C \tilde{\leq} G_B \Rightarrow F_A \tilde{\leq} G_B$
(3) If $K_D \tilde{\leq} F_A R_D G_B \tilde{\leq} H_C$, then $\exists L_M \in \mathscr{D} \ni \mathscr{K}_{\mathscr{D}} \tilde{\leq} \mathscr{F}_{\mathscr{A}} \tilde{\leq} \mathscr{L}_{\mathscr{M}} \tilde{\leq} \mathscr{G}_{\mathscr{B}} \tilde{\leq} \mathscr{H}_{\mathscr{C}}$ which implies that, $K_D R_{\mathscr{D}} H_C$.
So, $R_{\mathscr{D}}$ is a semi-topogenous soft fuzzy order.
(II) If $F_A R_{\mathscr{D}} G_B$ and $H_C R_{\mathscr{D}} K_M$, then $\exists L_N, Q_P \in \mathscr{D} \ni \mathscr{F}_{\mathscr{A}} \tilde{\leq} \mathscr{L}_{\mathscr{N}} \tilde{\leq} \mathscr{G}_{\mathscr{B}}$ and $H_C \tilde{\leq} Q_P \tilde{\leq} K_M$. so by the given condition $F_A \tilde{\vee} H_C \tilde{\leq} L_N \tilde{\vee} Q_P \tilde{\leq} G_B \tilde{\vee} K_M$ and $F_A \tilde{\wedge} H_C \tilde{\leq} L_N \tilde{\wedge} Q_P \tilde{\leq} G_B \tilde{\wedge} K_M$, which implies that $F_A \tilde{\vee} H_C R_{\mathscr{D}} G_B \tilde{\vee} K_M$ and also $F_A \tilde{\wedge} H_C R_{\mathscr{D}} G_B \tilde{\wedge} K_M$.
The proofs of, III and IV are similar.

Example 1. (1) The subset relation on $SFS(U,E)$ is a biperfect topogenous soft fuzzy order and is called discrete and defined by

$$R_{\tilde{\subset}} = \{(F_A, G_B) : F_A, G_B \in SFS(U,E), F_A \tilde{\leq} G_B\}.$$

(2) The relation $R_{\mathscr{D}}$ generated by the collection $\mathscr{D} = \{\tilde{\oplus}, \tilde{\mathscr{U}}\}$ which defined by

$$R_{\mathscr{D}} = \{(F_A, G_B) : F_A = \tilde{\Phi} \text{ or } G_B = \tilde{U}\}$$

is a topogenous soft fuzzy order and is called indiscrete.

Lemma 5. The composition of two $s.t.sfo.\underline{s}$ (respectively,$t.sfo.\underline{s}$, $p.sfo.\underline{s}$ and $b.sfo.\underline{s}$) R_1 and R_2 on $SFS(U,E)$ as a relation $R_1 \circ R_2$ is also s.t.sfo (respectively, t.sfo., p.sfo. and b.sfo.) where $R_1 \circ R_2$ is defined as follows for every $F_A, G_B \in SFS(U,E)$,
$F_A(R_1 \circ R_2) G_B \Leftrightarrow \exists H_C \in SFS(U,E) \ni F_A R_1 H_C$ and $H_C R_2 G_B$.

Proof Straightforward.

Lemma 6. Let $\{R_\alpha : \alpha \in \Delta\}$ be a family of semi-topogenous (resp. topogenous, perfect topogenous, biperfect topogenous) soft fuzzy orders on $SFS(U,E)$. Then
(1) $R = \bigcap_\alpha R_\alpha$ is also a semi-topogenous (resp. topogenous, prefect topogenous, biperfect topogenous) soft fuzzy order on $SFS(U,E)$, where

$$F_A R G_B \text{ iff } F_A R_\alpha G_B \ \forall\, \alpha \in \Delta$$

(2) $R = \bigcup_\alpha R_\alpha$ is also a semi-topogenous soft fuzzy order on $SFS(U,E)$, where

$$F_A R G_B \text{ iff } F_A R_{\alpha_0} G_B$$

for some $\alpha_0 \in \Delta$

Proof: Straightforward

Remark 7. The union of a family of topogenous soft fuzzy orders is not in general a topogenous soft fuzzy order, as we show in the following example.

Example 2. Let

$$U = \{a,b,c\}, E = \{e_1, e_2, e_3, e_4\}$$

$$D_1 = \{\tilde{\phi}, \tilde{U}, E_A\}, D_2 = \{\tilde{\phi}, \tilde{U}, G_B\}, where$$

$$F_A = \left\{ \begin{array}{l} F(e_1) = (a,0.4),(b,0.1),(c,0) \\ F(e_2) = (a,0.6),(b,0.5),(c,0.8) \\ F(e_3) = (a,0),(b,0),(c,0) \\ F(e_4) = (a,0.2),(b,0.6),(c,0.3) \end{array} \right\}$$

$$G_B = \left\{ \begin{array}{l} G(e_1) = (a,0),(b,0),(c,0) \\ G(e_2) = (a,0.7),(b,0.2),(c,0.1) \\ G(e_3) = (a,0),(b,0),(c,0) \\ G(e_4) = (a,0.5),(b,0.3),(c,0.9) \end{array} \right\}$$

$$R_{D_1} = \{(\tilde{\phi},\tilde{\phi}),(\tilde{U},\tilde{U})\} \cup \{(H_D,K_E) : H_D \tilde{\leq} F_A \tilde{\leq} K_E\}$$

$$R_{D_2} = \{(\tilde{\phi},\tilde{\phi})(\tilde{U},\tilde{U})\} \cup \{(H_D,K_E) : H_D \tilde{\leq} G_B \tilde{\leq} K_E\}$$

$$R_{D_1} \cup R_{D_2} = \{(\tilde{\phi},\tilde{\phi})(\tilde{U},\tilde{U})\} \cup \{(H_D,K_E) : H_D \tilde{\leq} F_A \tilde{\leq} K_E \text{ or } H_D \tilde{\leq} G_B \tilde{\leq} K_E\} = R_{D_1 \cup D_2}$$

It is clear that $F_A R_{D_1 \cup D_2} F_A$ and $G_B R_{D_1 \cup D_2} G_B$ but

$$((F_A \tilde{\wedge} G_B),(F_A \tilde{\wedge} G_B)) \notin R_{D_1 \cup D_2}$$

also

$$((F_A \tilde{\vee} G_B),(F_A \tilde{\vee} G_B)) \notin R_{D_1 \cup D_2}$$

then $R_{D_1 \cup D_2}$ is not a topogenous soft fuzzy order.

Definition 13. The complement of a soft fuzzy order R on $SFS(U,E)$ denoted by R_1^c is defined by

$$F_A R^c G_B \text{ iff } G_B^c R F_A^c, \forall F_A, G_B \in SFS(U,E).$$

R is called symmetric iff $R = R^c$.

Theorem 8. Let R_1 and R_2 be two semi-topogenous (respectively topogenous, perfect, biperfect) soft fuzzy orders on $SFS(U,E)$, then,

(1) R_1^c is a semi-topogenous (respectively,topogenous, perfect, biperfect) soft fuzzy order.

(2) $R_1^{cc} = R_1$

(3) $R_1 \tilde{\sqsubseteq} R_2 \Rightarrow R_1^c \tilde{\sqsubseteq} R_2^c$

(4) $(R_1 \circ R_2)^c = R_2^c \circ R_1^c$

Proof (1) Let R_1 be a semi-topogenous soft fuzzy order on $SFS(U,E)$.

(i) $\tilde{\phi} R_1 \tilde{\phi} \Rightarrow \tilde{U} R_1^c \tilde{U}$,
$\tilde{U} R_1 \tilde{U} \Rightarrow \tilde{\phi} R_1^c \tilde{\phi}$

(ii) $F_A R_1^c G_B \Rightarrow G_B^c R_1 F_A^c \Rightarrow G_B^c \tilde{\leq} F_A^c \Rightarrow F_A \tilde{\leq} G_B, \forall F_A, G_B \in SFS(U,E)$

(iii) Let $K_D \tilde{\leq} F_A R_1^c G_B \tilde{\leq} H_C \Rightarrow H_C^c \tilde{\leq} G_B^c R_1 F_A^c \tilde{\leq} K_D^c \Rightarrow H_C^c R_1 K_D^c \Rightarrow K_D R_1^c H_C \; \forall K_D, F_A, G_B, H_C \in SFS(U,E)$

then R_1^c is a semi-topogenous soft fuzzy order. The rest of (1) is similar.

(2) For any $F_A, G_B \in SFS(U,E)$,

$$F_A R_1^{cc} G_B \Leftrightarrow G_B^c R_1^c F_A^c \Leftrightarrow F_A R_1 G_B, \text{ i.e } R_1^{cc} = R_1.$$

(3) For any $F_A, G_B \in SFS(U,E)$, let $R_1 \tilde{\sqsubseteq} R_2$. So,

$$F_A R_1^c G_B \Rightarrow G_B^c R_1 F_A^c \Rightarrow G_B^c R_2 F_A^c \Rightarrow F_A R_2^c G_B.$$

Then $R_1^c \tilde{\sqsubseteq} R_2^c$.

(4) For any $F_A, G_B \in SFS(U,E)$,

$$\begin{aligned} F_A (R_1 \circ R_2)^c G_B &\Leftrightarrow G_B^c (R_1 \circ R_2) F_A^c \\ &\Leftrightarrow \exists H_D \text{ s.t } G_B^c R_1 H_D R_2 F_A^c \\ &\Leftrightarrow \exists H_D \text{ s.t } H_D^c R_1^c G_B \text{ and } F_A R_2^c H_D^c \\ &\Leftrightarrow \exists H_D \text{ s.t } F_A R_2^c H_D^c R_1^c G_B \\ &\Leftrightarrow F_A (R_2^c \circ R_1^c) G_B \end{aligned}$$

i.e $(R_1 \circ R_2)^c = R_2^c \circ R_1^c$

Definition 14. Let $f : U \to V$ be a function between sets and let E be any set of parameters. Using f we can determine two mappings $(\overrightarrow{f^*}, 1_E) : (U,E) \to (V,E)$ and $(\overleftarrow{f^*}, 1_E) : (V,E) \to (U,E)$as follows:

let $\overrightarrow{f^*} : I^U \to I^V$, $\overleftarrow{f^*} : I^V \to I^U$ are defined by $\overrightarrow{f^*}(\mu)(y) = \vee_{x \in f^{-1}(y)} \mu(x), \forall \mu \in I^U, y \in V$, and $\overleftarrow{f^*}(\gamma)(x) = \gamma(f(x)) \forall \gamma \in I^V, x \in U$. So,the image and the preimage of a soft set under a soft mapping is given as follows, for every soft fuzzy set $F_A \in SFS(U,E), (\overrightarrow{f^*}, I_E)(F_A) \in SFS(V,E)$ is given by

$$((\overrightarrow{f^*}, I_E)(F_A)(e))(y) = \vee_{x \in f^{-1}(y)} (F_A(e))(x).$$

Also for every soft fuzzy set $G_B \in SFS(V,E)$, the pre-image $(\overleftarrow{f^*}, I_E)(G_B) \in SFS(U,E))$ is given by $((\overleftarrow{f^*}, I_E)(G_B)(e))(x) = (G_B(e))(f(x))$.Denote $(\overrightarrow{f^*}, I_E)$ by $f^\diamond$ and $(\overleftarrow{f^*}, I_E)$ by $\overleftarrow{f^\diamond}$. In the following we define the inverse image of a semi-topogenous soft fuzzy order R under a function f.

Definition 15. Let $f : U \to V$ be a function and let R be a semi-topogenous (respectively, topogenous, perfect, biperfect) soft fuzzy order on (V,E). The inverse image of R under f denoted by $f^{-1}(R)$ defined by

$$F_A f^{-1}(R) G_B \text{ iff } f^\diamond(F_A) R (f^\diamond(G_B^c))^c$$

Proposition 9. The inverse image of a semi-topogenous soft fuzzy order on (V,E) is a semi-topogenous soft fuzzy order on (U,E).

Proof Straightforward

Proposition 10. Let $f : U \to V$ be a function and let R be a semi-topogenous (rep. topog., prefect, biperfect) soft fuzzy order on (V,E). Then for every $F_A, G_B \in SFS(U,E)$,
$F_A f^{-1}(R) G_B$ iff $\exists H_C, K_D \in SFS(V,E)$ s.t. $H_C R K_D$ and $F_A \tilde{\leq} \overleftarrow{f^\diamond}(H_C)$, $\overleftarrow{f^\diamond}(K_D) \tilde{\leq} G_B$. Also, $H_C R K_D \Rightarrow \overleftarrow{f^\diamond}(H_C)(f^{-1}(R)) \overleftarrow{f^\diamond}(K_D))$.

Proof In fact, $F_A f^{-1}(R) G_B$ iff $f^\diamond(F_A) R (f^\diamond(G_B^c))^c$, let $H_C = f^\diamond(F_A), K_D = (f^\diamond(G_B^c))^c$, then we get

$$H_C R K_D, H_C, K_D \in SFS(V,E)$$

and

$F_A \tilde{\leq} \overleftarrow{f^\diamond}(H_C)$, $\overleftarrow{f^\diamond}(K_D) = \overleftarrow{f^\diamond}((f(H_B^c))^c) = (\overleftarrow{f^\diamond} f^\diamond(G_B^c))^c \tilde{\leq} G_B^{cc} = G_B$.Conversely if $\exists H_C, K_D \in SFS(V,E)$ such that $H_C R K_D, F_A \tilde{\leq} \overleftarrow{f^\diamond}(H_C)$ and $\overleftarrow{f^\diamond}(K_D) \tilde{\leq} G_B$, then $f^\diamond(F_A) \tilde{\leq} H_C$ and $G_B^c \tilde{\leq} (\overleftarrow{f^\diamond}(K_D))^c = \overleftarrow{f^\diamond}(K_D{}^c)$. Consequently, $f^\diamond(G_B^c) \tilde{\leq} K_D^c$ which implies that $K_D \tilde{\leq} (f^\diamond(G_B^c))^c$. So , $H_C R K_D$ implies that $f^\diamond(F_A) \tilde{\leq} H_C R K_D \tilde{\leq} (f^\diamond(G_B^c))^c$. Which implies that $f^\diamond(F_A) R (f^\diamond(G_B^c))^c)$, and that $F_A(f^{-1}(R)) G_B$.

Theorem 11. Let $f:U\rightarrow V$ be a function, $R_1,R_2$and R be semi-topogenous (resp. topogenous perfect semi-topogenous, biperfect) soft fuzzy order on (V,E). Then
(I) $R_1\sqsubseteq R_2$ implies $f^{-1}(R_1)\sqsubseteq f^{-1}(R_2)$, and the converse is true if f is surjective.
(II) $f^{-1}(R^c)=(f^{-1}(R))^c$.

Proof Let $f:U\rightarrow V$ be a function and $R_1\sqsubseteq R_2$ are two semi-topogenous soft fuzzy orders on (V,E), and let $F_A,G_B\in SFS(U,E)$
Then

$F_A(f^{-1}(R_1))G_B\Rightarrow f^{\diamond}(F_A)R_1(f^{\diamond}(G_B^c))^c$
$\Rightarrow f^{\diamond}(F_A)R_2(f^{\diamond}(G_B^c))^c\Rightarrow F_A(f^{-1}(R_2))G_B$

Conversely, let f be a surjective function $f^{-1}(R_1)\sqsubseteq f^{-1}(R_2)$, and let $F_AR_1G_B$ for some $F_A,G_B\in SFS(V,E)$, so $\overleftarrow{f^{\diamond}}(F_A)(f^{-1}(R_1))(\overleftarrow{f^{\diamond}}(G_B))\Rightarrow(\overleftarrow{f^{\diamond}}(F_A)(f^{-1}(R_2))(\overleftarrow{f^{\diamond}}(G_B))\Rightarrow\exists H_C,K_DSFS(V,E)\ni$
$H_CR_2K_D,\overleftarrow{f^{\diamond}}(F_A)\tilde{\leq}\overleftarrow{f^{\diamond}}(H_C),\overleftarrow{f^{\diamond}}(K_D)\tilde{\leq}\overleftarrow{f^{\diamond}}(G_B)$

then $,f^{\diamond}(\overleftarrow{f^{\diamond}}(F_A))\tilde{\leq}f^{\diamond}(\overleftarrow{f^{\diamond}}(H_C))and,f^{\diamond}(\overleftarrow{f^{\diamond}}(K_D))\tilde{\leq}f^{\diamond}(\overleftarrow{f^{\diamond}}(G_B))$

since f is surjective, then $F_A\tilde{\leq}H_C,K_D\tilde{\leq}G_B$ and $H_CR_2K_D$
$\Rightarrow F_A\tilde{\leq}H_CR_1K_D\tilde{\leq}G_B\Rightarrow F_AR_1G_B$
Hence $R_1\tilde{\sqsubseteq}R_2$.
(II) For any $F_A,G_B\in SFS(U,E)$,
$F_A(f^{-1}R^c)G_B\Leftrightarrow(f^{\diamond}(F_A))R^c(f^{\diamond}(G_B^c))^c$
$\Leftrightarrow f^{\diamond}(G_B^c)R(f^{\diamond}(F_A))^c$......(1). Also,
$F_A(f^{-1}R)^cG_B\Leftrightarrow G_B^c(f^{-1}(R))F_A^c$
$\Leftrightarrow f^{\diamond}(G_B^c)R(f^{\diamond}(F_A))^c$
....(2).Consequently $f^{-1}(R^c)=(f^{-1}(R))^c$

Theorem 12. Let $f:U\rightarrow V$ and $g:V\rightarrow W$ be two functions, $g\circ f:V\rightarrow W$ is the composition of f,g then for any topogenous soft fuzzy order R on (W,E) we have

$$(g\circ f)^{-1}(R)=f^{-1}(G^{-1}(R))$$

Proof For any two soft fuzzy sets F_A and G_B of (U,E), $F_A((g\circ f)^{-1})R)G_B$
$\Leftrightarrow(g\circ f)^{\diamond}(F_A))R((g\circ f)^{\diamond})(G_B^c))^c$
$\Leftrightarrow g^{\diamond}(f^{\diamond}(F_A))Rg^{\diamond}(f^{\diamond}(G_B^c)))^c\Leftrightarrow f^{\diamond}(F_A)(g^{-1}(R))f^{\diamond}(G_B)$
$\Leftrightarrow(f^{\diamond}(F_A))(g^{-1}(R))(f^{\diamond}(G_B^c))^c\Leftrightarrow(F_A)(f^{-1}(g^{-1}(R)))(G_B)$

Theorem 13. Let $f:U\rightarrow V$ be a function, R_1 and R_2 two semi-topogenous soft fuzzy orders on (V,E) and $R=R_1\circ R_2$ then $f^{-1}(R)\sqsubseteq f^{-1}(R_1)\circ f^{-1}(R_2)$ and the equality holds if f is surjective.

Proof Straightforward.

4 The relation between soft fuzzy orders and ordinary fuzzy orders

It is clear that a soft set is a parameterized collection of sets. And a soft topological structure on a set is a parameterized collection of ordinary topological structures on the same set . Also a similar result will be proved for the soft fuzzy orders .

Theorem 14. Let R be a semi-topogenous (respectively, topogenous, perfect topogenous, biperfect topogenous) soft fuzzy order on (U,E). For every $e\in E$, consider the relation R_e on I^U given by, for $\mu,\nu\in I^U,\mu R_e\nu$ if $\exists F_A,G_B\in SFS(U,E)$ such that $\mu=F_A(e),\nu=G_B(e)$ and F_ARG_B. Then R_e is semi-topogenous (respectively, topogenous, perfect topogenous, biperfect topogenous) fuzzy order on U, for every $e\in E$.

Proof (1) $\tilde{\Phi}R\tilde{\Phi}\Rightarrow\underline{0}R_e\underline{0}$, also $\tilde{U}R\tilde{U}\Rightarrow\underline{1}R_e\underline{1}$ for every $e\in E$.
(2) Let $\mu R_e\nu\Rightarrow\exists F_A,G_B\in SFS(U,E)\ni\mu=F_A(e),\nu=G_B(e)$ and $F_ARG_B\Rightarrow F_A\tilde{\leq}G_B\Rightarrow F_A(e)\leq G_B(e)\Rightarrow\mu\leq\nu$.
(3) Let $\eta\leq\mu R_e\nu\leq\xi\Rightarrow\exists F_A,G_B\in SFS(U,E)\ni\mu=F_A(e),\nu=G_B(e)\Rightarrow\exists F_A^*,G_B^*\in SFS(U,E)$ defined by,

$$F_A^*(e)=\eta,F_A^*(t)=\underline{0}\,\forall\,t\in E-\{e\}$$

also,

$$G_B^*(e)=\xi,G_B^*(t)=\underline{1}\,\forall\,t\in G-\{e\}.$$

Consequently

$$F_A^*\tilde{\leq}F_ARG_B\tilde{\leq}G_B^*\Rightarrow F_A^*RG_B^*\Rightarrow\eta R_e\xi$$

The previous theorem show that that any semi-topogenous soft fuzzy order R on (U,E) generate a parameterized collection of semi-topogenous fuzzy orders $\{R_e:e\in E\}$ on U.

Theorem 15. Any topogenous (respectively, perfect topogenous, biperfect topogenous) soft fuzzy order on (U,E) is a parameterized collection of topogenous (respectively, perfect topogenous, biperfect topogenous) soft fuzzy orders $\{R_e:e\in E\}$ on U.

Proof Let R be a topogenous soft order on (U,E). for every $e\in E$ consider the semi-topogenous fuzzy order R_e given in the previous theorem.
Let $\mu R_e\nu$ and $\xi R_e\zeta$ for some fuzzy sets $\mu,\nu,\xi,\zeta\in I^U$. Then,there exist $F_A,G_B,M_C,N_D\in SFS(U,E)$ such that $F_A(e)=\mu$, $G_B(e)=\nu$, $M_C(e)=\xi$, $N_D(e)=\zeta$, F_ARG_B and M_CRN_D. This implies that $(F_A\tilde{\vee}M_C)R(G_B\tilde{\vee}N_D)$. Consequently

$$(F_A\tilde{\vee}M_C)(e)=F_A(e)\vee M_C(e)=\mu\vee\xi,$$

and

$$(G_B\tilde{\vee}N_D)(e)=G_B(e)\vee N_D(e)=\nu\vee\zeta.$$

So, $(\mu\tilde{\vee}\xi)R_e(\nu\tilde{\vee}\zeta)$.
The rest of the proof is similar.

Theorem 16. Every parameterized collection $\{R_e : e \in E\}$ of semi-topogenous (respectively, perfect topogenous, biperfect topogenous) fuzzy orders on a set U generate in a cononical correspondence a unique semi-topogenous (respectively, perfect topogenous, biperfect topogenous) soft fuzzy order R on (U,E).

Proof Let $\{R_e : e \in E\}$ be a collection of semi-topogenous fuzzy orders on a set U. Consider the relation R on $SFS(U,E)$ given as follows, for every two soft fuzzy sets $F_A, G_B \in SFS(U,E)$ $F_A R G_B$ if $F_A(e) R_e G_B(e) \forall e \in E$.
(1)Since R_e is a semi-topogenous order ,$\forall e \in E$, so, $0 R_e 0$ and $1 R_e 1 \forall e \in E \Rightarrow \tilde{\Phi} R \tilde{\Phi}$ and $\tilde{U} R \tilde{U}$.
(2) Let $F_A R G_B \Leftrightarrow F_A(e) R_e G_B(e) \forall e \in E \Rightarrow F_A(e) \leq G_B(e)$ for every $e \in E \Rightarrow F_A \tilde{\leq} G_B$.
(3) Let $M_C \tilde{\leq} F_A R G_B \tilde{\leq} N_D \Rightarrow M_C(e) \leq F_A(e) R_e G_B(e) \leq N_D(e) \forall e \in E \Rightarrow M_C(e) R_e N_D(e) \forall e \in E \Rightarrow M_C R N_D$.
The rest of the proof is by the same argument.

Remark 17.:It is clear that from theorem (2 ,12 , 13), both notions soft topogenous order and topogenous soft order are the same , also soft topological space and topological soft space are the same

5 The syntopogenous soft fuzzy structures

Definition 16. A syntopogenous soft fuzzy structure on a set (U,E) is a non-empty family S of topogenous soft fuzzy orders on (U,E) having the following two properties
(1) if $R_1, R_2 \in S \exists R \in S$ s.t $R_1 \tilde{\sqsubseteq} R, R_2 \tilde{\sqsubseteq} R$
(2) $\forall R_1 \in S \exists R_2 \in S$ s.t $R_1 \tilde{\sqsubseteq} R_2 \circ R_2$.

The pair (U,E,S) is called a syntopogenous soft fuzzy space. In case S consists of a single topogenous soft fuzzy order, it is called a simple syntopogenous soft fuzzy structure (or topogenous structure). If all orders of the space (U,E,S) are perfect or biperfect, then it is called perfect (or biperfect) syntopogenous soft fuzzy space.

Definition 17. A syntopogenous soft fuzzy structure S_1 on (U,E) is called finer than another one S_2 on the same space if $\forall R \in S_2 \exists R^* \in S_1 \ni R^*$ is finer than R, and is denoted by $S_2 \tilde{\subseteq} S_1$.

Lemma 18. Let (U,E,S) be a syntopogenous soft fuzzy space, then $S^c = \{R^c : R \in S\}$ is a syntopogenous soft fuzzy structure, and is called the complement of S.Also , S is called symmetric if $S^c = S$

Proof Straightforward.

Proposition 19. If R is a topogenous soft fuzzy order on (U,E), then $\{R\}$ is a topogenous soft fuzzy structure if it satisfies the condition ;for every $F_A, G_B \in SFS(U,E)$ if $F_A R G_B$, then there exists $H_C \in SFS(U,E) \ni F_A R H_C R G_B$

Proof Straightforward.

Proposition 20. Let f be a function from (U,E) into (V,E),S be a syntopogenous soft fuzzy structure on (V,E). Then the family $f^{-1}(S) = \{f^{-1}(R) : R \in S\}$ is a syntopogenous soft fuzzy structure of (U,E) and it is called the inverse image of S by the mapping f.

Proof (1) Let$f^{-1}(R_1), f^{-1}(R_2) \in (f^{-1}S)$ .Since S is syntopogenous soft fuzzy structure on (V,E) then $\exists R \in S$ s.t $R_1 \tilde{\sqsubseteq} R, R_2 \tilde{\sqsubseteq} R$
$\Rightarrow \exists f^{-1}(R) \in f^{-1}(S)$ s.t $f^{-1}(R_1) \tilde{\sqsubseteq} f^{-1}(R), f^{-1}(R_2) \tilde{\sqsubseteq} f^{-1}(R)$
(2)Let $f^{-1}(R) \in (f^{-1}S) \Rightarrow \exists R^\star \in S$ such that , $R \tilde{\sqsubseteq} R^\star \circ R^\star$.
Thus, $f^{-1}(R) \tilde{\sqsubseteq} f^{-1}(R^\star \circ R^\star) \tilde{\sqsubseteq} f^{-1}(R^\star) \circ f^{-1}(R^\star)$
. Thus $f^{-1}S$ is a syntopogenous soft fuzzy structure of (U,E)

Proposition 21. Let f be a function, $(f, I_E) : (U,E) \to (V,E)$, and let S, S' be two syntopogenous soft fuzzy structures on (V,E)
(1) if S is perfect (respectively biperfect, symmetric), then $f^{-1}(S)$ is also prefect (respectively biperfect, symmetric).
(2) if $S \tilde{\subset} S'$, then $f^{-1}(S) \tilde{\subset} f^{-1}(S')$.

Proof (i)It obvious
(ii)

$$f^{-1}(S) = \{f^{-1}(R) : R \in S\} \tilde{\subset} \{f^{-1}(R) : R \in S'\} = f^{-1}(S')$$

Definition 18.: Let S and S' be two syntopogenous soft fuzzy structures on (U,E) and (V,E) ,respectively, and let f be a function from (U,E) into (V,E). Then f is said to be (S,S') continuous iff $f^{-1}(S')$ is coarser than S (denoted by $f^{-1}(S') \tilde{\subset} S$) i.e $\forall R_1 \in S' \exists R_2 \in S$ which is finer then $f^{-1}(R_1)$ i.e. $f^{-1}(R_1) \tilde{\sqsubseteq} R_2$.

Theorem 22. Let $(U,S_1,E),(V,S_2,E),(W,S_3,E)$ be syntopogenous soft fuzzy spaces. If $(f, I_E) : (U,E) \to (V,E)$ is (S_1,S_2)-continuous and $(g, I_E) : (V,E) \to (W,E)$ is (S_2,S_3)-continuous. Then $(g \circ f, I_E) : (U,E) \to (W,E)$ is (S_1,S_3) continuous.

Proof The continuity of $f : (U,S_1,E) \to (V,S_2,E)$ and $g : (V,S_2,E) \to (W,S_3,E)$ implies that, for every $R \in S_3$, there exists $R_1 \in S_2$ such that $g^{-1}(R) \tilde{\leq} R_1$, also there exists $R_2 \in S_1$ such that $f^{-1}(R_1) \tilde{\leq} R_2$. Consequently , $f^{-1}(g^{-1}(R)) \tilde{\leq} R_2$ i.e. $(g \circ f)^{-1}(R) \tilde{\leq} R_2$. This implies that $g \circ f$ is continuous.

It is will known that there exists a one to one correspondence between the collection of all topological structures on a set and the collection of all perfect topogenous structures on the same set [9,10]. The following Theorem shows a similar result in the soft case.

Theorem 23. For any non-empty set U, there exists a one-to-one and onto correspondence between the collection of all soft fuzzy topological structures on (U,E) and the collection of all perfect topogenous soft fuzzy structures on the same space (U,E) with any set of parameters E.

Proof

For every perfect topogenous soft fuzzy structure $\{R\}$ on (U,E), consider the collection $\tau_R = \{G_A : G_A \in (U,E), G_A R G_A, A \subset E\}$. so, $\Phi_E, \tilde{U}_E \in \tau_R$. If $\{G^{\alpha}_{A_\alpha} : \alpha \in \Gamma\} \subset \tau_R$, R is perfect then $\tilde{\vee}_{\alpha\in\Gamma} G^{\alpha}_{A_\alpha} \in \tau_R$. Also if $G^1_{A_1}, G^2_{A_2} \in \tau_R$, R is topogenous then $G^1_{A_1} \tilde{\wedge} G^2_{A_2} \in \tau_R$. i.e. τ_R is a soft fuzzy topological structure on (U,E).
Also for every soft fuzzy topological structure τ on (U,E) consider the following order R_τ on (U,E), defined by

$$F_A R_\tau H_B \text{ if } \exists G_C \in \tau \ni F_A \tilde{\leq} G_C \tilde{\leq} H_B,$$
$$\forall F_A, G_B \in SFS(U,E)$$

It is clear that $\tilde{\Phi}, \tilde{U} \in \tau$, implies that $\tilde{\Phi} R_\tau \tilde{\Phi}$ and $\tilde{U} R_\tau \tilde{U}$.Also $F_A R_\tau H_B$ implies that $F_A \tilde{\leq} H_B$. If $F^{\alpha}_{A_\alpha} R_\tau H^{\alpha}_{B_\alpha}$,$\alpha \in \Gamma$, then $\exists G^{\alpha}_{C_\alpha} \in \tau$ such that $F^{\alpha}_{A_\alpha} \tilde{\leq} G^{\alpha}_{C_\alpha} \tilde{\leq} H^{\alpha}_{B_\alpha}$, so $\tilde{\vee}_{\alpha\in\Gamma} G^{\alpha}_{C_\alpha} \in \tau$ and,

$$\tilde{\vee}_{\alpha\in\Gamma} F^{\alpha}_{A_\alpha} \tilde{\leq} \tilde{\vee}_{\alpha\in\Gamma} G^{\alpha}_{C_\alpha} \tilde{\leq} \tilde{\vee}_{\alpha\in\Gamma} H^{\alpha}_{B_\alpha}$$

consequently, $(\tilde{\vee}_{\alpha\in\Gamma} F^{\alpha}_{A_\alpha}) R_\tau (\tilde{\vee}_{\alpha\in\Gamma} H^{\alpha}_{B_\alpha})$, i.e. R_τ is a perfect order.
Also, if $F^i_{A_i} R_\tau H^i_{B_i}$,i=1,2, then $\exists G^1_{C_1}, G^2_{C_2} \in \tau \ni F^i_{A_i} \tilde{\leq} G^i_{C_i} \tilde{\leq} H^i_{B_i}, i = 1,2$. This implies that $G^1_{C_1} \tilde{\vee} G^2_{C_2} \in \tau$ and $(F^1_{A_1} \tilde{\vee} F^2_{A_2}) R_\tau (H^1_{B_1} \tilde{\vee} H^2_{B_2})$. Consequently R_τ is a perfect topogenous soft fuzzy order on (U,E). Also it is clear that by the definition of R_τ, we have $R_\tau \circ R_\tau$ is coarser than R_τ, which implies that R_τ is a perfect topogenous soft fuzzy structure
Now, consider any soft fuzzy topological structure τ on (U,E) and consider the order R_τ and the topology τ_{R_τ}. For every $G_A \in \tau_{R_\tau}$, it follows that $G_A R_\tau G_A$, which implies that $G_A \in \tau$. Also if R is any perfect topogenous soft fuzzy order on (U,E), consider τ_R and R_{τ_R}. If $F_A R_{\tau_R} H_B$ for some $F_A, H_B \in SFS(U,E)$, then $\exists G_C \in \tau_R$ such that $F_A \tilde{\leq} G_C \tilde{\leq} H_B$, so $G_C R G_C$, which implies that $F_A R H_B$. Consequently the correspondence in the Theorem is one to one and onto.

Proposition 24. For any syntopogenous soft fuzzy structure Son the space (U,E), the collection $S^t = \{R_S\}$ is a topogenous soft fuzzy structure on (U,E), where $R_S = \cup\{R : R \in S\}$.

Proof Straightforward.

Corollary 25. Using the last theorem and proposition we can determine in a canonical correspondence, for any syntopogenous soft fuzzy structure S on a space (U,E), a soft fuzzy topological structure denoted τ_S which is τ_{S^t} or indeed it is $\tau_{R^p_S}$, where R^p_S is the coarsest perfect topogenous order finer than R_S.

Proposition 26. If S_1, S_2 are two syntopogenous soft fuzzy structures on (U,E) and $S_1 \tilde{\subset} S_2$, then $\tau_{S_1} \subset \tau_{S_2}$.

Proof Straightforward.

Theorem 27. Consider two collections of all soft fuzzy subsets (U,E) and (V,E), and let f be any surjective function $(f, I_E) : (U,E) \to (V,E)$. If S is a syntopogenous soft fuzzy structure on (V,E), then $\tau_{f^{-1}(S)} = f^{-1}(\tau_S)$.

Proof Let $F_A \in \tau_{f^{-1}(S)}$, so $F_A R^p F_A$, where R^p is the coarsest perfect topogenous structure finer than R and where $R = (f^{-1}(S))^t$. Consequently $f^{\diamond}(F_A) R^p_0 (f^{\diamond}(F^c_A))^c$ for some $R_0 \in S$, consequently , there exists $G_B \in \tau_S$,for which $f^{\diamond}(F_A) \tilde{\leq} G_B \tilde{\leq} (f^{\diamond}(F^c_A))^c$. This implies that$F_A \tilde{\leq} \overleftarrow{f^{\diamond}} (f^{\diamond}(F_A)) \tilde{\leq} \overleftarrow{f^{\diamond}} (G_B)) \tilde{\leq} \overleftarrow{f^{\diamond}} ((f^{\diamond}(F^c_A))^c) \tilde{\leq} F_A$ .So, $F_A = \overleftarrow{f^{\diamond}} (G_B)) \in f^{-1}(\tau_S)$, i.e$\tau_{f^{-1}(S)} = f^{-1}(\tau_S)$.
Conversley, let $F_A \in f^{-1}(\tau_s)$, so $f^{\diamond}(F_A) \in \tau_S$.Consequently, there exists$R_0 \in S$, such that$f^{\diamond}(F_A) R^p_0 f^{\diamond}(F_A)$, since f is surjective , then $f^{\diamond}(F_A) = (f^{\diamond}(F^c_A))^c$,so$f^{\diamond}(F_A) R^p_0 (f^{\diamond}(F^c_A))^c$which implies that $F_A (f^{-1} R^p_0) F_A$ i.e $F_A (f^{-1} R_0)^p F_A$ consequently, $F_A (f^{-1} S) F_A$,and this means that , $F_A \in \tau_{f^{-1}S}$.

Theorem 28. Let $(U,S_1,E),(V,S_2,E)$ be two syntopogenous soft fuzzy spaces ,$f : U \to V$ be a function. If (f, I_E) is (S_1,S_2)-continuous, then (f, I_E) is $\tau_{S_1} - \tau_{S_2}$continuous.

Proof Let $F_A \in \tau_{S_2}$, so $F_A R^p_{S^t_2} F_A$.So $(\overleftarrow{f^{\diamond}} (F_A)) f^{-1}(R^p_{S^t_2})(\overleftarrow{f^{\diamond}} (F_A))$. f is (S_1,S_2) continuous implies that$f^{-1}(R^p_{S^t_2}) \subset S_1{}^t$. Consequently ,$(\overleftarrow{f^{\diamond}} (F_A))(R^p_{S^t_1})(\overleftarrow{f^{\diamond}} (F_A))$ which implies that $(\overleftarrow{f^{\diamond}} (F_A)) \in \tau_{S_1}$.

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