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A Note on Coefficient Inequalities for Certain Classes of Ruscheweyh Type Analytic Functions in Conic Regions

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Abstract: A class of functions which provides an interesting transition from k- uniformly Janowski convex and k-Janowski starlike functions is defied by combing the concept of Ruscheweyh derivatives, Janowski functions and conic regions. Coefficient inequalities for functions in these classes are formulated which generalize the coefficient inequalities of Khalida Inayat Noor, Sarfraz Nawaz Malik and Latha.

Keywords: Analytic functions, Ruscheweyh derivative , Conic domains, Janowski functions, k-Starlike functions, k- Uniformaly convex functions

1 Introduction

Let $\mathscr{A}$ denote the class of functions

$$f(z)=z+\sum_{n=2}^{\infty}a_nz^n, \quad (1)$$

analytic in the open unit disk $\mathscr{U}=\{z:z\in\mathbb{C} \quad and \quad |z|<1\}$, normalized so that $f(0)=0$ and $f'(0)=1$. We designate $\mathscr{S}$ as the subclass $\mathscr{A}$ consisting of all function which are univalent in $\mathscr{U}$. In the following we shall give the basic properties of functions with positive real part in unit disc $\mathscr{U}$. Also we shall discuss the concept of subordination in the complex plane. Let P denote the class of analytic function p in $\mathscr{U}$ such that $p(0)=1$, $\Re\{p(z)\}>0$, any function p in P has the representation $p(z)=\frac{1+w(z)}{1-w(z)}$ where $w(0)=0$, $|w(z)|<1$ on $\mathscr{U}$, $z\in\mathscr{U}$ [2]. Given two functions f and g analytic in $\mathscr{U}$, we say that the function f is subordinate to g in $\mathscr{U}$ and write $f(z)\prec g(z)$, if there exists a Schwarz function w, which is analytic in $\mathscr{U}$ with $w(0)=0$ and $|w(z)|<1$, such that $f(z)=g(w(z))$, $z\in\mathscr{U}$. If g is univalent in $\mathscr{U}$ then $f\prec g$ if and only if $f(0)=g(0)$ and $f(\mathscr{U})\subset g(\mathscr{U})$.
The representation for functions with positive real part motivated Janowski to define the class $P[A,B]$.

Definition 1.*[1]Let $P[A,B]$, where $-1\le B<A\le 1$, denote the class of analytic function p defined on $\mathscr{U}$ with the representation $p(z)=\frac{1+Aw(z)}{1+Bw(z)}$, $z\in\mathscr{U}$,* $w(0)=0$, $|w(z)<1$.
$p\in P[A,B]$ if and only if $p(z)\prec\frac{1+Az}{1+Bz}$.

Geometrically, a function $p\in P[A,B]$ maps the opine unit onto the disk defined by the domain,

$$\Omega[A,B]=\left\{w:\left|w-\frac{1-AB}{1-B^2}\right|<\frac{A-B}{1-B^2}\right\}.$$

The class $P[A,B]$ is connected the class P of functions with positive real part by the relation,

$$p(z)\in P\Leftrightarrow\frac{(A+1)p(z)-(A-1)}{(B+1)p(z)-(B-1)}\in P[A,B].$$

Kanas and Wisniowska [3,10] introduced and studied the class $k-UCV$ of k-uniformly convex functions and the corresponding class $k-ST$ of k-starlike functions. These classes were defined subject to the conic region Ω_k, $k\ge 0$ given by as

$$\Omega_k=\{u+iv:u>k\sqrt{(u-1)^2+v^2}.$$

This domain represents the right half plane for $k=0$, hyperbola for $0<k<1$, a parabola for $k=1$ and ellipse

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for $k > 1$
The function p_k play the role of extremal functions for these conic regions where

$$p_k(z) = \begin{cases} \frac{1+z}{1-z}, & k=0 \\ 1+\frac{2}{\pi^2}\left(\log\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)^2, & k=1. \\ 1+\frac{2}{1-k^2}\sinh^2\left[\left(\frac{2}{\pi}\arccos k\right)\operatorname{arctan}h\sqrt{z}\right], & 0<k<1. \\ 1+\frac{2}{k^2-1}\sin\left[\frac{\pi}{2R(t)}\int_0^{\frac{u(z)}{\sqrt{t}}}\frac{1}{\sqrt{1-x^2}\sqrt{1-(tx)^2}}dx\right]+\frac{1}{k^2-1}, & k>1, \end{cases} \tag{2}$$

where $u(z)=\frac{z-\sqrt{t}}{1-\sqrt{tx}}$, $t\in(0,1)$, $z\in\mathscr{U}$ and z is chosen such that $k=\cosh\left(\frac{\pi R'(t)}{4R(t)}\right)$, $R(t)$ is the Legendre's complete elliptic integral of the first kind and $R'(t)$ is complementary integral $R(t)$; $p_k(z)=1+\delta_k z+....$, [9] where

$$\delta_k = \begin{cases} \frac{8(\arccos\ k)^2}{\pi^2(1-k^2)}, & 0\le k<1 \\ \frac{8}{\pi^2}, & k=1. \\ \frac{\pi^2}{4(k^2-1)\sqrt{t}(1+t)R^2(t)}, & k>1. \end{cases} \tag{3}$$

Now using the concepts of Janowski functions and the conic domain, we define the following.

Definition 2. *A function p is said to be in the class $k-P[A,B]$, if and only if,*

$$p(z) \prec \frac{(A+1)p_k(z)-(A-1)}{(B+1)p_k(z)-(B-1)},\ \ k\ge 0,$$

where $p_k(z)$ is defined by (2) and $-1\le B<A\le 1$.

Geometrically, the function $p(z)\in k-P[A,B]$ takes all values from the domain $\Omega_k[A,B]$, $-1\le B<A\le 1,\ k\ge 0$ which is defined as

$$\Omega_k[A,B] = \left\{ w:\Re\left(\frac{(B-1)w(z)-(A-1)}{(B+1)w(z)-(A+1)}\right) > k\left|\frac{(B-1)w(z)-(A-1)}{(B+1)w(z)-(A+1)}-1\right|, \right. \tag{4}$$

or equivalently

$$\Omega_k[A,B] =$$

$$\left\{ \begin{array}{l} w:\Re u+iv:[(B^2-1)(u^2+v^2)-2(AB-1)u+(A^2-1)]^2 \\ >k^2[(-2(B+1)(u^2+v^2)+2(A+B+2)u-2(A+1))^2] \\ \qquad +4k^2(A-B)^2v^2 \end{array} \right. \tag{5}$$

The domain $\Omega_k[A,B]$ retains the conic domain Ω_k inside the circular region defined by $\Omega[A,B]$. the impact of $\Omega[A,B]$ on the conic domain Ω_k changes the original shape of the conic regions . The ends of hyperbola and parabola get closer to each other but never meet anywhere and the ellipse gets the oval shape . When $A\to 1,\ B\to -1$, the radius of the circular disk defined by $\Omega[A,B]$ tends to infinity, consequently the arms of hyperbola and parabola expand and the oval turns into ellipse . It can be seen that $\Omega_k[1,-1]=\Omega_k$, the conic domain defined by Kanas and Wisniowska[10]. here are some basic facts about the class $k-P[A,B]$.
Now using the concept of Ruscheweyh derivative [7] we define the following

Definition 3. *A function $f\in\mathscr{A}$ is said to be in the class $k-\mathscr{V}_b^{\sigma}(A,B)$, $k\ge 0,\ -1\le B<A\le 1$, if and only if*

$$\Re\left(\frac{(B-1)J_f-(A-1)}{(B+1)J_f-(A+1)}\right) > k\left|\frac{(B-1)J_f-(A-1)}{(B+1)J_f-(A+1)}-1\right|, \tag{6}$$

where

$$J_f = 1-\frac{2}{b}+\frac{2}{b}\frac{D^{\sigma+1}f(z)}{D^{\sigma}f(z)},$$

and $b\neq 0, \sigma>-1$ and $D^{\sigma}f$ is the Rushceweyh derivative of f given by

$$D^{\sigma}f(z) = \frac{z}{(1-z)^{\sigma+1}} * f(z) = z+\sum_{n=2}^{\infty} a_n R_n(\sigma) z^n,$$

where $*$ stands for the convolution or Hadamard product of two power series and

$$R_n(\sigma) = \frac{(\sigma+1)(\sigma+2)...(\sigma+n-1)}{(n-1)!} = \frac{\Gamma(n+\sigma)}{(n-1)!.\Gamma(1+\sigma)}. \tag{7}$$

Or equivalently,

$$J_f \in k-P[A,B].$$

This class generalizes various classes studied earlier by Khalida Inyat Noor and Sarfraz Nawaz Malik [8], Kanas and Wisniowska[10], Latha [6], Janowski [1] and Shams [5].
We need the following lemmas to prove our main results

Lemma 1. *[11] Let $h(z)=1+\sum_{n=1}^{\infty}c_n z^n$ be subordinate to $H(z)=1+\sum_{n=1}^{\infty}b_n z^n$. If $H(z)$ is univalent in $\mathscr{U}$ and $H(z)$ is convex, then*

$$|c_n|\le|b_1|,\ n\ge 1.$$

Lemma 2. *[8] Let $h(z)=1+\sum_{n=1}^{\infty}c_n z^n\in P[A,B]$. Then*

$$|c_n|\le\frac{(A-B)|\delta_k|}{2},$$

where δ_k is defined by (3).

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2 Main results

Theorem 1. *A function $f\in\mathscr{A}$ and of the form (1) is in the class $k-\mathscr{V}_b^{\sigma}(A,B)$, if it satisfies the condition*

$$|B-A||b|(1+\sigma) >$$

$$\begin{cases} \sum_{n=2}^{\infty} |(B+1)[b(1+\sigma)+2(n-1)]-(A+1)(1+\sigma)b|R_n(\sigma)|a_n| \\ +\sum_{n=2}^{\infty} 4(k+1)(n-1)R_n(\sigma)|a_n|, \end{cases} \tag{8}$$

where $-1 \le B < A \le 1, b \neq 0, \sigma > -1, k \geq 0$ *and* $R_n(\sigma)$ *is defined by* (7).

Assuming that (**??**) holds, then it suffices to show that

$$1 > \begin{cases} k\left|\frac{(B-1)\left[1-\frac{2}{b}+\frac{2}{b}\frac{D^{\sigma+1}f(z)}{D^{\sigma}f(z)}\right]-(A-1)}{(B+1)\left[1-\frac{2}{b}+\frac{2}{b}\frac{D^{\sigma+1}f(z)}{D^{\sigma}f(z)}\right]-(A+1)}-1\right| \\ -\Re\left[\frac{(B-1)\left[1-\frac{2}{b}+\frac{2}{b}\frac{D^{\sigma+1}f(z)}{D^{\sigma}f(z)}\right]-(A-1)}{(B+1)\left[1-\frac{2}{b}+\frac{2}{b}\frac{D^{\sigma+1}f(z)}{D^{\sigma}f(z)}\right]-(A+1)}-1\right] \end{cases} \tag{9}$$

We get

$$\begin{cases} k\left|\frac{(B-1)\left[1-\frac{2}{b}+\frac{2}{b}\frac{D^{\sigma+1}f(z)}{D^{\sigma}f(z)}\right]-(A-1)}{(B+1)\left[1-\frac{2}{b}+\frac{2}{b}\frac{D^{\sigma+1}f(z)}{D^{\sigma}f(z)}\right]-(A+1)}-1\right| \\ -Re\left[\frac{(B-1)\left[1-\frac{2}{b}+\frac{2}{b}\frac{D^{\sigma+1}f(z)}{D^{\sigma}f(z)}\right]-(A-1)}{(B+1)\left[1-\frac{2}{b}+\frac{2}{b}\frac{D^{\sigma+1}f(z)}{D^{\sigma}f(z)}\right]-(A+1)}-1\right] \end{cases} \le \tag{10}$$

$$4(k+1)\left|\frac{D^{\sigma}f(z)-D^{\sigma+1}f(z)}{(B-A)bD^{\sigma}f(z)+2(B+1)[D^{\sigma+1}f(z)-D^{\sigma}f(z)]}\right|.$$

Since

$$D^{\sigma}f(z)-D^{\sigma+1}f(z)=\sum_{n=2}^{\infty}\frac{(1-n)}{(1+\sigma)}R_n(\sigma)a_nz^n,$$

and

$$(B-A)bD^{\sigma}f(z)+2(B+1)[D^{\sigma+1}f(z)-D^{\sigma}f(z)]=(B-A)bz$$

$$+\sum_{n=2}^{\infty}\frac{(B+1)[b(1+\sigma)+2(n-1)]-(A+1)b(1+\sigma)}{(1+\sigma)}R_n(\sigma)a_nz^n,$$

then

$$=4(k+1)\left|\frac{\sum_{n=2}^{\infty}(1-n)R_n(\sigma)a_nz^n}{(B-A)bz+\sum_{n=2}^{\infty}\kappa_n(B,b,\sigma)R_n(\sigma)a_nz^z}\right|$$

$$\le 4(k+1)\frac{\sum_{n=2}^{\infty}|1-n|R_n(\sigma)|a_n|}{|B-A||b|-\sum_{n=2}^{\infty}|\kappa_n(B,b,\sigma)|R_n(\sigma)|a_n|}.$$

where

$$\kappa_n(B,b,\sigma)=(B+1)[b(1+\sigma)+2(n-1)]-b(1+\sigma)(A+1),$$

The last expression is bounded above by 1, then

$$|B-A||b|(1+\sigma)>$$

$$\begin{cases} \sum_{n=2}^{\infty}\{|(B+1)[b(1+\sigma)+2(n-1)]-(A+1)(1+\sigma)b|\}R_n(\sigma)|a_n| \\ +\sum_{n=2}^{\infty}\{4(k+1)(n-1)\}R_n(\sigma)|a_n|, \end{cases} \tag{11}$$

and this completes the proof.

When $b=2, \sigma=0$ and $b=\sigma=1$, we have the following known results, proved by Khalida Inayat Noor and Sarfraz Nawaz Malik in [8].

Corollary 1. *A function* $f \in \mathscr{A}$ *and form* (1) *in the class* $k-ST[A,B]$, *if it satisfies the condition*

$$\sum_{n=2}^{\infty}\{2(k+1)(n-1)+|n(B+1)-(A-1)|\}|a_n|<|B-A|,$$

where $-1 \le B < A \le 1$ *and* $k \ge 0$.

Corollary 2. *A function* $f \in \mathscr{A}$ *and form* (1) *in the class* $k-UCV[A,B]$, *if it satisfies the condition*

$$\sum_{n=2}^{\infty} n\{2(k+1)(n-1)+|n(B+1)-(A-1)|\}|a_n|<|B-A|,$$

where $-1 \le B < A \le 1$ *and* $k \ge 0$.

Theorem 2. *Let* $f \in k-\mathscr{V}_b^{\sigma}(A,B)$ *and is of the form* (1). *Then for* $n \geq 2$.

$$|a_n| \le \frac{1}{R_n(\sigma)}\prod_{j=0}^{n-2}\frac{|\delta_k b(A-B)(1+\sigma)-4jB|}{4(j+1)}, \tag{12}$$

where δ_k *is defined* (3) *and* $R_n(\sigma)$ *is defined by* (7).

Proof. By the definition we have

$$1-\frac{2}{b}+\frac{2}{b}\frac{D^{\sigma+1}f(z)}{D^{\sigma}f(z)}=p(z), \tag{13}$$

where

$$p(z) \prec \frac{(A+1)p_k(z)-(A-1)}{(B+1)p_k(z)-(B-1)}$$

Now if $p(z)=1+\sum_{n=1}^{\infty}c_nz^n$, then by Lemma 1, we get

$$|c_n| \le \frac{1}{2}(A-B)\delta_k, \quad n \ge 1 \tag{14}$$

Now from (13), we have

$$2[D^{\sigma+1}f(z)-D^{\sigma}f(z)]=bD^{\sigma}f(z)\left(\sum_{n=1}^{\infty}c_nz^n\right).$$

Equating coefficients of z^n on both sides, we have

$$\frac{2R_n(\sigma)(n-1)a_n}{(\sigma+1)}=b\sum_{j=1}^{n-1}R_{n-j}(\sigma)a_{n-j}c_j, \qquad a_1=R_1(\sigma)=1.$$

By (14), we get

$$|a_n| \le \frac{|\delta_k||b|(1+\sigma)(A-B)}{4(n-1)R_n(\sigma)}\sum_{j=1}^{n-1}R_j(\sigma)|a_j|, \quad a_1=R_1(\sigma)=1. \tag{15}$$

Now we prove that

$$\frac{|\delta_k|(A-B)}{2(n-\lambda_N(n))}\sum_{j=1}^{n-1}R_j(\sigma)|a_j| \le$$

$$\frac{1}{R_n(\sigma)}\prod_{j=1}^{n-1}\frac{|\delta_k b(A-B)(1+\sigma)-4jB|}{4(j+1)}. \tag{16}$$

For this, we use the induction method.
For $n=2$: from (15), we have

$$|a_2|\le\frac{|\delta_k||b|(A-B)(1+\sigma)}{4R_2(\sigma)}.$$

From (12), we have

$$|a_2|\le\frac{|\delta_k||b|(A-B)(1+\sigma)}{4R_2(\sigma)}.$$

For $n=3$: from (15), we have

$$|a_3|\le\frac{|\delta_k||b|(A-B)(1+\sigma)}{8R_3(\sigma)}\left[1+\frac{|\delta_k||b|(A-B)(1+\sigma)}{4}\right].$$

From (12), we have

$$|a_3|\le\frac{1}{R_3(\sigma)}\left[\frac{|\delta_k||b|(A-B)(1+\sigma)}{4}\cdot\frac{|\delta_k b(A-B)(1+\sigma)-4B|}{8}\right]$$

$$\le\frac{|\delta_k||b|(A-B)(1+\sigma)}{4R_3(\sigma)}\left[1+\frac{|\delta_k||b|(A-B)(1+\sigma)}{4}\right].$$

Let the hypothesis be true for $n=m$. From (15), we have

$$|a_m|\le\frac{|\delta_k||b|(A-B)(1+\sigma)}{4(m-1)R_m(\sigma)}\sum_{j=1}^{m-1}R_j(\sigma)|a_j|,\qquad a_1=R_1(\sigma)=1.$$

From (12), we have

$$|a_m|\le\frac{1}{R_m(\sigma)}\prod_{j=0}^{m-2}\frac{|\delta_k b(A-B)(1+\sigma)-4jB|}{4(j+1)}.$$

$$\le\frac{1}{R_m(\sigma)}\prod_{j=0}^{m-2}\frac{|\delta_k||b|(A-B)(1+\sigma)+4j}{4(j+1)}.$$

By the induction hypothesis, we have

$$\frac{|\delta_k||b|(A-B)(1+\sigma)}{4(m-1)R_m(\sigma)}\sum_{j=1}^{m-1}R_j(\sigma)|a_j|$$

$$\le\frac{1}{R_m(\sigma)}\prod_{j=0}^{m-2}\frac{|\delta_k b(A-B)(1+\sigma)-4jB|}{4(j+1)}.$$

Multiplying both sides by

$$\frac{R_m(\sigma)}{R_{m+1}(\sigma)}\frac{|\delta_k||b|(A-B)(1+\sigma)+4(m-1)}{4m},$$

we have

$$\frac{1}{R_{m+1}}\prod_{j=0}^{m-1}\frac{|\delta_k||b|(A-B)(1+\sigma)+4j}{4(j+1)}\ge$$

$$\left\{\begin{array}{l}\frac{|\delta_k||b|(A-B)(1+\sigma)}{4(m-1)R_m(\sigma)}\cdot\frac{R_m(\sigma)}{R_{m+1}(\sigma)}\\ \times\\ \frac{|\delta_k||b|(A-B)(1+\sigma)+4(m-1)}{4m}\sum_{j=1}^{m-1}R_j(\sigma)|a_j|,,\end{array}\right. \tag{17}$$

$$=\left\{\begin{array}{l}\frac{|\delta_k||b|(A-B)(1+\sigma)}{4(m)R_{m+1}(\sigma)}\left[\frac{|\delta_k||b|(A-B)(1+\sigma)}{4(m-1)}\sum_{j=1}^{m-1}R_j(\sigma)|a_j|\right]\\ \\ +\frac{|\delta_k||b|(A-B)(1+\sigma)}{4(m)R_{m+1}(\sigma)}\sum_{j=1}^{m-1}R_j(\sigma)|a_j|,\end{array}\right. \tag{18}$$

$$\ge\frac{|\delta_k||b|(A-B)(1+\sigma)}{4(m)R_{m+1}(\sigma)}\left[R_m(\sigma)|a_m|+\sum_{j=1}^{m-1}R_j(\sigma)|a_j|\right],$$

$$=\frac{|\delta_k||b|(A-B)(1+\sigma)}{4(m)R_{m+1}(\sigma)}\sum_{j=1}^{m}R_j(\sigma)|a_j|.$$

That is

$$|a_{m+1}|\le\frac{|\delta_k||b|(A-B)(1+\sigma)}{4(m)R_{m+1}(\sigma)}\sum_{j=1}^{m}R_j(\sigma)|a_j|$$

$$\le\frac{1}{R_{m+1}}\prod_{j=0}^{m-1}\frac{|\delta_k||b|(A-B)(1+\sigma)+4j}{4(j+1)}.$$

Which shows that inequality (16) is true for $n=m+1$. Hence the required result.

When $b=2,\sigma=0$ and $b=\sigma=1$ we have the following results proved Khalida Inayat Noor and Sarfraz Nawaz Malik in [8]

Corollary 3. *Let $f\in k-ST[A,B]$, then*

$$|a_n|\le\prod_{j=0}^{n-2}\frac{|\delta_k(A-B)-2jB|}{2(j+1)},\quad -1\le B<A\le 1,\quad n\ge 2.$$

Corollary 4. *Let $f\in k-UCV[A,B]$, then*

$$|a_n|\le\frac{1}{n}\prod_{j=0}^{n-2}\frac{|\delta_k(A-B)-2jB|}{2(j+1)},\quad -1\le B<A\le 1,\quad n\ge 2.$$

For $A=1-2\alpha, B=-1$ we arrive to result proved by Latha in [6].

Corollary 5. *Let $f\in\mathscr{V}\mathscr{D}(\alpha,\beta,b,\delta)$, then*

$$|a_n|\le\frac{b(1-\beta)(\sigma+1)}{(n-1)|1-\alpha|R_n(\sigma)}\prod_{j=1}^{n-2}\left(1+\frac{b(\sigma+1)(1-\beta)}{j|1-\alpha|}\right),\quad n>2.$$

For $b=2,\sigma=0,\ A=1\ B=-1$ we arrive at Kanas and Wisniowska [3]

Corollary 6. *Let $f\in k-ST$, then*

$$|a_n|\le\prod_{j=0}^{n-2}\frac{|\delta_k+j|}{(j+1)},\qquad n\ge 2.$$

Also for $b=2,\sigma=0,\ k=0\ \delta_k=2$ we have the well-known result proved by Janowski [1].

Corollary 7. *Lat $f\in\mathscr{S}^*[A,B]$, then*

$$|a_n|\le\prod_{j=0}^{n-2}\frac{|(A-B)-jB|}{(j+1)},\quad -1\le B<A\le 1,\quad n\ge 2.$$

3 Conclusion

A modest attempt has been made in this paper to introduce the class $k-\mathscr{V}_b^{\sigma}(A,B)$ which provides an interesting transition from $k-$uniformly Janowski convex functions to $k-$Janowski starlike functions by combining the concept of Ruscheweyh derivatives, Janowski functions and conic regions. We derived condition for functions to be in this class and deduced interesting coefficient inequalities. There is further scope to improve using the generalized Janowski class and symmetric functions.

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