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Bayesian Estimation of Exponentiated Inverted Weibull Distribution under Asymmetric Loss Functions

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Abstract: In this paper, we obtained Bayesian estimators of the shape parameter of the Exponentiated Inverted Weibull distribution (EIWD) distribution using Bayesian method under Square error loss function, Entropy loss function and Precautionary loss function. In order to get better understanding of our Bayesian analysis we consider non informative prior for the shape parameter using Jeffery's prior Information, Extension of Jeffery's prior and Qausi prior. These Bayes estimators of the shape parameter of the EIWD distribution are compared with some classical estimators such as the Maximum likelihood estimator (MLE).

Keywords: Exponentiated Inverted Weibull distribution, Maximum Likelihood Estimator, Bayes estimation, Priors, Loss functions.

1. Introduction:

The inverted Weibull distribution is one the most popular probability distribution to analyze the life time data with some monotone failure rates. The two parameter exponentiated inverted Weibull distribution (EIWD) has been proposed by Flaih et al (2012). The EIWD is a generalization to the inverted Weibull distribution through adding a new shape parameter $\theta \in R^{+}$ by exponentiation to distribution function *F*. The two parameter EIWD has the following density function

$$f(x) = \theta\beta \, x^{-(\beta+1)} \left(e^{-x^{-\beta}}\right)^{\theta} \quad ; x > 0,\ \beta, \theta > 0 \tag{1}$$

For $\theta = 1$, it represents the standard inverted Weibull distribution and for $\beta = 1$, it represents the exponentiated inverted exponential distribution. Aljouharah (2013) estimates the parameters of an exponentiated inverted Weibull distribution under type-II censoring. Mudholka et al (2001) introduced the exponentiated Weibull distribution as a generalization of the standard Weibull distribution. Nadrajah and Gupta (2005) discussed in detail the moments of the exponentiated Weibull distribution. Maswadah (2003) has fitted inverse Weibull distribution to the flood data reported in Dumonceaux and Antle (1973). Soland (1968) discusses the Bayesian analysis of the Weibull process with unknown scale parameter and its application to acceptance sampling.

The aim of this paper is to propose the different methods of estimation of the parameters of the EIWD. In the next section, we obtain the MLE of the shape parameter θ in EIWD when β is known. We also discuss the procedures to obtain the Bayes estimators for the unkown parameters using Jeffery's prior, extension of Jeffery's and Quasi prior under entropy loss, square error and precautionary loss function.

2. Reliability Analysis

The reliability function *R*(*t*), which is the probability of an item not failing prior to sometime *t*, is defined by *R*(*t*) = 1 - *F*(*t*). The reliability function of EIWD is given by

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$$R(t,\theta,\beta)=1-(e^{-t^{-\beta}})^{\theta}$$

The other characteristic of interest of a random variable is the hazard rate function defined by

$$h(t)=\frac{f(t)}{1-F(t)}$$

which is an important quantity characterizing life phenomenon. It can be loosely interpreted as the conditional probability of failure, given it has survived to time t. The hazard rate function for exponentiated inverted random variable is given by

$$h(t,\theta,\beta)=\frac{\theta\beta t^{-(\beta+1)}(e^{-t^{-\beta}})\theta}{1-(e^{-t^{-\beta}})^{\theta}}$$

3. Estimation of the Shape Parameter θ

3.1 Maximum Likelihood Estimator

If $x_1, x_2, \ldots, x_n$ is a random sample from exponentiated inverted Weibull distribution given by (1), then the likelihood function becomes:

$$L(\beta,\theta)=(\theta\beta)^n \prod_{i=1}^{n}(x_i)^{-(\beta+1)}\left(\exp\left(-\sum_{i=1}^{n} x_i^{-\beta}\right)\right)^{\theta}$$

$$\log L(\beta,\theta)=n\log\theta+n\log\beta-(\beta+1)\sum_{i=1}^{n}\log x_i-\theta\sum_{i=1}^{n} x_i^{-\beta}$$

As parameter β is known, the MLE of θ which maximize the log likelihood must satisfy the normal equation given by:

$$\frac{\partial\log L(\beta,\theta)}{\partial\theta}=\frac{n}{\theta}-\sum_{i=1}^{n} x_i^{-\beta}=0$$

$$\Rightarrow \hat{\theta}=\frac{n}{\sum_{i=1}^{n} x_i^{-\beta}} \qquad (2)$$

3.2 Bayes Estimator: we now derive the Bayes estimator of the parameter θ in EIWD when the parameter β is known. We consider three different priors and three different loss functions.

(a) Jeffery's Prior: Jeffery's (1946) proposed a formal rule for obtaining a non-informative prior as

$$g(\theta)\propto\sqrt{I(\theta)}$$

Where θ is k-vector valued parameter and $I(\theta)$ is the Fishers information matrix of order k*k. Thus in our problem the prior distribution of θ to be

$$g_1(\theta)=k\frac{n}{\theta}$$

(b) Extension of Jeffery's Prior: The extended Jeffrey's prior proposed by Al-Kutubi (2005), is given as:

$$g(\theta)\propto[I(\theta)]^{c}\quad,\qquad c\varepsilon R^{+}$$

Where $[I(\theta)]=-nE\left[\frac{\partial^2\log f(y;\theta)}{\partial\theta^2}\right]$ is the Fisher's information matrix. For the model (1.1),

$$g_2(\theta)=k\left[\frac{n}{\theta^2}\right]^{c}$$

(c) Quasi Prior: when there is no information about the parameter θ, one may use the quasi density as given by:

$$g_3(\theta) = \frac{1}{\theta^d} \quad ,\theta > 0, d > 0$$

The quasi prior leads to diffuse prior when d=0 and to a non informative prior for a case when d=1.

(i) The square error loss function: A commonly used loss function is the square error loss function (SELF).

$$L(\hat{\theta},\theta) = c_1(\hat{\theta}-\theta)^2$$

which is symmetric loss function that assigns equal losses to over estimation and under estimation. The SELF is often used because it does not need extensive numerical computation. Basu and Ebrahimi (1992) derive Bayes estimator of the mean lifetime and reliability function in the exponential life testing model. Instead, the loss functions that they used are asymmetric to reflect that, in most situations of interest overestimating are most harmful than underestimating. Due to this reason, we use various asymmetric loss functions as follows:

(ii) The entropy loss function: In many practical situations, it appears to be more realistic to express the loss in terms of the ratio $\hat{\theta}/\theta$. In this case, Calabria and Pulcini (1994) point out that a useful asymmetric loss function is the entropy loss function:

$$L(\delta^p) \propto [\delta^p - p\log(\delta) - 1]$$

where $\delta = \frac{\hat{\theta}}{\theta}$ and p>0, whose minimum occurs at $\hat{\theta} = \theta$. Also, the loss function $L(\delta)$ has been used in Dey et al (1987) and Dey and Liu (1992), in the original form having p =1. Thus, $L(\delta)$ can be written as:

$$L(\delta) = b[\delta - \log(\delta) - 1]\ ; b > 0.$$

(iii) The precautionary loss function: Norstrom (1996) introduced an alternative asymmetric precautionary loss function, and also presented a general class of precautionary loss functions as a special case. These loss functions approach infinitely near the origin to prevent underestimation, thus giving conservative estimators, especially when low failure rates are being estimated. These estimators are very useful when underestimation may lead to serious consequences. A very useful and simple asymmetric precautionary loss function is

$$L(\hat{\theta},\theta) = \frac{(\hat{\theta}-\theta)^2}{\hat{\theta}}$$

3.3 Bayes estimator under $g_1(\theta)$

Under $g_1(\theta)$, using (1), the posterior distribution is given by

$$g_1(\theta|x) = k\theta^{n-1}\exp\left(-\theta\sum_{i=1}^{n} x_i^{-\beta}\right)$$

where k is independent of θ

and $$k^{-1} = \int_0^\infty \theta^{n-1}\exp\left(-\theta\sum_{i=1}^{n} x_i^{-\beta}\right)d\theta$$

$$\Rightarrow k^{-1} = \frac{\Gamma n}{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^n}$$

$$\Rightarrow k = \frac{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^n}{\Gamma n}$$

Thus posterior distribution is given by

$$g_1(\theta|x) = \frac{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^n}{\Gamma n} \theta^{n-1} \exp\left(-\theta \sum_{i=1}^{n} x_i^{-\beta}\right)$$

3.3.1 Estimator under SELF

By using SELF, the risk function $L(\hat{\theta}, \theta) = c_1(\hat{\theta} - \theta)^2$, the risk function is given by

$$R(\hat{\theta}) = \int_0^\infty c_1(\hat{\theta} - \theta)^2 \frac{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^n}{\Gamma n} \theta^{n-1} \exp\left(-\theta \sum_{i=1}^{n} x_i^{-\beta}\right) d\theta$$

$$R(\hat{\theta}) = c_1 \frac{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^n}{\Gamma n} \int_0^\infty (\hat{\theta} - \theta)^2 \theta^{n-1} \exp\left(-\theta \sum_{i=1}^{n} x_i^{-\beta}\right) d\theta$$

$$R(\hat{\theta}) = c_1 \left[\hat{\theta}^2 + \frac{n(n+1)}{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^2} - 2\hat{\theta} \frac{n}{\sum_{i=1}^{n} x_i^{-\beta}} \right]$$

Now solving $\frac{\partial R(\hat{\theta})}{\partial \hat{\theta}} = 0$, we obtain the Baye's estimator as

$$\hat{\theta}_s = \frac{n}{\sum_{i=1}^{n} x_i^{-\beta}} \tag{3}$$

3.3.2 Estimator under entropy loss function

By using entropy loss function $L(\delta) = b[\delta - \log(\delta) - 1]\ ; b > 0$, the risk function is given by

$$R(\hat{\theta}) = \int_0^\infty b[\delta - \log(\delta) - 1] \frac{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^n}{\Gamma n} \theta^{n-1} \exp\left(-\theta \sum_{i=1}^{n} x_i^{-\beta}\right) d\theta$$

$$R(\hat{\theta}) = b \frac{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^n}{\Gamma n} \int_0^\infty [\delta - \log(\delta) - 1] \theta^{n-1} \exp\left(-\theta \sum_{i=1}^{n} x_i^{-\beta}\right) d\theta$$

$$R(\hat{\theta}) = b \left[\frac{\hat{\theta} \sum_{i=1}^{n} x_i^{-\beta}}{(n-1)} - \log \hat{\theta} + \frac{\Gamma n'}{\Gamma n} - 1 \right]$$

Now solving $\frac{\partial R(\hat{\theta})}{\partial \hat{\theta}} = 0$, we obtain the Baye's estimator as

$$\hat{\theta}_e = \frac{n-1}{\sum_{i=1}^{n} x_i^{-\beta}} \tag{4}$$

3.3.3 Estimator under precautionary loss function

By using precautionary loss function $L(\hat{\theta},\theta) = \frac{(\hat{\theta}-\theta)^2}{\hat{\theta}}$, the risk function is given by

$$R(\hat{\theta}) = \int_0^\infty \frac{(\hat{\theta}-\theta)^2}{\hat{\theta}} \frac{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^n}{\Gamma n} \theta^{n-1} \exp\left(-\theta \sum_{i=1}^{n} x_i^{-\beta}\right) d\theta$$

$$R(\hat{\theta}) = \frac{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^n}{\hat{\theta}\,\Gamma n} \int_0^\infty (\hat{\theta}-\theta)^2 \theta^{n-1} \exp\left(-\theta \sum_{i=1}^{n} x_i^{-\beta}\right) d\theta$$

$$R(\hat{\theta}) = \left[\hat{\theta} + \frac{n(n+1)}{\hat{\theta}\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^2} - 2\frac{n}{\sum_{i=1}^{n} x_i^{-\beta}}\right]$$

Now solving $\frac{\partial R(\hat{\theta})}{\partial \hat{\theta}} = 0$, we obtain the Baye's estimator as

$$\hat{\theta}_p = \frac{[n(n+1)]^{\frac{1}{2}}}{\sum_{i=1}^{n} x_i^{-\beta}} \tag{5}$$

3.4 Bayes estimator under $g_2(\theta)$

Under $g_2(\theta)$, using (1), the posterior distribution is given by

$$g_2(\theta|x) = k\theta^{n-2c} \exp\left(-\theta \sum_{i=1}^{n} x_i^{-\beta}\right)$$

where k is independent of θ

and $k^{-1} = \int_0^\infty \theta^{n-2c} \exp\left(-\theta \sum_{i=1}^{n} x_i^{-\beta}\right) d\theta$

$$\Rightarrow k^{-1} = \frac{\Gamma(n-2c+1)}{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^{n-2c+1}}$$

$$\Rightarrow k = \frac{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^{n-2c+1}}{\Gamma(n-2c+1)}$$

Thus posterior distribution is given by

$$g_1(\theta|x) = \frac{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^{n-2c+1}}{\Gamma(n-2c+1)} \theta^{n-2c} \exp\left(-\theta \sum_{i=1}^{n} x_i^{-\beta}\right)$$

3.4.1 Estimator under SELF

By using SELF, the risk function $L(\hat{\theta},\theta) = c_1(\hat{\theta}-\theta)^2$, the risk function is given by

$$R(\hat{\theta}) = \int_0^{\infty} c_1(\hat{\theta}-\theta)^2 \frac{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^{n-2c+1}}{\Gamma(n-2c+1)} \theta^{n-2c} \exp\left(-\theta \sum_{i=1}^{n} x_i^{-\beta}\right) d\theta$$

$$R(\hat{\theta}) = c_1 \frac{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^{n-2c+1}}{\Gamma(n-2c+1)} \int_0^{\infty} (\hat{\theta}-\theta)^2 \theta^{n-2c} \exp\left(-\theta \sum_{i=1}^{n} x_i^{-\beta}\right) d\theta$$

$$R(\hat{\theta}) = c_1 \left[\hat{\theta}^2 + \frac{(n-2c+2)(n-2c+1)}{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^2} - 2\hat{\theta} \frac{(n-2c+1)}{\sum_{i=1}^{n} x_i^{-\beta}} \right]$$

Now solving $\frac{\partial R(\hat{\theta})}{\partial \hat{\theta}} = 0$, we obtain the Baye's estimator as

$$\hat{\theta}_s = \frac{n-2c+1}{\sum_{i=1}^{n} x_i^{-\beta}} \quad (6)$$

Remark 1.1: Replacing $c = 1/2$, in (6), we get the same Bayes estimator as obtained in (3) corresponding to the Jeffrey's prior and replace $c = 3/2$, we get Hartigan's prior.

3.4.2 Estimator under Entropy loss function

By using entropy loss function $L(\delta) = b[\delta - \log(\delta) - 1]\ ; b > 0$, the risk function is given by

$$R(\hat{\theta}) = \int_0^{\infty} b[\delta - \log(\delta) - 1] \frac{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^{n-2c+1}}{\Gamma(n-2c+1)} \theta^{n-2c} \exp\left(-\theta \sum_{i=1}^{n} x_i^{-\beta}\right) d\theta$$

$$R(\hat{\theta}) = b \frac{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^{n-2c+1}}{\Gamma(n-2c+1)} \int_0^{\infty} [\delta - \log(\delta) - 1] \theta^{n-2c} \exp\left(-\theta \sum_{i=1}^{n} x_i^{-\beta}\right) d\theta$$

$$R(\hat{\theta}) = b \left[\frac{\hat{\theta} \sum_{i=1}^{n} x_i^{-\beta}}{(n-2c)} - \log \hat{\theta} + \frac{\Gamma(n-2c+1)'}{\Gamma(n-2c+1)} - 1 \right]$$

Now solving $\frac{\partial R(\hat{\theta})}{\partial \hat{\theta}} = 0$, we obtain the Baye's estimator as

$$\hat{\theta}_e = \frac{n-2c}{\sum_{i=1}^{n} x_i^{-\beta}} \tag{7}$$

Remark 1.2: Replacing $c = 1/2$ in (7), we get the same Bayes estimator as obtained in (4) corresponding to the Jeffrey's prior and replace $c = 3/2$ we get Hartigan's prior.

3.4.3 Estimator under Precautionary loss function

By using precautionary loss function $L(\hat{\theta},\theta) = \frac{(\hat{\theta}-\theta)^2}{\hat{\theta}}$, the risk function is given by

$$R(\hat{\theta}) = \int_0^\infty \frac{(\hat{\theta}-\theta)^2}{\hat{\theta}} \frac{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^{n-2c+1}}{\Gamma(n-2c+1)} \theta^{n-2c} \exp\left(-\theta\sum_{i=1}^{n} x_i^{-\beta}\right) d\theta$$

$$R(\hat{\theta}) = \frac{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^{n-2c+1}}{\hat{\theta}\,\Gamma(n-2c+1)} \int_0^\infty (\hat{\theta}-\theta)^2 \theta^{n-2c} \exp\left(-\theta\sum_{i=1}^{n} x_i^{-\beta}\right) d\theta$$

$$R(\hat{\theta}) = \left[\hat{\theta} + \frac{(n-2c+2)(n-2c+1)}{\hat{\theta}\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^2} - 2\frac{(n-2c+1)}{\sum_{i=1}^{n} x_i^{-\beta}}\right]$$

Now solving $\frac{\partial R(\hat{\theta})}{\partial \hat{\theta}} = 0$, we obtain the Baye's estimator as

$$\hat{\theta}_p = \frac{\left[(n-2c+2)(n-2c+1)\right]^{\frac{1}{2}}}{\sum_{i=1}^{n} x_i^{-\beta}} \tag{8}$$

Remark 1.3: Replacing $c = 1/2$, in (8), we get the same Bayes estimator as obtained in (5) corresponding to the Jeffrey's prior and replace $c = 3/2$ we get Hartigan's prior.

3.5 Bayes estimator under $g_3(\theta)$

Under $g_3(\theta)$, using (1), the posterior distribution is given by

$$g_3(\theta|x) = k\theta^{n-d} \exp\left(-\theta\sum_{i=1}^{n} x_i^{-\beta}\right)$$

where k is independent of θ

and $$k^{-1} = \int_0^\infty \theta^{n-d} \exp\left(-\theta\sum_{i=1}^{n} x_i^{-\beta}\right) d\theta$$

$$\Rightarrow k^{-1} = \frac{\Gamma(n-d+1)}{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^{n-d+1}}$$

$$\Rightarrow k = \frac{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^{n-d+1}}{\Gamma(n-d+1)}$$

Thus posterior distribution is given by

$$g_1(\theta|x) = \frac{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^{n-d+1}}{\Gamma(n-d+1)} \theta^{n-d} \exp\left(-\theta \sum_{i=1}^{n} x_i^{-\beta}\right)$$

3.5.1 Estimator under SELF

By using SELF, the risk function $L(\hat{\theta}, \theta) = c_1(\hat{\theta} - \theta)^2$, the risk function is given by

$$R(\hat{\theta}) = \int_0^\infty c_1(\hat{\theta} - \theta)^2 \frac{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^{n-d+1}}{\Gamma(n-d+1)} \theta^{n-d} \exp\left(-\theta \sum_{i=1}^{n} x_i^{-\beta}\right) d\theta$$

$$R(\hat{\theta}) = c_1 \frac{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^{n-d+1}}{\Gamma(n-d+1)} \int_0^\infty (\hat{\theta} - \theta)^2 \theta^{n-d} \exp\left(-\theta \sum_{i=1}^{n} x_i^{-\beta}\right) d\theta$$

$$R(\hat{\theta}) = c_1 \left[\hat{\theta}^2 + \frac{(n-d+2)(n-d+1)}{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^2} - 2\hat{\theta} \frac{(n-d+1)}{\sum_{i=1}^{n} x_i^{-\beta}} \right]$$

Now solving $\frac{\partial R(\hat{\theta})}{\partial \hat{\theta}} = 0$, we obtain the Baye's estimator as

$$\hat{\theta}_s = \frac{n-d+1}{\sum_{i=1}^{n} x_i^{-\beta}} \tag{9}$$

3.5.2 Estimator under entropy loss function

By using entropy loss function $L(\delta) = b[\delta - \log(\delta) - 1]\ ; b > 0$, the risk function is given by

$$R(\hat{\theta}) = \int_0^\infty b[\delta - \log(\delta) - 1] \frac{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^{n-d+1}}{\Gamma(n-d+1)} \theta^{n-d} \exp\left(-\theta \sum_{i=1}^{n} x_i^{-\beta}\right) d\theta$$

$$R(\hat{\theta}) = b \frac{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^{n-d+1}}{\Gamma(n-d+1)} \int_0^\infty [\delta - \log(\delta) - 1] \theta^{n-d} \exp\left(-\theta \sum_{i=1}^{n} x_i^{-\beta}\right) d\theta$$

$$R(\hat{\theta}) = b\left[\frac{\hat{\theta}\sum_{i=1}^{n} x_i^{-\beta}}{(n-d)} - \log\hat{\theta} + \frac{\Gamma(n-d+1)'}{\Gamma(n-d+1)} - 1\right]$$

Now solving $\frac{\partial R(\hat{\theta})}{\partial \hat{\theta}} = 0$, we obtain the Baye's estimator as

$$\hat{\theta}_e = \frac{n-d}{\sum_{i=1}^{n} x_i^{-\beta}} \qquad (10)$$

3.5.3 Estimator under precautionary loss function

By using precautionary loss function $L(\hat{\theta},\theta) = \frac{(\hat{\theta}-\theta)^2}{\hat{\theta}}$, the risk function is given by

$$R(\hat{\theta}) = \int_0^\infty \frac{(\hat{\theta}-\theta)^2}{\hat{\theta}} \frac{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^{n-d+1}}{\Gamma(n-d+1)} \theta^{n-d} \exp\left(-\theta\sum_{i=1}^{n} x_i^{-\beta}\right) d\theta$$

$$R(\hat{\theta}) = \frac{\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^{ndc+1}}{\hat{\theta}\,\Gamma(n-d+1)} \int_0^\infty (\hat{\theta}-\theta)^2 \theta^{n-d} \exp\left(-\theta\sum_{i=1}^{n} x_i^{-\beta}\right) d\theta$$

$$R(\hat{\theta}) = \left[\hat{\theta} + \frac{(n-d+2)(n-d+1)}{\hat{\theta}\left(\sum_{i=1}^{n} x_i^{-\beta}\right)^2} - 2\frac{(n-d+1)}{\sum_{i=1}^{n} x_i^{-\beta}}\right]$$

Now solving $\frac{\partial R(\hat{\theta})}{\partial \hat{\theta}} = 0$, we obtain the Baye's estimator as

$$\hat{\theta}_p = \frac{[(n-d+2)(n-d+1)]^{\frac{1}{2}}}{\sum_{i=1}^{n} x_i^{-\beta}} \qquad (11)$$

3. 6 Concluding Remarks

In this article, we have primarily studied the Bayes estimator of the parameter of the Exponentiated Inverted Weibull distribution under three different priors by assuming three different loss functions. The extended Jeffrey's prior gives the opportunity of covering wide spectrum of priors to get Bayes estimates of the parameter - particular cases of which are Jeffrey's prior and Hartigan's prior.

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