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A Simulation Study for Unit Non Response under Double Sampling

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Abstract: This paper suggests a new class of estimators under the general framework of two phase sampling scheme using auxiliary information where there is non-response on study as well as auxiliary variables. The expressions of bias and mean squared error (MSE) of the suggested class of estimators were obtained. Some estimators are also derived from the proposed class of estimators by allocating the suitable values of constants used. It has been shown that the proposed estimator is more efficient than the usual unbiased estimator and other estimators. Moreover, these theoretical findings are supported by simulation.

Keywords: Efficiency, Exponential estimator, Mean squared error, Ratio estimator, Simulation.

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1 Introduction

The variable which is highly correlated with the study variable is popularly known as auxiliary variable which may be fruitfully utilized either at planning stage or at design stage or at the information stage to arrive at improved estimator compared to those without auxiliary information. Cochran (1942) pioneered the use of auxiliary information for forming ratio and regression method of estimation. When full information on auxiliary variable is available, and the auxiliary variable is highly correlated with the study variable, then it is well known that ratio and regression estimators provide more efficient estimates of the population mean of study variable. However, when the information on population mean $\overline{X}$ of the auxiliary variable x is unknown, it is sometimes relatively cheap to take a large preliminary sample in which auxiliary variable alone is measured. The aim of this sample is to obtain a good estimate of the population mean or total of the auxiliary variable or its frequency distribution (Prabhu-Ajgaonkar, 1975). This technique is well known as double sampling or two phase sampling.

In general, conducting a mail survey, the problem of non-response often happens due to the refusal of the subject, absenteeism and sometimes due to the lack of information. Hansen and Hurwitz (1946) pioneered the treatment of non-response, assumes that a sub sample of initial non respondents is recontacted with a more expensive method; they suggested the first attempt by mail questionnaire and the second attempt by a personal interview. Sodip and Obisesan (2007) have studied the problem of estimating the population mean in the presence of non-response, in sample survey with full response of an auxiliary variable x. In general, it has been observed that the population mean of the auxiliary variable is not known. In such cases two phase (Double) sampling can be used for estimating the population mean of the auxiliary character. Cochran (1977), Foradari (1961), Upadhyaya et. al (1985), Rao (1986, 1987), Srndal, C. E. and Swensson, B. (1987), Khare and Srivastava (1993, 1995, 1997), Okafor and Lee (2000), Khare and Sinha (2004, 2007), Tabasum and Khan (2004, 2006), Singh et. al (2006), Singh and Kumar (2008, 2009, 2010), Kumar et. al (2011) and Kumar (2012) have studied the problem of non-response under double (or two phase) sampling.

Consider a finite population $U=(U_1,\ U_2,\ \ldots,\ U_N)$, a large first phase sample of size n' is selected by simple random sampling without replacement (SRSWOR). From n' units, a second phase sample is selected of size n by SRSWOR. Non response occurs on the second phase sample of size n in which n_1 units respond and n_2 units do not respond. From the n_2

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non respondents, a sample of $r=(n_2/k)\,;\,;k>1$ units is selected by SRSWOR, where k is the inverse sampling rate at the second phase sample of size n. It is assumed that all the r units respond this time round.

Hansen and Hurwitz (1946) proposed an unbiased estimator for the population mean $\overline{Y}$ of the study variable y as

$$\overline{y}^{*}=w_1\overline{y}_1+w_2\overline{y}_{2r}, \tag{1}$$

where $w_i=(n_i/n)\,;\,;\ i=1,\ 2;\ \overline{y}_1=\frac{1}{n_1}\sum_{i=1}^{n_1}y_i$ and $\overline{y}_{2r}=\frac{1}{r}\sum_{i=1}^{r}y_i$.

The variance of $\overline{y}^{*}$ is given by

$$Var\left(\overline{y}^{*}\right)=\overline{Y}^{2}\left\{\left(\frac{1}{n}-\frac{1}{N}\right)C_y^2+\frac{W_2\,(k-1)}{n}C_{y(2)}^2\right\}, \tag{2}$$

where $S_y^2=\frac{1}{N-1}\sum_{i=1}^{N}\left(y_i-\overline{Y}\right)^2$; $S_{y(2)}^2=\frac{1}{N_2-1}\sum_{i=1}^{N_2}\left(y_i-\overline{Y}_2\right)^2$; $C_y=\frac{S_y}{\overline{Y}}$; $C_{y(2)}=\frac{S_{y(2)}}{\overline{Y}_2}$;

$\overline{Y}=\frac{1}{N}\sum_{i=1}^{N}y_i$; $\overline{Y}_2=\frac{1}{N_2}\sum_{i=1}^{N_2}y_i$; $W_i=\frac{N_i}{N}$; $i=1,\ 2$; N_1 and N_2 are the sizes of the responding and non-responding units from the finite population of size N.

Let x denote an auxiliary variable with population mean $\overline{X}=\frac{1}{N}\sum_{i=1}^{N}x_i$. Let $\overline{X}_1=\frac{1}{N_1}\sum_{i=1}^{N_1}x_i$ and $\overline{X}_2=\frac{1}{N_2}\sum_{i=1}^{N_2}x_i$ denote the population means of the responding and non-responding units. Let $\overline{x}=\frac{1}{n}\sum_{i=1}^{n}x_i$ denote the mean of all the n units. Let $\overline{x}_1=\frac{1}{n_1}\sum_{i=1}^{n_1}x_i$ and $\overline{x}_2=\frac{1}{n_2}\sum_{i=1}^{n_2}x_i$ are the means of the responding and non responding units. Further, let $\overline{x}_{2r}=\frac{1}{r}\sum_{i=1}^{r}x_i$ denotes the mean of the sub sampled units of size r. With this background, one can define an unbiased estimator of population mean as

$$\overline{x}^{*}=w_1\overline{x}_1+w_2\overline{x}_{2r}, \tag{3}$$

with variance

$$Var\left(\overline{x}^{*}\right)=\overline{X}^{2}\left\{\left(\frac{1}{n}-\frac{1}{N}\right)C_x^2+\frac{W_2\,(k-1)}{n}C_{x(2)}^2\right\}, \tag{4}$$

where $S_x^2=\frac{1}{N-1}\sum_{i=1}^{N}\left(x_i-\overline{X}\right)^2$; $S_{x(2)}^2=\frac{1}{N_2-1}\sum_{i=1}^{N_2}\left(x_i-\overline{X}_2\right)^2$; $C_x=\frac{S_x}{\overline{X}}$; $C_{x(2)}=\frac{S_{x(2)}}{\overline{X}}$.

In the present study, a new class of estimators for estimating the population mean of the study variable under the framework of two phase sampling scheme when there is non-response occurs on both the variables i.e. study as well as auxiliary variable is proposed which includes the estimators/classes of estimators and its properties are studied.

2 The proposed class of estimators

A class of estimators is define when there is non-response on the study and auxiliary variable as

$$t^{*}=\omega_1\overline{y}^{*}\left\{\frac{\overline{X}''}{\alpha\overline{x}^{1}+(1-\alpha)\overline{X}''}\right\}^{g}+\omega_2\overline{y}^{*}exp\left\{\frac{\delta\left(\overline{X}''-\overline{x}^{1}\right)}{\overline{X}''+\overline{x}^{1}}\right\}, \tag{5}$$

where $\overline{X}''=a\overline{x}'+b$; $\overline{x}^{1}=a\overline{x}^{*}+b$; and $a\,(\neq 0)$, b are either real numbers or functions of the known parameters of the auxiliary variable x such as the standard deviation (S_x), Coefficient of Variation (C_x), Coefficient of Skewness $\{\beta_1(x)\}$, Coefficient of Kurtosis $\{\beta_2(x)\}$, and Correlation coefficient (ρ_{yx}) of the population, α is a suitable constant, $(g,\ \delta)$ being constants which take values $(0,\ 1,\ -1)$ for deriving the different estimators, and $(\omega_1,\ \omega_2)$ are suitably chosen constants to be determined such that $MSE\,(t^{*})$ is minimum.

(i) for $(\omega_1,\ \omega_2)=(1,\ 0)$, the class of estimators t^{*} reduces to

$$t_1^{*}=\overline{y}^{*}\left\{\frac{\overline{X}''}{\alpha\overline{x}^{1}+(1-\alpha)\overline{X}''}\right\}^{g}; \tag{6}$$

(ii) for $(\omega_1,\ \omega_2)=(\omega_1,\ 0)$, the class of estimators t^{*} reduces to

$$t_2^{*}=\omega_1\overline{y}^{*}\left\{\frac{\overline{X}''}{\alpha\overline{x}^{1}+(1-\alpha)\overline{X}''}\right\}^{g}; \tag{7}$$

(iii) for $(\omega_1,\ \omega_2)=(0,\ 1)$, the class of estimators t^* reduces to

$$t_3^* = \overline{y}^* exp\left\{\frac{\delta\left(\overline{X}'' - \overline{x}^1\right)}{\overline{X}'' + \overline{x}^1}\right\}; \tag{8}$$

(iv) for $(\omega_1,\ \omega_2)=(0,\ \omega_2)$, the class of estimators t^* reduces to

$$t_4^* = \omega_2\overline{y}^* exp\left\{\frac{\delta\left(\overline{X}'' - \overline{x}^1\right)}{\overline{X}'' + \overline{x}^1}\right\}; \tag{9}$$

(v) for $(\omega_1,\ \omega_2,\ \delta)=(0,\ 1,\ 1)$, the class of estimators t^* reduces to

$$t_5^* = \overline{y}^* exp\left\{\frac{\left(\overline{X}'' - \overline{x}^1\right)}{\overline{X}'' + \overline{x}^1}\right\}; \tag{10}$$

The different estimators are defined in Appendix (A1-A8) are the members of the proposed estimators t^*, t_1^* , t_2^*, t_3^* , t_4^* and t_5^*, respectively. To obtain the bias and MSE of the proposed class of estimators, let $\overline{y}^* = \overline{Y}(1+\varepsilon_0)$, $\overline{x}^* = \overline{X}(1+\varepsilon_1)$ and $\overline{x}' = \overline{X}(1+\varepsilon_1')$ such that

$$E(\varepsilon_0) = E(\varepsilon_1) = E(\varepsilon_1') = 0;$$

and

$$E\left(\varepsilon_0^2\right) = \left\{\left(\frac{1}{n}-\frac{1}{N}\right)C_y^2 + \frac{W_2(k-1)}{n}C_{y(2)}^2\right\}; E\left(\varepsilon_1^2\right) = \left\{\left(\frac{1}{n}-\frac{1}{N}\right)C_x^2 + \frac{W_2(k-1)}{n}C_{x(2)}^2\right\};$$

$$E\left(\varepsilon_1'^2\right) = \left(\frac{1}{n'}-\frac{1}{N}\right)C_x^2; E\left(\varepsilon_1\varepsilon_1'\right) = \left(\frac{1}{n'}-\frac{1}{N}\right)C_x^2; E\left(\varepsilon_0\varepsilon_1\right) = \left(\frac{1}{n}-\frac{1}{N}\right)C_{yx} + \frac{W_2(k-1)}{n}C_{yx(2)};$$

$$E\left(\varepsilon_0\varepsilon_1'\right) = \left(\frac{1}{n'}-\frac{1}{N}\right)C_{yx},$$

where $S_{yx} = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i-\overline{Y}\right)\left(x_i-\overline{X}\right)$; $S_{yx(2)} = \frac{1}{N_2-1}\sum_{i=1}^{N_2}\left(y_i-\overline{Y}\right)\left(x_i-\overline{X}\right)$; $C_{yx} = \frac{S_{yx}}{\overline{Y}\overline{X}}$;

$$C_{yx(2)} = \frac{S_{yx(2)}}{\overline{Y}\overline{X}}.$$

Expand t^* in terms of $\varepsilon's$, we obtain

$$t^* = \omega_1\overline{Y}(1+\varepsilon_0)\left\{1+\frac{\alpha\nu\left(\varepsilon_1-\varepsilon_1'\right)}{\left(1+\nu\varepsilon_1'\right)}\right\}^{-g} + \omega_2\overline{Y}(1+\varepsilon_0)\,exp\left\{\frac{\nu\delta}{2}\left(\varepsilon_1-\varepsilon_1'\right)\left(1+\frac{\nu}{2}\left(\varepsilon_1+\varepsilon_1'\right)\right)^{-1}\right\} \tag{11}$$

where $\nu = \frac{a\overline{X}}{a\overline{X}+b}$.

Expanding the right hand side of (11), neglecting terms of $\varepsilon's$ having power greater than two and subtracting $\overline{Y}$ from both sides, we get

$$\left(t^*-\overline{Y}\right) = \overline{Y}\Big[\omega_1\left\{1-g\alpha\nu\left(\varepsilon_1-\varepsilon_1'-\nu\varepsilon_1\varepsilon_1'+\nu\varepsilon_1'^2\right)+\frac{g(g+1)}{2}\alpha^2\nu^2\left(\varepsilon_1^2+\varepsilon_1'^2-2\varepsilon_1\varepsilon_1'\right)+\varepsilon_0-g\alpha\nu\left(\varepsilon_0\varepsilon_1-\varepsilon_0\varepsilon_1'\right)\right\}+\omega_2$$
$$\left\{1+\frac{\nu\delta}{2}\left(\varepsilon_1'-\varepsilon_1\right)-\frac{\nu^2\delta}{4}\left(\varepsilon_1'^2-\varepsilon_1^2\right)+\frac{1}{2}\left(\frac{\nu\delta}{2}\left(\varepsilon_1'-\varepsilon_1\right)-\frac{\nu^2\delta}{2}\left(\varepsilon_1'^2-\varepsilon_1^2\right)\right)^2+\varepsilon_0+\frac{\nu\delta}{2}\left(\varepsilon_0\varepsilon_1'-\varepsilon_0\varepsilon_1\right)\right\}-1\Big] \tag{12}$$

The bias of t^* to the first degree of approximation is obtained by taking expectations on both sides of (12) as

$$B(t^*) = \overline{Y}\Big[\omega_1\left\{1+g\alpha\nu\left(\left(\lambda-\lambda'\right)\left((g+1)\alpha\nu C_x^2-C_{yx}\right)\right)-\theta C_{yx(2)}-\lambda(g+1)\alpha\nu C_X^2\right\}$$
$$+\omega_2\left\{1+\left(\lambda-\lambda'\right)\frac{\nu\delta}{8}\left(\nu(2-\delta)C_x^2-4C_{yx}\right)+\frac{\nu\delta}{8}\theta\left(\nu(2-\delta)C_{x(2)}^2-4C_{yx(2)}\right)\right\}\Big] \tag{13}$$

where $\lambda=\left(\frac{1}{n}-\frac{1}{N}\right)$; $\lambda'=\left(\frac{1}{n'}-\frac{1}{N}\right)$; $\theta=\frac{W_2(k-1)}{n}$.

By squaring and taking expectations of both sides of (12), we have the MSE of t^* to the first degree of approximation as

$$MSE\left(t^*\right)=\overline{Y}^2\left(1+\omega_1^2A+\omega_2^2C-2\omega_1B-2\omega_2E+2\omega_1\omega_2D\right), \tag{14}$$

where

$$A=1+\left(\lambda-\lambda'\right)\left\{C_y^2+\left(g^2\alpha^2v^2+g\left(g+1\right)\alpha^2v^2\right)C_x^2-4g\alpha vC_{yx}\right\}+\lambda'C_y^2$$
$$+\theta\left\{C_{y(2)}^2+\left(g^2\alpha^2v^2+g\left(g+1\right)\alpha^2v^2\right)C_{x(2)}^2-4g\alpha vC_{yx(2)}\right\};$$

$$B=1+\left(\lambda-\lambda'\right)\left\{\frac{g\left(g+1\right)}{2}\alpha^2v^2C_x^2-g\alpha vC_{yx}\right\}+\theta\left\{\frac{g\left(g+1\right)}{2}\alpha^2v^2C_{x(2)}^2-g\alpha vC_{yx(2)}\right\};$$

$$C=1+\left(\lambda-\lambda'\right)\left\{C_y^2+v^2\left(\frac{\delta^2+\delta}{2}\right)C_x^2-2v\delta C_{yx}\right\}+\lambda'C_y^2+\theta\left\{C_{y(2)}^2+v^2\left(\frac{\delta^2+\delta}{2}\right)C_{x(2)}^2-2v\delta C_{yx(2)}\right\};$$

$$D=1+\left(\lambda-\lambda'\right)\left\{C_y^2+\frac{v^2}{8}\left\{\left(2g\alpha+\delta\right)^2+2\left(2g\alpha^2+\delta\right)\right\}C_x^2-\left(2g\alpha+\delta\right)vC_{yx}\right\}+\lambda'C_y^2$$
$$+\theta\left\{C_{y(2)}^2+\frac{v^2}{8}\left\{\left(2g\alpha+\delta\right)^2+2\left(2g\alpha+\delta\right)\right\}C_{x(2)}^2-\left(2g\alpha+\delta\right)vC_{yx(2)}\right\};$$

$$E=1+\left(\lambda-\lambda'\right)\left\{v^2\left(\frac{\delta^2+2\delta}{8}\right)C_x^2-\frac{v\delta}{2}C_{yx}\right\}+\theta\left\{v^2\left(\frac{\delta^2+2\delta}{8}\right)C_{x(2)}^2-\frac{v\delta}{2}C_{yx(2)}\right\}.$$

The $MSE\left(t^*\right)$ is minimum, when

$$\omega_1=\frac{BC-ED}{AC-D^2}=\omega_{1(opt)} \tag{15}$$

and

$$\omega_2=\frac{AE-BD}{AC-D^2}=\omega_{2(opt)}. \tag{16}$$

By substituting the optimum values of ω_1 and ω_2 from (15) and (16), we get the minimum MSE of the class of estimators t^* as

$$min.MSE\left(t^*\right)=\overline{Y}^2\left\{1-\frac{B^2C-2BDE+AE^2}{AC-D^2}\right\}. \tag{17}$$

Thus, the following theorem to the first degree of approximation:

Theorem 1. $MSE\left(t^*\right)\geq\overline{Y}^2\left\{1-\frac{B^2C-2BDE+AE^2}{AC-D^2}\right\}$ *with equality holding if* $\omega_1=\frac{BC-ED}{AC-D^2}=\omega_{1(opt)}$ *and* $\omega_2=\frac{AE-BD}{AC-D^2}=\omega_{2(opt)}$.

The MSE of the estimators t_1^*, t_2^*, t_3^*, t_4^* and t_5^*, respectively obtained by substituting the different values of ω_1, ω_2 and δ, as follows:

$$MSE\left(t_1^*\right)=\overline{Y}^2\left[\left(\lambda-\lambda'\right)\left(C_y^2+g^2\alpha^2v^2C_x^2-2g\alpha vC_{yx}\right)+\lambda'C_y^2+\theta\left(C_{y(2)}^2+g^2\alpha^2v^2C_{x(2)}^2-2g\alpha vC_{yx(2)}\right)\right], \tag{18}$$

$$MSE\left(t_2^*\right)=\overline{Y}^2\left(1+\omega_1^2A-2\omega_1B\right), \tag{19}$$

$$MSE\left(t_3^*\right)=\overline{Y}^2\left[\left(\lambda-\lambda'\right)\left(C_y^2+\frac{v^2\delta^2}{4}C_x^2-v\delta C_{yx}\right)+\lambda'C_y^2+\theta\left(C_{y(2)}^2+\frac{v^2\delta^2}{4}C_{x(2)}^2-v\delta C_{yx(2)}\right)\right], \tag{20}$$

$$MSE\left(t_4^*\right)=\overline{Y}^2\left(1+\omega_2^2C-2\omega_2E\right), \tag{21}$$

$$MSE\left(t_5^*\right)=\overline{Y}^2\left[\left(\lambda-\lambda'\right)\left(C_y^2+\frac{v^2}{4}C_x^2-vC_{yx}\right)+\lambda'C_y^2+\theta\left(C_{y(2)}^2+\frac{v^2}{4}C_{x(2)}^2-vC_{yx(2)}\right)\right]. \tag{22}$$

The MSE of $t_i^*; i = 1(1)5$ are respectively, minimized for

$$\alpha = \frac{(\lambda - \lambda')C_{yx} + \theta C_{yx(2)}}{(\lambda - \lambda')C_x^2 + \theta C_{x(2)}^2} = \alpha_{(opt)}, \tag{23}$$

$$\omega_1 = \frac{B}{A} = \omega_{1(opt)}^*, \tag{24}$$

$$\delta = \frac{2\left\{(\lambda - \lambda')C_{yx} + \theta C_{yx(2)}\right\}}{v\left\{(\lambda - \lambda')C_x^2 + \theta C_{x(2)}^2\right\}} = \delta_{(opt)}, \tag{25}$$

$$\omega_2 = \frac{E}{C} = \omega_{2(opt)}^* \tag{26}$$

and

$$v = \frac{2\left\{(\lambda - \lambda')C_{yx} + \theta C_{yx(2)}\right\}}{\left\{(\lambda - \lambda')C_x^2 + \theta C_{x(2)}^2\right\}} = v_{(opt)}. \tag{27}$$

Substituting $\alpha_{(opt)}$, $\omega_{1(opt)}^*$, $\delta_{(opt)}$, $\omega_{2(opt)}^*$ and $v_{(opt)}$, from (23) to (27) in the expressions of MSE of $t_i^*; i = 1(1)5$ respectively, we obtain the $min.MSE$ of $t_i^*; i = 1(1)5$, to the first degree of approximation, as

$$min.MSE\left(t_1^*\right) = min.MSE\left(t_3^*\right) = min.MSE\left(t_5^*\right) =$$

$$\overline{Y}^2\left\{(\lambda - \lambda')C_y^2 + \theta C_{y(2)}^2 - \frac{\left\{(\lambda - \lambda')C_{yx} + \theta C_{yx(2)}\right\}^2}{(\lambda - \lambda')C_x^2 + \theta C_{x(2)}^2} + \lambda' C_y^2\right\}, \tag{28}$$

$$min.MSE\left(t_2^*\right) = \overline{Y}^2\left(1 - \frac{B^2}{A}\right), \tag{29}$$

$$min.MSE\left(t_4^*\right) = \overline{Y}^2\left(1 - \frac{E^2}{C}\right). \tag{30}$$

3 Particular Case: When $\omega_1 + \omega_2 = 1$

For $\omega_1 + \omega_2 = 1$, the proposed class of estimators t^* reduces to

$$t_c^* = \omega_1 \overline{y}^* \left\{\frac{\overline{X}''}{\alpha \overline{x}^1 + (1-\alpha)\overline{X}''}\right\}^g + (1-\omega_1)\overline{y}^* exp\left\{\frac{\delta\left(\overline{X}'' - \overline{x}^1\right)}{\left(\overline{X}'' + \overline{x}^1\right)}\right\}. \tag{31}$$

The expression of MSE of t_c^*, to the first degree of approximation, respectively, given as

$$MSE\left(t_c^*\right) = \overline{Y}^2\left\{1 + C - 2E + \omega_1^2(A + C - 2D) - 2\omega_1(C + B - E - D)\right\}, \tag{32}$$

which is minimum, when

$$\omega_1 = \frac{C + B - E - D}{A + C - 2D} = \omega_{1(opt)}^{**}.$$

Thus, the minimum MSE of t_c^* is given by

$$\begin{aligned} min.MSE\left(t_c^*\right) &= \overline{Y}^2\left\{1 + C - 2E - \frac{(C + B - E - D)^2}{A + C - 2D}\right\} \\ &= \overline{Y}^2\left[(\lambda - \lambda')C_y^2 + \theta C_{y(2)}^2 - \frac{\left\{(\lambda - \lambda')C_{yx} + \theta C_{yx(2)}\right\}^2}{(\lambda - \lambda')C_x^2 + \theta C_{x(2)}^2} + \lambda' C_y^2\right] \\ &= min.MSE\left(t_1^*\right) = min.MSE\left(t_3^*\right) = min.MSE\left(t_5^*\right). \end{aligned} \tag{33}$$

Thus, the following theorem to the first degree of approximation is stated as

Theorem 2. *$MSE\left(t_c^*\right) \geq \overline{Y}^2\left\{1 + C - 2E - \frac{(C+B-E-D)^2}{A+C-2D}\right\}$ with equality holding if $\omega_1 = \frac{C+B-E-D}{A+C-2D} = \omega_{1(opt)}^{**}$.*

Table 1: *PRE* of t^*_{Ri}, t^*_{Pi}, t^*_{Rei} and t^*_{Pei} $(i = 1, 2, \ldots, 13)$ with respect to $\bar{y}^*$.

Variance of $\bar{y}^* = 0.01514224$			
Ratio-type estimator	$PRE(\bar{y}^*, .)$	Product-type estimator	$PRE(\bar{y}^*, .)$
t^*_{R1}	114.2102	t^*_{P1}	116.2246
t^*_{R2}	118.3532	t^*_{P2}	109.9654
t^*_{R3}	116.7055	t^*_{P3}	114.1994
t^*_{R4}	109.6878	t^*_{P4}	97.4185
t^*_{R5}	117.3161	t^*_{P5}	113.1566
t^*_{R6}	104.8072	t^*_{P6}	104.7134
t^*_{R7}	115.7877	t^*_{P7}	115.2462
t^*_{R8}	117.7448	t^*_{P8}	111.914
t^*_{R9}	114.7163	t^*_{P9}	98.5811
t^*_{R10}	118.5042	t^*_{P10}	109.0759
t^*_{R11}	**118.9836**	t^*_{P11}	103.2461
t^*_{R12}	116.1492	t^*_{P12}	114.8503
t^*_{R13}	106.6251	t^*_{P13}	97.4487
Ratio-product-type exponential estimator		Product-ratio-type exponential estimator	
t^*_{Re1}	**119.2562**	t^*_{Pe1}	110.218
t^*_{Re2}	117.121	t^*_{Pe2}	99.7700
t^*_{Re3}	118.9385	t^*_{Pe3}	105.1443
t^*_{Re4}	104.1556	t^*_{Pe4}	97.5439
t^*_{Re5}	118.6169	t^*_{Pe5}	103.3413
t^*_{Re6}	101.3611	t^*_{Pe6}	102.0961
t^*_{Re7}	119.1775	t^*_{Pe7}	107.4332
t^*_{Re8}	118.0704	t^*_{Pe8}	101.6841
t^*_{Re9}	106.7859	t^*_{Pe9}	96.8283
t^*_{Re10}	116.5425	t^*_{Pe10}	99.1211
t^*_{Re11}	112.0671	t^*_{Pe11}	96.8456
t^*_{Re12}	119.0801	t^*_{Pe12}	106.5048
t^*_{Re13}	102.8637	t^*_{Pe13}	98.1200

4 Efficiency comparision

For comparison of the proposed class of estimators t^* with usual unbiased estimator $\bar{y}^*$, the usual ratio estimator $\bar{y}^*_R = \bar{y}^* \frac{\bar{x}'}{\bar{x}^*}$ and the usual product estimator $\bar{y}^*_P = \bar{y}^* \frac{\bar{x}^*}{\bar{x}'}$; the expression of MSE's to the first degree of approximation respectively, as

$$Var\left(\bar{y}^*\right) = \bar{Y}^2 \left\{ \left(\lambda - \lambda'\right) C_y^2 + \theta C_{y(2)}^2 + \lambda' C_y^2 \right\}, \tag{34}$$

$$MSE\left(\bar{y}^*_R\right) = \bar{Y}^2 \left\{ \left(\lambda - \lambda'\right) \left(C_y^2 + C_x^2 - 2C_{yx}\right) + \theta \left(C_{y(2)}^2 + C_{x(2)}^2 - 2C_{yx(2)}\right) + \lambda' C_y^2 \right\}, \tag{35}$$

$$MSE\left(\bar{y}^*_P\right) = \bar{Y}^2 \left\{ \left(\lambda - \lambda'\right) \left(C_y^2 + C_x^2 + 2C_{yx}\right) + \theta \left(C_{y(2)}^2 + C_{x(2)}^2 + 2C_{yx(2)}\right) + \lambda' C_y^2 \right\}. \tag{36}$$

From (17), (28), (29), (30), (33), (34), (35) and (36), we have

$$Var\left(\bar{y}^*\right) - min.MSE\left(t_1^* or\ t_3^*\ or\ t_5^*\ or\ t_c^*\right) = \frac{\left\{\left(\lambda - \lambda'\right) C_{yx} + \theta C_{yx(2)}\right\}^2}{\left(\lambda - \lambda'\right) C_x^2 + \theta C_{x(2)}^2} \geq 0, \tag{37}$$

$$MSE\left(\bar{y}^*_R\right) - min.MSE\left(t_1^* or\ t_3^*\ or\ t_5^*\ or\ t_c^*\right) = \frac{\left\{\left(\lambda - \lambda'\right)\left(C_x^2 - C_{yx}\right) + \theta\left(C_{x(2)}^2 - C_{yx(2)}\right)\right\}^2}{\left(\lambda - \lambda'\right) C_x^2 + \theta C_{x(2)}^2} \geq 0, \tag{38}$$

$$MSE\left(\bar{y}^*_P\right) - min.MSE\left(t_1^* or\ t_3^*\ or\ t_5^*\ or\ t_c^*\right) = \frac{\left\{\left(\lambda - \lambda'\right)\left(C_x^2 + C_{yx}\right) + \theta\left(C_{x(2)}^2 + C_{yx(2)}\right)\right\}^2}{\left(\lambda - \lambda'\right) C_x^2 + \theta C_{x(2)}^2} \geq 0, \tag{39}$$

Table 2: *PRE* of t^*_{RRi}, t^*_{PPi}, t^*_{RPi} and t^*_{PRi} $(i = 1,\ 2,\ \ldots,\ 13)$ with respect to $\bar{y}^*$.

Variance of $\bar{y}^*$ = 0.01514224			
Ratio-ratio type exponential estimator	$PRE(\bar{y}^*,\ .)$	Product-product type exponential estimator	$PRE(\bar{y}^*,\ .)$
t^*_{RR1}	72.88283	t^*_{PP1}	54.4895
t^*_{RR2}	**119.1121**	t^*_{PP2}	118.6979
t^*_{RR3}	118.8514	$\mathbf{t^*_{PP3}}$	**119.2595**
t^*_{RR4}	109.618	t^*_{PP4}	104.187
t^*_{RR5}	118.9539	t^*_{PP5}	119.1989
t^*_{RR6}	109.0434	t^*_{PP6}	108.6556
t^*_{RR7}	118.6964	t^*_{PP7}	119.2745
t^*_{RR8}	119.0066	t^*_{PP8}	119.029
t^*_{RR9}	114.808	t^*_{PP9}	108.951
t^*_{RR10}	**119.121**	t^*_{PP10}	118.4505
t^*_{RR11}	118.8057	t^*_{PP11}	115.5702
t^*_{RR12}	118.7521	t^*_{PP12}	119.2637
t^*_{RR13}	106.2542	t^*_{PP13}	101.913
Ratio-product-type exponential estimator		Product-ratio-type exponential estimator	
t^*_{RP1}	118.3666	t^*_{PR1}	119.1826
t^*_{RP2}	112.1377	t^*_{PR2}	70.6831
t^*_{RP3}	94.8842	t^*_{PR3}	62.2368
t^*_{RP4}	110.4339	t^*_{PR4}	90.9394
t^*_{RP5}	101.4054	t^*_{PR5}	64.8927
t^*_{RP6}	93.9754	t^*_{PR6}	98.1167
t^*_{RP7}	85.6730	t^*_{PR7}	58.8636
t^*_{RP8}	106.2526	t^*_{PR8}	67.3112
t^*_{RP9}	115.1801	t^*_{PR9}	86.7157
t^*_{RP10}	113.5166	t^*_{PR10}	71.8743
t^*_{RP11}	**119.1379**	t^*_{PR11}	79.6060
t^*_{RP12}	89.2789	t^*_{PR12}	60.1875
t^*_{RP13}	107.5185	t^*_{PR13}	93.3973

$$min.MSE\,(t_2^*) - min.MSE\,(t^*) = \bar{Y}^2 \frac{(AE-BD)^2}{A\,(AC-D^2)} \geq 0, \tag{40}$$

$$min.MSE\,(t_4^*) - min.MSE\,(t^*) = \bar{Y}^2 \frac{(BC-DE)^2}{C\,(AC-D^2)} \geq 0, \tag{41}$$

$$min.MSE\,(t_c^*) - min.MSE\,(t^*) = \bar{Y}^2 \frac{\{C\,(B-A)+D\,(D-B)+E\,(A-D)\}^2}{(AC-D^2)\,(A+C-2D)} \geq 0, \tag{42}$$

From (39)-(44), the following inequalities holds,

$$min.MSE\,(t^*) \leq min.MSE\,(t_2^*) \leq min.MSE\,(t_1^*)\,, \tag{43}$$

$$min.MSE\,(t^*) \leq min.MSE\,(t_4^*) \leq min.MSE\,(t_3^*)\,, \tag{44}$$

$$min.MSE\,(t^*) \leq min.MSE\,(t_1^*\ or\ t_3^*\ or\ t_5^*\ or\ t_c^*) \leq MSE\,(\bar{y}_R^*) \leq MSE\,(\bar{y}^*)\,. \tag{45}$$

From (43), (44) and (45), it is noted that the proposed class of estimator t^* at its optimum is the best among all the discussed estimators.

5 Simulation Study

In this section we demonstrate the performance of different types of estimators fallen under the proposed class of estimators mentioned in (5). Basically we compared all the estimators with the usual unbiased estimator $\bar{y}^*$ by an

Table 3: *PRE* of t^*_{1Ri}, t^*_{1Pi}, t^*_{2Ri} and t^*_{2Pi} $(i = 1,\ 2,\ \ldots,\ 13)$ with respect to $\bar{y}^*$.

Variance of $\bar{y}^*$ = 0.01514224			
Ratio- type estimator, $(\alpha,\ g) = (1,\ 1)$	$PRE(\bar{y}^*,\ .)$	Product-type estimator, $(\alpha,\ g) = (1,\ -1)$	$PRE(\bar{y}^*,\ .)$
t^*_{1R1}	32.8621	t^*_{1P1}	16.8928
t^*_{1R2}	96.0705	t^*_{1P2}	42.0280
t^*_{1R3}	58.4124	t^*_{1P3}	26.3190
t^*_{1R4}	111.9331	t^*_{1P4}	85.8688
t^*_{1R5}	70.0109	t^*_{1P5}	30.6321
t^*_{1R6}	82.2834	t^*_{1P6}	87.5777
t^*_{1R7}	45.7031	t^*_{1P7}	21.6785
t^*_{1R8}	80.6888	t^*_{1P8}	35.0751
t^*_{1R9}	116.4345	t^*_{1P9}	77.9212
t^*_{1R10}	100.1586	t^*_{1P10}	44.5980
t^*_{1R11}	**118.348**	t^*_{1P11}	62.4593
t^*_{1R12}	50.2727	t^*_{1P12}	23.3932
t^*_{1R13}	108.8982	t^*_{1P13}	90.0502
Ratio-type estimator, $(\alpha,\ g) = (1,\ 1)$		Product-type estimator, $(\alpha,\ g) = (1,\ -1)$	
t^*_{2R1}	0.2288	t^*_{2P1}	0.0835
t^*_{2R2}	0.5449	t^*_{2P2}	0.0802
t^*_{2R3}	0.2982	t^*_{2P3}	0.0805
t^*_{2R4}	1.9323	t^*_{2P4}	0.1676
t^*_{2R5}	0.3430	t^*_{2P5}	0.0799
t^*_{2R6}	0.2602	t^*_{2P6}	0.2611
t^*_{2R7}	0.2600	t^*_{2P7}	0.0817
t^*_{2R8}	0.4043	t^*_{2P8}	0.0796
t^*_{2R9}	3.2945	t^*_{2P9}	0.1255
t^*_{2R10}	0.6180	t^*_{2P10}	0.0809
t^*_{2R11}	1.9482	t^*_{2P11}	0.0924
t^*_{2R12}	0.2732	t^*_{2P12}	0.0812
t^*_{2R13}	1.2337	t^*_{2P13}	0.2065

exemplary simulation study. The whole simulation process, i.e. drawing a two-phase sample on interest variable Y and auxiliary variable X from the pseudo population and calculating the estimates was repeated 5000 times.

The pseudo population size $(N\ units)$ is 2000 and all the sample and sub-sample sizes are considered here to run this empirical study are $n' = 500,\ n = 100,\ n_1 = 70,\ n_2 = n - n_1 = 30,\ r = 20$. Now, we assume that the relationship between Y and X is linear with drift 3 and slope 1.5 and the auxiliary information on variable X has been generated from $N(0.5,\ 0.3)$ population. We have also employed an additive noise factor in the model $Y = 3 + 1.5X$, by random ε following $N(0,\ 1)$. This type of population is very relevant in mostly all socio-economic situations with one interest and one auxiliary variable.

We have computed the *PRE* of $t^*_{Ri},\ t^*_{Pi},\ t^*_{Rei},\ t^*_{Pei}, t^*_{RRi},\ t^*_{PPi},\ t^*_{RPi},\ t^*_{PRi},\ t^*_{1Ri},\ t^*_{1Pi},\ t^*_{2Ri},\ t^*_{2Pi},\ t^*_{3Rei},\ t^*_{3Pei},\ t^*_{4Rei},\ t^*_{4Pei}$ and $Var(\bar{y}^*)$, where $i = 1,\ 2,\ \ldots,\ 13$. The results are shown in tables (1-4).

Note: The estimator $t^*_5 : (\omega_1 = 0,\ \ \omega_2 = 1,\ \delta = 1\ )$ is same as t^*_3 for $\delta = 1$.

The following points are to be noted from tables (1-4) :

1. The ratio estimator $t^*_{R11},\ t^*_{Re1},\ t^*_{RR2},\ t^*_{RR10},\ t^*_{RP11},\ t^*_{1R11}$ and t^*_{3Re2} have the large efficiency with respect to the usual unbiased estimator $\bar{y}^*$ among the class of estimators $t^*_1,\ t^*_2,\ t^*_3$ and t^*_4 respectively.
2. Among all the proposed ratio-type class of estimators, the estimator t^*_{Re1} (i.e. Ratio type exponential estimator with parameters $g = 0,\ \delta = 1,\ a = 1$ *and* $b = 0$) is performing well in terms of *PRE* with respect to usual unbiased estimator $\bar{y}^*$.
3. The product-product type exponential estimator t^*_{PP3} $(\alpha = 1,\ g = -1,\ \delta = -1,\ a = \beta_2(x),\ b = C_x)$ is more efficient with respect to usual unbiased estimator $\bar{y}^*$ among all the proposed class of estimators.
4. The performance of $t^*_{1Ri},\ t^*_{1Pi},\ t^*_{2Ri},\ t^*_{2Pi},\ t^*_{3Ri},\ t^*_{3Pi},\ t^*_{4Ri}$ and t^*_{4Pi} $(i = 1,\ 2,\ \ldots,\ 13)$ estimators are worst compared to the usual unbiased estimator for population considered in the present simulation study.

Table 4: *PRE* of t^*_{3Ri}, t^*_{3Pi}, t^*_{4Ri} and t^*_{4Pi} $(i = 1,\ 2,\ \ldots,\ 13)$ with respect to $\bar{y}^*$.

Variance of $\bar{y}^*$ = 0.01514224			
Ratio-type exponential estimator $\delta = 1$	$PRE(\bar{y}^*,\ .)$	Product-type exponential estimator$\delta = -1$	$PRE(\bar{y}^*,\ .)$
t^*_{3Re1}	88.5738	t^*_{3Pe1}	38.4703
t^*_{3Re2}	**118.7231**	t^*_{3Pe2}	66.1256
t^*_{3Re3}	109.6319	t^*_{3Pe3}	50.9517
t^*_{3Re4}	106.4449	t^*_{3Pe4}	93.0202
t^*_{3Re5}	114.3062	t^*_{3Pe5}	55.6612
t^*_{3Re6}	93.9335	t^*_{3Pe6}	97.6524
t^*_{3Re7}	101.6158	t^*_{3Pe7}	45.2421
t^*_{3Re8}	116.7272	t^*_{3Pe8}	60.0671
t^*_{3Re9}	109.6907	t^*_{3Pe9}	88.9766
t^*_{3Re10}	118.5368	t^*_{3Pe10}	68.2052
t^*_{3Re11}	115.3762	t^*_{3Pe11}	80.3392
t^*_{3R12}	104.8377	t^*_{3Pe12}	47.4486
t^*_{3Re13}	104.6520	t^*_{3Pe13}	95.0889
Ratio-type exponential estimator, $\delta = 1$		Product-type exponential estimator, $\delta = -1$	
t^*_{4Re1}	2.3228	t^*_{4Pe1}	0.4348
t^*_{4Re2}	0.4557	t^*_{4Pe2}	0.2435
t^*_{4Re3}	1.1721	t^*_{4Pe3}	0.3466
t^*_{4Re4}	0.2290	t^*_{4Pe4}	0.1524
t^*_{4Re5}	0.8855	t^*_{4Pe5}	0.3142
t^*_{4Re6}	0.3927	t^*_{4Pe6}	0.3130
t^*_{4Re7}	1.6236	t^*_{4Pe7}	0.3867
t^*_{4Re8}	0.6653	t^*_{4Pe8}	0.2830
t^*_{4Re9}	0.1933	t^*_{4Pe9}	0.1364
t^*_{4Re10}	0.3949	t^*_{4Pe10}	0.2286
t^*_{4Re11}	0.2081	t^*_{4Pe11}	0.1597
t^*_{4Re12}	1.4280	t^*_{4Pe12}	0.3706
t^*_{4Re13}	0.2642	t^*_{4Pe13}	0.1793

Thus, we recommend that the proposed class of estimators t^* is the best in terms of having least MSE among all the estimators discussed here. Finally, the results of this simulation study provides an evidence that the proposed class of estimators t^* should be used when the study as well as auxiliary variables are linearly related in presence of non-response.

6 Conclusion

In this article, a class of estimators is proposed. The expressions of bias and MSE/VAR are derived under simple random sampling without replacement when non response occurs both on study and auxiliary variables. Proposed estimators are compared with other existing estimators by different authors and usual unbiased estimator. Theoretically, it is shown that the members of the proposed class of estimators are more efficient than already existing estimators.

Simulation study is conducted to show the performance of the proposed class of estimators under the assumption that the relationship between Y and X is linear with drift 3 and slope 1.5 and the auxiliary information on variable X has been generated from $N(0.5,\ 0.3)$ population. The results are shown in tables (9-12) which indicate that the proposed class of estimators is performing more efficiently than the usual unbiased estimator and the existing estimators. Finally we recommend for precise estimation of population mean of the study variable in presence of non-response under double sampling, the use of proposed family of estimators are recommended.

Appendix

Table A1: **Some members of the class of estimators t^***

Ratio type estimator $(\alpha,\ g,\ \delta) = (1,\ 1,\ 0)$	Product type estimator $(\alpha,\ g,\ \delta) = (1,\ -1,\ 0)$	a	b
$t^*_{R1} = \omega_1\bar{y}^*\left(\frac{\bar{x}'}{\bar{x}^*}\right) + \omega_2\bar{y}^*$	$t^*_{P1} = \omega_1\bar{y}^*\left(\frac{\bar{x}^*}{\bar{x}'}\right) + \omega_2\bar{y}^*$	1	0
$t^*_{R2} = \omega_1\bar{y}^*\left(\frac{\bar{x}'+C_x}{\bar{x}^*+C_x}\right) + \omega_2\bar{y}^*$	$t^*_{P2} = \omega_1\bar{y}^*\left(\frac{\bar{x}^*+C_x}{\bar{x}'+C_x}\right) + \omega_2\bar{y}^*$	1	C_x
$t^*_{R3} = \omega_1\bar{y}^*\left(\frac{\beta_2(x)\bar{x}'+C_x}{\beta_2(x)\bar{x}^*+C_x}\right) + \omega_2\bar{y}^*$	$t^*_{P3} = \omega_1\bar{y}^*\left(\frac{\beta_2(x)\bar{x}^*+C_x}{\beta_2(x)\bar{x}'+C_x}\right) + \omega_2\bar{y}^*$	$\beta_2(x)$	C_x
$t^*_{R4} = \omega_1\bar{y}^*\left(\frac{C_x\bar{x}'+\beta_2(x)}{C_x\bar{x}^*+\beta_2(x)}\right) + \omega_2\bar{y}^*$	$t^*_{P4} = \omega_1\bar{y}^*\left(\frac{C_x\bar{x}^*+\beta_2(x)}{C_x\bar{x}'+\beta_2(x)}\right) + \omega_2\bar{y}^*$	C_x	$\beta_2(x)$
$t^*_{R5} = \omega_1\bar{y}^*\left(\frac{\bar{x}'+S_x}{\bar{x}^*+S_x}\right) + \omega_2\bar{y}^*$	$t^*_{P5} = \omega_1\bar{y}^*\left(\frac{\bar{x}^*+S_x}{\bar{x}'+S_x}\right) + \omega_2\bar{y}^*$	1	S_x
$t^*_{R6} = \omega_1\bar{y}^*\left(\frac{\beta_1(x)\bar{x}'+S_x}{\beta_1(x)\bar{x}^*+S_x}\right) + \omega_2\bar{y}^*$	$t^*_{P6} = \omega_1\bar{y}^*\left(\frac{\beta_1(x)\bar{x}^*+S_x}{\beta_1(x)\bar{x}'+S_x}\right) + \omega_2\bar{y}^*$	$\beta_1(x)$	S_x
$t^*_{R7} = \omega_1\bar{y}^*\left(\frac{\beta_2(x)\bar{x}'+S_x}{\beta_2(x)\bar{x}^*+S_x}\right) + \omega_2\bar{y}^*$	$t^*_{P7} = \omega_1\bar{y}^*\left(\frac{\beta_2(x)\bar{x}^*+S_x}{\beta_2(x)\bar{x}'+S_x}\right) + \omega_2\bar{y}^*$	$\beta_2(x)$	S_x
$t^*_{R8} = \omega_1\bar{y}^*\left(\frac{\bar{x}'+\rho}{\bar{x}^*+\rho}\right) + \omega_2\bar{y}^*$	$t^*_{P8} = \omega_1\bar{y}^*\left(\frac{\bar{x}^*+\rho}{\bar{x}'+\rho}\right) + \omega_2\bar{y}^*$	1	ρ
$t^*_{R9} = \omega_1\bar{y}^*\left(\frac{\bar{x}'+\beta_2(x)}{\bar{x}^*+\beta_2(x)}\right) + \omega_2\bar{y}^*$	$t^*_{P9} = \omega_1\bar{y}^*\left(\frac{\bar{x}^*+\beta_2(x)}{\bar{x}'+\beta_2(x)}\right) + \omega_2\bar{y}^*$	1	$\beta_2(x)$
$t^*_{R10} = \omega_1\bar{y}^*\left(\frac{C_x\bar{x}'+\rho}{C_x\bar{x}^*+\rho}\right) + \omega_2\bar{y}^*$	$t^*_{P10} = \omega_1\bar{y}^*\left(\frac{C_x\bar{x}^*+\rho}{C_x\bar{x}'+\rho}\right) + \omega_2\bar{y}^*$	C_x	ρ
$t^*_{R11} = \omega_1\bar{y}^*\left(\frac{\rho\bar{x}'+C_x}{\rho\bar{x}^*+C_x}\right) + \omega_2\bar{y}^*$	$t^*_{P11} = \omega_1\bar{y}^*\left(\frac{\rho\bar{x}^*+C_x}{\rho\bar{x}'+C_x}\right) + \omega_2\bar{y}^*$	ρ	C_x
$t^*_{R12} = \omega_1\bar{y}^*\left(\frac{\beta_2(x)\bar{x}'+\rho}{\beta_2(x)\bar{x}^*+\rho}\right) + \omega_2\bar{y}^*$	$t^*_{P12} = \omega_1\bar{y}^*\left(\frac{\beta_2(x)\bar{x}^*+\rho}{\beta_2(x)\bar{x}'+\rho}\right) + \omega_2\bar{y}^*$	$\beta_2(x)$	ρ
$t^*_{R13} = \omega_1\bar{y}^*\left(\frac{\rho\bar{x}'+\beta_2(x)}{\rho\bar{x}^*+\beta_2(x)}\right) + \omega_2\bar{y}^*$	$t^*_{P13} = \omega_1\bar{y}^*\left(\frac{\rho\bar{x}^*+\beta_2(x)}{\rho\bar{x}'+\beta_2(x)}\right) + \omega_2\bar{y}^*$	ρ	$\beta_2(x)$

Table A2: **Some members of the class of estimators t^***

Ratio type exponential estimator $(g,\ \delta) = (0,\ 1)$	Product type exponential estimator $(g,\ \delta) = (0,\ -1)$	a	b
$t^*_{Re1} = \omega_1\bar{y}^*\left(\frac{\bar{x}'}{\bar{x}^*}\right) + \omega_2\bar{y}^*exp\left(\frac{\bar{x}'-\bar{x}^*}{\bar{x}'+\bar{x}^*}\right)$	$t^*_{Pe1} = \omega_1\bar{y}^* + \omega_2\bar{y}^*exp\left(\frac{\bar{x}^*-\bar{x}'}{\bar{x}^*+\bar{x}'}\right)$	1	0
$t^*_{Re2} = \omega_1\bar{y}^* + \omega_2\bar{y}^*exp\left\{\frac{\bar{x}'-\bar{x}^*}{(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	$t^*_{Pe2} = \omega_1\bar{y}^* + \omega_2\bar{y}^*exp\left\{\frac{\bar{x}^*-\bar{x}'}{(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	1	C_x
$t^*_{Re3} = \omega_1\bar{y}^* + \omega_2\bar{y}^*exp\left\{\frac{\beta_2(x)(\bar{x}'-\bar{x}^*)}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	$t^*_{Pe3} = \omega_1\bar{y}^* + \omega_2\bar{y}^*exp\left\{\frac{\beta_2(x)(\bar{x}^*-\bar{x}')}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	$\beta_2(x)$	C_x
$t^*_{Re4} = \omega_1\bar{y}^* + \omega_2\bar{y}^*exp\left\{\frac{C_x(\bar{x}'-\bar{x}^*)}{C_x(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	$t^*_{Pe4} = \omega_1\bar{y}^* + \omega_2\bar{y}^*exp\left\{\frac{C_x(\bar{x}^*-\bar{x}')}{C_x(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	C_x	$\beta_2(x)$
$t^*_{Re5} = \omega_1\bar{y}^* + \omega_2\bar{y}^*exp\left\{\frac{(\bar{x}'-\bar{x}^*)}{(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	$t^*_{Pe5} = \omega_1\bar{y}^* + \omega_2\bar{y}^*exp\left\{\frac{(\bar{x}^*-\bar{x}')}{(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	1	S_x
$t^*_{Re6} = \omega_1\bar{y}^* + \omega_2\bar{y}^*exp\left\{\frac{\beta_1(x)(\bar{x}'-\bar{x}^*)}{\beta_1(x)(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	$t^*_{Pe6} = \omega_1\bar{y}^* + \omega_2\bar{y}^*exp\left\{\frac{\beta_1(x)(\bar{x}^*-\bar{x}')}{\beta_1(x)(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	$\beta_1(x)$	S_x
$t^*_{Re7} = \omega_1\bar{y}^* + \omega_2\bar{y}^*exp\left\{\frac{\beta_2(x)(\bar{x}'-\bar{x}^*)}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	$t^*_{Pe7} = \omega_1\bar{y}^* + \omega_2\bar{y}^*exp\left\{\frac{\beta_2(x)(\bar{x}^*-\bar{x}')}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	$\beta_2(x)$	S_x
$t^*_{Re8} = \omega_1\bar{y}^* + \omega_2\bar{y}^*exp\left\{\frac{(\bar{x}'-\bar{x}^*)}{(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	$t^*_{Pe8} = \omega_1\bar{y}^* + \omega_2\bar{y}^*exp\left\{\frac{(\bar{x}^*-\bar{x}')}{(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	1	ρ
$t^*_{Re9} = \omega_1\bar{y}^* + \omega_2\bar{y}^*exp\left\{\frac{(\bar{x}'-\bar{x}^*)}{(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	$t^*_{Pe9} = \omega_1\bar{y}^* + \omega_2\bar{y}^*exp\left\{\frac{(\bar{x}^*-\bar{x}')}{(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	1	$\beta_2(x)$
$t^*_{Re10} = \omega_1\bar{y}^* + \omega_2\bar{y}^*exp\left\{\frac{C_x(\bar{x}'-\bar{x}^*)}{C_x(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	$t^*_{Pe10} = \omega_1\bar{y}^* + \omega_2\bar{y}^*exp\left\{\frac{C_x(\bar{x}^*-\bar{x}')}{C_x(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	C_x	ρ
$t^*_{Re11} = \omega_1\bar{y}^* + \omega_2\bar{y}^*exp\left\{\frac{\rho(\bar{x}'-\bar{x}^*)}{\rho(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	$t^*_{Pe11} = \omega_1\bar{y}^* + \omega_2\bar{y}^*exp\left\{\frac{\rho(\bar{x}^*-\bar{x}')}{\rho(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	ρ	C_x
$t^*_{Re12} = \omega_1\bar{y}^* + \omega_2\bar{y}^*exp\left\{\frac{\beta_2(x)(\bar{x}'-\bar{x}^*)}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	$t^*_{Pe12} = \omega_1\bar{y}^* + \omega_2\bar{y}^*exp\left\{\frac{\beta_2(x)(\bar{x}^*-\bar{x}')}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	$\beta_2(x)$	ρ
$t^*_{Re13} = \omega_1\bar{y}^* + \omega_2\bar{y}^*exp\left\{\frac{\rho(\bar{x}'-\bar{x}^*)}{\rho(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	$t^*_{Pe13} = \omega_1\bar{y}^* + \omega_2\bar{y}^*exp\left\{\frac{\rho(\bar{x}^*-\bar{x}')}{\rho(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	ρ	$\beta_2(x)$

Table A3: **Some members of the class of estimators** t^*

Ratio-ratio type exponential estimator $(\alpha,\ g,\ \delta)=(1,\ 1,\ 1)$	Product-product type exponential estimator $(\alpha,\ g,\ \delta)=(1,\ -1,\ -1)$	a	b
$t^*_{RR1}=\omega_1\bar{y}^*\left(\frac{\bar{x}'}{\bar{x}^*}\right)+\omega_2\bar{y}^*exp\left(\frac{\bar{x}'-\bar{x}^*}{\bar{x}'+\bar{x}^*}\right)$	$t^*_{PP1}=\omega_1\bar{y}^*\left(\frac{\bar{x}^*}{\bar{x}'}\right)+\omega_2\bar{y}^*exp\left(\frac{\bar{x}^*-\bar{x}'}{\bar{x}^*+\bar{x}'}\right)$	1	0
$t^*_{RR2}=\omega_1\bar{y}^*\left(\frac{\bar{x}'+C_x}{\bar{x}^*+C_x}\right)+\omega_2\bar{y}^*exp\left\{\frac{\bar{x}'-\bar{x}^*}{(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	$t^*_{PP2}=\omega_1\bar{y}^*\left(\frac{\bar{x}^*+C_x}{\bar{x}'+C_x}\right)+\omega_2\bar{y}^*exp\left\{\frac{\bar{x}^*-\bar{x}'}{(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	1	C_x
$t^*_{RR3}=\omega_1\bar{y}^*\left(\frac{\beta_2(x)\bar{x}'+C_x}{\beta_2(x)\bar{x}^*+C_x}\right)+\omega_2\bar{y}^*exp\left\{\frac{\beta_2(x)(\bar{x}'-\bar{x}^*)}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	$t^*_{PP3}=\omega_1\bar{y}^*\left(\frac{\beta_2(x)\bar{x}^*+C_x}{\beta_2(x)\bar{x}'+C_x}\right)+\omega_2\bar{y}^*exp\left\{\frac{\beta_2(x)(\bar{x}^*-\bar{x}')}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	$\beta_2(x)$	C_x
$t^*_{RR4}=\omega_1\bar{y}^*\left(\frac{C_x\bar{x}'+\beta_2(x)}{C_x\bar{x}^*+\beta_2(x)}\right)+\omega_2\bar{y}^*exp\left\{\frac{C_x(\bar{x}'-\bar{x}^*)}{C_x(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	$t^*_{PP4}=\omega_1\bar{y}^*\left(\frac{C_x\bar{x}^*+\beta_2(x)}{C_x\bar{x}'+\beta_2(x)}\right)+\omega_2\bar{y}^*exp\left\{\frac{C_x(\bar{x}^*-\bar{x}')}{C_x(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	C_x	$\beta_2(x)$
$t^*_{RR5}=\omega_1\bar{y}^*\left(\frac{\bar{x}'+S_x}{\bar{x}^*+S_x}\right)+\omega_2\bar{y}^*exp\left\{\frac{(\bar{x}'-\bar{x}^*)}{(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	$t^*_{PP5}=\omega_1\bar{y}^*\left(\frac{\bar{x}^*+S_x}{\bar{x}'+S_x}\right)+\omega_2\bar{y}^*exp\left\{\frac{(\bar{x}^*-\bar{x}')}{(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	1	S_x
$t^*_{RR6}=\omega_1\bar{y}^*\left(\frac{\beta_1(x)\bar{x}'+S_x}{\beta_1(x)\bar{x}^*+S_x}\right)+\omega_2\bar{y}^*exp\left\{\frac{\beta_1(x)(\bar{x}'-\bar{x}^*)}{\beta_1(x)(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	$t^*_{PP6}=\omega_1\bar{y}^*\left(\frac{\beta_1(x)\bar{x}^*+S_x}{\beta_1(x)\bar{x}'+S_x}\right)+\omega_2\bar{y}^*exp\left\{\frac{\beta_1(x)(\bar{x}^*-\bar{x}')}{\beta_1(x)(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	$\beta_1(x)$	S_x
$t^*_{RR7}=\omega_1\bar{y}^*\left(\frac{\beta_2(x)\bar{x}'+S_x}{\beta_2(x)\bar{x}^*+S_x}\right)+\omega_2\bar{y}^*exp\left\{\frac{\beta_2(x)(\bar{x}'-\bar{x}^*)}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	$t^*_{PP7}=\omega_1\bar{y}^*\left(\frac{\beta_2(x)\bar{x}^*+S_x}{\beta_2(x)\bar{x}'+S_x}\right)+\omega_2\bar{y}^*exp\left\{\frac{\beta_2(x)(\bar{x}^*-\bar{x}')}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	$\beta_2(x)$	S_x
$t^*_{RR8}=\omega_1\bar{y}^*\left(\frac{\bar{x}'+\rho}{\bar{x}^*+\rho}\right)+\omega_2\bar{y}^*exp\left\{\frac{(\bar{x}'-\bar{x}^*)}{(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	$t^*_{PP8}=\omega_1\bar{y}^*\left(\frac{\bar{x}^*+\rho}{\bar{x}'+\rho}\right)+\omega_2\bar{y}^*exp\left\{\frac{(\bar{x}^*-\bar{x}')}{(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	1	ρ
$t^*_{RR9}=\omega_1\bar{y}^*\left(\frac{\bar{x}'+\beta_2(x)}{\bar{x}^*+\beta_2(x)}\right)+\omega_2\bar{y}^*exp\left\{\frac{(\bar{x}'-\bar{x}^*)}{(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	$t^*_{PP9}=\omega_1\bar{y}^*\left(\frac{\bar{x}^*+\beta_2(x)}{\bar{x}'+\beta_2(x)}\right)+\omega_2\bar{y}^*exp\left\{\frac{(\bar{x}^*-\bar{x}')}{(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	1	$\beta_2(x)$
$t^*_{RR10}=\omega_1\bar{y}^*\left(\frac{C_x\bar{x}'+\rho}{C_x\bar{x}^*+\rho}\right)+\omega_2\bar{y}^*exp\left\{\frac{C_x(\bar{x}'-\bar{x}^*)}{C_x(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	$t^*_{PP10}=\omega_1\bar{y}^*\left(\frac{C_x\bar{x}^*+\rho}{C_x\bar{x}'+\rho}\right)+\omega_2\bar{y}^*exp\left\{\frac{C_x(\bar{x}^*-\bar{x}')}{C_x(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	C_x	ρ
$t^*_{RR11}=\omega_1\bar{y}^*\left(\frac{\rho\bar{x}'+C_x}{\rho\bar{x}^*+C_x}\right)+\omega_2\bar{y}^*exp\left\{\frac{\rho(\bar{x}'-\bar{x}^*)}{\rho(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	$t^*_{PP11}=\omega_1\bar{y}^*\left(\frac{\rho\bar{x}^*+C_x}{\rho\bar{x}'+C_x}\right)+\omega_2\bar{y}^*exp\left\{\frac{\rho(\bar{x}^*-\bar{x}')}{\rho(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	ρ	C_x
$t^*_{RR12}=\omega_1\bar{y}^*\left(\frac{\beta_2(x)\bar{x}'+\rho}{\beta_2(x)\bar{x}^*+\rho}\right)+\omega_2\bar{y}^*exp\left\{\frac{\beta_2(x)(\bar{x}'-\bar{x}^*)}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	$t^*_{PP12}=\omega_1\bar{y}^*\left(\frac{\beta_2(x)\bar{x}^*+\rho}{\beta_2(x)\bar{x}'+\rho}\right)+\omega_2\bar{y}^*exp\left\{\frac{\beta_2(x)(\bar{x}^*-\bar{x}')}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	$\beta_2(x)$	ρ
$t^*_{RR13}=\omega_1\bar{y}^*\left(\frac{\rho\bar{x}'+\beta_2(x)}{\rho\bar{x}^*+\beta_2(x)}\right)+\omega_2\bar{y}^*exp\left\{\frac{\rho(\bar{x}'-\bar{x}^*)}{\rho(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	$t^*_{PP13}=\omega_1\bar{y}^*\left(\frac{\rho\bar{x}^*+\beta_2(x)}{\rho\bar{x}'+\beta_2(x)}\right)+\omega_2\bar{y}^*exp\left\{\frac{\rho(\bar{x}^*-\bar{x}')}{\rho(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	ρ	$\beta_2(x)$

Table A4: **Some members of the class of estimators** t^*

Ratio-product type exponential estimator $(\alpha,\ g,\ \delta)=(1,\ 1,\ -1)$	Product-ratio type exponential estimator $(\alpha,\ g,\ \delta)=(1,\ -1,\ 1)$	a	b
$t^*_{RP1}=\omega_1\bar{y}^*\left(\frac{\bar{x}'}{\bar{x}^*}\right)+\omega_2\bar{y}^*exp\left(\frac{\bar{x}^*-\bar{x}'}{\bar{x}^*+\bar{x}'}\right)$	$t^*_{PR1}=\omega_1\bar{y}^*\left(\frac{\bar{x}^*}{\bar{x}'}\right)+\omega_2\bar{y}^*exp\left(\frac{\bar{x}'-\bar{x}^*}{\bar{x}'+\bar{x}^*}\right)$	1	0
$t^*_{RP2}=\omega_1\bar{y}^*\left(\frac{\bar{x}'+C_x}{\bar{x}^*+C_x}\right)+\omega_2\bar{y}^*exp\left\{\frac{\bar{x}^*-\bar{x}'}{(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	$t^*_{PR2}=\omega_1\bar{y}^*\left(\frac{\bar{x}^*+C_x}{\bar{x}'+C_x}\right)+\omega_2\bar{y}^*exp\left\{\frac{\bar{x}'-\bar{x}^*}{(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	1	C_x
$t^*_{RP3}=\omega_1\bar{y}^*\left(\frac{\beta_2(x)\bar{x}'+C_x}{\beta_2(x)\bar{x}^*+C_x}\right)+\omega_2\bar{y}^*exp\left\{\frac{\beta_2(x)(\bar{x}^*-\bar{x}')}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	$t^*_{PR3}=\omega_1\bar{y}^*\left(\frac{\beta_2(x)\bar{x}^*+C_x}{\beta_2(x)\bar{x}'+C_x}\right)+\omega_2\bar{y}^*exp\left\{\frac{\beta_2(x)(\bar{x}'-\bar{x}^*)}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	$\beta_2(x)$	C_x
$t^*_{RP4}=\omega_1\bar{y}^*\left(\frac{C_x\bar{x}'+\beta_2(x)}{C_x\bar{x}^*+\beta_2(x)}\right)+\omega_2\bar{y}^*exp\left\{\frac{C_x(\bar{x}^*-\bar{x}')}{C_x(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	$t^*_{PR4}=\omega_1\bar{y}^*\left(\frac{C_x\bar{x}^*+\beta_2(x)}{C_x\bar{x}'+\beta_2(x)}\right)+\omega_2\bar{y}^*exp\left\{\frac{C_x(\bar{x}'-\bar{x}^*)}{C_x(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	C_x	$\beta_2(x)$
$t^*_{RP5}=\omega_1\bar{y}^*\left(\frac{\bar{x}'+S_x}{\bar{x}^*+S_x}\right)+\omega_2\bar{y}^*exp\left\{\frac{(\bar{x}^*-\bar{x}')}{(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	$t^*_{PR5}=\omega_1\bar{y}^*\left(\frac{\bar{x}^*+S_x}{\bar{x}'+S_x}\right)+\omega_2\bar{y}^*exp\left\{\frac{(\bar{x}'-\bar{x}^*)}{(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	1	S_x
$t^*_{RP6}=\omega_1\bar{y}^*\left(\frac{\beta_1(x)\bar{x}'+S_x}{\beta_1(x)\bar{x}^*+S_x}\right)+\omega_2\bar{y}^*exp\left\{\frac{\beta_1(x)(\bar{x}^*-\bar{x}')}{\beta_1(x)(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	$t^*_{PR6}=\omega_1\bar{y}^*\left(\frac{\beta_1(x)\bar{x}^*+S_x}{\beta_1(x)\bar{x}'+S_x}\right)+\omega_2\bar{y}^*exp\left\{\frac{\beta_1(x)(\bar{x}'-\bar{x}^*)}{\beta_1(x)(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	$\beta_1(x)$	S_x
$t^*_{RP7}=\omega_1\bar{y}^*\left(\frac{\beta_2(x)\bar{x}'+S_x}{\beta_2(x)\bar{x}^*+S_x}\right)+\omega_2\bar{y}^*exp\left\{\frac{\beta_2(x)(\bar{x}^*-\bar{x}')}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	$t^*_{PR7}=\omega_1\bar{y}^*\left(\frac{\beta_2(x)\bar{x}^*+S_x}{\beta_2(x)\bar{x}'+S_x}\right)+\omega_2\bar{y}^*exp\left\{\frac{\beta_2(x)(\bar{x}'-\bar{x}^*)}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	$\beta_2(x)$	S_x
$t^*_{RP8}=\omega_1\bar{y}^*\left(\frac{\bar{x}'+\rho}{\bar{x}^*+\rho}\right)+\omega_2\bar{y}^*exp\left\{\frac{(\bar{x}^*-\bar{x}')}{(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	$t^*_{PR8}=\omega_1\bar{y}^*\left(\frac{\bar{x}^*+\rho}{\bar{x}'+\rho}\right)+\omega_2\bar{y}^*exp\left\{\frac{(\bar{x}'-\bar{x}^*)}{(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	1	ρ
$t^*_{RP9}=\omega_1\bar{y}^*\left(\frac{\bar{x}'+\beta_2(x)}{\bar{x}^*+\beta_2(x)}\right)+\omega_2\bar{y}^*exp\left\{\frac{(\bar{x}^*-\bar{x}')}{(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	$t^*_{PR9}=\omega_1\bar{y}^*\left(\frac{\bar{x}^*+\beta_2(x)}{\bar{x}'+\beta_2(x)}\right)+\omega_2\bar{y}^*exp\left\{\frac{(\bar{x}'-\bar{x}^*)}{(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	1	$\beta_2(x)$
$t^*_{RP10}=\omega_1\bar{y}^*\left(\frac{C_x\bar{x}'+\rho}{C_x\bar{x}^*+\rho}\right)+\omega_2\bar{y}^*exp\left\{\frac{C_x(\bar{x}^*-\bar{x}')}{C_x(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	$t^*_{PR10}=\omega_1\bar{y}^*\left(\frac{C_x\bar{x}^*+\rho}{C_x\bar{x}'+\rho}\right)+\omega_2\bar{y}^*exp\left\{\frac{C_x(\bar{x}'-\bar{x}^*)}{C_x(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	C_x	ρ
$t^*_{RP11}=\omega_1\bar{y}^*\left(\frac{\rho\bar{x}'+C_x}{\rho\bar{x}^*+C_x}\right)+\omega_2\bar{y}^*exp\left\{\frac{\rho(\bar{x}^*-\bar{x}')}{\rho(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	$t^*_{PR11}=\omega_1\bar{y}^*\left(\frac{\rho\bar{x}^*+C_x}{\rho\bar{x}'+C_x}\right)+\omega_2\bar{y}^*exp\left\{\frac{\rho(\bar{x}'-\bar{x}^*)}{\rho(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	ρ	C_x
$t^*_{RP12}=\omega_1\bar{y}^*\left(\frac{\beta_2(x)\bar{x}'+\rho}{\beta_2(x)\bar{x}^*+\rho}\right)+\omega_2\bar{y}^*exp\left\{\frac{\beta_2(x)(\bar{x}^*-\bar{x}')}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	$t^*_{PR12}=\omega_1\bar{y}^*\left(\frac{\beta_2(x)\bar{x}^*+\rho}{\beta_2(x)\bar{x}'+\rho}\right)+\omega_2\bar{y}^*exp\left\{\frac{\beta_2(x)(\bar{x}'-\bar{x}^*)}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	$\beta_2(x)$	ρ
$t^*_{RP13}=\omega_1\bar{y}^*\left(\frac{\rho\bar{x}'+\beta_2(x)}{\rho\bar{x}^*+\beta_2(x)}\right)+\omega_2\bar{y}^*exp\left\{\frac{\rho(\bar{x}^*-\bar{x}')}{\rho(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	$t^*_{PR13}=\omega_1\bar{y}^*\left(\frac{\rho\bar{x}^*+\beta_2(x)}{\rho\bar{x}'+\beta_2(x)}\right)+\omega_2\bar{y}^*exp\left\{\frac{\rho(\bar{x}'-\bar{x}^*)}{\rho(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	ρ	$\beta_2(x)$

Table A5: **Some members of the class of estimators t_1^***

Ratio-type estimators $(\alpha,\ g)=(1,\ 1)$	Product-type estimators $(\alpha,\ g)=(1,\ -1)$	a	b
t^*_{1R1} [1] $=\bar{y}^*\left(\frac{\bar{x}'}{\bar{x}^*}\right)$	t^*_{1P1} [2] $=\bar{y}^*\left(\frac{\bar{x}^*}{\bar{x}'}\right)$	1	0
$t^*_{1R2}=\bar{y}^*\left(\frac{\bar{x}'+C_x}{\bar{x}^*+C_x}\right)$ Singh and Kumar (2009)	$t^*_{1P2}=\bar{y}^*\left(\frac{\bar{x}^*+C_x}{\bar{x}'+C_x}\right)$	1	C_x
$t^*_{1R3}=\bar{y}^*\left(\frac{\beta_2(x)\bar{x}'+C_x}{\beta_2(x)\bar{x}^*+C_x}\right)$	$t^*_{1P3}=\bar{y}^*\left(\frac{\beta_2(x)\bar{x}^*+C_x}{\beta_2(x)\bar{x}'+C_x}\right)$	$\beta_2(x)$	C_x
$t^*_{1R4}=\bar{y}^*\left(\frac{C_x\bar{x}'+\beta_2(x)}{C_x\bar{x}^*+\beta_2(x)}\right)$	$t^*_{1P4}=\bar{y}^*\left(\frac{C_x\bar{x}^*+\beta_2(x)}{C_x\bar{x}'+\beta_2(x)}\right)$	C_x	$\beta_2(x)$
$t^*_{1R5}=\bar{y}^*\left(\frac{\bar{x}'+S_x}{\bar{x}^*+S_x}\right)$	$t^*_{1P5}=\bar{y}^*\left(\frac{\bar{x}^*+S_x}{\bar{x}'+S_x}\right)$	1	S_x
$t^*_{1R6}=\bar{y}^*\left(\frac{\beta_1(x)\bar{x}'+S_x}{\beta_1(x)\bar{x}^*+S_x}\right)$	$t^*_{1P6}=\bar{y}^*\left(\frac{\beta_1(x)\bar{x}^*+S_x}{\beta_1(x)\bar{x}'+S_x}\right)$	$\beta_1(x)$	C_x
$t^*_{1R7}=\bar{y}^*\left(\frac{\beta_2(x)\bar{x}'+S_x}{\beta_2(x)\bar{x}^*+S_x}\right)$	$t^*_{1P7}=\bar{y}^*\left(\frac{\beta_2(x)\bar{x}^*+S_x}{\beta_2(x)\bar{x}'+S_x}\right)$	$\beta_2(x)$	S_x
t^*_{1R8} [3] $=\bar{y}^*\left(\frac{\bar{x}'+\rho}{\bar{x}^*+\rho}\right)$	$t^*_{1P8}=\bar{y}^*\left(\frac{\bar{x}^*+\rho}{\bar{x}'+\rho}\right)$	1	ρ
$t^*_{1R9}=\bar{y}^*\left(\frac{\bar{x}'+\beta_2(x)}{\bar{x}^*+\beta_2(x)}\right)$	$t^*_{1P9}=\bar{y}^*\left(\frac{\bar{x}^*+\beta_2(x)}{\bar{x}'+\beta_2(x)}\right)$	1	$\beta_2(x)$
$t^*_{1R10}=\bar{y}^*\left(\frac{C_x\bar{x}'+\rho}{C_x\bar{x}^*+\rho}\right)$	$t^*_{1P10}=\bar{y}^*\left(\frac{C_x\bar{x}^*+\rho}{C_x\bar{x}'+\rho}\right)$	C_x	ρ
$t^*_{1R11}=\bar{y}^*\left(\frac{\rho\bar{x}'+C_x}{\rho\bar{x}^*+C_x}\right)$	$t^*_{1P11}=\bar{y}^*\left(\frac{\rho\bar{x}^*+C_x}{\rho\bar{x}'+C_x}\right)$	ρ	C_x
$t^*_{1R12}=\bar{y}^*\left(\frac{\beta_2(x)\bar{x}'+\rho}{\beta_2(x)\bar{x}^*+\rho}\right)$	$t^*_{1P12}=\bar{y}^*\left(\frac{\beta_2(x)\bar{x}^*+\rho}{\beta_2(x)\bar{x}'+\rho}\right)$	$\beta_2(x)$	ρ
$t^*_{1R13}=\bar{y}^*\left(\frac{\rho\bar{x}'+\beta_2(x)}{\rho\bar{x}^*+\beta_2(x)}\right)$	$t^*_{1P13}=\bar{y}^*\left(\frac{\rho\bar{x}^*+\beta_2(x)}{\rho\bar{x}'+\beta_2(x)}\right)$	ρ	$\beta_2(x)$

Table A6: **Some members of the class of estimators t_2^***

Ratio-type estimators $(\alpha,\ g)=(1,\ 1)$	Product-type estimators $(\alpha,\ g)=(1,\ -1)$	a	b
$t^*_{2R1}=\omega_1\bar{y}^*\left(\frac{\bar{x}'}{\bar{x}^*}\right)$	$t^*_{2P1}=\omega_1\bar{y}^*\left(\frac{\bar{x}^*}{\bar{x}'}\right)$	1	0
$t^*_{2R2}=\omega_1\bar{y}^*\left(\frac{\bar{x}'+C_x}{\bar{x}^*+C_x}\right)$	$t^*_{2P2}=\omega_1\bar{y}^*\left(\frac{\bar{x}^*+C_x}{\bar{x}'+C_x}\right)$	1	C_x
$t^*_{2R3}=\omega_1\bar{y}^*\left(\frac{\beta_2(x)\bar{x}'+C_x}{\beta_2(x)\bar{x}^*+C_x}\right)$	$t^*_{2P3}=\omega_1\bar{y}^*\left(\frac{\beta_2(x)\bar{x}^*+C_x}{\beta_2(x)\bar{x}'+C_x}\right)$	$\beta_2(x)$	C_x
$t^*_{2R4}=\omega_1\bar{y}^*\left(\frac{C_x\bar{x}'+\beta_2(x)}{C_x\bar{x}^*+\beta_2(x)}\right)$	$t^*_{2P4}=\omega_1\bar{y}^*\left(\frac{C_x\bar{x}^*+\beta_2(x)}{C_x\bar{x}'+\beta_2(x)}\right)$	C_x	$\beta_2(x)$
$t^*_{2R5}=\omega_1\bar{y}^*\left(\frac{\bar{x}'+S_x}{\bar{x}^*+S_x}\right)$	$t^*_{2P5}=\omega_1\bar{y}^*\left(\frac{\bar{x}^*+S_x}{\bar{x}'+S_x}\right)$	1	S_x
$t^*_{2R6}=\omega_1\bar{y}^*\left(\frac{\beta_1(x)\bar{x}'+S_x}{\beta_1(x)\bar{x}^*+S_x}\right)$	$t^*_{2P6}=\omega_1\bar{y}^*\left(\frac{\beta_1(x)\bar{x}^*+S_x}{\beta_1(x)\bar{x}'+S_x}\right)$	$\beta_1(x)$	C_x
$t^*_{2R7}=\omega_1\bar{y}^*\left(\frac{\beta_2(x)\bar{x}'+S_x}{\beta_2(x)\bar{x}^*+S_x}\right)$	$t^*_{2P7}=\omega_1\bar{y}^*\left(\frac{\beta_2(x)\bar{x}^*+S_x}{\beta_2(x)\bar{x}'+S_x}\right)$	$\beta_2(x)$	S_x
$t^*_{2R8}=\omega_1\bar{y}^*\left(\frac{\bar{x}'+\rho}{\bar{x}^*+\rho}\right)$	$t^*_{2P8}=\omega_1\bar{y}^*\left(\frac{\bar{x}^*+\rho}{\bar{x}'+\rho}\right)$	1	ρ
$t^*_{2R9}=\omega_1\bar{y}^*\left(\frac{\bar{x}'+\beta_2(x)}{\bar{x}^*+\beta_2(x)}\right)$	$t^*_{2P9}=\omega_1\bar{y}^*\left(\frac{\bar{x}^*+\beta_2(x)}{\bar{x}'+\beta_2(x)}\right)$	1	$\beta_2(x)$
$t^*_{2R10}=\omega_1\bar{y}^*\left(\frac{C_x\bar{x}'+\rho}{C_x\bar{x}^*+\rho}\right)$	$t^*_{2P10}=\omega_1\bar{y}^*\left(\frac{C_x\bar{x}^*+\rho}{C_x\bar{x}'+\rho}\right)$	C_x	ρ
$t^*_{2R11}=\omega_1\bar{y}^*\left(\frac{\rho\bar{x}'+C_x}{\rho\bar{x}^*+C_x}\right)$	$t^*_{2P11}=\omega_1\bar{y}^*\left(\frac{\rho\bar{x}^*+C_x}{\rho\bar{x}'+C_x}\right)$	ρ	C_x
$t^*_{2R12}=\omega_1\bar{y}^*\left(\frac{\beta_2(x)\bar{x}'+\rho}{\beta_2(x)\bar{x}^*+\rho}\right)$	$t^*_{2P12}=\omega_1\bar{y}^*\left(\frac{\beta_2(x)\bar{x}^*+\rho}{\beta_2(x)\bar{x}'+\rho}\right)$	$\beta_2(x)$	ρ
$t^*_{2R13}=\omega_1\bar{y}^*\left(\frac{\rho\bar{x}'+\beta_2(x)}{\rho\bar{x}^*+\beta_2(x)}\right)$	$t^*_{2P13}=\omega_1\bar{y}^*\left(\frac{\rho\bar{x}^*+\beta_2(x)}{\rho\bar{x}'+\beta_2(x)}\right)$	ρ	$\beta_2(x)$

Table A7: **Some members of the class of estimators t_3^***

Ratio-type estimators $\delta = 1$	Product-type estimators $\delta = -1$	a	b
$t_{3Re1}^* = \bar{y}^* exp\left(\frac{\bar{x}'-\bar{x}^*}{\bar{x}'+\bar{x}^*}\right)$ Singh et. al (2010)	$t_{3Pe1}^* = \bar{y}^* exp\left(\frac{\bar{x}^*-\bar{x}'}{\bar{x}^*+\bar{x}'}\right)$ Singh et. al (2010)	1	0
$t_{3Re2}^* = \bar{y}^* exp\left\{\frac{\bar{x}'-\bar{x}^*}{(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	$t_{3Pe2}^* = \bar{y}^* exp\left\{\frac{\bar{x}^*-\bar{x}'}{(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	1	C_x
$t_{3Re3}^* = \bar{y}^* exp\left\{\frac{\beta_2(x)(\bar{x}'-\bar{x}^*)}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	$t_{3Pe3}^* = \bar{y}^* exp\left\{\frac{\beta_2(x)(\bar{x}^*-\bar{x}')}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	$\beta_2(x)$	C_x
$t_{3Re4}^* = \bar{y}^* exp\left\{\frac{C_x(\bar{x}'-\bar{x}^*)}{C_x(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	$t_{3Pe4}^* = \bar{y}^* exp\left\{\frac{C_x(\bar{x}^*-\bar{x}')}{C_x(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	C_x	$\beta_2(x)$
$t_{3Re5}^* = \bar{y}^* exp\left\{\frac{(\bar{x}'-\bar{x}^*)}{(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	$t_{3Pe5}^* = \bar{y}^* exp\left\{\frac{(\bar{x}^*-\bar{x}')}{(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	1	S_x
$t_{3Re6}^* = \bar{y}^* exp\left\{\frac{\beta_1(x)(\bar{x}'-\bar{x}^*)}{\beta_1(x)(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	$t_{3Pe6}^* = \bar{y}^* exp\left\{\frac{\beta_1(x)(\bar{x}^*-\bar{x}')}{\beta_1(x)(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	$\beta_1(x)$	C_x
$t_{3Re7}^* = \bar{y}^* exp\left\{\frac{\beta_2(x)(\bar{x}'-\bar{x}^*)}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	$t_{3Pe7}^* = \bar{y}^* exp\left\{\frac{\beta_2(x)(\bar{x}^*-\bar{x}')}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	$\beta_2(x)$	S_x
$t_{3Re8}^* = \bar{y}^* exp\left\{\frac{(\bar{x}'-\bar{x}^*)}{(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	$t_{3Pe8}^* = \bar{y}^* exp\left\{\frac{(\bar{x}^*-\bar{x}')}{(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	1	ρ
$t_{3Re9}^* = \bar{y}^* exp\left\{\frac{(\bar{x}'-\bar{x}^*)}{(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	$t_{3Pe9}^* = \bar{y}^* exp\left\{\frac{(\bar{x}^*-\bar{x}')}{(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	1	$\beta_2(x)$
$t_{3Re10}^* = \bar{y}^* exp\left\{\frac{C_x(\bar{x}'-\bar{x}^*)}{C_x(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	$t_{3Pe10}^* = \bar{y}^* exp\left\{\frac{C_x(\bar{x}^*-\bar{x}')}{C_x(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	C_x	ρ
$t_{3Re11}^* = \bar{y}^* exp\left\{\frac{\rho(\bar{x}'-\bar{x}^*)}{\rho(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	$t_{3Pe11}^* = \bar{y}^* exp\left\{\frac{\rho(\bar{x}^*-\bar{x}')}{\rho(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	ρ	C_x
$t_{3Re12}^* = \bar{y}^* exp\left\{\frac{\beta_2(x)(\bar{x}'-\bar{x}^*)}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	$t_{3Pe12}^* = \bar{y}^* exp\left\{\frac{\beta_2(x)(\bar{x}^*-\bar{x}')}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	$\beta_2(x)$	ρ
$t_{3Re13}^* = \bar{y}^* exp\left\{\frac{\rho(\bar{x}'-\bar{x}^*)}{\rho(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	$t_{3Pe13}^* = \bar{y}^* exp\left\{\frac{\rho(\bar{x}^*-\bar{x}')}{\rho(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	ρ	$\beta_2(x)$

Table A8: **Some members of the class of estimators t_4^***

Ratio type exponential estimator $\delta = 1$	Product type exponential estimator $\delta = -1$	a	b
$t_{4Re1}^* = \omega_2\bar{y}^* exp\left(\frac{\bar{x}'-\bar{x}^*}{\bar{x}'+\bar{x}^*}\right)$	$t_{4Pe1}^* = \omega_2\bar{y}^* exp\left(\frac{\bar{x}^*-\bar{x}'}{\bar{x}^*+\bar{x}'}\right)$	1	0
$t_{4Re2}^* = \omega_2\bar{y}^* exp\left\{\frac{\bar{x}'-\bar{x}^*}{(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	$t_{4Pe2}^* = \omega_2\bar{y}^* exp\left\{\frac{\bar{x}^*-\bar{x}'}{(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	1	C_x
$t_{4Re3}^* = \omega_2\bar{y}^* exp\left\{\frac{\beta_2(x)(\bar{x}'-\bar{x}^*)}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	$t_{4Pe3}^* = \omega_2\bar{y}^* exp\left\{\frac{\beta_2(x)(\bar{x}^*-\bar{x}')}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	$\beta_2(x)$	C_x
$t_{4Re4}^* = \omega_2\bar{y}^* exp\left\{\frac{C_x(\bar{x}'-\bar{x}^*)}{C_x(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	$t_{4Pe4}^* = \omega_2\bar{y}^* exp\left\{\frac{C_x(\bar{x}^*-\bar{x}')}{C_x(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	C_x	$\beta_2(x)$
$t_{4Re5}^* = \omega_2\bar{y}^* exp\left\{\frac{(\bar{x}'-\bar{x}^*)}{(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	$t_{4Pe5}^* = \omega_2\bar{y}^* exp\left\{\frac{(\bar{x}^*-\bar{x}')}{(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	1	S_x
$t_{4Re6}^* = \omega_2\bar{y}^* exp\left\{\frac{\beta_1(x)(\bar{x}'-\bar{x}^*)}{\beta_1(x)(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	$t_{4Pe6}^* = \omega_2\bar{y}^* exp\left\{\frac{\beta_1(x)(\bar{x}^*-\bar{x}')}{\beta_1(x)(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	$\beta_1(x)$	S_x
$t_{4Re7}^* = \omega_2\bar{y}^* exp\left\{\frac{\beta_2(x)(\bar{x}'-\bar{x}^*)}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	$t_{4Pe7}^* = \omega_2\bar{y}^* exp\left\{\frac{\beta_2(x)(\bar{x}^*-\bar{x}')}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2S_x}\right\}$	$\beta_2(x)$	S_x
$t_{4Re8}^* = \omega_2\bar{y}^* exp\left\{\frac{(\bar{x}'-\bar{x}^*)}{(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	$t_{4Pe8}^* = \omega_2\bar{y}^* exp\left\{\frac{(\bar{x}^*-\bar{x}')}{(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	1	ρ
$t_{4Re9}^* = \omega_2\bar{y}^* exp\left\{\frac{(\bar{x}'-\bar{x}^*)}{(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	$t_{4Pe9}^* = \omega_2\bar{y}^* exp\left\{\frac{(\bar{x}^*-\bar{x}')}{(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	1	$\beta_2(x)$
$t_{4Re10}^* = \omega_2\bar{y}^* exp\left\{\frac{C_x(\bar{x}'-\bar{x}^*)}{C_x(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	$t_{4Pe10}^* = \omega_2\bar{y}^* exp\left\{\frac{C_x(\bar{x}^*-\bar{x}')}{C_x(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	C_x	ρ
$t_{4Re11}^* = \omega_2\bar{y}^* exp\left\{\frac{\rho(\bar{x}'-\bar{x}^*)}{\rho(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	$t_{4Pe11}^* = \omega_2\bar{y}^* exp\left\{\frac{\rho(\bar{x}^*-\bar{x}')}{\rho(\bar{x}'+\bar{x}^*)+2C_x}\right\}$	ρ	C_x
$t_{4Re12}^* = \omega_2\bar{y}^* exp\left\{\frac{\beta_2(x)(\bar{x}'-\bar{x}^*)}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	$t_{4Pe12}^* = \omega_2\bar{y}^* exp\left\{\frac{\beta_2(x)(\bar{x}^*-\bar{x}')}{\beta_2(x)(\bar{x}'+\bar{x}^*)+2\rho}\right\}$	$\beta_2(x)$	ρ
$t_{4Re13}^* = \omega_2\bar{y}^* exp\left\{\frac{\rho(\bar{x}'-\bar{x}^*)}{\rho(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	$t_{4Pe13}^* = \omega_2\bar{y}^* exp\left\{\frac{\rho(\bar{x}^*-\bar{x}')}{\rho(\bar{x}'+\bar{x}^*)+2\beta_2(x)}\right\}$	ρ	$\beta_2(x)$

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