Journal of Statistics Applications & Probability Letters
An International Journal

http://dx.doi.org/10.12785/jsapl/010305

Finite-Time Ruin Probabilities for Risk Models with Sequences of Independent and Continously Distributed Random Variables

Bui Khoi Dam[1] *and Nguyen Thi Thuy Hong*[2,*]

[1]Applied Mathematics and Informatics School, Hanoi University of Science and Technology, Ha Noi,Viet Nam
[2]Hanoi College of Education, Ha Noi, Viet Nam

Received: 29 Apr. 2014, Revised: 5 Jun. 2014, Accepted: 9 Jun. 2014
Published online: 1 Sep. 2014

Abstract: In this article, we proved the estimation formula for the ruin probability for risk model with sequences of independent and continuously distributed random variables. We generalized the Picard-Lefvre formula (see [7]) for the ruin probability for risk models as well as the results of Claude Lefvre and Stephane Loisel (see [2]). In their studies, the authors gave only the formula of ruin probability for classical risk model while in our study, we established the formula for continuously distributed random variables. Otherwise, we extended the results for the model with sequences which are dependent of Markov type.

Keywords: Ruin probability, non-ruin probability, premiums, claims.

1 Introduction

In recent years, the finance services in general and insurance industry in particular have played a crucial role in economy helping to adjust and promote all other economic activities. This area has also become a platform for launching various ideas and intellectual applications. Over the past few decades, questions in insurance and finance have been among the focus of many mathematicians in the area of probability theory and statistics.

Financial investment is an integral part of any business and sector that wants to make a profit. However, investments always carry risk elements in itself, which might even result in loss or bankruptcy. While the nature of insurance is to share risks, the underlying part of an insurance company is also an investment, and therefore carries risk itself. Assessment of risk level and likelihood of occurrence is apparently a much needed research to mitigate risk and minimize possible loss. The risk theory has been widely studied recently, particularly on insurance and financial risks. One of the key issues that the theory deals with is the Ruin Probability in the risk models over discrete or continuous-time horizon.

This paper aims to provide an estimation formula for the ruin probability for risk models with sequences of independent and continuously distributed random variables. Picard and Lefvre [7] recently derived an explicit formula (hereinafter referred to P-L formula) for the finite non-ruin probability in a compound Poisson model where the claim amounts are integer-valued. Such a case is important because discretization of claim amounts is often required for numerical calculations in practice. Many (see De Vylder [3], [4] and Ignatov [**?**,**?**, 5] [6]) have pointed out the importance of the P-L formula as well as its wide applications. Claude Lefvre and Stephane Loisel (see [2]) has recently extended P-L formula for the compound binomial and compound Poisson risk models. Moreover, the ruin probability formula is also given explicitly. The key idea in the two authors proof is the use of ballot theorem. Bui Khoi Dam and Phung Duy Quang [1] recently derived an explicit formula for the finite time Ruin Probabilities In a Generalized Risk Processes under Interest Force.

Now we consider the risk models with sequences of Markov dependent random variables.

* Corresponding author e-mail: ntthong05@gmail.com

© 2014 NSP
Natural Sciences Publishing Cor.

2 The exact formula for the ruin probability for the model with sequences are dependent Markov

Let us consider an insurance company that evaluates its revenues, expenses and costs, losses, profits at periodic times fixed in advance (for example month, quarter or year).

Assume that the insurance company's initial capital is $u > 0$. At the end of each period $t, t = 1,2,3\dots$ we denote:

X_t are claim size in the t^{th} period
Y_t are premiums in the t^{th} period.

The reserves U_t of the company at time t are given by

$$U_t \equiv u + \sum_{i=1}^{t} Y_i - \sum_{i=1}^{t} X_i \tag{1}$$

At the end of each period $t, t = 1,2,3,\dots$ the ruin would occur if $U_t < 0$.

The ruin probability within finite time $\Psi(u,t)$ is defined by:

$$\Psi(u,t) = P(U_t < 0 \ \text{ for some } \ t = 1,2,3,\dots,T; 1 \leq T \leq +\infty).$$

T_u stands for the first time of ruin. $T_u \geq 1$ necessarily and for $t \geq 1$,

$$T_u \geq t+1 \ \text{ means } S_i < u + i \ \text{ for } \ 1 \leq i \leq t.$$

We have

$$\Psi(u,t) = 1 - P(T_u \geq t+1),$$

where $P(T_u \geq t+1)$ denotes the probability of non-ruin within finite time.

We give the formula for calculating the probability of non-ruin within finite time $P(T_u \geq t+1)$ for the model with $(X_i)_{i\geq 1}$ and $(Y_i)_{i\geq 1}$ sequences are dependent Markov as follows.

Theorem 2.1. Assume that the initial capital of a company is $u \in \mathbb{N}^*$, the insurer's capital at the end of period $t, t = 1,2,3,\dots$ is a random variable U_t defined by

$$U_t \equiv u + \sum_{i=1}^{t} Y_i - \sum_{i=1}^{t} X_i$$

Where X_i, Y_i are the claim size and premiums in the ith period, respectively.

Also we can assume that:

There exists a positive integer $M < \infty$ such that $P(Y_1 \leq M) = 1$ and $P(X_1 \leq M) = 1$ (because the total premiums and claim size are finite).

Claim size process $(X_i)_{i\geq 1}$ and premiums proccess $(Y_i)_{i\geq 1}$ are the non-negative integer-valued homogeneous discrete-time Markov chain with the initial distribution and the transition probability matrix are:

$$P(X_1 = k) = p_k, \sum_{k=0}^{M} p_k = 1, \ [p_{ij}] \text{ matrix }, p_{ij} = P(X_{n+1} = j \,|X_n \ = i)$$
$$P(Y_1 = k) = q_k, \sum_{k=0}^{M} q_k = 1, \ [q_{ij}] \text{ matrix }, q_{ij} = P(Y_{n+1} = j \,|Y_n \ = i), \text{ respectively.}$$

Then we have the exact formula for the probability of non-ruin within finite time:

$$P(T_u \geq t+1) = \left(\sum_{\substack{0 \leq (k_i - k_{i-1}) \leq M \\ 1 \leq i \leq t \\ k_0 = 0}} q_{k_1} q_{k_1,k_2-k_1} \dots q_{k_{t-1}-k_{t-2},k_t-k_{t-1}} \left(\sum_{\substack{0 \leq i_1 < k_1 + u \\ 0 \leq i_1 + i_2 < k_2 + u \\ \dots \\ 0 \leq i_1 + \dots + i_t < k_t + u}} p_{i_1} p_{i_1,i_2} \dots p_{i_{t-1},i_t} \right) \right) \tag{2}$$

© 2014 NSP
Natural Sciences Publishing Cor.

Proof. For convenience, we denote the formula (1) in a particular way

$$U_t = u + V_t - S_t$$

where

$V_t = \sum_{i=1}^{t} Y_i$ are total premiums (at the end of the ith period),

$S_t = \sum_{i=1}^{t} X_i$ are total claim size (at the end of the ith period),

T_u stands for the first time of ruin.

The goal is to calculate the non ruin probability $P(T_u \geq t+1)$: That is, until the end of period t, the company is not at risk. Obviously, the first we have

$$\{T_u \geq t+1\} = \{U_i > 0, i = 1, 2, ..., t\}.$$

We have

$$\begin{aligned}(T_u \geq t+1) &= (U_i > 0, 1 \leq i \leq t) = (S_i < V_i + u, 1 \leq i \leq t) \\ &= \bigcap_{i=1}^{t} \bigcup_{k=1}^{iM} (S_i < k+u)(V_i = k) \end{aligned} \tag{3}$$

since

$$P(0 \leq Y_i \leq M) = 1 \Rightarrow P(0 \leq V_i = Y_1 + Y_2 + ... + Y_i \leq iM) = 1, \forall i : 1 \leq i \leq t.$$

From (3) we get the formula:

$$\begin{aligned}P(T_u \geq t+1) = P\big\{&[(S_1 < u)(V_1 = 0) \cup (S_1 < 1+u)(V_1 = 1) \cup ... \cup (S_1 < M+u)\ (V_1 = M)] \cap \\ &\cap [(S_2 < u)(V_2 = 0) \cup (S_2 < 1+u)(V_2 = 1) \cup ... \cup (S_2 < 2M+u)(V_2 = 2M)] \cap ... \\ &... \cap \big[(S_t < u)(V_t = 0) \cup (S_t < 1+u)(V_t = 1) \cup ... \cup (S_t < tM+u)\ (V_t = tM)\big]\big\} \\ = P\big[&(S_1 < u) \cap (S_2 < u) \cap ... \cap (S_t < u) \cap (V_1 = 1) \cap (V_2 = 2) \cap (V_t = t)\big] \cup ... \cup ... \\ = &\sum_{\substack{0 \leq k_i - k_{i-1} \leq M \\ 1 \leq i \leq t \\ k_0 = 0}} P\big\{[(S_1 < k_1 + u) \cap (S_2 < k_2 + u) \cap ... \\ &\cap (S_t < k_t + u)]\,(V_1 = k_1)(V_2 = k_2)(V_t = k_t)\big\}. \end{aligned} \tag{4}$$

We have (4) due to the following properties of V_i, noting that all Y_i are non-negative integers.
If $i < j$ and $k_i \geq k_j$ then

$$\begin{aligned}P\big[(V_i = k_i)(V_j = k_j)\big] &= P\big[(Y_1 + Y_2 + ... + Y_i = k_i)(Y_1 + Y_2 + ... + Y_i + ... + Y_j = k_j)\big] \\ &= P\big[(V_i = k_i)(Y_{i+1} + ... + Y_j = k_j - k_i)\big] \\ &= 0.\end{aligned}$$

We have

$$\begin{aligned}&P(T_u \geq t+1) \\ &= \sum_{\substack{0 \leq (k_i - k_{i-1}) \leq M \\ 1 \leq i \leq t \\ k_0 = 0}} P\big[(S_1 < k_1 + u)(S_2 < k_2 + u)...(S_t < k_t + u)\big] P\big[(Y_1 = k_1)(Y_2 = k_2 - k_1)...(Y_t = k_t - k_{t-1})\big].\end{aligned}$$

According to the multiplication principle in probability, we have

$$\begin{aligned}&P\big[(Y_1 = k_1)(Y_2 = k_2 - k_1)...(Y_t = k_t - k_{t-1})\big] \\ &= P(Y_1 = k_1) P(Y_2 = k_2 - k_1 \,|Y_1 = k_1) P(Y_3 = k_3 - k_2 \,|Y_1 = k_1, Y_2 = k_2 - k_1)... \\ &\qquad ...P(Y_t = k_t - k_{t-1} \,|Y_1 = k_1, Y_2 = k_2 - k_1, ..., Y_{t-1} = k_{t-1} - k_{t-2}).\end{aligned}$$

© 2014 NSP
Natural Sciences Publishing Cor.

By the Markov property, we have

$$\begin{aligned}
&P\left[(Y_1 = k_1)(Y_2 = k_2 - k_1)\dots(Y_t = k_t - k_{t-1})\right] \\
&= P(Y_1 = k_1)P(Y_2 = k_2 - k_1\,|Y_1 = k_1)P(Y_3 = k_3 - k_2\,|Y_2 = k_2 - k_1)\dots. \\
&\qquad\dots P(Y_t = k_t - k_{t-1}\,|Y_{t-1} = k_{t-1} - k_{t-2}) \\
&= q_{k_1} q_{k_1,k_2-k_1} q_{k_2-k_1,k_3-k_2} \dots q_{k_{t-1}-k_{t-2},k_t-k_{t-1}}.
\end{aligned} \tag{5}$$

Continuing to calculate the right side of (5),we have

$$\begin{aligned}
&P\left[(S_1 < k_1 + u)(S_2 < k_2 + u)\dots(S_t < k_t + u)\right] \\
&= P(S_1 < k_1 + u)P(S_2 < k_2 + u\,|S_1 < k_1 + u)\,P(S_3 < k_3 + u\,|S_1 < k_1 + u, S_2 < k_2 + u)\dots \\
&\qquad\dots P(S_t < k_t + u\,|S_1 < k_1 + u\,, S_2 < k_2 + u, \dots, S_{t-1} < k_{t-1} + u)
\end{aligned}$$

Similarly, by the Markov property, we have

$$\begin{aligned}
&P\left[(S_1 < k_1 + u)(S_2 < k_2 + u)\dots(S_t < k_t + u)\right] \\
&= P(S_1 < k_1 + u)P(S_2 < k_2 + u\,|S_1 < k_1 + u)\,P(S_3 < k_3 + u\,|S_2 < k_2 + u)\dots \\
&\qquad\dots P(S_t < k_t + u\,|S_{t-1} < k_{t-1} + u) \\
&= p_{i_1} p_{i_1,i_2} p_{i_2,i_3} \dots p_{i_{t-1},i_t}.
\end{aligned} \tag{6}$$

Combining the results (4), (5) and (6) we have the formula (2).

The proof of Theorem 2.1 is thus complete. □

Remark 2.1 In considering the risk model (1) with interest appearencce, Bui Khoi Dam and Phung Duy Quang [1] introduced a formula calculating exactly the ruin probability for the mode when the premiums and the claim size are sequences of non-negative random variables (not necessarily take integer values in a finite set). In the case of no interest, we obtain the result as in the following theorem.

Theorem 2.2. Assume that the initial capital of a company is $u \in \mathbb{N}^*$,the insurer's capital at the end of the period $t, t = 1, 2, 3, \dots$ is a random variable U_t defined by

$$U_t \equiv u + \sum_{i=1}^{t} Y_i - \sum_{i=1}^{t} X_i$$

X_i are claim size in the ith period
Y_i are premiums in the ith period

$Y = \{Y_i\}_{i=1}^t$ is a sequence of independent and identically distributed random variables, Y_i take values in a finite set of positive numbers $E_Y = \{y_1, y_2, \dots, y_N\}$, $(0 < y_1 < y_2 < \dots < y_N)$ with $q_k = P(Y_1 = y_k)\,(y_k \in E_Y)$, $0 \le q_k \le 1$, $\sum\limits_{k=1}^{N} q_k = 1$; $X = \{X_i\}_{i=1}^t$ is a sequence of independent and identically distributed random variables, X_i take values in a finite set of positive numbers, with $p_k = P(X_1 = x_k)\,(x_k \in E_X)$, $0 \le p_k \le 1$, $\sum\limits_{k=1}^{M} p_k = 1$. The sequences $X = \{X_i\}_{i=1}^t$ and $Y = \{Y_i\}_{i=1}^t$ are assumed to be independent.

Then we have the exact formula for the probability of non-ruin within finite time:

$$P(T_u \ge t+1) = \sum_{n_1,n_2,\dots,n_t=1}^{N} q_{n_1} q_{n_2} \dots q_{n_t} \left(\sum_{1 \le m_1 \le g_1} \sum_{1 \le m_2 \le g_2} \dots \sum_{1 \le m_t \le g_t} p_{m_1} p_{m_2} \dots p_{m_t} \right) \tag{7}$$

with

$$\begin{aligned}
g_1 &= \max\{m_1 : x_{m_1} \le \min\{u + y_{n_1}, x_M\}\}, \\
g_2 &= \max\left\{ m_2 : x_{m_2} \le \min\{u + \sum_{k=1}^{1} (y_{n_k} - x_{m_k}) + y_{n_2}, x_M\} \right\} \\
&\dots\dots\dots\dots\dots \\
g_t &= \max\left\{ m_t : x_{m_t} \le \min\{u + \sum_{k=1}^{t-1} (y_{n_k} - x_{m_k}) + y_{n_k}, x_M\} \right\}
\end{aligned}$$

© 2014 NSP
Natural Sciences Publishing Cor.

In the classical risk model $U_t = u + ct - S_t$, the authors Claude Lefvre and Stephane Loisel [2] considered the models in which the premiums are linear time while the claim size are sequences of binomial distributed random variables. However in fact, the are many cases when the claim size have continuous distribution. Therefore how to estimate the ruin probability for such cases is a significant problem (until now there is no exact formula). In the followings, we will resolve this problem with arbitrarily small error.

3 Estimating the ruin probability for risk models with sequences of continuously distributed random variables

Now we consider the risk model:

$$U_t = u + ct - S_t \ \text{ with } \ S_t = \sum_{k=1}^{t} X_k \tag{8}$$

where $X_1, X_2, \dots, X_t$ are sequences of non-negative, independent, identically and continuously random variables with the distribution function F. Then the sum S_t has the distribution $F_s = \overset{t}{\circledast} F$ (the t-times convolution of the distribution function F). Since F is continuous, F_s is also continuous.

For every arbitrarily small positive number ε, there exists a positive integer $M = M(\varepsilon)$ such that

$$P(S_t \geq M) \leq \varepsilon. \tag{9}$$

For continuous F on $[0,M]$, F_s is regularly continuous on $[0,M]$. For all given ε, there exists a positive number $\delta = \delta(\varepsilon)$ and for all partition of $[0,M]$ satisfying

$$0 = a_0 < a_1 < .... < a_i < .... < a_n = M \ \text{ with } \max_{0 \leq i \leq n-1} (a_{i+1} - a_i) \leq \delta,$$

we have

$$[F_s(a_{n+1}) - F_s(a_n)] < \varepsilon. \tag{10}$$

Now we estimate the ruin probability for the previous risk models with arbitrarily small error. This is shown in the following theorem.

Theorem 3.1. Assume that the initial capital of a company is $u \in \mathbb{N}^*$, the insurer's capital at the end of period $t, t = 1, 2, 3, \dots$, is a random variable U_t defined by

$$U_t = u + ct - S_t \ \text{ with } \ S_t = \sum_{k=1}^{t} X_k.$$

Let us mention that if the premium per time unit is equal to a positive constant $c \neq 1$, X_k is the claim size in the kth period. Also we suppose that the numbers $\varepsilon, M = M(\varepsilon), \delta = \delta(\varepsilon)$ satisfy the conditions (9), (10) for all partitions $0 = a_0 < a_1 < .... < a_i < .... < a_n = M$.

Denote by

$$B = \{\omega : S_t(\omega) > M\}$$
$$A_{i,n} := \{\omega : a_i \leq S_t(\omega) < a_{i+1}\}, \ n \text{ is a given fixed number}$$

and consider the random variables

$$S_t^{(1)}(\omega) = \sum_{i=0}^{M-1} a_i 1_{A_{i,n}} + M1_B, S_t^{(2)}(\omega) = \sum_{i=0}^{M-2} a_{i+1} 1_{A_{i,n}} + M1_B$$

where

$$1_{A_{i,n}}(\omega) = \begin{cases} 0 & \text{when } \ \omega \notin A_{i,n} \\ 1 & \text{when } \ \omega \in A_{i,n} \end{cases} \ \text{ and } \ 1_B(\omega) = \begin{cases} 0 & \text{when } \ \omega \notin B \\ 1 & \text{when } \ \omega \in B. \end{cases}$$

Then we have the estimation formula for the ruin probability on finite-time

$$\Psi^{(1)}(u,t) \leq \Psi(u,t), \ \text{ with } \Psi^{(1)}(u,t) = P\left(\bigcup_{j=1}^{t} U_t^{(1)}(u) < 0\right) \tag{11}$$

© 2014 NSP
Natural Sciences Publishing Cor.

and we have

$$\lim_{\substack{\varepsilon \to 0 \\ n \to \infty}} \left(\Psi(u,t) - \Psi^{(1)}(u,t)\right) = 0. \tag{12}$$

Proof. We have $S_t^{(1)}(\omega) \le S_t(\omega)$ with $\forall \omega \in \Omega$ and $S_t(\omega) \le S_t^{(2)}(\omega)$ $\forall \omega \in \sum_{i=0}^{M-1} A_{i,n}$.

On the other hand, we observe:

$$\begin{aligned} &U_t^{(1)} < 0 \Leftrightarrow u + ct - S_t^{(1)} < 0 \\ &U_t^{(2)} < 0 \Leftrightarrow u + ct - S_t^{(2)} < 0 \\ &U_t < 0 \Leftrightarrow u + ct - S_t < 0 \end{aligned}$$

this leads to

$$\left\{U_t^{(1)} < 0\right\} \subset \{U_t < 0\} \Rightarrow P\left\{U_t^{(1)} < 0\right\} \le P\{U_t < 0\}.$$

Hence we obtain

$$\Psi^{(1)}(u,t) \le \Psi(u,t)$$

with

$$\Psi^{(1)}(u,t) = P\left(\bigcup_{j=1}^{t} U_t^1(u) < 0\right).$$

So (11) is proved.

To prove (12), we see that:

$$\begin{aligned} \Psi(u,t) - \Psi^{(1)}(u,t) &= P(u+ct < S_t) - P(u+ct < S_t^{(1)}) = P(S_t^{(1)} \le u+ct < S_t) \\ &= \sum_{i=0}^{M-1} P\left[A_{n,i}(S_t^{(1)} \le u+ct < S_t)\right] + P\left[B(S_t^{(1)} \le u+ct < S_t)\right] \\ &\le \sum_{i=0}^{M-1} P\left[A_{n,i}(S_t^{(1)} \le u+ct < S_t)\right] + P(B) \\ &\le \sum_{i=0}^{M-1} P\left[A_{n,i}(S_t^{(1)} \le u+ct < S_t)\right] + \varepsilon \\ &\le \sum_{i=0}^{M-1} P\left[A_{n,i}(S_t^{(1)} \le u+ct < S_t^{(2)})\right] + \varepsilon. \end{aligned} \tag{13}$$

Since for $\omega \in A_{n,i}$, $(S_t^{(1)} \le u+ct < S_t) \subset (S_t^{(1)} \le u+ct < S_t^{(2)})$.

If $u + ct < M$, there exists uniquely index j such that

$$a_j < u + ct < a_{j+1}.$$

From (13) we have

$$\begin{aligned} \Psi(u,t) - \Psi^{(1)}(u,t) &\le P\left[A_{n,j}(S_t^{(1)} < u+ct < S_t^{(2)})\right] + \varepsilon \\ &\le P(A_{n,j}) + \varepsilon = P(a_j < S_t < a_{j+1}) + \varepsilon \\ &= P(S_t < a_{j+1}) - P(S_t \le a_j) + \varepsilon \\ &= F_s(a_{j+1}) - F_s(a_j) + \varepsilon, \end{aligned}$$

(By the properties of regular continuity of F on $[0,M]$, $F_s(a_{j+1}) - F_s(a_j) < \varepsilon$), hence

$$0 \le \Psi(u,t) - \Psi^{(1)}(u,t) \le \varepsilon + \varepsilon = 2\varepsilon,$$

© 2014 NSP
Natural Sciences Publishing Cor.

which yields

$$\lim_{\substack{\varepsilon\to 0\\ n\to\infty}} \left(\Psi(u,t)-\Psi^{(1)}(u,t)\right)=0.$$

If $u+ct\geq M$, then in a similar way we have

$$0\leq \Psi(u,t)-\Psi^{(1)}(u,t)\leq P\left[A_{n,j}(S_t^{(1)}<u+ct<S_t^{(2)})\right]+\varepsilon=P\left(\emptyset\right)+\varepsilon=\varepsilon$$

therefore

$$\lim_{\substack{\varepsilon\to 0\\ n\to\infty}} \left(\Psi(u,t)-\Psi^{(1)}(u,t)\right)=0.$$

So (12) is proved. □

Remark 3.1 From (11), we can estimate the probability $\Psi(u,t)$ with arbitrarily small error. We can evaluate $\Psi^1(u,t)$ exactly due to formula (7), because $S_t^{(1)}(\omega)$ only takes finite values since $P\left(S_t^{(1)}=a_i\right)=F_s\left(a_{i+1}\right)-F_s\left(a_i\right)$.

References

[1] Bui Khoi Dam and Phung Duy Quang. Finite time ruin probabilities in a generalized risk processes under interest force. Mathematica Aeterna, **4**, 198-215 (2014).

[2] Claude lefvre and Stephane loisel. On finite time Ruin probabilities for classical risk models. Scandinavian Actuarial Journal, **1**, 41-60 (2008).

[3] De Vylder, F. E., La formule de Picard et Lefvre pour la probabilit de ruine en temps fini. Bulletin Francais dActuariat, **1**, 30-41 (1997).

[4] De Vylder, F. E., Numerical finite-time ruin probabilities by the Picard-Lefvre formula. Scandinavian Actuarial Journal, **2**, 375-386 (1999).

[5] Ignatov, Z. G., Kaishev, V. K. and Krachunov, R. S., An improved finite-time ruin probability formula and its Mathematica implementation. Insurance: Mathematics and Economics, **29**, 375-386 (2001).

[6] Ignatov, Z. G., and Kaishev, V. K., A finite-time ruin probability formula for continuous claim severities. Journal of Applied Probability, **41**, 570-578 (2004).

[7] Picard, Ph. and Lefvre, Cl., The probability of ruin in finite time with discrete claim size distribition. Scandinavian Actuarial Journal, **1**, 5869 (1997).

Bui Khoi Dam is Professor of Mathematical at Applied Mathematics and Informatics School, Hanoi University of Science and Technology (Viet Nam). He is refereeand Editor of several international journals in the frame of pure and applied mathematics, applied economics. His main research interests are: Mathematical economics, game theory, risk theory, and applied economics.

Nguyen Thi Thuy Hong is a lecturer at the Hanoi College of Education (Viet Nam). She main research interests are: Probability and mathematical statistics, risk theory and Mathematical economics.