Dual to Separate Ratio Type Exponential Estimator in Post-Stratification

Hilal A. Lone [*] *and Rajesh Tailor*

School of Studies in Statistics, Vikram University, Ujjain-456010, M.P. India

Abstract: This paper deals with the problem of estimation of finite population mean in case of post-stratification. Following the approach of Srivenkataramana [16] and Bondyopadhyayh [2], we have proposed dual to separate ratio type exponential estimator. It has been shown that the proposed estimator is more efficient than usual unbiased estimator, usual dual to separate ratio type estimator and other estimators under certain given conditions. The bias and mean squared error of the proposed estimator is obtained up to the first degree of approximation. An empirical study has been carried out to demonstrate the performance of the suggested estimator.

Keywords: Finite Population mean, Post-stratification, Bias, Mean squared error

1 Introduction

Use of auxiliary information has been in practice for improving the efficiency of estimator(s) of parameter(s).Usually, auxiliary information is easily available with study variate with little extra cost and efforts. Auxiliary information may be used in various ways like at pre - selection stage, selection stage or design stage, post–selection or estimation stage and selection stage as well as estimation stages. In stratified random sampling, it is assumed that the size of the strata as well as sampling frame is available. But in many situations sampling frame is not available. For example in households surveys it is possible to know the number of families aided in a locality but which families belong to which locality i.e. up to date sampling frame from different strata may not be available. In this type of situation post-stratification technique, in which first a sample of size n is selected using simple random sampling then units of the sample is stratified into different stratum according to a factor is used.

Bhal and Tuteja [1] envisaged ratio and product type exponential estimators using exponential function which were defined in stratified random sampling by Singh et al. [15]. Later these estimators have been defined by Chouhan [3] in case of post stratification. Motivated by Srivenkataramana [16], Tailor and Tailor [21] proposed dual to Bhal and Tuteja [1] ratio and product type exponential estimators in simple random sampling. Tailor and Tailor [21] lead authors to propose dual to separate ratio type exponential estimator in case of post stratification. Cochran [4] and Robson [11] envisaged classical ratio and product estimators which were studied in case of post stratification by Ige and Tripathi [6].Various authors including Lone et al.[9],Lone et al.[10],Tailor and Lone [18], Tailor and Lone [20],Singh et al.[12], Singh et al.[13] and Singh et al.[14] developed various exponential estimators to improve the classical ratio and product estimators.

When information of parameters of auxiliary variate is available in each stratum, separate ratio type estimators may be constructed easily and perform better as compared to combined estimators. Yadav et al. [23] proposed the improved separate ratio type exponential estimator for population mean. Vishwakarma and Singh [22] defined separate ratio-

---

[*] Corresponding author e-mail: hilalstat@gmail.com

product estimators for estimating population mean using auxiliary information. Tailor and Lone [19] discussed some separate type estimators in the line of kadilar and Cingi [8] estimators. Motivated by Srivenkataramana [16] and Tailor and Tailor [21], we propose dual to separate ratio type exponential estimator in case of post stratification. The bias and mean squared error of the proposed estimator is obtained up to the first degree of approximation. The proposed separate ratio type exponential estimator has been compared with usual unbiased estimator, usual dual to separate ratio type estimator and other considered estimators.

Let us consider a finite population $U = (U_1, U_2, ..., U_N)$. A sample of size n is drawn from population U using simple random sampling without replacement. After selecting the sample, it is observed that which units belong to h^{th} stratum. Let n_h be the size of the sample falling in h^{th} stratum such that $\sum_{h=1}^{L} n_h = n$. Here it is assumed that n is so large that possibility of n_h being zero is very small.

Let y_{hi} be the observation on i^{th} unit that fall in h^{th} stratum for study variate y and x_{hi} be the observation on i^{th} unit that fall in n_h stratum for auxiliary variate x, then

$$\bar{Y}_h = \frac{1}{N_h} \sum_{i=1}^{N_h} y_{hi} : h^{th} \text{ stratum mean for study variate } y,$$

$$\bar{X}_h = \frac{1}{N_h} \sum_{i=1}^{N_h} x_{hi} : h^{th} \text{ stratum mean for auxiliary variate } x,$$

$$\bar{Y} = \frac{1}{N} \sum_{h=1}^{L} \sum_{i=1}^{N_h} y_{hi} = \sum_{h=1}^{L} W_h \bar{Y}_h : \text{Population mean of the study variate } y \text{ and}$$

$$\bar{X} = \frac{1}{N} \sum_{h=1}^{L} \sum_{i=1}^{N_h} x_{hi} = \sum_{h=1}^{L} W_h \bar{X}_h : \text{Population mean of the auxiliary variate } x.$$

In case of post-stratification, usual unbiased estimator of population mean $\bar{Y}$ is defined as

$$\bar{y}_{PS} = \sum_{h=1}^{L} W_h \bar{y}_h . \tag{1.1}$$

where

$W_h = \frac{N_h}{N}$ is the weight of the hth stratum and $\bar{y}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} y_{hi}$ is sample mean of n_h sample units that fall in the hth stratum.

Using the results from Stephen [17], the variance of $\bar{y}_{PS}$ to the first degree of approximation is obtained as

$$Var(\bar{y}_{PS}) = \left(\frac{1}{n} - \frac{1}{N}\right) \sum_{h=1}^{L} W_h S_{yh}^2 + \frac{1}{n^2} \sum_{h=1}^{L} (1 - W_h) S_{yh}^2 , \tag{1.2}$$

where $S_{yh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (y_{hi} - \bar{Y}_h)^2$.

Utilizing the information on population mean $\overline{X}$ of auxiliary variate x, Ige and Tripathi [9] proposed a ratio type estimator in case of post-stratification as

$$\hat{\overline{Y}}_{RPS} = \overline{y}_{PS}\left(\frac{\overline{X}}{\overline{x}_{PS}}\right). \tag{1.3}$$

Here we define separate ratio type estimator for estimating the population mean $\overline{Y}$ of the variable under study y in case of post-stratification as

$$\hat{\overline{Y}}_{RPS}^{S} = \sum_{h=1}^{L} W_h\, \overline{y}_h\left(\frac{\overline{X}_h}{\overline{x}_h}\right) \tag{1.4}$$

Upto the first degree of approximation, mean squared error of $\hat{\overline{Y}}_{RPS}^{S}$ is obtained as

$$MSE\left(\hat{\overline{Y}}_{RPS}^{S}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\left[\sum_{h=1}^{L} W_h S_{yh}^2 + \sum_{h=1}^{L} W_h R_h^2 S_{xh}^2 - 2\sum_{h=1}^{L} W_h R_h S_{yxh}\right]. \tag{1.5}$$

To have a survey estimate of the population mean $\overline{Y}$ of the variable under study y, assuming the knowledge of the population mean $\overline{X}_h$ of the hth stratum $(h=1,2,3,...,L)$ of the auxiliary variable x, we define usual dual to separate ratio type estimator in case of post-stratification as

$$\hat{\overline{Y}}_{RPS}^{*} = \sum_{h=1}^{L} W_h \overline{y}_h\left(\frac{\overline{x}_h^{*}}{\overline{X}_h}\right), \tag{1.6}$$

where $\overline{x}_h^{*} = \dfrac{N_h \overline{X}_h - n_h \overline{x}_h}{N_h - n_h}$.

To the first degree of approximation, bias and mean squared error of the usual separate ratio type estimator are obtained as

$$B(\hat{\overline{Y}}_{RPS}^{*}) = -\left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} a_h \overline{Y}_h\, \rho_{yxh}\, C_{yh}\, C_{xh}, \tag{1.7}$$

and

$$MSE\left(\hat{\overline{Y}}_{RPS}^{*}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\left[\sum_{h=1}^{L} W_h\, S_{yh}^2 + \sum_{h=1}^{L} W_h\, R_h^2 a_h^2 S_{xh}^2 - 2\sum_{h=1}^{L} W_h\, a_h R_h S_{yxh}\right] \tag{1.8}$$

where $a_h = \dfrac{n_h}{N_h - n_h}$ and $R_h = \dfrac{\overline{Y}_h}{\overline{X}_h}$.

2. Separate Ratio Type Exponential Estimator
Bhal and Tuteja [1] defined the ratio type exponential estimator for population mean $\overline{Y}$ as

$$\hat{\overline{Y}}_1 = \overline{y}\exp\left(\frac{\overline{X} - \overline{x}}{\overline{X} + \overline{x}}\right). \tag{2.1}$$

Singh et al. [15] defined Bhal and Tuteja [1] ratio type exponential estimator for population mean in stratified random sampling as

$$\hat{\bar{Y}}_{St}^{Re} = \bar{y}_{st} \exp\left(\frac{\bar{X} - \bar{x}_{st}}{\bar{X} + \bar{x}_{st}} \right).$$ (2.2)

Chouhan [3] proposed ratio type exponential estimator for population mean in case of post-stratification as

$$\hat{\bar{Y}}_{PS}^{Re} = \bar{y}_{PS} \exp\left(\frac{\bar{X} - \bar{x}_{PS}}{\bar{X} + \bar{x}_{PS}} \right).$$ (2.3)

Up to the first degree of approximation, the bias and mean squared error of estimator $\hat{\bar{Y}}_{PS}^{Re}$ are obtained as

$$B\left(\hat{\bar{Y}}_{PS}^{Re}\right) = \left(\frac{1}{n} - \frac{1}{N} \right) \sum_{h=1}^{L} \frac{1}{\bar{X}} \left(\frac{3}{8} R S_{xh}^2 - \frac{1}{2} S_{yxh} \right),$$ (2.4)

and

$$MSE\left(\hat{\bar{Y}}_{PS}^{Re}\right) = \left(\frac{1}{n} - \frac{1}{N} \right) \sum_{h=1}^{L} W_h \left(S_{yh}^2 + \frac{1}{4} R^2 S_{xh}^2 - R S_{yxh} \right)$$ (2.5)

where $R = \dfrac{\bar{Y}}{\bar{X}}$.

The separate version of Chouhan [3] ratio type exponential estimator can be written as

$$\hat{\bar{Y}}_{PS}^{S\,Re} = \sum_{h=1}^{L} W_h\, \bar{y}_h \left(\frac{\bar{X}_h - \bar{x}_h}{\bar{X}_h + \bar{x}_h} \right).$$ (2.6)

Mean squared error of $\hat{\bar{Y}}_{PS}^{S\,Re}$ upto the first degree of approximation is obtained as

$$MSE\left(\hat{\bar{Y}}_{PS}^{S\,Re}\right) = \left(\frac{1}{n} - \frac{1}{N} \right) \sum_{h=1}^{L} W_h \left(S_{yh}^2 + \frac{1}{4} R_h^2 S_{xh}^2 - R_h S_{yxh} \right) .$$ (2.7)

Srivenkataramana [16] and Bandhyopadhyayh [2] defined dual to classical ratio estimator using transformation $x_i^* = \dfrac{N\bar{X} - n\,x_i}{N - n}$ on auxiliary variate x as

$$\hat{\bar{Y}}_2^* = \bar{y} \left(\frac{\bar{x}^*}{\bar{X}} \right) .$$ (2.8)

where $\bar{x}^* = \dfrac{N\bar{X} - n\,\bar{x}}{N-n}$ is unbiased estimator of population mean $\bar{X}$.

Tailor and Tailor [21] proposed dual to Bhal and Tuteja [1] ratio type exponential estimator using the transformation $\bar{x}^* = \dfrac{N\bar{X} - n\bar{x}}{N-n}$ on auxiliary variate x as

$$\hat{\bar{Y}}_3 = \bar{y}\left(\frac{\bar{x}^* - \bar{X}}{\bar{x}^* + \bar{X}}\right). \tag{2.9}$$

Following Srivenkataramana [16] and Bondyopadhyayh [2] transformation, we propose dual to separate ratio type exponential estimator for population mean $\bar{Y}$ in case of post-stratification as

$$\hat{\bar{Y}}_{PS}^{*\mathrm{Re}} = \sum_{h=1}^{L} W_h \bar{y}_h \exp\left(\frac{\bar{x}_h^* - \bar{X}_h}{\bar{x}_h^* + \bar{X}_h}\right), \tag{2.10}$$

where $\bar{x}_h^* = \dfrac{N_h \bar{X}_h - n_h \bar{x}_h}{N_h - n_h}$.

To obtain the bias and mean squared error of the proposed estimator $\hat{\bar{Y}}_{PS}^{*\mathrm{Re}}$, we write

$\bar{y}_h = \bar{Y}_h \left(1 + e_{0h}\right)$, $\bar{x}_h = \bar{X}_h \left(1 + e_{1h}\right)$ such that

$E(e_{0h}) = E(e_{1h}) = 0$,

$E(e_{0h}^2) = \left(\dfrac{1}{nW_h} - \dfrac{1}{N_h}\right) C_{yh}^2$,

$E(e_{1h}^2) = \left(\dfrac{1}{nW_h} - \dfrac{1}{N_h}\right) C_{xh}^2$ and

$E(e_{0h} e_{1h}) = \left(\dfrac{1}{nW_h} - \dfrac{1}{N_h}\right) \rho_{yxh} C_{yh} C_{xh}$.

Expressing (2.10) in terms of e_{ih}'s, we have

$$\hat{\bar{Y}}_{PS}^{*\mathrm{Re}} = \sum_{h=1}^{L} W_h \bar{Y}_h \left(1 + e_{0h}\right) \exp\left(\frac{-a_h \bar{X}_h e_{1h}}{2\bar{X}_h - a_h \bar{X}_h e_{1h}}\right)$$

$$\hat{\bar{Y}}_{PS}^{*\mathrm{Re}} = \sum_{h=1}^{L} W_h \bar{Y}_h \left(1 + e_{0h}\right) \exp\left\{\frac{-a_h e_{1h}}{2}\left(1 - \frac{a_h e_{1h}}{2}\right)^{-1}\right\}$$

$$\hat{\bar{Y}}_{PS}^{*\mathrm{Re}} = \sum_{h=1}^{L} W_h \bar{Y}_h \left(1 + e_{0h}\right) \left[1 - \frac{1}{2}a_h e_{1h}\left(1 - \frac{a_h e_{1h}}{2}\right)^{-1} + \frac{1}{2}\left\{\frac{a_h^2 e_{1h}^2}{4}\left(1 - \frac{a_h e_{1h}}{2}\right)^{-2}\right\} + \dots\right]$$

$$\hat{\bar{Y}}_{PS}^{*\text{Re}} = \sum_{h=1}^{L} W_h \bar{Y}_h (1 + e_{0h}) \left[1 - \frac{a_h e_{1h}}{2} - \frac{a_h^2 e_{1h}^2}{8} + \dots \right]$$

$$\left(\hat{\bar{Y}}_{PS}^{*\text{Re}} - \bar{Y} \right) = \sum_{h=1}^{L} W_h \bar{Y}_h \left[e_{0h} - \frac{a_h e_{1h}}{2} - \frac{a_h^2 e_{1h}^2}{8} - \frac{a_h e_{0h} e_{1h}}{2} \right] \tag{2.11}$$

Now taking expectation of both sides of (2.11), the bias of the proposed estimator $\hat{\bar{Y}}_{PS}^{*\text{Re}}$ to the first degree of approximation is obtained as

$$B(\hat{\bar{Y}}_{PS}^{*\text{Re}}) = \left(\frac{1}{n} - \frac{1}{N} \right) \sum_{h=1}^{L} \frac{1}{4\,\bar{X}_h} \left[-\frac{1}{2} R_h \, a_h^2 S_{xh}^2 - 2 a_h S_{yxh} \right]. \tag{2.12}$$

Squaring both sides of (2.11) and then taking expectation, we get the mean squared error of the proposed estimator $\hat{\bar{Y}}_{PS}^{*\text{Re}}$ up the first degree of approximation as

$$MSE\left(\hat{\bar{Y}}_{PS}^{*\text{Re}} \right) = \left(\frac{1}{n} - \frac{1}{N} \right) \left[\sum_{h=1}^{L} W_h S_{yh}^2 + \frac{1}{4} \sum_{h=1}^{L} W_h a_h^2 R_h^2 S_{xh}^2 - \sum_{h=1}^{L} W_h a_h R_h S_{yxh} \right] \tag{2.13}$$

where $S_{xh}^2 = \dfrac{1}{N_h - 1} \sum_{i=1}^{N_h} \left(x_{hi} - \bar{X}_h \right)^2$ and $S_{yxh} = \dfrac{1}{N_h - 1} \sum_{i=1}^{N_h} \left(y_{hi} - \bar{Y}_h \right)\left(x_{hi} - \bar{X}_h \right)$.

3. Efficiency Comparisons of the Proposed Dual to Separate Ratio Type Exponential Estimator $\hat{\bar{Y}}_{PS}^{*\text{Re}}$ with $\bar{y}_{PS}, \hat{\bar{Y}}_{RPS}^{S}, \hat{\bar{Y}}_{RPS}^{*}$ and $\hat{\bar{Y}}_{PS}^{S\,\text{Re}}$.

Comparison of (1.2), (1.5), (1.8), (2.7) and (2.13) shows that the proposed estimator $\hat{\bar{Y}}_{PS}^{*\text{Re}}$ would be more efficient than

(i) $\bar{y}_{PS}$ if

$$\alpha < 4 \sum_{h=1}^{L} W_h a_h R_h S_{yxh}. \tag{3.1}$$

(ii) $\hat{\bar{Y}}_{RPS}^{S}$ if

$$\alpha < 4 \left[\sum_{h=1}^{L} W_h R_h^2 S_{xh}^2 + \sum_{h=1}^{L} W_h R_h S_{yxh} (a_h - 2) \right] \tag{3.2}$$

(iii) $\hat{\bar{Y}}_{RPS}^{*}$ if

J. Stat. Appl. Pro. **3**, No. 3, 425-432 (2014) / www.naturalspublishing.com/Journals.asp

431

$$\alpha > \frac{4}{3}\sum_{h=1}^{L} W_h a_h R_h S_{yxh}. \tag{3.3}$$

(iv) $\hat{\bar{Y}}_{PS}^{S\,\text{Re}}$ if

$$\alpha < \left[\sum_{h=1}^{L} W_h R_h^2 S_{xh}^2 + 4\sum_{h=1}^{L} W_h R_h S_{yxh}(a_h - 1)\right] \tag{3.4}$$

where $\alpha = \sum_{h=1}^{L} W_h a_h^2 R_h^2 S_{xh}^2$.

4. Empirical Study

To exhibit the performance of the proposed estimator in comparison to other estimators, two population data sets are being considered. The descriptions of populations are given below:

Population I [Source: Johnston [7], p. 171]

Constants	n_1	N_1	$\overline{X}_1$	$\overline{Y}_1$	$S_{x_1}^2$	$S_{y_1}^2$	ρ_{yx_1}	S_{yx_1}
Stratum I	2	5	37.60	45.60	5.04	21.04	0.39	4.04
Stratum II	2	5	46.40	58.40	10.24	15.84	0.27	3.44

Population II [Source: Japan meteorological society [5]]

Constants	n_1	N_1	$\overline{X}_1$	$\overline{Y}_1$	$S_{x_1}^2$	$S_{y_1}^2$	ρ_{yx_1}	S_{yx_1}
Stratum I	4	10	149.70	142.80	181.17	37.08	0.22	18.44
Stratum II	4	10	91.00	102.60	43.16	158.76	0.28	23.30

Table 4.1 Percent relative Efficiency of $\overline{y}_{PS}, \hat{\bar{Y}}_{RPS}^{S}, \hat{\bar{Y}}_{RPS}^{*}, \hat{\bar{Y}}_{PS}^{S\,\text{Re}}$ and $\hat{\bar{Y}}_{PS}^{*\text{Re}}$ with respect to $\overline{y}_{PS}$

Estimators	$\overline{y}_{PS}$	$\hat{\bar{Y}}_{RPS}^{S}$	$\hat{\bar{Y}}_{RPS}^{*}$	$\hat{\bar{Y}}_{PS}^{S\,\text{Re}}$	$\hat{\bar{Y}}_{PS}^{*\text{Re}}$
Population I	100.00	87.69	105.20	109.89	110.59
Population II	100.00	59.73	83.33	94.64	102.60

5. Conclusion

From table 4.1, it is observed that the proposed dual to separate ratio type exponential estimator $\hat{\bar{Y}}_{PS}^{*\text{Re}}$ is

proved to be best estimator in the sense of having highest percent relative efficiency than usual unbiased estimator $\bar{y}_{PS}$, $\hat{\bar{Y}}_{RPS}^{S}$ $\hat{\bar{Y}}_{RPS}^{*}$ and $\hat{\bar{Y}}_{PS}^{S\,\mathrm{Re}}$ for both population data sets. Therefore the proposed estimator is recommended for use in practice if the conditions defined in section 3 are satisfied.

References

[1] Bahl, S. and Tuteja, R. K. (1991).Ratio and product type exponential estimators. Infor. & Optimiz. Sci., 12, 159-163.

[2] Bandyopadhyay, S. (1980). Improved ratio and product estimators. Sankhya, 42, C, 45-49.

[3] Chouhan, S. (2012). Improved estimation of parameters using auxiliary information in sample surveys. Ph. D. Thesis, Vikram University, Ujjain, M.P., India.

[4] Cochran, W. G. (1940).The estimation of the yields of the cereal experiments by sampling for the ratio of grain to total produce. J. Agri. Sci., 30, 262-275.

[5] http://www.data.jma.gr.jb/obd/stats/data/en/index.html (Official website of Japan Meteorological society).

[6] Ige, A. F. and Tripathi, T. P. (1989). Estimation of population mean using post-stratification and auxiliary information. Abacus, 18, 2, 265-276.

[7] Johnston, J. (1972). Econometric Methods, 2nd ed., Tokyo: McGraw-Hill.

[8] Kadilar, C. and Cingi, H. (2003). Ratio estimators in stratified random sampling. Biom. J., 45, 2, 218-225.

[9] Lone, H. A., Tailor, R. and Singh, H. P. (2014a). Generalized Ratio-Cum-Product Type Exponential Estimator in Stratified Random Sampling. Comm. statist. Theo. & meth., 10.1080/03610926.2014.901379 (in press).

[10] Lone, H. A., Tailor, R. and Singh, H. P. and Verma, M. R.(2014b). New alternatives to ratio estimators of population variance in sample surveys, Appl. Math. Comput. (2014), http://dx.doi.org/10.1016/j.amc.2014.08.093

[11] Robson, D. S. (1957). Application of multivariate polykays to the theory of unbiased ratio- type estimation. J. Am. Statist. Assoc., 59, 1225-1226.

[12] Singh, H. P., Sharma, N. and Tarray, T. A. (2014a): An efficient class of two-phase exponential ratio and product – type estimators for estimating the finite population mean. J. Statist. Appl. Prob. Lett..(in press)

[13] Singh, H. P., Goli, R. and Tarray, T. A (2014b): Study of Yadav and Kadilar's improved exponential type ratio estimator of population variance in two-phase sampling. Hacettepe Journal of Mathematics and Statristics. (in press)

[14] Singh, H. P., Gehlot, S. and Tarray, T. A. (2014c): Improved exponential type ratio estimator for population variance in two occasion rotation patterns. J. Appl. Statist. Sci. (in press)

[15] Singh, R., Kumar, M., Singh, R. D., and Chaudhary, M. K. (2008). Exponential ratio type estimators in stratified random sampling. Presented in International Symposium on Optimization and Statistics (I.S.O.S) at A. M. U., Aligarh, India, during 29-31 Dec 2008.

[16] Srivenkataramana, T. (1980). A dual of ratio estimator in sample surveys. Biom. J., 67, 1, 199-204.

[17]Stephan, F. (1945). The expected value and variance of the reciprocal and other negative powers of a positive Bernoullian variate. Ann. Math. Statist. 16, 50-61.

[18] Tailor R. and Lone, H. A. (2014a). Ratio –cum- product estimator of finite population mean in double sampling for stratification. J. of Rel. & Statist. Stds. 7, 1, 93-101.

[19] Tailor, R. and Lone, H. A. (2014b). Separate ratio type estimators of population mean in stratified random sampling. J. Mod. Appl. Statist. Meth. 13, 1, 223-233

[20] Tailor, R. and Lone, H. A. (2014c).Improved ratio-cum-product type exponential estimators for ratio of two population means in sample surveys. Modl. Assist. Statist. Appls.,10.3233/MAS- 140300

[21] Tailor, R. and Tailor, R. (2012). Dual to ratio and product type exponential estimators of finite population mean. Presented in National conference on Statistical Inference, at A.M.U., Aligarh, India, Aligarh-202002, Feb 11-12, 2012.

[22] Vishwakarma, G. K. and Singh, H. P. (2011). Separate ratio-product estimators for estimating population mean using auxiliary information. J. statist. Theo. & Appl., 10, 4, 653-664

[23] Yadav, R., Upadhyaya, L. N., Singh, H. P. and Chatterjee, S. (2011). Improved separate ratio exponential estimator for population mean using auxiliary information. Statist. Transn. - New series, 12, 2, 401-412.