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Fuzzy Soft Semi Connected Properties in Fuzzy Soft Topological Spaces

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Abstract: In the present paper, we continue the study on fuzzy soft topological spaces and investigate the properties of fuzzy soft semi connected sets, fuzzy soft semi separated sets and fuzzy soft semi *s*-connected sets and have established several interesting properties supported by examples. Moreover, we show that a fuzzy soft semi disconnectedness property is not hereditary property in general. Finally, we show that the fuzzy irresolute surjective soft image of fuzzy soft semi connected (resp. fuzzy soft semi *s*-connected) is also a fuzzy soft semi *s*-connected. We hope that the findings in this paper will help researcher enhance and promote the further study on fuzzy soft topology to carry out a general framework for their applications in practical life.

Keywords: Soft set, Fuzzy soft set, Fuzzy soft topological space, Fuzzy semi soft interior, Fuzzy semi soft closure, Fuzzy semi open soft, Fuzzy semi closed soft, Fuzzy semi continuous soft functions, Fuzzy soft connected, Fuzzy soft semi connected, Fuzzy soft semi *s*-connected.

1 Introduction

The concept of soft sets was first introduced by Molodtsov [29] in 1999 as a general mathematical tool for dealing with uncertain objects. In [29,30], Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on. After presentation of the operations of soft sets [27], the properties and applications of soft set theory have been studied increasingly [4,22,30]. Xiao et al.[39] and Pei and Miao [33] discussed the relationship between soft sets and information systems. They showed that soft sets are a class of special information systems. In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [2,3,5,8, 25,26,27,28,30,31,42]. To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [9].
Recently, in 2011, Shabir and Naz [36] initiated the study of soft topological spaces. They defined soft topology on the collection τ of soft sets over X. Consequently, they defined basic notions of soft topological spaces such as open soft and closed soft sets, soft subspace, soft closure, soft nbd of a point, soft separation axioms, soft regular spaces and soft normal spaces and established their several properties. Min in [38] investigate some properties of these soft separation axioms. Kandil et al. [19] introduce the notion of soft semi separation axioms. In particular they studied the properties of the soft semi regular spaces and soft semi normal spaces. Maji et. al. [25] initiated the study involving both fuzzy sets and soft sets. The notion of soft ideal was initiated for the first time by Kandil et al.[15]. They also introduced the concept of soft local function. These concepts are discussed with a view to find new soft topologies from the original one, called soft topological spaces with soft ideal $(X,\tau,E,\tilde{I})$. Applications to various fields were further investigated by Kandil et al. [13,14,16,17,18].
In [6] the notion of fuzzy set soft set was introduced as a fuzzy generalization of soft sets and some basic properties of fuzzy soft sets are discussed in detail. Then many scientists such as X. Yang et. al. [40], improved the concept of fuzziness of soft sets. In [1], Karal and Ahmed

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defined the notion of a mapping on classes of fuzzy soft sets, which is fundamental important in fuzzy soft set theory, to improve this work and they studied properties of fuzzy soft images and fuzzy soft inverse images of fuzzy soft sets. Tanay et.al. [37] introduced the definition of fuzzy soft topology over a subset of the initial universe set while Roy and Samanta [35] gave the definition f fuzzy soft topology over the initial universe set. Chang [10] introduced the concept of fuzzy topology on a set X by axiomatizing a collection $\mathfrak{T}$ of fuzzy subsets of X.

In the present paper, we continue the study on fuzzy soft topological spaces and investigate the properties of fuzzy soft semi connected sets, fuzzy soft semi separated sets and fuzzy soft semi s-connected sets and have established several interesting properties supported by examples. Moreover, we show that a fuzzy soft semi disconnectedness property is not hereditary property in general. Finally, we show that the fuzzy irresolute surjective soft image of fuzzy soft semi connected (resp. fuzzy soft semi s-connected) is also a fuzzy soft semi s-connected. Since the authors introduced topological structures on fuzzy soft sets [6,11,37], so the semi topological properties, which introduced by Mahanta et al.[24], is generalized here to the fuzzy soft sets which will be useful in the fuzzy systems. Because there exists compact connections between soft sets and information systems [33,39], we can use the results deducted from the studies on fuzzy soft topological space to improve these kinds of connections.

2 Preliminaries

In this section, we present the basic definitions and results of soft set theory which will be needed in the sequel.

Definition 2.1.[41] A fuzzy set A of a non-empty set X is characterized by a membership function $\mu_A : X \longrightarrow [0,1] = I$ whose value $\mu_A(x)$ represents the "degree of membership" of x in A for $x \in X$.
Let I^X denotes the family of all fuzzy sets on X. If $A,B \in I^X$, then some basic set operations for fuzzy sets are given by Zadeh [41], as follows:

(1)$A \leq B \Leftrightarrow \mu_A(x) \leq \mu_B(x) \; \forall\, x \in X$.
(2)$A = B \Leftrightarrow \mu_A(x) = \mu_B(x) \; \forall\, x \in X$.
(3)$C = A \vee B \Leftrightarrow \mu_C(x) = \mu_A(x) \vee \mu_B(x) \; \forall\, x \in X$.
(4)$D = A \wedge B \Leftrightarrow \mu_D(x) = \mu_A(x) \wedge \mu_B(x) \; \forall\, x \in X$.
(5)$M = A' \Leftrightarrow \mu_M(x) = 1 - \mu_A(x) \; \forall\, x \in X$.

Definition 2.2.[25] Let $A \subseteq E$. A pair (f,A), denoted by f_A, is called fuzzy soft set over X , where f is a mapping given by $f : A \to I^X$ defined by $f_A(e) = \mu^e_{f_A}$, where $\mu^e_{f_A} = \overline{0}$ if $e \notin A$ and $\mu^e_{f_A} \neq \overline{0}$ if $e \in A$, where $\overline{0}(x) = 0 \; \forall\, x \in X$. The family of all these fuzzy soft sets over X denoted by $FSS(X)_A$.

Proposition 2.1.[3] Every fuzzy set may be considered a soft set.

Definition 2.3.[34] The complement of fuzzy soft set (f,A), denoted by $(f,A)'$, is defined by $(f,A)' = (f',A)$, $f'_A : E \to I^X$ is a mapping given by $\mu^e_{f'_A} = \overline{1} - \mu^e_{f_A} \; \forall e \in A$, where $\overline{1}(x) = 1 \; \forall\, x \in X$. Clearly, $(f'_A)' = f_A$.

Definition 2.4.[27] A fuzzy soft set f_A over X is said to be a NULL fuzzy soft set, denoted by $\tilde{0}_A$, if for all $e \in A$, $f_A(e) = \overline{0}$.

Definition 2.5.[27] A fuzzy soft set f_A over X is said to be an absolute fuzzy soft set, denoted by $\tilde{1}_A$, if for all $e \in A$, $f_A(e) = \overline{1}$. Clearly we have $(\tilde{1}_A)' = \tilde{0}_A$ and $(\tilde{0}_A)' = \tilde{1}_A$.

Definition 2.6.[34] Let $f_A, g_B \in FSS(X)_E$. Then f_A is fuzzy soft subset of g_B, denoted by $f_A \sqsubseteq g_B$, if $A \subseteq B$ and $\mu^e_{f_A} \subseteq \mu^e_{g_B} \; \forall \; e \in A$, i.e. $\mu^e_{f_A}(x) \leq \mu^e_{g_B}(x) \; \forall\, x \in X \; and \; \forall\, e \in A$.

Definition 2.7.[34] The union of two fuzzy soft se ts f_A and g_B over the common universe X is also a fuzzy soft set h_C, where $h_C(e) = \mu^e_{h_c} = \mu^e_{f_A} \cup \mu^e_{g_B} \; \forall e \in E$. Here, we write $h_C = f_A \sqcup g_B$.

Definition 2.8.[34] The intersection of two fuzzy soft sets f_A and g_B over the common universe X is also a fuzzy soft set h_C, where $h_C(e) = \mu^e_{h_c} = \mu^e_{f_A} \cap \mu^e_{g_B} \; \forall e \in E$. Here, we write $h_C = f_A \sqcap g_B$.

Theorem 2.1.[2] Let $\{(f,A)_j : j \in J\} \subseteq FSS(X)_E$. Then the following statements hold,

(1)$[\sqcup_{j\in J}(f,A)_j]' = \sqcap_{j\in J}(f,A)'_j$,
(2)$[\sqcap_{j\in J}(f,A)_j]' = \sqcup_{j\in J}(f,A)'_j$.

Definition 2.9.[34] Let $\mathfrak{T}$ be a collection of fuzzy soft sets over a universe X with a fixed set of parameters E, then $\mathfrak{T} \subseteq FSS(X)_E$ is called fuzzy soft topology on X if

(1)$\tilde{1}_E, \tilde{0}_E \in \mathfrak{T}$, where $\tilde{0}_E(e) = \overline{0}$ and $\tilde{1}_E(e) = \overline{1}$, $\forall e \in E$,
(2)the union of any members of $\mathfrak{T}$ belongs to $\mathfrak{T}$,
(3)the intersection of any two members of $\mathfrak{T}$ belongs to $\mathfrak{T}$.

The triplet $(X,\mathfrak{T},E)$ is called fuzzy soft topological space over X. Also, each member of $\mathfrak{T}$ is called fuzzy open soft in $(X,\mathfrak{T},E)$. We denote the set of all open soft sets by $FOS(X,\mathfrak{T},E)$, or $FOS(X)$.

Definition 2.10.[34] Let $(X,\mathfrak{T},E)$ be a fuzzy soft topological space. A fuzzy soft set f_A over X is said to be fuzzy closed soft set in X, if its relative complement f'_A is fuzzy open soft set. We denote the set of all fuzzy closed soft sets by $FCS(X,\mathfrak{T},E)$, or $FCS(X)$.

Definition 2.11.[32] Let $(X,\mathfrak{T},E)$ be a fuzzy soft topological space and $f_A \in FSS(X)_E$. The fuzzy soft closure of f_A, denoted by $Fcl(f_A)$ is the intersection of all fuzzy closed soft super sets of f_A. i.e.,
$Fcl(f_A) = \sqcap\{h_D : h_D \; is \; fuzzy \; closed \; soft \; set \; and \; f_A \sqsubseteq h_D\}$.
The fuzzy soft interior of g_B, denoted by $Fint(f_A)$ is the fuzzy soft union of all fuzzy open soft subsets of f_A.i.e.,
$Fint(g_B) = \sqcup\{h_D : h_D \; is \; fuzzy \; open \; soft \; set \; and \; h_D \sqsubseteq g_B\}$.

Definition 2.12.[24] The fuzzy soft set $f_A \in FSS(X)_E$ is called fuzzy soft point if there exist $x \in X$ and $e \in E$ such that $\mu^e_{f_A}(x) = \alpha$ $(0 < \alpha \leq 1)$ and $\mu^e_{f_A}(y) = \overline{0}$ for each $y \in X - \{x\}$, and this fuzzy soft point is denoted by x^e_α or f_e.

Definition 2.13.[24] The fuzzy soft point x^e_α is said to be belonging to the fuzzy soft set (g,A), denoted by $x^e_\alpha \tilde{\in} (g,A)$, if for the element $e \in A$, $\alpha \leq \mu^e_{g_A}(x)$.

Theorem 2.2.[24] Let $(X,\mathfrak{T},E)$ be a fuzzy soft topological space and f_e be a fuzzy soft point. Then the following properties hold:

(1)If $f_e \tilde{\in} g_A$, then $f_e \tilde{\notin} g'_A$;
(2)$f_e \tilde{\in} g_A \nRightarrow f'_e \tilde{\in} g'_A$;
(3)Every non-null fuzzy soft set f_A can be expressed as the union of all the fuzzy soft points belonging to f_A.

Definition 2.14.[24] A fuzzy soft set g_B in a fuzzy soft topological space $(X,\mathfrak{T},E)$ is called fuzzy soft neighborhood of the fuzzy soft point x^e_α if there exists a fuzzy open soft set h_C such that $x^e_\alpha \tilde{\in} h_C \sqsubseteq g_B$. A fuzzy soft set g_B in a fuzzy soft topological space $(X,\mathfrak{T},E)$ is called fuzzy soft neighborhood of the soft set f_A if there exists a fuzzy open soft set h_C such that $f_A \sqsubseteq h_C \sqsubseteq g_B$. The fuzzy soft neighborhood system of the fuzzy soft point x^e_α, denoted by $N_{\mathfrak{T}}(x^e_\alpha)$, is the family of all its fuzzy soft neighborhoods.

Definition 2.15.[24] Let $(X,\mathfrak{T},E)$ be a fuzzy soft topological space and $Y \subseteq X$. Let h^Y_E be a fuzzy soft set over (Y,E) such that $h^Y_E : E \to I^Y$ such that $h^Y_E(e) = \mu^e_{h^Y_E}$,

$$\mu^e_{h^Y_E}(x) = \begin{cases} 1 \; x \in Y, \\ 0, \; x \notin Y. \end{cases}$$

Let $\mathfrak{T}_Y = \{h^Y_E \sqcap g_B : g_B \in \mathfrak{T}\}$, then the fuzzy soft topology $\mathfrak{T}_Y$ on $(Y,E)$is called fuzzy soft subspace topology for (Y,E) and $(Y,\mathfrak{T}_Y,E)$ is called fuzzy soft subspace of $(X,\mathfrak{T},E)$. If $h^Y_E \in \mathfrak{T}$ (resp. $h^Y_E \in \mathfrak{T}'$), then $(Y,\mathfrak{T}_Y,E)$ is called fuzzy open (resp. closed) soft subspace of $(X,\mathfrak{T},E)$.

Definition 2.16.[32] Let $FSS(X)_E$ and $FSS(Y)_K$ be families of fuzzy soft sets over X and Y, respectively. Let $u : X \to Y$ and $p : E \to K$ be mappings. Then the map f_{pu} is called fuzzy soft mapping from X to Y and denoted by $f_{pu} : FSS(X)_E \to FSS(Y)_K$ such that,

(1)If $f_A \in FSS(X)_E$. Then the image of f_A under the fuzzy soft mapping f_{pu} is the fuzzy soft set over Y defined by $f_{pu}(f_A)$, where $\forall\, k \in p(E)$, $\forall\, y \in Y$,
$$f_{pu}(f_A)(k)(y) = \begin{cases} \bigvee_{u(x)=y} \; [\vee_{p(e)=k}(f_A(e))](x) & if\; x \in u^{-1}(y), \\ 0 & otherwise. \end{cases}$$

(2)If $g_B \in FSS(Y)_K$, then the pre-image of g_B under the fuzzy soft mapping f_{pu} is the fuzzy soft set over X defined by $f^{-1}_{pu}(g_B)$, where $\forall\, e \in p^{-1}(K)$, $\forall\, x \in X$,
$$f^{-1}_{pu}(g_B)(e)(x) = \begin{cases} g_B(p(e))(u(x)) & for\; p(e) \in B, \\ 0 & otherwise. \end{cases}$$

The fuzzy soft mapping f_{pu} is called surjective (resp. injective) if p and u are surjective (resp. injective), also it is said to be constant if p and u are constant.

Definition 2.17.[32] Let $(X,\mathfrak{T}_1,E)$ and $(Y,\mathfrak{T}_2,K)$ be two fuzzy soft topological spaces and $f_{pu} : FSS(X)_E \to FSS(Y)_K$ be a fuzzy soft mapping. Then f_{pu} is called

(1)Fuzzy continuous soft if $f^{-1}_{pu}(g_B) \in \mathfrak{T}_1 \;\forall\, (g_B) \in \mathfrak{T}_2$.
(2)Fuzzy open soft if $f_{pu}(g_A) \in \mathfrak{T}_2 \forall\, (g_A) \in \mathfrak{T}_1$.

Theorem 2.3.[1] Let $FSS(X)_E$ and $FSS(Y)_K$ be two families of fuzzy soft sets. For the fuzzy soft function $f_{pu} : FSS(X)_E \to FSS(Y)_K$, the following statements hold,

(a)$f^{-1}_{pu}((g,B)') = (f^{-1}_{pu}(g,B))' \forall\, (g,B) \in FSS(Y)_K$.
(b)$f_{pu}(f^{-1}_{pu}((g,B))) \sqsubseteq (g,B) \forall\, (g,B) \in FSS(Y)_K$. If f_{pu} is surjective, then the equality holds.
(c)$(f,A) \sqsubseteq f^{-1}_{pu}(f_{pu}((f,A))) \forall\, (f,A) \in FSS(X)_E$. If f_{pu} is injective, then the equality holds.
(d)$f_{pu}(\tilde{0}_E) = \tilde{0}_K$, $f_{pu}(\tilde{1}_E) \sqsubseteq \tilde{1}_K$. If f_{pu} is surjective, then the equality holds.
(e)$f^{-1}_{pu}(\tilde{1}_K) = \tilde{1}_E$ and $f^{-1}_{pu}(\tilde{0}_K) = \tilde{0}_E$.
(f)If $(f,A) \sqsubseteq (g,A)$, then $f_{pu}(f,A) \sqsubseteq f_{pu}(g,A)$.
(g)If $(f,B) \sqsubseteq (g,B)$, then $f^{-1}_{pu}(f,B) \sqsubseteq f^{-1}_{pu}(g,B) \;\forall\, (f,B),(g,B) \in FSS(Y)_K$.
(h)$f^{-1}_{pu}(\sqcup_{j\in J}(f,B)_j) = \sqcup_{j\in J} f^{-1}_{pu}(f,B)_j$ and $f^{-1}_{pu}(\sqcap_{j\in J}(f,B)_j) = \sqcap_{j\in J} f^{-1}_{pu}(f,B)_j, \forall\; (f,B)_j \in FSS(Y)_K$.
(I)$f_{pu}(\sqcup_{j\in J}(f,A)_j) = \sqcup_{j\in J} f_{pu}(f,A)_j$ and $f_{pu}(\sqcap_{j\in J}(f,A)_j) \sqsubseteq \sqcap_{j\in J} f_{pu}(f,A)_j \;\forall\; (f,A)_j \in FSS(X)_E$. If f_{pu} is injective, then the equality holds.

Definition 2.18.[24] Let $(X,\mathfrak{T},E)$ be a fuzzy soft topological space. A fuzzy soft separation of $\tilde{1}_E$ is a pair of non null proper fuzzy open soft sets g_B, h_C such that $g_B \sqcap h_C = \tilde{0}_E$ and $\tilde{1}_E = g_B \sqcup h_C$.

Definition 2.19.[24] A fuzzy soft topological space $(X,\mathfrak{T},E)$ is said to be fuzzy soft connected if and only if there is no fuzzy soft separations of $\tilde{X}$. Otherwise, $(X,\mathfrak{T},E)$ is said to be fuzzy soft disconnected space.

Definition 2.20.[12] Let (X,τ,E) be a soft topological space and $F_A \in SS(X)_E$. If $F_A \tilde{\subseteq} cl(int(F_A))$, then F_A is called semi open soft set. We denote the set of all semi open soft sets by $SOS(X,\tau,E)$, or $SOS(X)$ and the set of all semi closed soft sets by $SCS(X,\tau,E)$, or $SCS(X)$.

Definition 2.21.[7] Let (X,τ,E) be a soft topological space. A soft semi separation on $\tilde{X}$ is a pair of non null proper semi open soft sets F_A, G_B such that $F_A \tilde{\cap} G_B = \tilde{\phi}$ and $\tilde{X} = F_A \tilde{\cup} G_B$.

Definition 2.22.[7] A soft topological space (X,τ,E) is said to be soft semi connected if and only if there is no soft semi separations of $\tilde{X}$. Otherwise, (X,τ,E) is said to be soft semi disconnected space.

Definition 2.23.[20] Let $(X,\mathfrak{T},E)$ be a fuzzy soft topological space and $f_A \in FSS(X)_E$. If $f_A \sqsubseteq Fcl(Fint(f_A))$, then f_A is called fuzzy semi open soft set. We denote the set of all fuzzy semi open soft sets by $FSOS(X,\mathfrak{T},E)$, or $FSOS(X)$ and the set of all fuzzy semi closed soft sets by $FSCS(X,\mathfrak{T},E)$, or $FSCS(X)$.

Definition 2.24.[20] Let $(X,\mathfrak{T},E)$ be a fuzzy soft topological space, $f_A \in FSS(X)_E$ and $f_e \tilde{\in} FSS(X)_E$. Then

(1) f_e is called fuzzy semi interior soft point of f_A if $\exists$ $g_B \in FSOS(X)$ such that $f_e \tilde{\in} g_B \sqsubseteq f_A$. The set of all fuzzy semi interior soft points of f_A is called the fuzzy semi soft interior of f_A and is denoted by $FSint(f_A)$ consequently,
$FSint(f_A) = \sqcup\{g_B : g_B \sqsubseteq f_A,\ g_B \in FSOS(X)\}$.

(2) f_e is called fuzzy semi closure soft point of f_A if $f_A \sqcap h_C \neq \tilde{0}_E\ \forall\ h_D \in FSOS(X)$. The set of all fuzzy semi closure soft points of f_A is called fuzzy semi soft closure of f_A and denoted by $FScl(f_A)$. Consequently, $FScl(f_A) = \sqcap\{h_D : h_D \in FSCS(X),\ f_A \sqsubseteq h_D\}$.

Definition 2.25.[20] Let $(X,\mathfrak{T}_1,E)$, $(Y,\mathfrak{T}_2,K)$ be fuzzy soft topological spaces and $f_{pu} : FSS(X)_E \to FSS(Y)_K$ be a soft function. Then f_{pu} is called;

(1) Fuzzy semi continuous soft function if $f_{pu}^{-1}(g_B) \in FSOS(X) \forall g_B \in \mathfrak{T}_2$.
(2) Fuzzy fuzzy semi open soft if $f_{pu}(g_A) \in FSOS(Y) \forall g_A \in \mathfrak{T}_1$.
(3) Fuzzy semi closed soft if $f_{pu}(f_A) \in FSCS(Y) \forall f_A \in \mathfrak{T}_1'$.
(4) Fuzzy irresolute soft if $f_{pu}^{-1}(g_B) \in FSOS(X) \forall g_B \in FSOS(Y)$.
(5) Fuzzy irresolute open soft if $f_{pu}(g_A) \in FSOS(Y) \forall g_A \in FSOS(X)$.
(6) Fuzzy irresolute closed soft if $f_{pu}(f_A) \in FSCS(Y) \forall f_A \in FSCS(Y)$.

3 Fuzzy soft semi connectedness

Connectedness is one of the important notions of topology. F. Lin [23] introduced the notions of soft connectedness in soft topological spaces. Mahanta and Das [24] introduce the notions of fuzzy soft connectedness in fuzzy soft topological spaces. In this section, we introduce the notions of fuzzy soft semi connectedness in fuzzy soft topological space and examine its basic properties.

Definition 3.1. Two fuzzy soft sets f_A and g_B are said to be disjoint, denoted by $f_A \sqcap g_B = \tilde{0}_E$, if $A \cap B = \varphi$ and $\mu^e_{f_A} \cap \mu^e_{g_B} = \overline{0}\ \forall e \in E$.

Definition 3.2. Let $(X,\mathfrak{T},E)$ be a fuzzy soft topological space. A fuzzy soft semi separation on $\tilde{1}_E$ is a pair of non null proper fuzzy semi open soft sets f_A, g_B such that $f_A \sqcap g_B = \tilde{0}_E$ and $\tilde{1}_E = f_A \sqcup g_B$.

Definition 3.3. A fuzzy soft topological space $(X,\mathfrak{T},E)$ is said to be fuzzy soft semi connected if and only if there is no fuzzy soft semi separations of $\tilde{1}_E$. Otherwise, $(X,\mathfrak{T},E)$ is said to be fuzzy soft semi disconnected space.

Examples 3.1.

(1) Let $X = \{a,b\}$, $E = \{e_1,e_2\}$ and $\mathfrak{T}$ be the discrete fuzzy soft topology on X. Then $(X,\mathfrak{T},E)$ is not fuzzy soft semi connected.
(2) Let $X = \{a,b\}$, $E = \{e_1,e_2\}$ and $\mathfrak{T}$ be the indiscrete fuzzy soft topology on X. Then $\mathfrak{T}$ is always fuzzy soft semi connected.

Definition 3.4. A fuzzy soft subspace $(Y,\mathfrak{T}_Y,E)$ of fuzzy soft topological space $(X,\mathfrak{T},E)$ is said to be fuzzy semi open soft (resp. semi closed soft, soft semi connected) subspace if $h^Y_E \in FSOS(X)$ (resp. $h^Y_E \in FSCS(X)$, h^Y_E is fuzzy soft semi connected).

Theorem 3.1. Let $(Y,\mathfrak{T}_Y,E)$ be a fuzzy soft semi connected subspace of fuzzy soft topological space $(X,\mathfrak{T},E)$ such that $h^Y_E \sqcap g_A \in FSOS(X)\ g_A \in FSOS(X)$. If $\tilde{1}_E$ has a fuzzy soft semi separations f_A, g_B, then either $h^Y_E \sqsubseteq f_A$, or $h^Y_E \sqsubseteq g_B$.

Proof. Let f_A, g_B be fuzzy soft semi separation on $\tilde{1}_E$. By hypothesis, $f_A \sqcap h^Y_E \in FSOS(X)$, $g_B \sqcap h^Y_E \in FSOS(X)$ and $[g_B \sqcap h^Y_E] \sqcup [f_A \sqcap h^Y_E] = h^Y_E$. Since h^Y_E is fuzzy soft semi connected. Then either $g_B \sqcap h^Y_E = \tilde{0}_E$, or $f_A \sqcap h^Y_E = \tilde{0}_E$. Therefore, either $h^Y_E \sqsubseteq f_A$, or $h^Y_E \sqsubseteq g_B$.

Theorem 3.2. If $(X,\mathfrak{T}_2,E)$ is a fuzzy soft semi connected space and $\mathfrak{T}_1$ is fuzzy soft coarser than $\mathfrak{T}_2$, then $(X,\mathfrak{T}_1,E)$ is also a fuzzy soft semi connected.

Proof. Let f_A, g_B be fuzzy soft semi separation on $(X,\mathfrak{T}_1,E)$. Then $f_A, g_B \in \mathfrak{T}_1$. Since $\mathfrak{T}_1 \subseteq \mathfrak{T}_2$. Then $f_A, g_B \in \mathfrak{T}_2$ such that f_A, g_B is fuzzy soft semi separation on $(X,\mathfrak{T}_2,E)$, which is a contradiction with the fuzzy soft semi connectedness of $(X,\mathfrak{T}_2,E)$. Hence, $(X,\mathfrak{T}_1,E)$ is fuzzy soft semi connected.

Remark 3.1 The converse of Theorem 3.2 is not true in general, as shown in the following example.

Example 3.1. Let $X = \{a,b,c\}$, $E = \{e_1,e_2,e_3,e_4\}$ and $A,B \subseteq E$ where $A = \{e_1,e_2\}$ and $B = \{e_3,e_4\}$. Let $\mathfrak{T}_1$ be the indiscrete fuzzy soft semi topology, then $\mathfrak{T}_1$ is fuzzy soft semi connected, on the other hand, let $\mathfrak{T}_2 = \{\tilde{1}_E, \tilde{0}_E, f_A, g_A, k_B, h_B, s_E, v_E\}$ where $f_A, g_A, k_B, h_B, s_E, v_E$ are fuzzy soft sets over X defined as follows:
$\mu^{e_1}_{f_A} = \{a_1,b_1,c_1\}$, $\mu^{e_2}_{f_A} = \{a_1,b_1,c_1\}$,
$\mu^{e_1}_{g_A} = \{a_{0.2},b_{0.5},c_{0.8}\}$, $\mu^{e_2}_{g_A} = \{a_{0.1},b_{0.6},c_{0.7}\}$,
$\mu^{e_3}_{k_B} = \{a_1,b_1,c_1\}$, $\mu^{e_4}_{k_B} = \{a_1,b_1,c_1\}$,
$\mu^{e_3}_{h_B} = \{a_{0.5},b_0,c_{0.3}\}$, $\mu^{e_4}_{h_B} = \{a_1,b_{0.8},c_{0.3}\}$,
$\mu^{e_1}_{s_E} = \{a_{0.2},b_{0.5},c_{0.8}\}$, $\mu^{e_2}_{s_E} = \{a_{0.1},b_{0.6},c_{0.7}\}$,
$\mu^{e_3}_{s_E} = \{a_1,b_1,c_1\}$, $\mu^{e_4}_{s_E} = \{a_1,b_1,c_1\}$,
$\mu^{e_1}_{v_E} = \{a_1,b_1,c_1\}$, $\mu^{e_2}_{v_E} = \{a_1,b_1,c_1\}$,
$\mu^{e_3}_{v_E} = \{a_{0.5},b_0,c_{0.3}\}$, $\mu^{e_4}_{v_E} = \{a_1,b_{0.8},c_{0.3}\}$,
Then $\mathfrak{T}_2$ defines a fuzzy soft topology on X such that $\mathfrak{T}_1 \subseteq \mathfrak{T}_2$. Now, f_A and k_B are fuzzy semi open soft sets in which form a fuzzy soft semi separation of $(X,\mathfrak{T}_2,E)$ where $f_A \sqcap k_B = \tilde{0}_E$ and $\tilde{1}_E = f_A \sqcup k_B$. Hence, $(X,\mathfrak{T}_2,E)$ is fuzzy soft semi disconnected.

Theorem 3.3. A fuzzy soft subspace $(Y,\mathfrak{T}_Y,E)$ of fuzzy soft semi disconnectedness space $(X,\mathfrak{T},E)$ is fuzzy soft semi disconnected if $h^Y_E \sqcap g_A \in FSOS(X)\ \forall\ g_A \in FSOS(X)$.

Proof. Let $(Y,\mathfrak{T}_Y,E)$ be fuzzy soft semi connected space. Since $(X,\mathfrak{T},E)$ is fuzzy soft semi disconnected. Then there exist fuzzy soft semi separation f_A, g_B on $(X,\mathfrak{T},E)$. By hypothesis, $f_A \sqcap h_E^Y \in FSOS(X)$, $g_B \sqcap h_E^Y \in FSOS(X)$ and $[g_B \sqcap h_E^Y] \sqcup [f_A \sqcap h_E^Y] = h_E^Y$, which is a contradiction with the fuzzy soft semi connectedness of $(Y,\mathfrak{T}_Y,E)$. Therefore, $(Y,\mathfrak{T}_Y,E)$ is fuzzy soft semi disconnected.

Remark 3.2 A fuzzy soft semi disconnectedness property is not hereditary property in general, as in the following example.

Example 3.2. In Example 3.1, let $Y = \{a,b\} \subseteq X$. We consider the fuzzy soft set h_E^Y over (Y,E) defined as follows:
$\mu_{h_E^Y}^{e_1} = \{a_1,b_1,c_0\}$, $\mu_{h_E^Y}^{e_2} = \{a_1,b_1,c_0\}$, $\mu_{h_E^Y}^{e_3} = \{a_1,b_1,c_0\}$, $\mu_{h_E^Y}^{e_4} = \{a_1,b_1,c_0\}$.
Then we find $\mathfrak{T}_Y$ as follows, $\mathfrak{T}_Y = \{h_E^Y \sqcap z_E : z_E \in \mathfrak{T}\}$ where
$h_E^Y \sqcap \tilde{0}_E = \tilde{0}_E$, $h_E^Y \sqcap \tilde{1}_E = h_E^Y$, $h_E^Y \sqcap f_A = h_C$, where
$\mu_{h_C}^{e_1} = \{a_1,b_1,c_0\}$, $\mu_{h_C}^{e_2} = \{a_1,b_1,c_0\}$,

$h_E^Y \sqcap g_A = h_W$, where
$\mu_{h_W}^{e_1} = \{a_{0.2},b_{0.5},c_0\}$, $\mu_{h_W}^{e_2} = \{a_{0.1},b_{0.6},c_0\}$,

$h_E^Y \sqcap k_B = h_R$, where
$\mu_{h_R}^{e_3} = \{a_1,b_1,c_0\}$, $\mu_{h_R}^{e_4} = \{a_1,b_1,c_0\}$,

$h_E^Y \sqcap h_B = h_T$, where
$\mu_{h_T}^{e_3} = \{a_{0.5},b_0,c_0\}$, $\mu_{h_T}^{e_4} = \{a_1,b_{0.8},c_0\}$,

$h_E^Y \sqcap s_E = h_P$, where
$\mu_{h_P}^{e_1} = \{a_{0.2},b_{0.5},c_0\}$, $\mu_{h_P}^{e_2} = \{a_{0.1},b_{0.6},c_0\}$,
$\mu_{h_P}^{e_3} = \{a_1,b_1,c_0\}$, $\mu_{h_P}^{e_4} = \{a_1,b_1,c_0\}$.
Thus, the collection $\mathfrak{T}_Y = \{h_E^Y \sqcap z_E : z_E \in \mathfrak{T}\}$ is a fuzzy soft topology on (Y,E) in which there is no fuzzy soft semi separation on $(Y,\mathfrak{T}_Y,E)$. Therefore, $(Y,\mathfrak{T}_Y,E)$ is fuzzy soft semi connected at the time that $(X,\mathfrak{T},E)$ is fuzzy soft semi disconnected as shown in Example 3.1.

Theorem 3.4. Let $(X_1,\mathfrak{T}_1,E)$ and $(X_2,\mathfrak{T}_2,K)$ be fuzzy soft topological spaces and $f_{pu} : (X_1,\mathfrak{T}_1,E) \to (X_2,\mathfrak{T}_2,K)$ be a fuzzy irresolute surjective soft function. If $(X_1,\mathfrak{T}_1,E)$ is fuzzy soft semi connected, then $(X_2,\mathfrak{T}_2,K)$ is also a fuzzy soft semi connected.

Proof. Let $(X_2,\mathfrak{T}_2,K)$ be a fuzzy soft semi disconnected space. Then there exist f_A, g_B pair of non null proper fuzzy semi open soft subsets of $\tilde{1}_K$ such that $f_A \sqcap g_B = \tilde{0}_K$ and $\tilde{1}_K = f_A \sqcup g_B$. Since f_{pu} is fuzzy irresolute soft function, then $f_{pu}^{-1}(f_A), f_{pu}^{-1}(g_B)$ are pair of non null proper fuzzy semi open soft subsets of $\tilde{1}_E$ such that $f_{pu}^{-1}(f_A) \sqcap f_{pu}^{-1}(g_B) = f_{pu}^{-1}(f_A \sqcap g_B) = f_{pu}^{-1}(\tilde{0}_K) = \tilde{0}_E$ and $f_{pu}^{-1}(f_A) \sqcup f_{pu}^{-1}(g_B) = f_{pu}^{-1}(f_A \sqcup g_B) = f_{pu}^{-1}(\tilde{1}_K) = \tilde{1}_E$ from Theorem 2.3. This means that, $f_{pu}^{-1}(f_A), f_{pu}^{-1}(g_B)$ forms a fuzzy soft semi separation of $\tilde{1}_E$, which is a contradiction with the fuzzy soft semi connectedness of $(X_1,\mathfrak{T}_1,E)$. Therefore, $(X_2,\mathfrak{T}_2,K)$ is fuzzy soft semi connected.

4 Fuzzy soft semi *s*-connected spaces

In this section, we introduce the notions of fuzzy soft semi separated sets and use it to introduce the notions of fuzzy semi *s*-connectedness in fuzzy soft topological spaces and study its basic properties. **Definition 4.1.** A non null fuzzy soft subsets f_A, g_B of fuzzy soft topological space $(X,\mathfrak{T},E)$ are said to be fuzzy soft semi separated sets if $FScl(f_A) \sqcap g_B = FScl(g_B) \sqcap f_A = \tilde{0}_E$.

Theorem 4.1. Let $f_A \sqsubseteq g_B$, $h_C \sqsubseteq k_D$ and g_B, k_D are soft fuzzy soft semi separated subsets of fuzzy soft topological space $(X,\mathfrak{T},E)$. Then f_A, h_C are fuzzy soft semi separated sets.

Proof. Let $f_A \sqsubseteq g_B$, then $FScl(f_A) \sqsubseteq FScl(g_B)$. It follows that,
$FScl(f_A) \sqcap h_C \sqsubseteq FScl(f_A) \sqcap k_D \sqsubseteq FScl(g_B) \sqcap k_D = \tilde{0}_E$. Also, since $h_C \sqsubseteq k_D$. Then $FScl(h_C) \sqsubseteq FScl(k_D)$. Hence, $f_A \sqcap FScl(h_C) \sqsubseteq FScl(k_D) \sqcap g_B = \tilde{0}_E$. Thus, f_A, h_C are fuzzy soft semi separated sets.

Theorem 4.2. Two fuzzy semi closed soft subsets of fuzzy soft topological space $(X,\mathfrak{T},E)$ are fuzzy soft semi separated sets if and only if they are disjoint.

Proof. Let f_A, g_B are fuzzy soft semi separated sets. Then $FScl(g_B) \sqcap f_A = g_B \sqcap FScl(f_A) = \tilde{0}_E$. Since f_A, g_B are fuzzy semi closed soft sets. Then $f_A \sqcap g_B = \tilde{0}_E$. Conversely, let f_A, g_B are disjoint fuzzy semi closed soft sets. Then $g_B \sqcap FScl(f_A) = f_A \sqcap g_B = \tilde{0}_E$ and $FScl(g_B) \sqcap f_A = f_A \sqcap g_B = \tilde{0}_E$. It follows that, f_A, g_B are fuzzy soft semi separated sets.

Definition 4.2. A fuzzy soft topological space $(X,\mathfrak{T},E)$ is said to be fuzzy soft semi *s*-connected if and only if $\tilde{1}_E$ can not expressed as the fuzzy soft union of two fuzzy soft semi separated sets in $(X,\mathfrak{T},E)$.

Theorem 4.3. Let $(Z,\mathfrak{T}_Z,E)$ be a fuzzy soft subspace of fuzzy soft topological space $(X,\mathfrak{T},E)$ and f_A, $g_B \sqsubseteq z_E \sqsubseteq \tilde{1}_E$. Then f_A and g_B are fuzzy soft semi separated on $\mathfrak{T}_Z$ if and only if f_A and g_B are fuzzy soft semi separated on $\mathfrak{T}$, where $\mathfrak{T}_Z$ is the fuzzy soft subspace for z_E.

Proof. Suppose that f_A and g_B are fuzzy soft semi separated on $\mathfrak{T}_Z$ $\Leftrightarrow$ $FScl_{\mathfrak{T}_Z}(f_A) \sqcap g_B = \tilde{\phi}$ and $f_A \sqcap FScl_{\mathfrak{T}_Z}(g_B) = \tilde{0}_E$ $\Leftrightarrow$ $[FScl_{\mathfrak{T}}(f_A) \sqcap z_E] \sqcap g_B = FScl_{\mathfrak{T}}(f_A) \sqcap g_B = \tilde{0}_E$ and $[FScl_{\mathfrak{T}}(g_B) \sqcap z_E] \sqcap f_A = FScl_{\mathfrak{T}}(g_B) \sqcap f_A = \tilde{0}_E$ $\Leftrightarrow$ f_A and g_B are fuzzy soft semi separated sets on $\mathfrak{T}$.

Theorem 4.4. Let z_E be a fuzzy soft subset of fuzzy soft topological space $(X,\mathfrak{T},E)$. Then z_E is fuzzy soft semi *s*-connected w.r.t $(X,\mathfrak{T},E)$ if and only if z_E is fuzzy soft semi *s*-connected w.r.t $(Z,\mathfrak{T}_Z,E)$.

Proof. Suppose that z_E is not fuzzy soft semi *s*-connected w.r.t $(Z,\mathfrak{T}_Z,E)$. Then, $z_E = f_{1A} \sqcup f_{2B}$, where f_{1A} and f_{2B} are fuzzy soft semi separated sets on $\mathfrak{T}_Z$ $\Leftrightarrow$ $z_E = f_{1A} \sqcup f_{2B}$, where f_{1A} and f_{2B} are fuzzy soft semi separated on $\mathfrak{T}_Z$ from Theorem 4.3 $\Leftrightarrow$ z_E is not fuzzy soft semi *s*-connected w.r.t $(X,\mathfrak{T},E)$.

Theorem 4.5. Let $(Z,\mathfrak{T}_Z,E)$ be a fuzzy soft semi s-connected subspace of fuzzy soft topological space $(X,\mathfrak{T},E)$ and f_A, g_B be fuzzy soft semi separated of $\tilde{1}_E$ with $z_E \sqsubseteq f_A \sqcup g_B$, then either $z_E \sqsubseteq f_A$, or $z_E \sqsubseteq g_B$.

Proof. Let $z_E \sqsubseteq f_A \sqcup g_B$ for some fuzzy soft semi separated subsets f_A, g_B of $\tilde{1}_E$. Since $z_E = (z_E \sqcap f_A) \sqcup (z_E \sqcap g_B)$. Then $(z_E \sqcap f_A) \sqcap FScl_{\mathfrak{T}}(z_E \sqcap g_B) \sqsubseteq (f_A \sqcap FScl_{\mathfrak{T}} g_B) = \tilde{0}_E$. Also, $FScl_{\mathfrak{T}}(z_E \sqcap f_A) \sqcap (z_E \sqcap g_B) \sqsubseteq FScl_{\mathfrak{T}}(f_A) \sqcap g_B = \tilde{0}_E$. Since $(Z,\mathfrak{T}_Z,E)$ is fuzzy soft semi s-connected. Thus, either $z_E \sqcap f_A = \tilde{0}_E$ or $z_E \sqcap g_B = \tilde{0}_E$. It follows that, $z_E = z_E \sqcap f_A$ or $z_E = z_E \sqcap g_B$. This implies that, $z_E \sqsubseteq f_A$ or $z_E \sqsubseteq g_B$.

Theorem 4.6. Let $(Z,\mathfrak{T}_Z,N)$ and $(Y,\mathfrak{T}_Y,M)$ be fuzzy soft semi s-connected subspaces of fuzzy soft topological space $(X,\mathfrak{T},E)$ such that none of them is fuzzy soft semi separated. Then $z_N \sqcup y_M$ is fuzzy soft semi s-connected.

Proof. Let $(Z,\mathfrak{T}_Z,N)$ and $(Y,\mathfrak{T}_Y,M)$ be fuzzy soft semi s-connected subspaces of $\tilde{1}_E$ such that $z_N \sqcup y_M$ is not fuzzy soft semi s-connected. Then, there exist two non null fuzzy soft semi separated sets k_D and h_C of $\tilde{1}_E$ such that $z_N \sqcup y_M = k_D \sqcup h_C$. Since z_N, y_M are fuzzy soft semi s-connected, $z_N, y_M \sqsubseteq z_N \sqcup f_A = k_D \sqcup h_C$. By Theorem 4.5, either $z_N \sqsubseteq k_D$ or $z_N \sqsubseteq h_C$, also, either $y_M \sqsubseteq k_D$ or $y_M \sqsubseteq h_C$. If $z_N \sqsubseteq k_D$ or $z_N \sqsubseteq h_C$. Then $z_N \sqcap h_C \sqsubseteq k_D \sqcap h_C = \tilde{0}_E$ or $z_N \sqcap k_D \sqsubseteq z_N \sqcap k_D = \tilde{0}_E$. Therefore, $[z_N \sqcup y_M] \sqcap k_D = [z_N \sqcap k_D] \sqcup [y_M \sqcup k_D] = [y_M \sqcap k_D] \sqcup \tilde{0}_E = y_M \sqcap k_D = y_M$ since $y_M \sqsubseteq k_D$. Similarly, if $y_M \sqsubseteq k_D$ or $y_M \sqsubseteq h_C$. we get $[z_N \sqcup y_M] \sqcap h_C = z_N$.

Now, $[(z_N \sqcup y_M) \sqcap h_C] \sqcap FScl[(z_N \sqcup y_M) \sqcap k_D] \sqsubseteq [(z_N \sqcup y_M) \sqcap h_C] \sqcap [FScl(z_N \sqcup y_M) \sqcap FScl(k_D)] = [z_N \sqcup y_M] \sqcap [h_C \sqcap FScl(k_D)] = \tilde{0}_E$ and $FScl[(z_N \sqcup y_M) \sqcap h_C] \sqcap [(z_N \sqcup y_M) \sqcap k_D] \sqsubseteq [FScl(z_N \sqcup y_M) \sqcap FScl(h_C)] \sqcap [(z_N \sqcup y_M) \sqcap k_D] = [z_N \sqcup y_M] \sqcap [FScl(h_C) \sqcap k_D] = \tilde{0}_E$. It follows that, $[z_N \sqcup y_M] \sqcap k_D = z_N$ and $[z_N \sqcup y_M] \sqcap h_C = y_M$ are fuzzy soft semi separated, which is a contradiction. Hence, $z_N \sqcup y_M$ is fuzzy soft semi s-connected.

Theorem 4.7. Let $(Z,\mathfrak{T}_Z,N)$ be a fuzzy soft semi s-connected subspace of fuzzy soft topological space $(X,\mathfrak{T},E)$ and $S_M \in SS(X)_E$. If $z_N \sqsubseteq S_M \sqsubseteq FScl(z_N)$. Then $(S,\mathfrak{T}_S,M)$ is fuzzy soft semi s-connected subspace of $(X,\mathfrak{T},E)$.

Proof. Suppose that $(S,\mathfrak{T}_S,M)$ is not fuzzy soft semi s-connected subspace of $(X,\mathfrak{T},E)$. Then, there exist fuzzy soft semi separated sets f_A and g_B on $\mathfrak{T}$ such that $S_M = f_A \sqcup g_B$. So, we have z_N is fuzzy soft semi s-connected subset of fuzzy soft semi s-disconnected space. By Theorem 4.5, either $z_N \sqsubseteq f_A$ or $z_N \sqsubseteq g_B$. If $z_N \sqsubseteq f_A$. Then $FScl(z_N) \sqsubseteq FScl(f_A)$. It follows $FScl(z_N) \sqcap g_B \sqsubseteq FScl(f_A) \sqcap g_B = \tilde{0}_E$. Hence, $g_B = FScl(z_N) \sqcap g_B = \tilde{0}_E$ which is a contradiction. If $z_N \sqsubseteq g_B$. By a similar way, we can get $f_A = \tilde{0}_E$, which is a contradiction. Hence, $(S,\mathfrak{T}_S,M)$ is fuzzy soft semi s-connected subspace of $(X,\mathfrak{T},E)$.

Corollary 4.1. If $(Z,\mathfrak{T}_Z,N)$ is fuzzy soft semi s-connected subspace of fuzzy soft topological space $(X,\mathfrak{T},E)$. Then $FScl(z_N)$ is fuzzy soft semi s-connected.

Proof. It obvious from Theorem 4.7.

Theorem 4.8. If for all pair of distinct fuzzy soft point f_e, g_e, there exists a fuzzy soft semi s-connected set $z_N \sqsubseteq \tilde{1}_E$ with $f_e, g_e \tilde{\in} z_N$, then $\tilde{1}_E$ is fuzzy soft semi s-connected.

Proof. Suppose that $\tilde{1}$ is fuzzy soft semi s-disconnected. Then $\tilde{1}_E = f_A \sqcup g_B$, where f_A, g_B are fuzzy soft semi separated sets. It follows $f_A \sqcap g_B = \tilde{0}_E$. So, $\exists\ f_e \tilde{\in} f_A$ and $g_e \tilde{\in} g_B$. Since $f_A \sqcap g_B = \tilde{0}_E$. Then f_e, g_e are distinct fuzzy soft point in $\tilde{1}_E$. By hypothesis, there exists a fuzzy soft semi s-connected set z_N such that $f_e, g_e \tilde{\in} z_N \sqsubseteq \tilde{1}_E$ and $f_e, g_e \tilde{\in} z_N$. Moreover, we have z_N is fuzzy soft semi s-connected subset of a a fuzzy soft semi s-disconnected space. It follows by Theorem 4.5, either $z_N \sqsubseteq f_A$ or $z_N \sqsubseteq g_B$ and both cases is a contradiction with the hypothesis. Therefore, $\tilde{1}_E$ is fuzzy soft semi s-connected.

Theorem 4.9. Let $\{(Z_j,\mathfrak{T}_{Z_j},N) : j \in J\}$ be a non null family of fuzzy soft semi s-connected subspaces of fuzzy soft topological space $(X,\mathfrak{T},E)$. If $\sqcap_{j\in J}(z_j,N) \neq \tilde{0}_E$, then $(\sqcup_{j\in J} Z_j, \mathfrak{T}_{\sqcup_{j\in J} Z_j}, N)$ is also a fuzzy soft semi s-connected fuzzy subspace of $(X,\mathfrak{T},E)$.

Proof. Suppose that $(Z,\mathfrak{T}_Z,N) = (\sqcup_{j\in J} Z_j, \mathfrak{T}_{\sqcup_{j\in J} Z_j}, N)$ is fuzzy soft semi s-disconnected. Then $z_N = f_A \sqcup g_B$ for some fuzzy soft semi separated subsets f_A, g_B of $\tilde{1}_E$. Since $\sqcap_{j\in J}(z_j,N) \neq \tilde{0}_E$. Then $\exists\ f_e \tilde{\in} \sqcap_{j\in J}(z,N)_j$. It follows that, $f_e \tilde{\in} z_N$. So, either $f_e \tilde{\in} f_A$ or $f_e \tilde{\in} g_B$. Suppose that $f_e \tilde{\in} f_A$. Since $f_e \tilde{\in} (z,N)_j\ \forall j \in J$ and $(z,N)_j \sqsubseteq z_N$. So, we have $(z,N)_j$ is fuzzy soft semi s-connected subset of fuzzy soft semi s-disconnected set z_N. By Theorem 4.5, either $(z,N)_j \sqsubseteq f_A$ or $(z,N)_j \sqsubseteq g_B\ \forall j \in J$. If $(z,N)_j \sqsubseteq f_A\ \forall j \in J$. Then, $z_N \sqsubseteq f_A$. This implies that, $g_B = \tilde{0}_E$, which is a contradiction. Also, if $(z,N)_j \sqsubseteq g_B\ \forall j \in J$. Also, if $f_e \tilde{\in} g_B$, by a similar way, we get $f_A = \tilde{0}_E$, which is a contradiction. Therefore, $(Z,\mathfrak{T}_Z,N) = (\sqcup_{j\in J} Z_j, \mathfrak{T}_{\sqcup_{j\in J} Z_j}, N)$ is fuzzy soft semi s-connected.

Theorem 4.10. Let $\{(Z_j,\mathfrak{T}_{Z_j},N) : j \in J\}$ be a family of fuzzy soft semi s-connected subspaces of fuzzy soft topological space $(X,\mathfrak{T},E)$ such that one of the members of the family fuzzy soft intersects every other members, then $(\sqcup_{j\in J} Z_j, \mathfrak{T}_{\sqcup_{j\in J} Z_j}, N)$ is fuzzy subspace of $(X,\mathfrak{T},E)$.

Proof. Let $(Z,\mathfrak{T}_Z,N) = (\sqcup_{j\in J} Z_j, \mathfrak{T}_{\sqcup_{j\in J} Z_j}, N)$ and $(z,N)_{jo} \in \{(z,N)_j : j \in J\}$ such that $(z,N)_{jo} \sqcap (z,N)_j \neq \tilde{0}_E\ \ \forall j \in J$. Then $(z,N)_{jo} \sqcup (z,N)_j$ is fuzzy soft semi s-connected $\forall j \in J$ by Theorem 4.6. Hence, the collection $\{(z,N)_{jo} \sqcup (z,N)_j : j \in J\}$ is a collection of fuzzy soft semi s-connected subsets of $\tilde{1}$, which having a non null fuzzy soft intersection. Therefore, $(Z,\mathfrak{T}_Z,N) = (\sqcup_{j\in J} Z_j, \mathfrak{T}_{\sqcup_{j\in J} Z_j}, N)$ is fuzzy soft semi s-connected subspace of $(X,\mathfrak{T},E)$ by Theorem 4.6.

Theorem 4.11. Let $(X_1,\mathfrak{T}_1,E)$ and $(X_2,\mathfrak{T}_2,K)$ be fuzzy soft topological spaces and $f_{pu} : (X_1,\mathfrak{T}_1,E) \to (X_2,\mathfrak{T}_2,K)$ be a fuzzy irresolute surjective soft function. If $(X_1,\mathfrak{T}_1,E)$ is fuzzy soft semi s-connected, then $(X_2,\mathfrak{T}_2,K)$ is also a fuzzy soft semi s-connected.

Proof. Let $(X_2,\mathfrak{T}_2,K)$ be fuzzy soft semi disconnected space. Then there exist f_A, g_B pair of non null proper

fuzzy soft semi separated sets such that $\tilde{1}_K = f_A \sqcup g_B$, $FScl(f_A) \sqcap g_B = FScl(g_B) \sqcap f_A = \tilde{0}_E$. Since f_{pu} is fuzzy irresolute soft function, then $f_{pu}^{-1}(f_A), f_{pu}^{-1}(g_B)$ are pair of non null proper fuzzy semi open soft subsets of $\tilde{1}_E$ such that
$FScl(f_{pu}^{-1}(f_A)) \sqcap f_{pu}^{-1}(g_B) \sqsubseteq f_{pu}^{-1}(FScl(f_A)) \sqcap f_{pu}^{-1}(g_B) = f_{pu}^{-1}(f_A \sqcap g_B) = f_{pu}^{-1}(\tilde{0}_K) = \tilde{0}_E$, $f_{pu}^{-1}(f_A) \sqcap FScl(f_{pu}^{-1}(g_B)) \sqsubseteq f_{pu}^{-1}(f_A) \sqcap f_{pu}^{-1}(FScl(g_B)) = f_{pu}^{-1}(f_A \sqcap g_B) = f_{pu}^{-1}(\tilde{0}_K) = \tilde{0}_E$ and $f_{pu}^{-1}(f_A) \sqcup f_{pu}^{-1}(g_B)) = f_{pu}^{-1}(f_A \sqcup g_B) = f_{pu}^{-1}(\tilde{1}_K) = \tilde{1}_E$ from Theorem 2.3 and [[20] Theorem 4.2]. This means that, $f_{pu}^{-1}(f_A), f_{pu}^{-1}(g_B)$ are pair of non null proper fuzzy soft semi separated sets of $\tilde{1}_E$, which is a contradiction of the fuzzy soft semi s-connectedness of $(X_1, \mathfrak{T}_1, E)$. Therefore, $(X_2, \mathfrak{T}_2, K)$ is fuzzy soft semi s-connected.

5 Conclusion

Topology is an important and major area of mathematics and it can give many relationships between other scientific areas and mathematical models. Recently, many scientists have studied the soft set theory, which is initiated by Molodtsov [29] and easily applied to many problems having uncertainties from social life. In the present work, we have continued to study the properties of fuzzy soft topological spaces. We introduce the some new concepts in fuzzy soft topological spaces such as fuzzy soft semi connected sets, fuzzy soft semi separated sets and fuzzy soft semi s-connected sets and have established several interesting properties. Since the authors introduced topological structures on fuzzy soft sets [6,11,37], so the semi topological properties, which introduced by Mahanta et al.[24], is generalized here to the fuzzy soft sets which will be useful in the fuzzy systems. Because there exists compact connections between soft sets and information systems [33,39], we can use the results deducted from the studies on fuzzy soft topological space to improve these kinds of connections. We hope that the findings in this paper will help researcher enhance and promote the further study on fuzzy soft topology to carry out a general framework for their applications in practical life. This paper, not only can form the theoretical basis for further applications of topology on soft sets, but also lead to the development of information systems.

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