Journal of Statistics Applications & Probability
An International Journal

http://dx.doi.org/10.12785/jsap/030306

On the Use of One-Parameter Family of Synthetic Estimators for Small Areas

V. K. Singh and S. K. Seth*

Department of Statistics, Banaras Hindu University, Varanasi 221005, India

Received: 12 Aug. 2014, Revised: 9 Sep. 2014, Accepted: 12 Sep. 2014
Published online: 1 Nov. 2014

Abstract: Small area estimation (SAE) has received a lot of concern of sample survey statisticians in last few years in want of reliable small area statistics which is generally not possible to get by traditional area-specific direct estimators due to very small size of samples in small area. This paper has been devoted to the development of an efficient sampling strategy by suggesting a one-parameter class of estimators for the estimation of parameters of small area using one auxiliary characteristic. The suggested class provides a class of synthetic estimators. Certain important properties have been discussed with. In order to demonstrate the superiority of the suggested estimators over some existing synthetic estimators, a simulation study has been carried out with the help of an empirical data.

Keywords: Small domain, synthetic estimators, one - parameter family of estimators, absolute relative bias, simulated relative standard error.

1 Introduction

Sample surveys are generally considered to be a cost-effective mean of obtaining information on a member of characteristics of a population. They are widely used to provide estimates not only for the entire population but also for a variety of sub-populations (domains). These sub-populations (domains) may be the geographical areas or some socio-demographic or ethnic groups of the population; for instance state/province, county, municipality, school district, unemployment insurance region, village panchayats in India, socially deprived classes of the population, etc. Usually, in the context of sample surveys, a direct estimator of such a domain is based only on the domain-specific sample data, but due to cost and other operational considerations, the sample from domains are quite small and hence a traditional direct estimator may have unacceptably high variability. In this context, the term "Small Area" is generally used to denote any domain for which direct estimators of adequate precision can not be produced.

In recent years, small area estimation has received a lot of attention due to growing demand for reliable small area statistics for formulating policies and programs not only for the population of a country but also for small sub-sections of the population. In other words, the thrust of planning process has shifted from macro to micro level. Accordingly, an offshoot of this change is that various small area estimation(SAE) techniques are being proposed by the researchers (see Rao, 2003 for a review of available methods).

Among other SAE technique, one of the technique is Indirect Estimation Method (or Synthetic Estimation Method). The U.S. National Centre for Health Statistics (1968) pioneered the use of synthetic estimation to obtain state estimates of long and short term physical disabilities from the National Health Interview Survey (NHIS) data. Gonzalez (1973) described synthetic estimates as follows:

"An unbiased estimate is obtained from a sample survey for a large area; when this estimate is used to derive estimates for sub-areas under the assumption that the small areas have the same characteristics as the large area, we identify these estimates as "Synthetic Estimates."

In recent past, being a simple common sense approach for SAE, many of the authors developed a variety of synthetic estimators under various realistic conditions and sampling schemes. Some of the works are by Tikkiwal and Ghiya (2000), Singh et al (2002), Tikkiwal and Pandey(2007), Pandey and Tikkiwal (2010) and Rai and Pandey (2013). Rao(2003) has presented a good deal of synthetic estimators with and without auxiliary information.

* Corresponding author e-mail: vijay_usha_2000@yahoo.com

The present work deals with the development of a one-parameter family of synthetic estimators for domain mean which includes a number of classical estimators as particular cases. The class exhibits some nice properties which makes it preferable over other synthetic estimators. In order to demonstrate the applicability of the suggested class of estimators, a simulation study has also been made using an empirical data.

2 Formulation of the Problem and Notations

Let us consider a finite population $U = \{Y_1, Y_2, ..., Y_N\}$ of size N, where Y represents the characteristic under study. Let the population be divided into $'A'$ non-overlapping small domains U_a of size $N_a (a = 1, 2, 3, ..., A)$ for which estimates are required. Let X be an auxiliary variable for which the information are available in the population. Let a simple random sample of size n be selected from the population U such that n_a units in the sample comes from small domain U_a. Obviously then we have

$\sum_{a=1}^{A} N_a = N$ and $\sum_{a=1}^{A} n_a = n$.

We define the following population and sample values for the characteristics X and Y :

$\overline{Y}(\overline{X})$: mean of the variable $Y(X)$ in the population.

$\overline{Y_a}(\overline{X_a})$: mean of the variable $Y(X)$ for the domain U_a.

$\overline{y}(\overline{x})$: mean of the variable $Y(X)$ in the sample of size n.

$\overline{y_a}(\overline{x_a})$: mean of the variable $Y(X)$ in the sample of size n_a.

In the similar fashion, we can define mean squares and coefficient of variations in the population, in the domain $'a'$ and in the samples as :

$S_Y^2(S_X^2), C_Y(C_X), S_{XY}, C_{XY}, S_{Y_a}^2(S_{X_a}^2), C_{Y_a}(C_{X_a}), C_{Y_aX_a}, S_{Y_aX_a}$

where

$S_Z^2 = \frac{1}{N-1} \sum_{i=1}^{N} (z_i - \overline{Z})^2$

$S_{XY} = \frac{1}{N-1} \sum_{i=1}^{N} (y_i - \overline{Y})(x_i - \overline{X})$

$S_{Z_a}^2 = \frac{1}{N_a-1} \sum_{i=1}^{N_a} (z_{a_i} - \overline{Z}_a)^2$

$S_{Y_aX_a} = \frac{1}{N_a-1} \sum_{i=1}^{N_a} (y_{a_i} - \overline{Y}_a)(x_{a_i} - \overline{X}_a)$

$C_Z = S_Z/\overline{Z}, \qquad C_{XY} = S_{XY}/\overline{X}\overline{Y}, \qquad C_{Z_a} = S_{Z_a}/\overline{Z}_a,$

$C_{Y_aX_a} = S_{Y_aX_a}/\overline{Y}_a\overline{X}_a; \qquad Z = X, Y;$

z_{a_i} being the i^{th} observation of the domain $'a'$.

3 The Proposed Family of Synthetic Estimators :

As mentioned earlier, our aim is to propose synthetic estimator for the mean $\overline{Y}_a$, based on auxiliary information X. Let us define an estimator.

$$T_{\alpha,a} = \overline{y}\psi(\alpha, \overline{X}_a, \overline{x}); \tag{1}$$

where

$$\psi(\alpha,\bar{X}_a,\bar{x}) = \frac{\eta\{\phi_1(\alpha)\}}{\eta\{\phi_2(\alpha)\}};$$

$$\eta\{\phi_i(\alpha)\} = \phi_i(\alpha) + \{1-\phi_i(\alpha)\}\frac{\bar{X}_a}{\bar{x}}; \qquad i = 1,2,..$$

$$\phi_1(\alpha) = \frac{fQ}{P+fQ+R} \quad , \quad \phi_2(\alpha) = \frac{R}{P+fQ+R} \quad , \quad f = \frac{n}{N}$$

and

$$P = (\alpha-1)(\alpha-2), \qquad Q = (\alpha-1)(\alpha-4), \qquad R = (\alpha-2)(\alpha-3)(\alpha-4),$$

α being a constant such that $\alpha > 0$.

Remark 1. It is obvious that $T_{\alpha,a}$ define a one parameter family of synthetic estimator for estimating the mean of the domain $'a'$.

Remark 2. In fact, $T_{\alpha,a}$ is a synthetic version of factor-type estimator (FTE), propsed by Singh and Shukla (1987) and Shukla(1988).

4 Particular Cases of $T_{\alpha,a}$:

It is easy to observe that the class $T_{\alpha,a}$ includes some well-known synthetic estimators as particular cases. For example,

(i) $T_{1,a}$, is ratio synthetic estimator :

$$\bar{y}_{RS,a} = \frac{\bar{y}}{\bar{x}}\bar{X}_a \tag{2}$$

discussed by Rao(2003) and Tikkiwal and Ghiya(2000) and others.

(ii) $T_{2,a}$ is product synthetic estimator :

$$\bar{y}_{PS,a} = \bar{y}\frac{\bar{x}}{\bar{X}_a} \tag{3}$$

(iii) $T_{3,a}$ is synthetic estimator based on the concept of dual to ratio estimator proposed by Srivenkataramana(1980) :

$$\bar{y}_{DS,a} = \bar{y}\left[\frac{N\bar{X}_a - n\bar{x}}{(N-n)\bar{X}_a}\right] \tag{4}$$

(iv) $T_{4,a}$ is simple synthetic estimator :

$$\bar{y}_{SS,a} = \bar{y} \tag{5}$$

5 Important Properties of $T_{\alpha,a}$:

(i) The estimator $T_{\alpha,a}$ asymptotically converges to $\bar{y}_{RS,a}$ as α becomes infinitely large, that is

$$\lim_{\alpha\to\infty} T_{\alpha,a} = \bar{y}_{RS,a} \tag{6}$$

This, in fact, guarantees the existence of a finite value of the estimator even if one arbitrarily selects a large value of the parameter in any particular situation.

(ii) Since $T_{\alpha,a}$ is a function of the parameter α and the constant R is a cubic function of α, the expression of mean square error(MSE) would also be at least a cubic function of α. Thus, while minimizing the MSE expression of $T_{\alpha,a}$ with respect to α, so as to obtain the minimum MSE, one gets more than one optimum values of α for which minimum MSE are equal. This, in fact, provides a method of selecting least bias of the estimator with minimum MSE. Singh and Shukla(1987) considered it as an extra advantage of the class of factor type estimators, since one can select the least bias of the estimator, obtained for all the possible values of optimum α, thus, putting a control on the bias while obtaining minimum MSE, Such a property of controlling the bias is generally not exhibited by other one-parameter families of estimators.

(iii) The structure and the properties of the estimator remains unchanged even if the values of the constant P, Q and R are re-structured as.

$$P=(\alpha-k)(\alpha-k-1)\,,\; Q=(\alpha-k)(\alpha-k-3)$$
$$R=(\alpha-k-1)(\alpha-k-2)(\alpha-k-3)\,; \qquad k=1,2,3....$$

Remark 4. All the above mentioned properties of the estimator $T_{\alpha,a}$ can be derived with the help of the factor type estimator (Singh and Shukla, 1987). In fact Shukla(1988) has discussed these properties in detail.

Remark 5. It is to be mentioned here that the properties of the class of estimators, $T_{\alpha,a}$, as discussed above, makes it superior to other one-parameter families of synthetic estimator. As an example, the family

$$\bar{y}_{syn,a}=\bar{y}\left(\frac{\bar{x}}{X_a}\right)^{\beta}, \tag{7}$$

defined by Tikkiwal and Ghiya(2000) and the family.

$$\bar{y}_{syn,a}=W_1\bar{y}\left(\frac{\bar{x}_1}{X_{1a}}\right)^{\beta_1}+W_2\bar{y}\left(\frac{\bar{x}_2}{X_{2a}}\right)^{\beta_2}, \tag{8}$$

defined by Rai and Pandey(2013) do not exhibit the above mentioned properties.

6 Design-Bias and MSE of $T_{\alpha,a}$:

The bias and MSE of $T_{\alpha,a}$ can easily be obtained using the large sample approximation theory.

Let us write

$$\bar{y}=\bar{Y}(1+e_1)\,, \qquad \bar{x}=\bar{X}(1+e_2)\,,$$

so that

$$E(e_1)=E(e_2)=0 \qquad \text{and} \qquad E\left(e_1^2\right)=\frac{N-n}{Nn}C_Y^2,$$
$$E\left(e_2^2\right)=\frac{N-n}{Nn}C_X^2, \qquad E(e_1e_2)=\frac{N-n}{Nn}C_{YX}.$$

Then expressing $T_{\alpha,a}$ in terms of e_1 and e_2, expending the expression and retaining the terms up to the order $O\left(n^{-1}\right)$, we get

$$B[T_{\alpha,a}]=\left(\frac{p_1}{p_2}\bar{Y}-\bar{Y}_a\right)+\frac{p_1}{p_2}\bar{Y}\left[V_{11}-\frac{R\bar{X}}{p_2}V_{02}\right]\bar{X}\left[\frac{fQ}{p_1}-\frac{R}{p_2}\right] \tag{9}$$

where

$$p_1=(P+R)\bar{X}_a+fQ\bar{X}, \qquad p_2=(P+fQ)\bar{X}_a+R\bar{X} \qquad \text{and}$$

$$V_{ij}=E\left[\left(\frac{y-\overline{Y}}{Y}\right)^{i}\left(\frac{x-\overline{X}}{X}\right)^{j}\right];\qquad i,j=0,1,2,.....$$

Further, the MSE of $T_{\alpha,a}$ would be

$$M\left[T_{\alpha,a}\right]=E\left[T_{\alpha,a}-\overline{Y}_a\right]^2. \tag{10}$$

Assuming that the contribution of terms involving powers of e_1 and e_2 higher than the second are negligible, we get

$$\begin{aligned}M\left[T_{\alpha,a}\right]=&\left\{\frac{p_1}{p_2}\overline{Y}-\overline{Y}_a\right\}^2+\left(\frac{p_1}{p_2}\overline{Y}\right)^2V_{20}+\overline{X}^2\left(\frac{p_1}{p_2}\overline{Y}\right)\left(\frac{fQ}{p_1}-\frac{R}{p_2}\right)\\&\left\{\frac{p_1}{p_2}\overline{Y}\left(\frac{fQ}{p_1}-\frac{R}{p_2}\right)-2\left(\frac{p_1}{p_2}\overline{Y}\right)\frac{R}{p_2}+2\overline{Y}_a\frac{R}{p_2}\right\}V_{02}\\&+2\overline{X}\left(\frac{p_1}{p_2}\overline{Y}\right)\left(\frac{fQ}{p_1}-\frac{R}{p_2}\right)\left\{2\left(\frac{p_1}{p_2}\overline{Y}\right)-\overline{Y}_a\right\}V_{11}.\end{aligned} \tag{11}$$

7 Bias and MSE of $T_{1,a}$ and $T_{4,a}$:

As particular cases of the general class of synthetic estimator, $T_{\alpha,a}$, we shall discuss here the ratio-synthetic estimator, proposed by Rao(2003) and the simple synthetic estimator, assuming that Y and X are positively correlated.

7.1 Letting $\alpha=1$ in (1)

, we have

$$T_{1,a}=\overline{y}\left(\frac{\overline{X}_a}{\overline{x}}\right)=\overline{y}_{RS,a} \tag{12}$$

which is ratio-synthetic estimator.

Now from(9) and (11), we get

$$B\left[\overline{y}_{RS,a}\right]=\left\{\overline{Y}\left(\frac{\overline{X}_a}{\overline{X}}\right)-\overline{Y}_a\right\}+\overline{Y}\left(\frac{\overline{X}_a}{\overline{X}}\right)\frac{N-n}{Nn}\left\{C_X^2-C_{XY}\right\} \tag{13}$$

and

$$\begin{aligned}M\left[\overline{y}_{RS,a}\right]=&\left\{\overline{Y}\left(\frac{\overline{X}_a}{\overline{X}}\right)-\overline{Y}_a\right\}^2+\frac{N-n}{Nn}\overline{Y}\left(\frac{\overline{X}_a}{\overline{X}}\right)\\&\left[\overline{Y}\left(\frac{\overline{X}_a}{\overline{X}}\right)\left\{3C_X^2+C_Y^2-4C_{XY}\right\}-2\overline{Y}_a\left\{C_X^2-C_{XY}\right\}\right]\end{aligned} \tag{14}$$

These expressions are similar to the expressions obtained in Tikkiwal and Ghiya(2000) for the estimator.

$$\overline{y}_{syn,a}=\overline{y}\left(\frac{\overline{X}_a}{\overline{x}}\right)^{\beta}\ for\ \beta=\ -1.$$

Remark 6.

From the expression (13) it can be seen that the estimator $T_{1,a}$ may be heavily biased unless the synthetic assumption

$$\left(\frac{\overline{Y}_a}{\overline{X}_a}\right)=\left(\frac{\overline{Y}}{\overline{X}}\right),\ for\ a\in A$$

is satisfied, that is, area specific ratio $\overline{Y}_a/\overline{X}_a$ is close to the overall ratio $\overline{Y}/\overline{X}$

Under this assumption (13) reduces to

$$B\left[\overline{y}_{RS,a}\right]_{SA}=\overline{Y}_a\left(\frac{N-n}{Nn}\right)\left[C_X^2-C_{XY}\right]. \tag{15}$$

Similarly, under the above synthetic assumption, the MSE of $\overline{y}_{RS,a}$ reduces to

$$M\left[\overline{y}_{RS,a}\right]_{SA}=\frac{N-n}{Nn}\overline{Y}_a^2\left[C_Y^2+C_X^2-2C_{XY}\right] \tag{16}$$

7.2 Now let us consider the simple synthetic estimator $T_{4,a}=\overline{y}_{SS,a}$.

From (9), the bias is obtained as :

$$B\left[\overline{y}_{SS,a}\right]=\left(\overline{Y}-\overline{Y}_a\right) \tag{17}$$

and from (11) the MSE is

$$M\left[\overline{y}_{SS,a}\right]=\left(\overline{Y}-\overline{Y}_a\right)^2+\frac{N-n}{Nn}S_Y^2. \tag{18}$$

8 Optimum Values of α and Minimum MSE.

Since the proposed class of synthetic estimators involves a parameter α and consequently the bias and MSE are functions of the parameter, the optimum estimator within the class could be obtained by minimizing $M\left[T_{\alpha,a}\right]$ with respect to α. As it is evident from the expression (11), it would not be possible to derive an explicit expression for $\frac{\partial M\left[T_{\alpha,a}\right]}{\partial\alpha}=0$, so as to get the optimum values of α. However, the equation can be solved by iteration method and very close approximations of optimum values of α can be obtained.

It is further noted from the expression(11) that the equation $\frac{\partial M\left[T_{\alpha,a}\right]}{\partial\alpha}=0$ would yield more than one optimum α, some of which would be real, some negative and some imaginary. Since $\alpha>0$, only the real possible values of optimum α would be considered.

9 Efficiency Comparison

It is advisable to compare the performance of the general class of estimators, $T_{\alpha,a}$ in terms of its precision with other synthetic estimators under similar conditions. Since the ratio synthetic estimator $T_{1,a}$ is a particular member of the family and the optimum estimator within the class could be obtained, we have compared the performance of $T_{1,a}$ and $T_{\alpha,a}|opt\alpha$, with that of direct estimator, where these estimators are:

Direct Estimator (Direct ratio estimator):

$$T_{D,a}=\overline{y}_a\left(\frac{\overline{X}_a}{\overline{x}_a}\right)$$

Indirect Estimators :

Ratio synthetic estimator :

$$T_{1,a} = \bar{y}\left(\frac{\bar{X}_a}{\bar{x}}\right)$$

Optimum synthetic estimator :

$$T_{\alpha_0,a} = \bar{y}\psi(\alpha_0, \bar{X}_a, \bar{x})$$

10 Simulation Study :

The performance of the above estimators and their comparison have been done through a simulation study, by considering an empirical data set which is described below:

10.1 For the application purpose

we have made use of MU284 population given in Sarndal et al(1992), **Appendix B**. The population of Sweden is divided into 284 municipalities spread over four major regions : North, South, East and West.

Considering only the east, central and south regions (region indicators: 1,2,3,6,7 and 8), we treated it as a population with $N = 190$. Now in the population regions 1,2,3,6,,7,and 8 were considered separately to be small areas with sizes 25, 48, 32, 41, 15 and 24 respectively. Our aim is here to estimate the mean of the study variable Y for all these six small domains using synthetic method of estimation, where

Y: is the total number of seats in municipal council.

X: is the number of conservative seat in municipal council.

For the entire population of size 190 and the six small area, the following values were obtained:

Table 1: Population and Domain Values.

Population Values

N	$\bar{Y}$	$\bar{X}$	S_Y^2	S_X^2	S_{XY}	ρ_{XY}
190	47.69	8.3	137.71	26.82	41.94	0.69

Domain value	**Domain**					
	(1)	(2)	(3)	(6)	(7)	(8)
N_a	25	48	32	41	15	24
$\bar{Y}_a$	51.16	47.66	50.25	46.56	54.2	40.17
$\bar{X}_a$	16	8.1	9.5	6.73	6.06	4.04
$S_{Y_a}^2$	197.97	166.35	106.77	67.7	130.17	99.29
$S_{X_a}^2$	36	23.2	9.35	8.8	8.06	4.85
$S_{X_aY_a}$	61.25	55.75	26.38	14.87	25.91	18.48
$\rho_{Y_aX_a}$	0.726	0.898	0.835	0.609	0.799	0.842

Since the synthetic assumption plays an important role in the efficiency of a ratio synthetic estimator, we have examined the absolute difference between $\frac{\bar{Y}_a}{\bar{X}_a}$ and $\frac{\bar{Y}}{\bar{X}}$ for all the six domains in the following table:

Table 2: Absolute difference under synthetic assumption of ratio synthetic estimator for various domains

Domain	$\left(\frac{\bar{Y}_a}{\bar{X}_a}\right)$	$\left(\frac{\bar{Y}}{\bar{X}}\right)$	$\lvert\frac{\bar{Y}_a}{\bar{X}_a}-\frac{\bar{Y}}{\bar{X}}\rvert$
1	3.197	5.743	2.546
2	5.882	5.743	0.139
3	5.289	5.743	0.454
6	6.917	5.743	1.174
7	8.934	5.743	3.191
8	9.873	5.743	4.13

From the table, it is apparent that in comparison to other domains, the synthetic assumption closely meet in domains 2,3 and 6.

10.2 Now for the purpose of simulation study

we selected 500 independent simple random samples of size 19 from the population of size 190. Further, to assess the relative performance of the estimators under consideration, their Absolute relative bias (ARB) and Simulated relative standard error (SRSE) were obtained for each domain on the basis of the selected samples as follows:

$$ARB\left(T_{k,a}\right)=\frac{\lvert\frac{1}{500}\sum_{s=1}^{500}T_{k,a}^{s}-\bar{Y}_a\rvert}{Y_a}\times 100 \tag{19}$$

$$SRSE\left(T_{k,a}\right)=\frac{\sqrt{SMSE\left(T_{k,a}\right)}}{Y_a}\times 100 \tag{20}$$

where

$$SMSE\left(T_{k,a}\right)=\frac{1}{500}\sum_{s=1}^{500}\left(T_{k,a}^{s}-\bar{Y}_a\right)^2 \tag{21}$$

$T_{k,a}$ denotes a particular synthetic estimator for domain $'a'$ and $T_{k,a}^{s}$ stands for the value of $T_{k,a}$ for domain $'a'$ for the s^{th} sample, where $a=1,2,3,6,7$ and 8.
The values of ARB and SRSE of the estimators $T_{D,a}$, $T_{1,a}$ and $T_{\alpha_0,a}$ along with the value of α_0 for each domain are presented in Table 3

11 Conclusions

From the above analysis certain important conclusions come up. Firstly, it is clear that the estimator $T_{1,a}$, which is one of the member of the suggested class, $T_{\alpha,a}$, possesses smaller values of ARB for the domains 2,3 and 6 as compared to other three domains. This is because of the fact that for these domains, the synthetic assumption is closely met. Moreover, due to the same reason, SRSE are also smaller in these domains.

Further, it was observed that for optimum values of parameter α, in each domain, the values of SRSE are drastically reduced and these are quit smaller than that of direct estimator $T_{D,a}$. This suggests that, in general the suggested class of estimators is always preferable under optimality condition over other synthetic estimators. However, the process of

Table 3: ARB and SRSE(in percent) of various Estimators

Estimator		**Domain**					
		(1)	(2)	(3)	(6)	(7)	(8)
$T_{D,a}$	SRSE	68.27	80.61	27.8	78.21	150.2	111.53
	ARB	3.05	3.6	1.24	3.5	6.72	4.99
$T_{1,a}$	SRSE	1814.56	34.85	212.55	363.94	787.94	927.02
	ARB	81.15	1.56	9.51	16.28	35.24	41.56
$T_{\alpha_0,a}$	SRSE	1.88	5.16	7.92	16.77	105.85	1.81
	ARB	0.08	0.23	0.35	0.75	4.73	0.08
	α_0	1.89	1.85	1.83	2.95	1.9	1.86

finding optimum value of the parameter is quite cumbersome, but it could be resolved with the aid of sophisticated computers and software.

References

[1] Gonzalez, M.E.(1973): Use and evaluation of synthetic estimates, *Proceeding of the Social Statistics*, *American Statistical Association*, 33-36.

[2] National Centre for Health Statistics(1968): Synthetic State Estimates of Disability, P.H.S. Publication 1759, Washington, DC: U.S. Government printing Office.

[3] Pandey,K.K. and Tikkiwal, G.C.(2010): Generalized class of synthetic estimators for small areas under systematic sampling scheme, *Statistics in Transition*, 11(1), 75-89.

[4] Rai, P.K. and Pandey, K.K.(2013): Synthetic estimators using auxiliary information in small domains, *Statistics in Transition*, 14(1), 31-44.

[5] Rao, J.N.K.(2003): Small Area Estimation, Wiley Inter-science, John Wiley and Sons, New Jersey.

[6] Sarndal,C.E., Swensson, B. and Wretman, J.H.(1992): Model Assisted Survey Sampling, Springer-Verlog, New York.

[7] Shukla, D.(1988): On Some Factor-Type Estimators for Population Parameter in Sample Surveys, *Unpublished* Ph.D. Thesis submitted to Banaras Hindu University, Varanasi, India.

[8] Singh, S.P., Srivastava, A.K. and Sisodia, B.V.S.(2002): Evaluation of a synthetic method of estimation for small area, *Journal of Applied Statistics*, 29(8), 1147-1151.

[9] Singh, V.K. and Shukla, D.(1987): One parameter family of factor-type ratio estimation, *Metron*, 45(1-2),273-283.

[10] Srivenkataramana, T.(1980): A Dual to ratio estimator in sample surveys, *Biometrika*, 67(1), 199-204.

[11] Tikkiwal, G.C. and Ghiya, A.(2000): A generalised class of synthetic estimators with application to crop acreage estimation for small domains, *Biometrical Journal*, 42(7), 865-876.

[12] Tikkiwal, G.C. and Pandey, K.K.(2007): On synthetic and composite estimators for small area estimation under Lahiri-Midzuno sampling scheme, *Statistics in Transition*, 8(1), 111-123.