Applied Mathematics & Information Sciences **3**(3) (2009), 367–373
– An International Journal

Heavy Quark Contributions for Longitudinal Proton Structure Function

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We study the heavy-quark contribution to the longitudinal proton structure function $F_L(x, Q^2)$. In our QCD calculations we extract light and asymptotic heavy flavor contributions for the non-singlet, singlet and gluon parts due to charm to F_L. Our calculations for longitudinal proton structure function based on the Jacobi polynomials method are in good agreement with very recent H1 experimental data.

Keywords: Proton structure function, heavy quarks, Jacobi polynomials.

1 Introduction

Deep inelastic scattering (DIS) of leptons off nucleons has been the key for our understanding of the structure of the nucleon. HERA at DESY is a unique facility for colliding electron(or positron) with protons. The cross section of the neutral-current (NC) DIS interaction, $e^{\pm}p \to e^{\pm}X$, can be written as [1]

$$\frac{d^2\sigma(e^{\pm}p \to e^{\pm}X)}{dxdQ^2} = \frac{2\pi\alpha^2}{xQ^4}\{Y_+F_2 \mp Y_-xF_3 - y^2F_L\}, \tag{1.1}$$

where α is the fine structure constant, $Y_{\pm} = 1 \pm (1-y)^2$ with $y = Q^2/xs$ being the inelasticity, and F_2, xF_3 and F_L are the structure functions of the proton.

In the framework of the perturbative QCD inspired quark parton model, the structure functions can be directly related to the parton distribution functions (PDF's) which are probability densities of partons existing inside proton.

All calculations of high energy processes with initial hadrons, whether within the standard model or exploring new physics, require PDF's as an essential input. The reliability of these calculations, which underpins both future theoretical and experimental progress,

A.N.K. thanks Semnan university for partial financial support of this project. We acknowledge the Institute for Studies in Theoretical Physics and Mathematics (IPM) for financially supporting this project.

depends on understanding the uncertainties of the PDF's. The assessment of PDF's, their uncertainties and extrapolation to the kinematics relevant for future colliders such as the LHC is an important challenge to high energy physics in recent years.

At low Q^2, dominantly contributing to the cross section is F_2 which is an electric-charge squared weighted sum of all flavor quark PDF's. In the low-x region, F_2 is dominated by sea-quark PDF's, and the DGLAP evolution of QCD ascribes the Q^2-dependence of F_2 ("scaling violation") as largely owing to gluon splitting into $q\overline{q}$-pairs. At large Q^2, xF_3 becomes significant, and gives information on valence quark PDF's. The structure function F_L is zero in the naive quark-parton model, i.e. without QCD, but in leading order QCD, a finite value of F_L is expected in the small x region by being directly related to the gluon PDF.

Since very recently, the first direct measurement of the F_L at H1 using the lowered proton beam energy is extracted [2], so we have enough motivation to study the longitudinal structure function $F_L(x, Q^2)$. In this paper we use the massive operator matrix elements, which contribute to the heavy flavor Wilson coefficients in unpolarized deeply inelastic scattering in the region $Q^2 >> m^2$ [3]. The results of the present analysis is based on the Jacobi polynomials expansion of the non-singlet structure function. This method was developed and applied for QCD analysis [4–17]. The same method has also been applied in polarized case [18–23]. Also in [24] the proton structure functions are derived in the QCD dipole picture of BFKL dynamics.

The plan of the paper is to give an introduce of theoretical formalism in Section 2. The method of the QCD analysis of longitudinal structure function, based on Jacobi polynomials are written down in the section 3. Our conclusions are summarized in Section 4.

2 Theoretical Formalism

The nucleon structure functions $F_i(x, Q^2)$ are described as Mellin convolutions between the parton densities f_j and the Wilson coefficients C_i^j

$$F_i(x, Q^2) = \sum_j C_i^j\left(x, \frac{Q^2}{\mu^2}\right) \otimes f_j(x, \mu^2)\ , \tag{2.1}$$

to all orders in perturbation theory. Here μ^2 denotes the factorization scale and the Mellin convolution is given by the integral

$$[C \otimes f](x) = \int_0^1 dx_1 \int_0^1 dx_2\ \delta(x - x_1 x_2)\ C(x_1) f(x_2)\ . \tag{2.2}$$

Since the distributions f_j refer to massless partons, the heavy flavor effects are contained in the Wilson coefficients only. In the perturbative predictions and in Mellin n-space, the longitudinal structure function F_L consists of the light and heavy flavor contributions [25]

$$F_L(n, Q^2) = F_L^{\rm light}(n, Q^2) + F_L^{\rm heavy}(n, Q^2)$$

$$
\begin{aligned}
&= \left[C_L^{NS,\text{light}}(n,a_s) + H_L^{NS}\left(n,a_s,\frac{Q^2}{m^2}\right)\right] q_{\text{NS}}(n,\mu^2) \\
&+ \left[C_L^{S,\text{light}}(n,a_s) + H_L^{S}\left(n,a_s,\frac{Q^2}{m^2}\right)\right] q_{\text{S}}(n,\mu^2) \\
&+ \left[C_L^{g,\text{light}}(n,a_s) + H_L^{NS}\left(n,a_s,\frac{Q^2}{m^2}\right)\right] g(n,\mu^2)\ . \qquad (2.3)
\end{aligned}
$$

We choose $Q^2 = \mu^2$ as uniform factorization scale in this paper.

In the limit $Q^2 \gg m^2$ the massive Wilson coefficients H_L^i up to $O(\alpha_s^2)$ are given by [3]

$$
H_{L,q}^{\text{NS}}\left(x,a_s,\frac{Q^2}{m^2}\right) = a_s^2\left[-\beta_{0,Q}C_{L,q}^{(1)}\ln\left(\frac{Q^2}{m^2}\right) + \widehat{C}_{L,q}^{\text{NS},(2)}\right]\ , \qquad (2.4)
$$

$$
H_{L,q}^{\text{PS}}\left(x,a_s,\frac{Q^2}{m^2}\right) = a_s^2\widehat{C}_{L,q}^{\text{PS},(2)}\ , \qquad (2.5)
$$

$$
H_{L,g}^{\text{S}}\left(x,a_s,\frac{Q^2}{m^2}\right) = a_s\widehat{C}_{L,g}^{(1)} + a_s^2\left[\frac{1}{2}\widehat{P}_{qg}^{(0)}C_{L,q}^{(1)}\ln\left(\frac{Q^2}{m^2}\right) + \widehat{C}_{L,g}^{(2)}\right]\ , \qquad (2.6)
$$

where $H_{L,q}^{\text{S}} = H_{L,q}^{\text{NS}} + H_{L,q}^{\text{PS}}$. The $\overline{\text{MS}}$ coefficient functions, in the massless limit, are denoted by

$$
\widehat{C}_{L,q}^{i}\left(\frac{Q^2}{\mu^2}\right) = C_{L,q}^{i}\left(\frac{Q^2}{\mu^2}, N_L + N_H\right) - C_{L,q}^{i}\left(\frac{Q^2}{\mu^2}, N_L\right)\ , \qquad (2.7)
$$

where N_H, N_L are the number of heavy and light flavors, respectively. In the following we will consider the case of a single heavy quark, i.e. $N_H = 1$.

At $N^m LO$ the scale dependence of a_s is given by

$$
\frac{d\,a_s}{d\ln Q^2} = \beta_{N^mLO}(a_s) = -\sum_{k=0}^{m} a_s^{k+2}\,\beta_k\ . \qquad (2.8)
$$

The expansion coefficients β_k of the β-function of QCD are known up to $k = 2$, i.e., N^2LO [26, 27]

$$
\begin{aligned}
\beta_0 &= 11 - 2/3\,n_f\ , \\
\beta_1 &= 102 - 38/3\,n_f\ , \\
\beta_2 &= 2857/2 - 5033/18\,n_f + 325/54\,n_f^2\ , \qquad (2.9)
\end{aligned}
$$

where n_f stands for the number of effectively massless quark flavors. The strong coupling constant up to NNLO is as following [28]

$$
a_s(Q^2) = \frac{1}{\beta_0 L_\Lambda} - \frac{1}{(\beta_0 L_\Lambda)^2}b_1\ \ln L_\Lambda + \frac{1}{(\beta_0 L_\Lambda)^3}\left[b_1^2\left(\ln^2 L_\Lambda - \ln L_\Lambda - 1\right) + b_2\right], \qquad (2.10)
$$

where $L_\Lambda \equiv ln(Q^2/\Lambda^2)$, $b_k \equiv \beta_k/\beta_0$, and Λ is the QCD scale parameter.

Now the moments of non-singlet, singlet and also gluon parts of longitudinal structure function F_L is available by using Eq. (2.3).

3 The Method

In this work we want to calculate the non-singlet, singlet and also gluon contributions of longitudinal structure function by using the Jacobi polynomials method. One of the simplest and fastest possibilities in the structure function reconstruction from the QCD predictions for its Mellin moments is Jacobi polynomials expansion. According to this method one can relate the structure function with its Mellin moments [5–9]

$$F_L(x,Q^2) = x^\beta (1-x)^\alpha \sum_{n=0}^{N_{max}} \Theta_n^{\alpha,\beta}(x) \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta) F_L(j+2,Q^2) \,, \tag{3.1}$$

where N_{max} is the number of polynomials and $\Theta_n^{\alpha,\beta}(x)$ are the Jacobi polynomials of order n. $F_L(j+2,Q^2)$ is the moments of the longitudinal structure function which introduced in the previous section. The same method has been applied to calculate the non-singlet structure function F_2 [14–17] and xF_3 from their moments [10–13] and for polarized structure function xg_1 [18–20]. Now it is possible to determine $F_L(x,Q^2)$ by having the information of massless and massive parts of $F_L(n,Q^2)$ in n-moment space.

4 Conclusion

For extraction non-singlet part of longitudinal structure function F_L we choose our very recently parametrization for the valence quark densities [14]. To extract singlet and gluon parts of longitudinal structure function F_L we used the reported results for singlet and gluon distributions from [3].

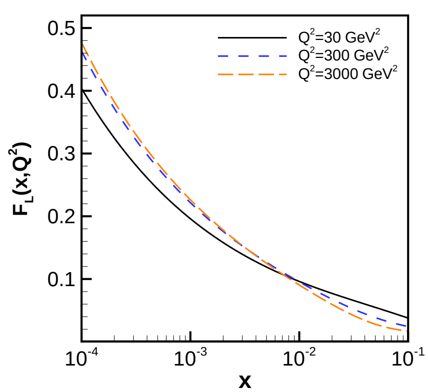

Figure 4.1: The longitudinal structure function $F_L(x,Q^2)$ as a function of x and for fixed $Q^2 = 30, 300, 3000$ GeV^2 values.

In Fig. (4.1) we show the longitudinal structure function $F_L(x,Q^2)$ as a function of x

and for $Q^2 = 30, 300, 3000$ GeV^2. Longitudinal structure function F_L as a function of x and for $Q^2 \sim 25$ GeV^2 is presented in Fig. (4.2). In this figure we compared our results with the available experimental data [2, 29, 30]. In Fig. (4.3) we present our QCD analysis for longitudinal structure function F_L as a function of Q^2. The H1 data [2, 29] are at fixed $W = 276$ GeV. We found that the Jacobi polynomial approach can describe the behavior of light and asymptotic heavy flavor contributions due to charm to F_L.

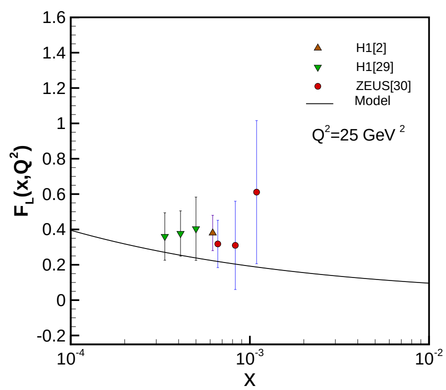

Figure 4.2: Longitudinal structure function F_L as a function of x and for fixed Q^2 value compared with the available experimental data [2, 29, 30].

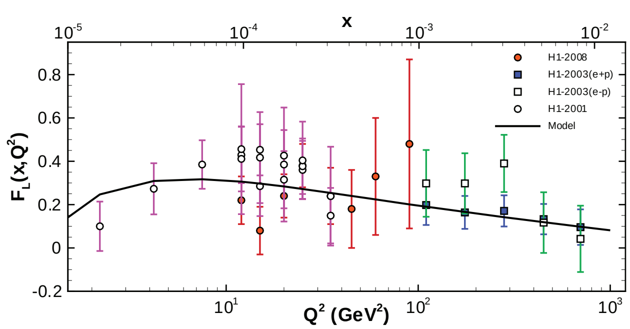

Figure 4.3: Longitudinal structure function F_L as a function of Q^2. The H1 data [2, 29] are at fixed $W = 276$ GeV.

We hope our results of QCD analysis of structure functions in terms of Jacobi polynomials could be able to describe more complicated hadron structure functions. We also hope to be able to consider the massive quark contributions for $F_2(x, Q^2)$ by using the structure function expansion in terms of the Jacobi polynomials.

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