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Finite Element Analysis of Convective Flow through Porous Medium with Variable Suction

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Abstract: The investigation has been carried out to analyse the convective flow through a porous medium with variable suction. The finite element analysis has been used for computational results. The effect of different dimensionless numbers on the convective flow has been done using Matlab. The effect of magnetic parameter on the flow has been analysed. The fluid behavior has been also observed with the variation of the permeability parameter.

Keywords: Porous medium, convective flow, finite element method

1 Introduction

The problem of free convection flow through a porous medium of variable permeability is one of the interesting problems which has applications in the design of steam displacement process in an oil recovery and various geothermal systems. Chandrasekhar *et al.* [1], Vedha Nayagam *et al.* [2], Sreekanth *et al.*[3] have studied the hydro magnetic free convective flow through a porous medium with variable permeability. Chamkha *et al.*[4] studied computationally the influence of mass transfer and radiation flux on natural convection flows. Abd-El-Naby *et al.* [5] studied the effects of radiation on unsteady free convective flow past a semi-infinite vertical plate with a variable surface temperature. Chamkha *et al.*[6] used the Rosseland diffusion flux model to analyze the buoyancy-driven dissipative natural convection-radiation boundary layer flow from a wedge in a porous medium. They showed that an increase in Boltzmann-Rosseland radiation-conduction number and negative Eckert number enhances heat transfer gradients at the wedge face considerably.

The MHD fluctuating free convective flow with radiation embedded in porous medium having variable permeability and heat source / sink has been studied by Sharma *et al.* [7]. Humera *et al.* [8] have studied the hydromagnetic free convective Revlin-Ericksen flow through a porous medium with variable permeability. Das *et al.* [9] have studied the mass transfer effects on unsteady hydro magnetic convective flow past a vertical porous plate in a porous medium with heat source. Reddy and Reddy [10] studied the mass transfer and heat generation effects on MHD free convective flow past an inclined vertical surface in a porous medium. Pullepu and Chamkha [11] investigated the unsteady free convection from a vertical cone with non-uniform surface heat flux. The solutions are obtained numerically. The local as well as average skin-friction and Nusselt number are also presented and analyzed graphically. The present results are compared with available results in literature and are found to be in good agreement.

Hussaini et al. [12] has investigated a free convective unsteady visco-elastic flow through porous medium of variable permeability bounded by an infinite vertical porous plate with variable suction, constant heat flux under the influence of transverse uniform magnetic field. Approximate solutions for mean velocity, transient velocity, mean temperature and transient temperature of non-Newtonian flow and skin friction are obtained. The effects of various parameters such as Pr (Prandtl number), Gr (Grashof number), M (Hartmann number), ω (frequency parameter) and k_0 (mean permeability parameter) on the above are depicted, skin friction, amplitude and phase are shown graphically. Our interest in the present investigation is to study the memory convective flow through porous medium with variable suction using finite element analysis and simulating the results.

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2 Formulation of the Problem

We consider the flow of convective memory fluid through a porous medium bounded by an infinite vertical porous plate with constant heat flux under the influence of uniform transverse magnetic field. The x - axis is taken along the plate in the upward direction and y - axis normal to it. All the fluid properties are assumed to be constant, except that influence of the density variations with temperature is considered only in the body force term. The magnetic field of small intensity H_0 is induced in the y - direction. Since the fluid is slightly conducting, the magnetic Reynolds number is far lesser than unity hence the induced magnetic field is neglected in comparison with the applied magnetic field. The equations of motion are:

$$\upsilon \frac{\partial^2 u}{\partial y^2} - \frac{\upsilon u}{k} - \beta \upsilon \frac{\partial^3 u}{\partial y^3} - \frac{\sigma \mu e^2 H_0^2}{\rho} u + g\beta_1 (T - T_\infty) - v\frac{\partial u}{\partial y} = 0, \quad (1)$$

$$\frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} - v\frac{\partial T}{\partial y} = 0. \quad (2)$$

The boundary conditions are:

$$y = 0; u = 0, \frac{\partial T}{\partial y} = -\frac{q}{k} \quad (3)$$

$$y \to \infty; u = 0, T = T_\infty \quad (4)$$

where u and v are velocity components along x and y - directions, ω is the frequency parameter is the coefficient of volume expansion, is kinematic visco-elasticity, is electrical conductivity, is magnetic permeability, H_0 is magnetic intensity, is thermal conductivity, C_p specific heat at constant pressure, q is heat flux and the permeability of porous medium is k

Introducing the following non-dimensional quantities:

$$y' = \frac{y v_0}{\vartheta},\ \omega' = \frac{4\vartheta\omega}{v_0^2}, u' = \frac{u}{v_0}, Gr = \frac{g\beta_1 q \vartheta^2}{k v_0^4}, \Pr = \frac{\mu C_p}{k}, Rm = \frac{\beta \upsilon_0^2}{\vartheta^2}$$

$$k_0' = \frac{k_0 v_0^2}{\vartheta^2}, M = \frac{\sigma \mu_e^2 H_0^2 v_0}{q\vartheta}, \theta = \frac{(T - T_\infty) k v_0}{q\vartheta} \quad (5)$$

Equations (1) and equation (2) in view of (6) and (7) are transformed to the following equations:

$$\frac{\partial u}{\partial y} + Gr\theta = \frac{u}{k_0} + Mu - \frac{\partial^2 u}{\partial y^2} - Rm\frac{\partial^3 u}{\partial y^3}, \quad (6)$$

$$\frac{\partial \theta}{\partial y} = -\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} \quad (7)$$

The corresponding boundary conditions are:

$$y = 0 : u = 0, \frac{\partial \theta}{\partial y} = -1 \quad (8)$$

$$y \to \infty : u = 0, \theta = 0. \quad (9)$$

3 Method of Solution

By applying Galerkin finite element method over the element (e)

$$z_j \le z \le z_k$$

We have

$$\int_{z_j}^{z_k} N^{(e)T} \begin{bmatrix} \frac{\partial^2 u^{(e)}}{\partial y^3} + \frac{\partial^2 u^{(e)}}{\partial y^2} + \frac{\partial u^{(e)}}{\partial y} \\ -Au^{(e)} + P \end{bmatrix} dy \quad (10)$$

$$where\ A = \frac{u}{k_0} + M,\ P = G_r \theta_i^j$$

Integrating the first and second term by parts one obtains:

$$N^{(e)T}\left(\frac{\partial^2 u^{(e)}}{\partial y^2}\right)_{z_j}^{z_k} + N^{(e)T}\left(\frac{\partial u^{(e)}}{\partial y}\right)_{z_j}^{z_k} - \int_{z_j}^{z_k} \frac{\partial N^{(e)T}}{\partial y}\frac{\partial^2 u^{(e)}}{\partial y^2} dy + \frac{\partial N^{(e)T}}{\partial y}\frac{\partial u^{(e)}}{\partial y} + N^{(e)T}\left(-\frac{\partial u^{(e)}}{\partial y} + Au^{(e)} - P\right) dy = 0 \quad (11)$$

Neglecting the first two terms one gets:

$$\int_{z_j}^{z_k} \frac{\partial N^{(e)T}}{\partial y}\frac{\partial^2 u^{(e)}}{\partial y^2} + \frac{\partial N^{(e)T}}{\partial y}\frac{\partial u^{(e)}}{\partial y} + N^{(e)T}\left(-\frac{\partial u^{(e)}}{\partial y} + Au^{(e)} - P\right) dy = 0 \quad (12)$$

Let $u^{(e)} = N^{(e)}\phi^{(e)}$ is the linear piecewise approximation over the element (e) where

$N^{(e)} = [N_j\ N_k]$, $\phi^{(e)} = [u_j\ u_k]^T$, $N_j = \frac{z_k - z}{z_k - z_j}$, $N_k = \frac{z - z_j}{z_k - z_j}$ are basis functions

The integrant of the first term will be zero. The equation reduces to

$$\int_{z_j}^{z_k} \begin{bmatrix} N_j' N_j' & N_j' N_k' \\ N_j' N_k' & N_k' N_k' \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} dy - \int_{z_j}^{z_k} \begin{bmatrix} N_j' N_j' & N_j N_k' \\ N_j N_k & N_k' N_k \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} dy + A\int_{z_j}^{z_k} \begin{bmatrix} N_j N_j & N_j N_k \\ N_j N_k & N_k N_k \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} dy$$

$$= P\int_{z_j}^{z_k} \begin{bmatrix} N_j \\ N_k \end{bmatrix} \quad (13)$$

On simplification we get,

$$\frac{1}{l_e^2}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} - \frac{1}{2l_e}\begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} + \frac{A}{6}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} = \frac{P}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (14)$$

Now put row corresponding to the node i to zero, from equation (14) the deformation scheme with $l^{(e)} = h$ is

$$\frac{1}{h^2}[-u_{i-1} + 2u_i + u_{i+1}] - \frac{1}{2h}[-u_{i-1} + u_{i+1}] + \frac{A}{6}[u_{i-1} + 4u_i + u_{i+1}] = P \quad (15)$$

Applying the trapezoidal rule following system of equations in Crank Nicolson method are obtained.

$$A_1 u_{i-1}^{n+1} + A_2 u_i^{n+1} + A_3 u_{i+1}^{n+1} = A_4 u_{i-1}^n + A_5 u_i^n + A_6 u_{i+1}^n + P^*. \quad (16)$$

Where

A_1= Ah+3h-6

A_2 = 12+4Ah

A_3 = Ah+6-3h

A_4 = -Ah-3h+6

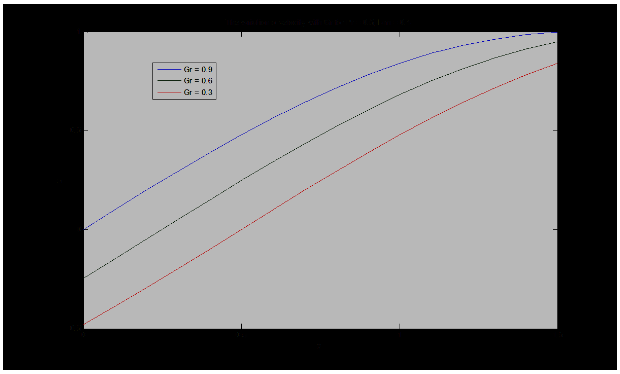

Fig. 1: The variation of velocity with Gr for Pr = 0.5, Rm= 0.4

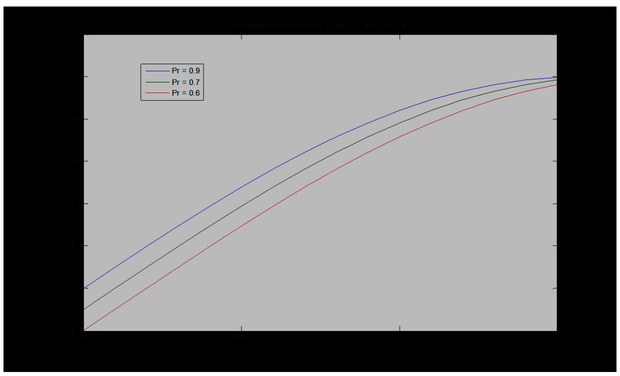

Fig. 2: The variation of velocity with Pr for Gr = 0.5, Rm = 0.4

A_5 = 8- 4Ak- 12r

A_6 = 2- Ah + 3rh +6r

P* = 12Ah+$G_r\theta_i^j$

Here h is the mesh size along y- direction and index ‘i’ refers to space. In the equation (15) taking i = 1(1) n, and using the boundary conditions (8) and (9) following system of equations are obtained.

$$A_iX_i = B_i$$

where $A_{i's}$ *are* matrices of order n, X_i and $B_{i's}$are column matrices of order n.The solutions of above system of equations are obtained by using Thomas algorithm for primary velocity and secondary velocity. Also, numerical solutions for these equations

are obtained by Matlab – programme. In order to prove the convergence and stability of Galerkin finite element method, the same Matlab – programme was run with smaller values of h and no significant change was observed in the values of u and v. Hence the Galerkin finite element method is stable and convergent.

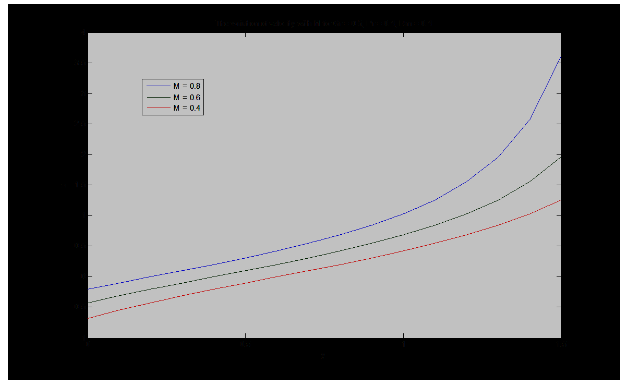

Fig. 3: The variation of velocity with M for Gr = 0.5, Pr = 0.4, Rm= 0.4

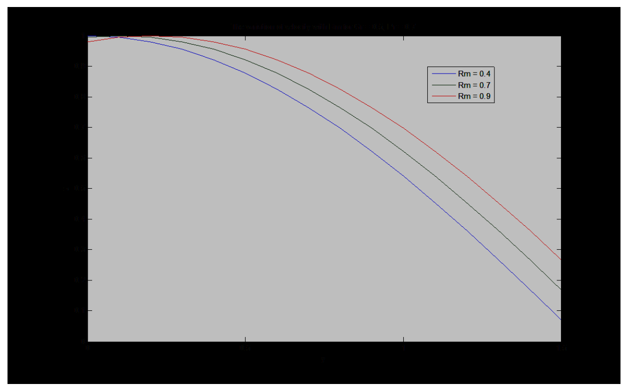

Fig. 4: The variation of velocity with Rm for Gr = 0.5, Pr = 0.7

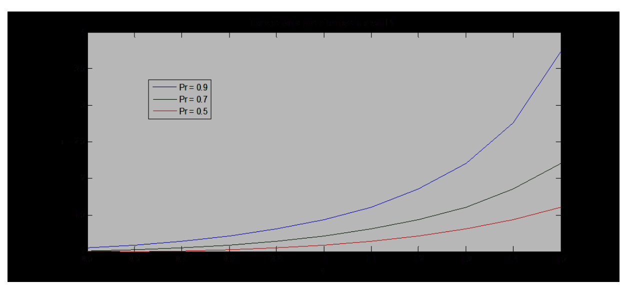

Fig. 5: The variation of mean temperature with Pr

4 Discussions and conclusions

The graphs has been plotted for variation of parameters Gr, Pr and Rm. Different range of for set of parameters have been considered to study the behavior of velocity. The graphs are plotted by suitable programming in

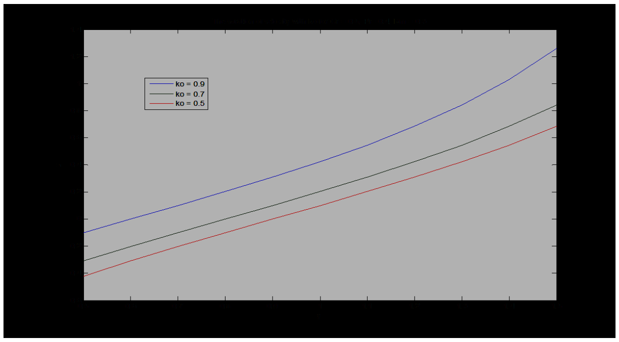

Fig. 6: The variation of velocity with ko for Gr = 0.5, Pr = 0.4, Rm = 0.3

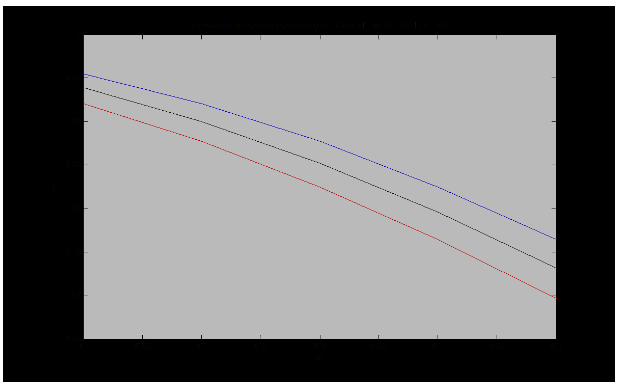

Fig. 7: The variation of mean temperature against ko with Pr for Gr = 0.4, Rm = 0.6.

Matlab. The results can be viewed and compared through different graphs.

The effect of Magnetic field has been studied by varying the parameter M is the given form of governing equations. In Fig.1 the variation in the velocity has been studied with Grashof number and it is observed that for this analysis the velocity increases with the increase Grashof number while in fig. 2 the analysis has been carried out with increasing Pr. The velocity profile has been obtained which was different and it shows that the velocity increases very slowly with the increase of Pr.In Fig. 3 the velocity behavior has been observed with Magnetic field. With the increase of magnetic field the graph goes smoothly with the slight increase and as the distance increases the change in the velocity is observable.

In Fig. 4 the effect of magnetic Reynolds number has been shown on velocity. The velocity decreases with the increase of Rm and this behavior is smooth for different values of Rm.In Fig. 5 the variation of mean temperature with Pr has been shown. For small values of distance it is not significant. But of large values of y, it is observed that the mean temperature increases sharply.Fig. 6 the variation of velocity with permeability has been plotted to show the effect of the presence of porous medium.Due to the presence of porous medium the velocity increases with distance sharply. The varation is quite smooth for different values of $k_{o.}$The variation of mean temperature against k_o has been plotted for different values of Pr, the behavior is observed that with the increase of permeability the Temperature profile decreases. The results which are obtained may be useful for the scientific purposes.

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