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A Study on Ricci Solitons in almost $C(\lambda)$ Manifolds

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Abstract: We show that $C(\lambda)$ manifold is cone if the Ricci Solitons (g,V,λ_1), $n \geq 3$ is expanding and τ-curvature tensor is zero where τ is a generalized curvature tensor and consists of Riemannian, Conformal, quasi-conformal, Conharmonic, Concircular, Pseudoprojetive, Projective and M-Projective etc., curvature tensors. Also it is shown that Ricci Solitons of $C(\lambda)$ manifolds are shrinking when $C-$Bochner curvature tensor is Zero.

Keywords: Almost $C(\lambda)$ manifolds, τ-curvature tensor, C-Bochner curvature tensor, η-Einstein

1 Introduction

In 1982 Hamilton [4] introduced an a excellent tool for simplifying the structure of manifolds which smooth out the topology of the manifolds and to make them more symmetric.

$$\frac{\partial g}{\partial t} = -2Ricg \tag{1}$$

known as Hemalton Ricci flow equation and this is nothing but one type of heat equation.

A Ricci soliton is a natural generalization of an Einstein metric which moves under the Ricci flow simply by diffeomorphism of the initial metric [10]. A Ricci soliton is a triple (g,V,λ_1) with g a Riemannian metric, V a vector fileld and λ_1 a real scalar such that

$$\mathscr{L}_V g + 2S + 2\lambda_1 g = 0, \tag{2}$$

where S is a Ricci tensor of M and $\mathscr{L}_V$ denotes the Lie derivative operator along the vector field V. The Ricci soliton is said to be shrinking, steady and expanding according as λ_1 is negative, zero and positive respectively.

In 1981, D. Janssen and L. Vanhecke [11] introduced the notion of almost $C(\lambda)$ manifolds and they have neatly explained the different types of the manifolds depending on the value λ. Further many authors Z. Olszak, R. Rosca [18] and S. V. Kharitonava [15] studied the flatness of curvature tensors in $C(\lambda)$ manifolds and Ali Akbar [2] has obtained results on Ricci tensor and quasi conformal curvature tensor of $C(\lambda)$ manifolds. Further G. Zhen, J. L. Cabrerizo, L. M. Fernandez and M. Fernandez [12] have studied ξ conhormonic flat generalized Sasakian space forms on $C(\lambda)$ manifolds. In this paper we study the Ricci solitons in $C(\lambda)$ manifolds using the flatness condition on τ-curvature tensor, C-Bochner curvature tensor, W_2-curvature tensor, $\tilde{P}$ Pseudo Qusai conformal curvature tensor.

2 Preliminaries

Let M be a n-dimensional connected differentiable manifold endowed with an almost contact metric structure (ϕ,ξ,η,g), where ϕ is a tensor field of type $(1,1)$, ξ is a vector field, η is an 1-form and g is a Riemannian metric on M such that [5].

$$\eta(\xi) = 1, \tag{3}$$
$$\phi^2 = -I + \eta \otimes \xi, \tag{4}$$
$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \tag{5}$$
$$g(X,\xi) = \eta(X), \tag{6}$$
$$\phi\xi = 0, \quad \eta(\phi X) = 0. \tag{7}$$

In [11] D. Janssen and L. Vanhecke introduced the almost $C(\lambda)$ manifolds, where λ is a real number. Further Z. Olszak, R. Rosca [18] and others investigated such manifolds.

Definition 21[*11*]: *An almost $C(\lambda)$-Manifold M is an almost contact manifold, if the Riemann curvature tensor*

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satisfies the following property:

$$R(X,Y,Z,W) = R(X,Y,\phi Z,\phi W) + \lambda[-g(X,Z)g(Y,W) \quad (8)$$
$$+g(X,W)g(Y,Z) + g(X,\phi Z)g(Y,\phi W) \quad (9)$$
$$-g(X,\phi W)g(Y,\phi Z)], \quad (10)$$
$$R(X,Y)Z = R(\phi X,\phi Y)Z - \lambda[Xg(Y,Z) - g(X,Z)Y \quad (11)$$
$$-\phi Xg(\phi Y,Z) + g(\phi X,Z)\phi Y]. \quad (12)$$

for a real number λ and $X,Y,Z,W \in T(M)$

Definition 22[11]: *A normal almost $C(\lambda)$-manifold is called a $C(\lambda)$ manifold. The authors [11] prived that cosymplectic, Sasakian, Kenmotsu manifolds are respectively $C(0), C(1)$ and $C(-1)$ manifolds. For Kenmotsu manifold the following holds*

$$(\nabla_X\phi)Y = g(\phi X,Y)\xi - \eta(Y)\phi X. \quad (13)$$

From (13), we have

$$\nabla_X\xi = X - \eta(X)\xi. \quad (14)$$

Remark 1*Let (g,ξ,λ) be a Ricci soliton in an n-dimensional Kenmotsu manifold M. From (14) we have following identity*

$$(\mathscr{L}_\xi g)(X,Y) = 2[g(X,Y) - \eta(X)\eta(Y)]. \quad (15)$$

From (2) and (15), we get

$$S(X,Y) = -(\lambda_1+1)g(X,Y) + \eta(X)\eta(Y). \quad (16)$$

The above equation yields:

$$QX = -(\lambda_1+1)X + \eta(X)\xi, \quad (17)$$
$$S(X,\xi) = -\lambda_1\eta(X), \quad (18)$$
$$r = -\lambda_1 n - (n-1), \quad (19)$$

where S is the Ricci tensor, Q is the Ricci operator and r is the scalar curvature on M.

3 Ricci solitons on $C(\lambda)$-Manifolds with $\tau(X,Y)Z = 0$.

Definition 31*The τ-curvature tensor [16] is given by*

$$\tau(X,Y)Z = a_0R(X,Y)Z + a_1S(Y,Z)X + a_2S(X,Z)Y + a_3S(X,Y)Z + a_4g(Y,Z)QX + a_5g(X,Z)QY + a_6g(X,Y)QZ + a_7[g(Y,Z)X - g(X,Z)Y], \quad (20)$$

where $a_0,\dots,a_7$ are some smooth functions on M. For different values of $a_0,\dots,a_7$ the τ-curvature tensor reduces to the curvature tensor R, quasi-conformal curvature tensor, conformal curvature tensor, conharmonic curvature tensor, concircular curvature tensor, pseudo-projective curvature tensor, projective curvature tensor, M-projective curvature tensor, W_i-curvature tensors $(i = 0,\dots,9)$, W_j^*-curvature tensors $(j = 0,1)$.

If τ curvature tensor vanishes identically then we say that the manifold is τ flat. Thus for a τ flat $C(\lambda)$ manifold, we get

$$a_0R(X,Y)Z = -a_1S(Y,Z)X - a_2S(X,Z)Y - a_3S(X,Y)Z - a_4g(Y,Z)QX - a_5g(X,Z)QY - a_6g(X,Y)QZ - a_7[g(Y,Z)X - g(X,Z)Y], \quad (21)$$

In view of (11) we have from (21)

$$a_0R(\phi X,\phi Y)Z = -a_0[g(Y,Z)X - g(X,Z)Y - g(\phi Y,Z)\phi X + g(\phi X,Z)\phi Y] - a_1S(Y,Z)X - a_2S(X,Z)Y - a_3S(X,Y)Z - a_4g(Y,Z)QX - a_5g(X,Z)QY - a_6g(X,Y)QZ - a_7[g(Y,Z)X - g(X,Z)Y], \quad (22)$$

Take innerproduct with respect to ξ and $Y = \xi$ in (22). By virtue of (7), (16), (17), (18) and on simplification, we get

$$a_2S(X,Z) = [a_0 + \lambda_1a_5 - a_7r]g(X,Z) + [\lambda_1a_1 - a_0 + \lambda_1a_3 + \lambda_1a_4 + \lambda_1a_6 - a_7r]\eta(X)\eta(Z) \quad (23)$$

Taking $X = Z = e_i$ in (23) and summing over $\{e_i : i = 1,2,\dots,n\}$. Then we get on simplification

$$\lambda_1 = \frac{a_7r(n+1) - a_0(n-1) + a_2r}{a_5n + a_1 + a_3 + a_4 + a_6} \quad (24)$$

From the definition (31) we have the following:
The quasi conformal curvature tensor $\bar{C}$ if

$$a_1 = -a_2 = a_4 = -a_5, a_3 = a_6 = 0,$$

$$a_7 = -\frac{1}{n}\left(\frac{a_0}{n-1} + 2a_1\right),$$

The conformal curvature tensor C if

$$a_0 = 1, a_1 = -a_2 = a_4 = -a_5 = -\frac{1}{n-2},$$

$$a_3 = a_6 = 0, a_7 = \frac{1}{(n-1)(n-2)},$$

The conharmonic curvature tensor N if

$$a_0 = 1, a_1 = -a_2 = a_4 = -a_5 = -\frac{1}{n(n-1)},$$

$$a_3 = a_6 = 0 = a_7 = 0,$$

The concircular curvature tensor if

$$a_0 = 1, a_1 = a_2 = a_3 = a_4 = a_5 = a_6 = 0,$$

$$a_7 = -\frac{1}{n(n-1)},$$

The pseudo-projective curvature tensor $\bar{P}$ if

$$a_1 = -a_2, a_3 = a_4 = a_5 = a_6 = 0,$$

$$a_7 = -\frac{1}{n}\left(\frac{a_0}{n-1} + a_1\right),$$

The projective curvature tensor P if

$$a_0 = 1, a_1 = -a_2 = -\frac{1}{(n-1)},$$

$$a_3 = a_4 = a_5 = a_6 = a_7 = 0,$$

The M-projective curvature tensor if

$$a_0 = 1, a_3 = a_6 = a_7 = 0,$$

$$a_1 = -a_2 = a_4 = -a_5 = -\frac{1}{2(n-1)},$$

The W_0-curvature tensor if

$$a_0 = 1, a_2 = a_3 = a_4 = a_6 = 0 = a_7 = 0,$$

$$a_1 = -a_5 = -\frac{1}{n-1},$$

The W_0^*-curvature tensor if

$$a_0 = 1, a_2 = a_3 = a_4 = a_5 = a_6 = 0 = a_7 = 0,$$

$$a_1 = -a_5 = \frac{1}{n-1},$$

The W_1-curvature tensor if

$$a_0 = 1, a_3 = a_4 = a_5 = a_6 = a_7 = 0,$$

$$a_1 = -a_2 = \frac{1}{n-1},$$

The W_1^*-curvature tensor if

$$a_0 = 1, a_3 = a_4 = a_5 = a_6 = 0 = a_7 = 0,$$

$$a_1 = -a_2 = -\frac{1}{n-1},$$

The W_2-curvature tensor if

$$a_0 = 1, a_1 = a_2 = a_3 = a_6 = a_7 = 0,$$

$$a_4 = -a_5 = -\frac{1}{n-1},$$

The W_3-curvature tensor if

$$a_0 = 1, a_1 = a_3 = a_5 = a_6 = a_7 = 0,$$

$$a_2 = -a_4 = -\frac{1}{n-1},$$

The W_4-curvature tensor if

$$a_0 = 1, a_1 = a_2 = a_3 = a_4 = a_7 = 0,$$

$$a_5 = -a_6 = \frac{1}{n-1},$$

The W_5-curvature tensor if

$$a_0 = 1, a_1 = a_3 = a_4 = a_6 = a_7 = 0,$$

$$a_2 = -a_5 = -\frac{1}{n-1},$$

The W_6-curvature tensor if

$$a_0 = 1, a_2 = a_3 = a_4 = a_5 = a_7 = 0,$$

$$a_1 = -a_6 = -\frac{1}{n-1},$$

The W_7-curvature tensor if

$$a_0 = 1, a_2 = a_3 = a_5 = a_6 = a_7 = 0,$$

$$a_1 = -a_4 = -\frac{1}{n-1},$$

The W_8-curvature tensor if

$$a_0 = 1, a_2 = a_4 = a_5 = a_6 = a_7 = 0,$$

$$a_1 = -a_3 = -\frac{1}{n-1},$$

The W_9-curvature tensor if

$$a_0 = 1, a_1 = a_2 = a_5 = a_6 = a_7 = 0,$$

$$a_3 = -a_4 = \frac{1}{n-1},$$

Also we have the following table by virtue of (24) and flat curvature tensors:

Curvature Tensor	λ_1	Ricci soliton
$\bar{C} = 0$	$\lambda_1 = \frac{-(n-1)[a_0n+a_1(n-1)(3n+2)]}{a_1(3n^3-n^2-n-2)+a_0n(n+1)}$	Shrinking
$C = 0$	$\lambda_1 = \frac{-(n-1)(n^2-n+2)}{(3n^2-3n+2)}$	Shrinking
$N = 0$	$\lambda_1 = \frac{-(n-1)}{2}$	Shrinking
$\bar{P} = 0$	$\lambda_1 = \frac{-(n-1)[a_0(-n^2+2n+1)+a_1(n-1)(2n+1)]}{n[2na_1(n-1)+a_0(n+1)]}$	Shrinking
$P = 0$	$\lambda_1 = -n$	Shrinking
$M = 0$	$\lambda_1 = -(n-1)$	Shrinking
$W_0 = 0$	$\lambda_1 = -(n-1)$	Shrinking
$W_0^* = 0$	$\lambda_1 = (n-1)$	Expanding
$W_1 = 0$	$\lambda_1 = (n-2)$	Expanding
$W_1^* = 0$	$\lambda_1 = -n$	Shrinking
$W_2 = 0$	$\lambda_1 = -(n-1)$	Shrinking
$W_3 = 0$	$\lambda_1 = (n-2)$	Expanding
$W_4 = 0$	$\lambda_1 = -(n-1)$	Shrinking
$W_6 = 0$	$\lambda_1 = \infty$	Expanding
$W_7 = 0$	$\lambda_1 = \infty$	Expanding
$W_8 = 0$	$\lambda_1 = \infty$	Expanding
$W_9 = 0$	$\lambda_1 = \infty$	Expanding

(25)

We use the following results:

Definition 3.1.[8] **Asymptotic curvature ratio:** The asymptotic curvature ratio of a complete nocompact Riemannian manifold (M^n, g) is defined by

$$A(g) := \limsup_{r_p(x)\to+\infty} r_p(x)^2 |Rm(g)(x)|.$$

Noted that it is well-defined since it does not depend on the reference point $p \in M^n$. Moreover, it is invariant under scalings. This geometric invariant has generated a lot of interest: See for example for a static study of the asymptotic curvature ratio and linking this invariant with the Ricci flow. Note also that Gromov and Lott-Shen have shown that any paracompact manifolds can support a complete metric g with finite $A(g)$. Therefore, the only geometric constraint is the Ricci solitons structure.

Theorem 3.1.[8] [Cone structure at infinity] Let $(M^n, g, \nabla f)$, $n \geq 3$, be a complete expanding gradient Ricci soliton with finite $A(g)$.

For $p \in M^n$, $(M^n, t^{-2}g, p)_t$ Gromov-Hausdroff converges to a metric cone $(C(S_\infty).d_\infty, x_\infty)$ over a compact length space S_∞. Moreover,

1. $C(S_\infty) \setminus \{x_\infty\}$ is a smooth manifold with a C^1, α metric g_∞ compatible with d_∞ and the convergence is C^1, α outside the apex x_∞.
2. (S_∞, gs_∞) where gs_∞ is the metric induced by g_∞ on S_∞, is the C^1, α limit of the rescaled levels of the potential function f.
$(f^-1(t^2), t^-2g_{t^2/4})$ where $g_{t^2/4}$ is the metric induced by g on $f_-1(t^2/4)$.

Finally we can ensure that

$$|K_g s_\infty| \leq A(g).in\ Alexandrov\ sense \tag{26}$$

$$\frac{Vol(A_\infty, gS_\infty)}{n} = \lim_r \rightarrow +\infty \frac{VolB(q,r)}{r^n}.\ q \in M^n \tag{27}$$

As direct consequence of Theorem3. in case of vanishing asymptotic curvature ratio, we get the following:

Corollary 3.2.[8](Asymptotically flatness). Let $(M^n, g, \nabla f), n \geq 3$, be a complete expanding gradient Ricci soliton. Assume

$$A(g) = 0.$$

Then, with the notations of Theorem 3, $(S_\infty, gS_\infty) \quad =_i \in \quad I(S^n \quad - \quad 1/\Gamma_i, g_s td)$ and $(C(S_\infty), d_\infty, x_\infty) = (C(S_\infty), eucl, 0)$ where Γ_i are finite groups of Euclidean isometries and $|I|$ is the (finite) number of ends of M^n.
Moreover, for $p \in M^n$,

$$\sum \frac{\omega_n}{|\Gamma_i|} = \lim_r \rightarrow +\infty \frac{VolB(p,r)}{r^n} \tag{28}$$

where ω_n is the volume of the unit Euclidean ball.

From (24), (25), Theorem 3 and corollary 3 we have

Theorem 3.3. If the Ricci soliton (g, V, λ_1), $n \geq 3$ is for zero τ-curvature expanding at ∞ then it has cone structure at ∞, provided asymptotic curvature $A(G)$ is finite or otherwise it is asymptotically flat.

Remark 3.4. The independent calculations for different curvature tensors which are particular curves of $\tau-$curvature tensor will yield the same results of Theorem 3.

4 Ricci solitons on $C(\lambda)$-Manifolds with $B(X,Y)Z = 0$.

S. Bochner introduced a Kähler analogue of the Weyl conformal curvature tensor by purely formal considerations, which is now well known as the Bochner curvature tensor [7]. A geometric meaning of the Bochner curvature tensor is given by D.E. Blair in [6] by using the Boothby-Wang's fibration. In 1969, Matsumoto and Chuman [14] constructed the notion of C-Bochner curvature tensor in a Sasakian manifold and studied its several properties.

The C-Bochner curvature tensor [13] B in M is defined by

$$\begin{aligned}
B(X,Y)Z = {} & R(X,Y)Z + \frac{1}{n+3}[g(X,Z)QY - S(Y,Z)X \\
& - g(Y,Z)QX + S(X,Z)Y + g(\phi X,Z)Q\phi Y \\
& - S(\phi Y,Z)\phi X - g(\phi Y,Z)Q\phi X + S(\phi X,Z)\phi Y \\
& + 2S(\phi X,Y)\phi Z + 2g(\phi X,Y)Q\phi Z + \eta(Y)\eta(Z)QX \\
& - \eta(Y)S(X,Z)\xi + \eta(X)S(Y,Z)\xi - \eta(X)\eta(Z)QY] \\
& - \frac{D+n-1}{n+3}[g(\phi X,Z)\phi Y - g(\phi Y,Z)\phi X \\
& + 2g(\phi X,Y)\phi Z] \\
& + \frac{D}{n+3}[\eta(Y)g(X,Z)\xi - \eta(Y)\eta(Z)X \\
& + \eta(X)\eta(Z)Y - \eta(X)g(Y,Z)\xi] \\
& - \frac{D-4}{n+3}[g(X,Z)Y - g(Y,Z)X],
\end{aligned} \tag{29}$$

where $D = \frac{r+n-1}{n+1}$.

If B vanishes identically then we say that the manifold is C-Bochnerly flat. Thus for a C-Bochnerly flat $C(\lambda)$ manifold, we get

$$\begin{aligned}
R(X,Y)Z = {} & -\frac{1}{n+3}[g(X,Z)QY - S(Y,Z)X - g(Y,Z)QX \\
& + S(X,Z)Y + g(\phi X,Z)Q\phi Y - S(\phi Y,Z)\phi X \\
& - g(\phi Y,Z)Q\phi X + S(\phi X,Z)\phi Y + 2S(\phi X,Y)\phi Z \\
& + 2g(\phi X,Y)Q\phi Z + \eta(Y)\eta(Z)QX - \eta(Y)S(X,Z)\xi \\
& + \eta(X)S(Y,Z)\xi - \eta(X)\eta(Z)QY] \\
& + \frac{D+n-1}{n+3}[g(\phi X,Z)\phi Y - g(\phi Y,Z)\phi X \\
& + 2g(\phi X,Y)\phi Z] \\
& - \frac{D}{n+3}[\eta(Y)g(X,Z)\xi - \eta(Y)\eta(Z)X \\
& + \eta(X)\eta(Z)Y - \eta(X)g(Y,Z)\xi] \\
& + \frac{D-4}{n+3}[g(X,Z)Y - g(Y,Z)X],
\end{aligned} \tag{30}$$

In view of (11) we get from (30)

$$\begin{aligned}
R(\phi X,\phi Y)Z = {} & g(X,Z)Y - Xg(Y,Z) + \phi Xg(\phi Y,Z) \\
& - g(\phi X,Z)\phi Y - \frac{1}{n+3}[g(X,Z)QY \\
& - S(Y,Z)X - g(Y,Z)QX + S(X,Z)Y \\
& + g(\phi X,Z)Q\phi Y - S(\phi Y,Z)\phi X - g(\phi Y,Z)Q\phi X \\
& + S(\phi X,Z)\phi Y + 2S(\phi X,Y)\phi Z + 2g(\phi X,Y)Q\phi Z \\
& + \eta(Y)\eta(Z)QX - \eta(Y)S(X,Z)\xi + \eta(X)S(Y,Z)\xi \\
& - \eta(X)\eta(Z)QY] \\
& + \frac{D+n-1}{n+3}[g(\phi X,Z)\phi Y - g(\phi Y,Z)\phi X \\
& + 2g(\phi X,Y)\phi Z] \\
& - \frac{D}{n+3}[\eta(Y)g(X,Z)\xi - \eta(Y)\eta(Z)X \\
& + \eta(X)\eta(Z)Y - \eta(X)g(Y,Z)\xi] \\
& + \frac{D-4}{n+3}[g(X,Z)Y - g(Y,Z)X] \qquad (31)
\end{aligned}$$

Taking innerproduct with respect to ξ and $Y=\xi$ in (31)By virtue of (7) (16), (17), (18) and on simplification, we get

$$\left[1+\frac{D-4}{n+3}-\frac{D}{n+3}+\frac{\lambda_1}{n+3}\right][g(X,Z)-\eta(X)\eta(Z)] = 0. \qquad (32)$$

Taking $X=Z=e_i$ in (32) and summing over $i=1,2,\dots,n$. Then we get

$$\left[1+\frac{D-4}{n+3}-\frac{D}{n+3}+\frac{\lambda_1}{n+3}\right](n-1) = 0. \qquad (33)$$

On simplification, we get

$$\lambda_1 = -(n-1). \qquad (34)$$

Thus we can state the following:

Theorem 4.1. A Ricci soliton in $C(\lambda)$-manifolds satisfying $B=0$ is shrinking.

5 Conclusion

We use concept of asymptotic curvature $A(G)$ and results on cone structure at ∞ of an expanding gradient Ricci Soliton of [8]. It is shown that $C(\lambda)$-manifold looks like cone at ∞ prosvided asymptotic curvature $A(G)$ is finite and τ-curvature is zero. If $A(G)$ is not finite at ∞ then $C(\lambda)$ is asymptotically flat. Further it is shown that Ricci Solitin of $C(\lambda)$ manifolds is shrinking, when $B=0$.

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