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Contribution of Coherent Neutral-Pion Photoproduction Channel to the Gerasimov-Drell-Hearn Sum Rule for the Deuteron*

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The helicity dependence of the $\vec{\gamma}\vec{d} \rightarrow \pi^0 d$ reaction channel is studied for incident photon energies from threshold up to the $\Delta(1232)$-resonance using an enhanced elementary pion photoproduction operator and a realistic high-precision potential for the deuteron wave function. The doubly polarized total cross sections for parallel and antiparallel helicity states are predicted. Then the contribution to the deuteron spin asymmetry is calculated. In addition, the contribution to the Gerasimov-Drell-Hearn (GDH) integral is evaluated by explicit integration up to a photon lab-energy of 350 MeV. The sensitivity of the results to the elementary pion photoproduction amplitude and the potential model used for the deuteron wave function is also investigated. Considerable dependence of the results on both the elementary amplitude and the deuteron wave function is found. We expect that these results may be useful to interpret the recent measurements from A2 and GDH@MAMI Collaborations.

Keywords: Sum rules, meson production, polarization phenomena in reactions, photoproduction reactions, spin observables.

1 Introduction

The Gerasimov-Drell-Hearn (GDH) sum rule [1,2] gives a fundamental relation for the study of the particle spin structure via real photon absorption. It was derived by Gerasimov [1] in 1965 and independently by Drell and Hearn [2] in 1966. The GDH sum rule links

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the anomalous magnetic moment of a particle to the energy weighted integral of the spin asymmetry of the photoabsorption cross sections $(\sigma^P - \sigma^A)$ with respect to circularly polarized photons and a polarized target. For a particle of mass M, charge eQ, anomalous magnetic moment κ, and spin S it reads

$$I^{GDH}(\infty) = \int_0^\infty dE'_\gamma \frac{\sigma^P(E'_\gamma) - \sigma^A(E'_\gamma)}{E'_\gamma} = 4\pi^2\kappa^2 \frac{e^2}{M^2} S\,, \tag{1.1}$$

where $\sigma^{P(A)}(E'_\gamma)$ denote the total photoabsorption cross sections for circularly polarized photons on a target with spin parallel (P) and antiparallel (A) to the photon spin, respectively, and the anomalous magnetic moment is defined by the total magnetic moment operator of the particle $\vec{M} = e\,(Q+\kappa)\,\vec{S}/M$. The GDH sum rule provides a very interesting relation between a magnetic ground state property (κ) of a particle and an integral property of its whole excitation spectrum. Apart from the general assumption that the integral in (1.1) converges, its derivation is based solely on first principles like Lorentz and gauge invariances, unitarity, crossing symmetry, and causality of the Compton scattering amplitude of a particle. Consequently, from the experimental and theoretical points of view, a test for various targets becomes very important.

In the case of the nucleon, proton or neutron, the total particle spin $S = 1/2$, and there are two possible spin configurations in combination with the photon spin: $3/2$ and $1/2$. The GDH sum rule has been derived in the center-of-mass (c.m.) frame and, therefore, the spin configuration of 'parallel' in laboratory frame is 'antiparallel' in the c.m. one. Thus, the parallel (P) is really spin of $1/2$ and the antiparallel is spin of $3/2$. When the GDH sum rule is derived using the imaginary part of the forward Compton scattering amplitude, the GDH sum rule of the nucleon can be written as

$$I_N^{GDH}(\infty) = \int_0^\infty dE'_\gamma \frac{\sigma^{1/2}(E'_\gamma) - \sigma^{3/2}(E'_\gamma)}{E'_\gamma} = -2\pi^2\kappa^2 \frac{e^2}{M_N^2}\,. \tag{1.2}$$

The cross section is again integrated over all photon energies. In the case of the proton $\kappa_p = 1.97\,\mu_N$ and in the case of neutron $\kappa_n = -1.91\,\mu_N$. Since proton and neutron have large anomalous magnetic moments, one finds correspondingly large GDH sum rule predictions for them, i.e. $I_p^{GDH}(\infty) = 204.8\,\mu$b for the proton and $I_n^{GDH}(\infty) = 233.2\,\mu$b for the neutron.

Obviously, when $\kappa \neq 0$ the particle possesses an internal structure with excited states. However, the opposite is not in general true. A particle having a vanishing or very small anomalous magnetic moment κ is not necessarily pointlike or nearly pointlike. In this respect, the deuteron is a particularly instructive example because it has a very small anomalous magnetic moment $\kappa_d = -0.143\,\mu_N$. This gives a very small GDH sum rule value for the deuteron: $I_d^{GDH}(\infty) = 0.65\,\mu$b, which is close to zero and more than two orders of magnitude smaller than values predicted for the nucleon. On the other hand, it is well known that the deuteron has quite an extended spatial structure due to its small binding energy. The smallness of κ_d arises from an almost complete cancellation of the anomalous magnetic moments of the neutron and the proton whose spins are parallel and predominantly aligned along the deuteron spin direction.

If we naively assume that the deuteron is formed by almost-free nucleons, then the deuteron sum rule value should be approximatively equal to the sum of the proton and neutron sum rule values: $I_d^{GDH}(\infty) \simeq I_p^{GDH}(\infty) + I_n^{GDH}(\infty) = 204.8\,\mu$b $+233.2\,\mu$b $= 438\,\mu$b. But, due to the cancellation of the anomalous magnetic moments of the neutron and the proton, it is expected that a similar cancellation of different contributions should occur also for the sum rule integral. The reproduction of this is then a challenge for any microscopic nuclear theory of the deuteron. It has been shown in [3] that a large negative contribution to the sum rule of about $-381.52\,\mu$b arises from the photodisintegration channel $\vec{\gamma}\vec{d} \to pn$, which has its origin in a large negative spin asymmetry right above break-up threshold ($E_\gamma \sim$ 2.2 MeV). This contribution (in absolute magnitude) is almost equal to the sum of the neutron and proton GDH sum rule values. The resulting total predicted $27.31\,\mu$b value of the full GDH integral of the deuteron still overshoots the sum rule value. Thus, there is room for improvements of the theoretical framework which will allow to close the gap between the model-dependent evaluations and the sum rule prediction.

Most recently, an improved calculation of the spin asymmetry and the GDH integral of incoherent single-pion photoproduction on the deuteron has been performed in [4] in which final-state interactions are included completely in the NN- and πN-subsystems and an enhanced elementary pion photoproduction operator taken from [5] has been used. The influence of the elementary operator on the spin asymmetry and the GDH integral for both the neutral and the charged pion production channels has been investigated and was found to be very important. In many cases the deviation among results obtained using different operators is very large. In particular, the total value of the GDH integral $I^{GDH}_{\gamma d \to \pi NN}(350\,MeV) = 41.29\,\mu$b [4] has been obtained using the effective Lagrangian approach (ELA) [5], whereas the value $I^{GDH}_{\gamma d \to \pi NN}(350\,MeV) = 84.61\,\mu$b has been obtained using the MAID-2003 model [6]. The total value $I^{GDH}_{\gamma d \to \pi NN}(350\,MeV) = 41.29\,\mu$b from all three pion channels of the $\vec{\gamma}\vec{d} \to \pi NN$ reaction has been found [4]. This value is relatively small compared to other contributions. For instance in [3] a value of about $240\,\mu$b for the total GDH integral up to 350 MeV was found. It has been mentioned in [4] that the rest should be either coherent pion production or two-nucleon break-up.

The ultimate goal of the present paper is, therefore, to extend the model, recently presented in [4], to make theoretical predictions for the helicity dependence in photoabsorption cross sections of the process $\vec{\gamma}\vec{d} \to \pi^0 d$ in the energy range from threshold up to the Δ(1232)-resonance. For the elementary $\gamma N \to \pi^0 N$ amplitude, an enhanced elementary pion photoproduction operator taken from [5] is used. This model displays chiral symmetry, gauge invariance, and crossing symmetry, as well as a consistent treatment of the interaction with spin-3/2 particles. It also provides a reliable description of the threshold region. In this work we restrict ourselves to the impulse approximation (IA), the primary process against which all other effects will be gauged, in order to investigate as a first step the influence of the elementary pion photoproduction operator and the sensitivity of the deuteron wave function. We explicitly evaluate the contribution from the $\vec{\gamma}\vec{d} \to \pi^0 d$ channel to the spin asymmetry and the GDH integral for the deuteron. Furthermore, we investigate whether the differences found between the ELA [5] and MAID [6] elementary operators for the predictions of the spin asymmetry and the GDH integral in the $\vec{\gamma}\vec{d} \to \pi NN$ reaction channels are also seen in the coherent pion photoproduction channel $\vec{\gamma}\vec{d} \to \pi^0 d$. The calculation is of theoretical interest because it provides an important test of our understanding of the elementary neutron amplitude in the absence of a neutron target.

A direct experimental verification of the fundamental GDH sum rule requires the measurement of photoabsorption cross sections for circularly polarized real photons impinging on longitudinally polarized targets. This measurement has been carried out or planned at different laboratories such as MAMI, ELSA, LEGS, JLab, and HIγS. The first experimental check of the GDH sum rule for the proton was carried out at MAMI and ELSA, where the proton GDH integral was experimentally evaluated in the photon energy range 200 MeV $< E_\gamma <$ 2.9 GeV [7]. The helicity dependent total photoabsorption cross sections on the deuteron have been measured at ELSA [8] in the photon energy range from 815 to 1825 MeV using circularly polarized photons impinging on a longitudinally polarized deuterons. A measurement of the deuteron GDH integrand in the energy region from 200 to 800 MeV was carried out at MAMI [9] in order to test experimentally the behavior of the GDH integral. The present focus of the HIγS facility [10] is the measurement of the GDH integral from the near photodisintegration threshold up to the pion threshold [11].

In section 2, a brief review of the framework for the reaction $\gamma d \to \pi^0 d$, in which the transition matrix elements are calculated, is given. Results for the spin asymmetry and the GDH integral for the deuteron are presented and discussed in section 3, focusing on the sensitivity of results to the elementary pion photoproduction operator and the deuteron wave function. Finally, we provide conclusions in section 4. Throughout the paper we use natural units $\hbar = c = 1$.

2 Theoretical Model

2.1 Kinematics and cross section

As a starting point, we will first consider the formalism for coherent pion photoproduction from the deuteron which contains only two particles in the initial and in the final states. The general form of the two-body reaction is

$$a(p_a) + b(p_b) \to c(p_c) + d(p_d)\,, \tag{2.1}$$

where $p_i = (E_i, \vec{p}_i)$ denotes the four-momentum of particle "i" with $i \in \{a, b, c, d\}$.

Following the conventions of Bjorken and Drell [12] the general form for the differential cross section of a two-particle reaction in the center-of-mass (c.m.) system is given by

$$\frac{d\sigma}{d\Omega_c} = \frac{1}{(2\pi W)^2} \frac{p_c}{p_a} \frac{E_a E_b E_c E_d}{F_a F_b F_c F_d} \frac{1}{s} \sum_{\mu_d \mu_c \mu_b \mu_a} |T_{\mu_d \mu_c \mu_b \mu_a}(\vec{p}_d, \vec{p}_c, \vec{p}_b, \vec{p}_a)|^2 \tag{2.2}$$

with $T_{\mu_d \mu_c \mu_b \mu_a}$ as reaction matrix, μ_i denoting the spin projection of particle "i" on some quantization axis, and F_i is a factor arising from the covariant normalization of the states and its form depends on whether the particle is a boson ($F_i = 2E_i$) or a fermion ($F_i = E_i/m_i$), where E_i and m_i are its energy and mass, respectively. The factor $s = (2s_a + 1)(2s_b + 1)$ takes into account the averaging over the initial spin states, where s_a and s_b denote the spins of the incoming particles a and b, respectively. All momenta are functions of the invariant mass of the two-body system W, i.e. $p_i = p_i(W)$, where $W = E_a + E_b = E_c + E_d$.

Focusing on coherent pion photoproduction from the deuteron and choosing the photon-deuteron c.m. frame with the z-axis along the photon momentum $\vec{k}$ and the x-axis in the direction of maximal linear photon polarization, the reaction (2.1) becomes

$$\gamma(E_\gamma, \vec{k}, \lambda) + d(E_d, -\vec{k}) \to \pi^0(E_{\pi^0}, \vec{q}) + d(E_d', -\vec{q}) \,, \tag{2.3}$$

where energy and momenta of the participating particles are given in the parentheses, and λ stands for the circular photon polarization. The F_i factor is given by

$$F_a = 2E_\gamma, \qquad F_b = 2E_d, \qquad F_c = 2E_{\pi^0}, \qquad F_d = 2E_d' \tag{2.4}$$

and therefore one finds $s = 6$. The differential cross section of the reaction $\gamma d \to \pi^0 d$ in the c.m. system is then given by

$$\frac{d\sigma}{d\Omega_{\pi^0}} = \frac{1}{(8\pi W_{\gamma d})^2} \frac{|\vec{q}|}{|\vec{k}|} \frac{1}{6} \sum_{m_d m_d' \lambda} \left| T_{m_d m_d' \lambda}(\vec{k}, \vec{q}) \right|^2 \,, \tag{2.5}$$

where m_d' (m_d) is the spin projection of the outgoing (incoming) deuteron and $\vec{q}$ and $\vec{k}$ are the c.m. momenta of the pion and photon, respectively. Moreover, the invariant energy of the γd system is given as

$$\begin{aligned} W_{\gamma d} &= E_\gamma + \sqrt{\vec{k}^2 + M_d^2}, \qquad E_\gamma = |\vec{k}| \,, \\ &= E_{\pi^0} + \sqrt{\vec{q}^{\,2} + M_d^2}, \qquad E_{\pi^0} = \sqrt{\vec{q}^{\,2} + m_{\pi^0}^2} \,, \end{aligned} \tag{2.6}$$

where M_d and m_{π^0} are the deuteron and neutral-pion masses, respectively.

2.2 The scattering T-matrix

The scattering amplitude of coherent pion photoproduction on the deuteron is given in the impulse approximation by

$$T_{m_d m'_d \lambda}(\vec{k},\vec{q}) = 2\int \frac{d^3p}{(2\pi)^3}\,\phi^{\dagger}_{m'_d}(\vec{p}\,')\,t^{\lambda}_{\gamma\pi^0}(\vec{k},\vec{p}_i,\vec{q},\vec{p}_f)\phi_{m_d}(\vec{p}) \tag{2.7}$$

with $t^{\lambda}_{\gamma\pi^0}$ standing for the corresponding elementary amplitude $\gamma N \to \pi^0 N$. Furthermore, the vectors $\vec{p}_i$ and $\vec{p}_f$ denote initial and final momenta of the active nucleon in the deuteron, for which we have $\vec{p}_i = \vec{p} - \vec{k}/2$ and $\vec{p}_f = \vec{p} - \vec{q} + \vec{k}/2$, and $\vec{p}\,' = \vec{p} + (\vec{k} - \vec{q})/2$ denotes the relative momentum in the final deuteron state. Diagrammatic representation of the scattering matrix is shown in Fig. 2.1. As already mentioned in the Introduction, we restrict ourselves to the IA only in order to study in details the sensitivity of results on both the elementary pion photoproduction operator on the free nucleon and the deuteron wave function.

Introducing a partial wave decomposition, one finds for the scattering matrix the relation

$$T_{m_d m'_d \lambda}(\vec{k},\vec{q}) = e^{i(m_d+\lambda)\phi_{\pi^0}}\,t_{m_d m'_d \lambda}(W_{\gamma d},\theta_{\pi^0})\,, \tag{2.8}$$

where the reduced t-matrix elements are the basic quantities that determine cross sections and polarization observables. If parity is conserved, the reduced t-matrix obeys the symmetry relation

$$t_{-m_d - m'_d - \lambda} = (-)^{1+m_d+m'_d+\lambda}\,t_{m_d m'_d \lambda}\,. \tag{2.9}$$

For the deuteron wave function we use the familiar ansatz

$$\phi_{m_d}(\vec{p}) = \sum_{L=0,2}\sum_{m_L m_S}(L m_L 1 m_S|1 m_d)u_L(p)Y_{L m_L}(\hat{p})\chi_{m_S}\zeta_0\,, \tag{2.10}$$

where the last two terms denote spin and isospin wave functions, respectively. In the present work, we compute the radial deuteron wave function $u_L(p)$ using different realistic high-precision potential models. These are the CD-Bonn [13], the Bonn (full model) [14], and the Paris [15] potentials.

For the elementary pion photoproduction operator on the free nucleon, $\gamma N \to \pi^0 N$, we use in this work the ELA model elaborated in [5], which has been applied successfully from threshold up to 1 GeV of photon energy in the laboratory reference system and succeeds to reconcile [16] pion photoproduction experiments in the Δ(1232) region [17, 18] with the latest Lattice QCD calculations of the quadrupole deformation of the Δ(1232) [19]. Recently, the model has also been applied successfully to eta photoproduction from the proton [20]. This model is based upon an effective Lagrangian approach which from a theoretical point of view is a very appealing, reliable, and formally well-established approach in the energy region of the mass of the nucleon. It displays chiral symmetry, gauge invariance, and crossing symmetry as well as a consistent treatment of the spin-3/2 interaction.

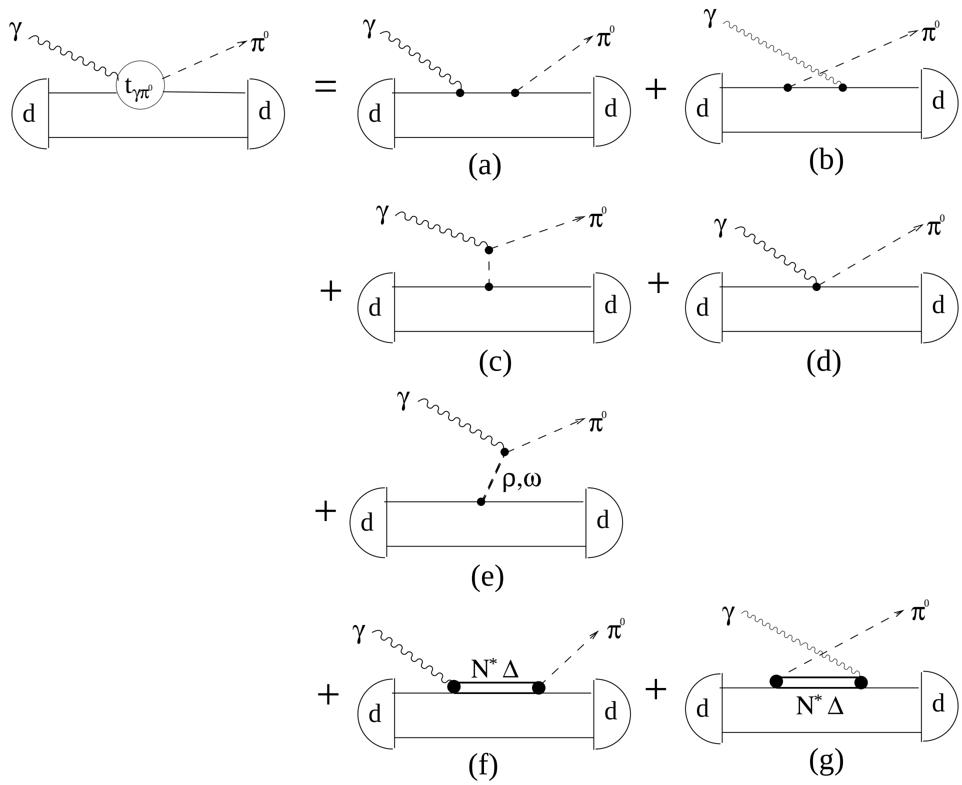

Figure 2.1: Time-ordered graphs taken into account in the present work for the amplitude of coherent pion photoproduction from the deuteron. Born terms: (a) direct nucleon pole, (b) crossed nucleon pole, (c) pion pole, and (d) Kroll-Rudermann contact term; (e) vector-meson exchange (ρ and ω); resonance excitations contribution: (f) direct and (g) crossed.

The model includes Born terms (diagrams (A)-(D) in Fig. 2.2), vector-meson exchanges (ρ and ω, diagram (E) in Fig. 2.2), and all the four star resonances in Particle Data Group (PDG) [17] up to 1.7 GeV and up to spin-3/2: Δ(1232), N(1440), N(1520), Δ(1620), N(1650), and Δ(1700) (diagrams (F) and (G) in Fig. 2.2).

In the pion photoproduction model from free nucleons [5] it was assumed that FSI factorize and can be included through the distortion of the πN final state wave function (pion-nucleon rescattering). πN-FSI was included by adding a phase δ_{FSI} to the electromagnetic multipoles. This phase is set so that the total phase of the multipole matches the total phase of the energy dependent solution of SAID [21]. In this way it was possible to isolate the contribution of the bare diagrams to the physical observables. The parameters of the resonances were extracted fitting the data to the electromagnetic multipoles from the energy independent solution of SAID [21] applying modern optimization techniques based upon a genetic algorithm combined with gradient based routines [22] which provides reli-

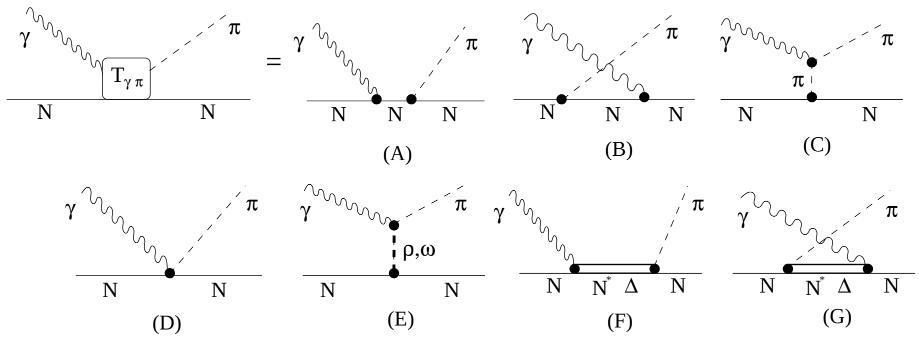

Figure 2.2: Feynman diagrams for pion photoproduction from a single nucleon. Born terms: (A) direct nucleon pole, (B) crossed nucleon pole, (C) pion in flight, and (D) Kroll-Rudermann contact term; (E) vector-meson exchange (ρ and ω); resonance excitations contribution: (F) direct and (G) crossed.

able values for the parameters of the nucleon resonances. Once the parameters, including phase shifts, are fitted to data we can distinguish between bare and dressed photo-pion production amplitudes on the nucleon. In what follows we call bare amplitudes to the ones provided by our model using the fitted values for all the parameters except those of the phase shifts which are set to zero.

3 Results and Discussion

In this section we explore the dependence of the results for the observables in the $\vec{\gamma}\vec{d} \to \pi^0 d$ reaction in the impulse approximation (IA) on the input elementary pion photoproduction operator and the potential model used for the deuteron wave function. We show results for the doubly polarized total photoabsorption cross sections σ^P for circularly polarized photons on a target with spin parallel to the photon spin and σ^A the same for antiparallel spins of photon and deuteron target, spin asymmetry $\sigma^P - \sigma^A$, and the GDH integral for the deuteron in the energy region from near threshold to the Δ(1232)-resonance, using as elementary reaction amplitudes the ones provided by the ELA model from [5] and those obtained using MAID model [6]. For the deuteron wave function, we use the CD-Bonn [13], Bonn (full model) [14], and Paris [15] potential models. The results of this comparison are collected in Figs. 3.3 and 3.4.

We would like to explain carefully what we call IA and how we compute it. Our IA calculation does not employ directly the amplitudes that fit the data on electromagnetic multipoles for the $\gamma N \to \pi^0 N$ process. This is due to the fact that πN-rescattering is unavoidably included in the amplitude in these fits to data. We call IA to the bare contribution to the observables. Therefore, if we wish to calculate the contribution coming from the pure IA, the bare IA contribution to the amplitude has to be extracted from the analysis of the $\gamma N \to \pi^0 N$, where the final state interaction has to be removed. This was done in [5]. We name IA* to the calculations where the πN-rescattering is included in the elementary reaction.

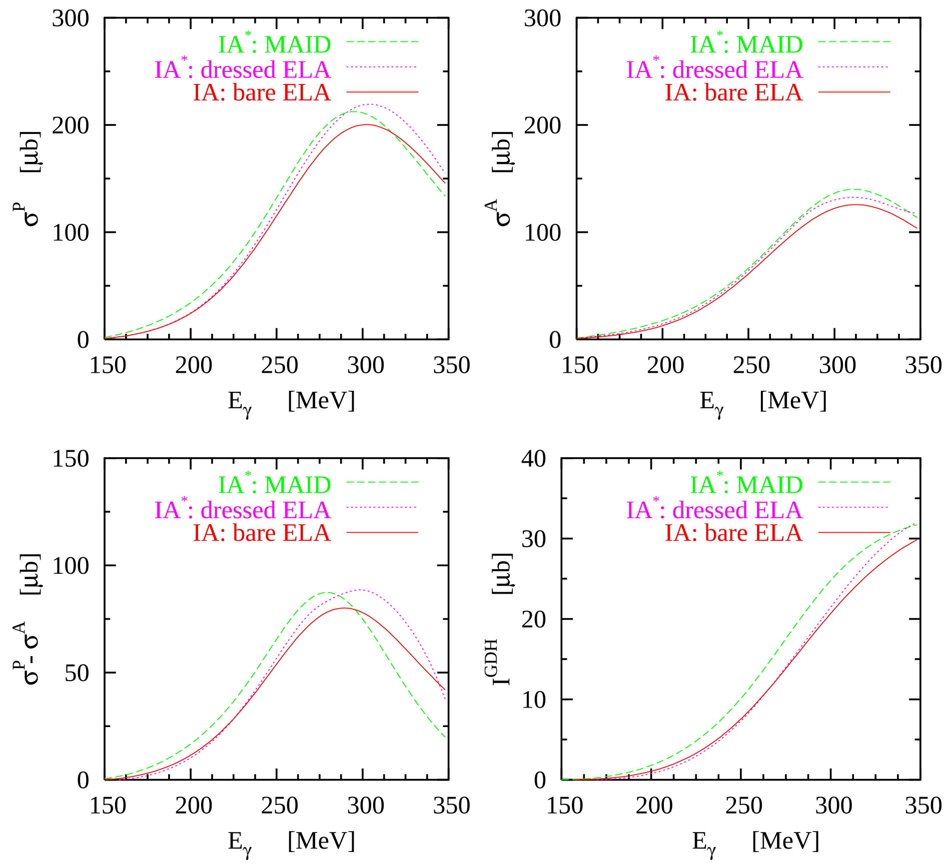

Figure 3.3: (Color online) Contribution of coherent neutral-pion photoproduction to the spin asymmetry and the GDH integral for the deuteron using different elementary pion photoproduction operators and the CD-Bonn potential [13] for the deuteron wave function. Upper panels show the helicity dependent total photoabsorption cross sections for circularly polarized photons on a longitudinally polarized deuteron with spin parallel σ^P (upper left panel) and antiparallel σ^A (upper right panel) to the photon spin. Lower left panel: the deuteron spin asymmetry of total photoabsorption cross section $\sigma^P - \sigma^A$; lower right panel: the GDH integral as a function of the upper integration limit. Curve conventions: dashed, IA* using MAID-2003 [6]; dotted, IA* using the dressed multipoles of ELA [5]; solid, IA using the bare electromagnetic multipoles of ELA [5]. IA* denotes the calculation when the πN-rescattering is included in the elementary reaction (see text).

The first comparison (Fig. 3.3) shows the sensitivity of the results for the doubly polarized total cross sections σ^P (upper left panel) and σ^A (upper right panel), spin asymmetry $\sigma^P - \sigma^A$ (lower left panel), and the GDH integral for the deuteron as a function of the upper integration limit (lower right panel) on the elementary pion photoproduction operator using the CD-Bonn potential [13] for the deuteron wave function. The dashed curve in Fig. 3.3 shows the results of IA* using the MAID-2003 model [6], whereas the dotted (solid) curve shows the results of IA* (IA) using the dressed (bare) electromagnetic multipoles of ELA [5]. As already mentioned, IA* denotes the calculation when the πN-rescattering is included in the elementary reaction.

We find that the doubly polarized total cross sections σ^P and σ^A as well as the spin asymmetry $\sigma^P - \sigma^A$ present qualitative similar behaviors for different elementary operators. One sees that σ^P, σ^A and their difference $\sigma^P - \sigma^A$ have a peak at photon energy of about 300 MeV. The maximum of this peak is greater in σ^P than in σ^A and therefore the spin symmetry $\sigma^P - \sigma^A$ and in turn the GDH integral have positive contributions. It is also clear that the computations with different elementary amplitudes are quite different. For example, at the peak position we obtain larger values using MAID than using ELA. This discrepancy shows up the differences among elementary operators. This means that the doubly polarized total cross sections, spin asymmetry and the GDH integral are sensitive to the choice of the elementary amplitude. The difference between the dotted (dressed ELA) and solid (bare ELA) curves shows the effect of πN-rescattering in the elementary $\gamma N \to \pi^0 N$ amplitude, which is also found to be important.

In what follows, we study the influence of results on the potential model used for the deuteron wave function as displayed in Fig. 3.4. The contribution of $\vec{\gamma}\vec{d} \to \pi^0 d$ to the doubly polarized total cross sections σ^P (upper left panel) and σ^A (upper right panel), spin asymmetry $\sigma^P - \sigma^A$ (lower left panel), and the GDH integral (lower right panel) is depicted in this figure using the bare electromagnetic multipoles of ELA [5] and different realistic models for the deuteron wave function. For the latter, the CD-Bonn [13], Bonn (full model) [14], and Paris [15] potentials are used. The dashed curve in Fig. 3.4 shows the results of IA using the Bonn potential (full model) [14] for the deuteron wave function, whereas the dotted and solid curves show the results of IA using the Paris [15] and the CD-Bonn [13] potentials, respectively.

In general, one sees qualitatively a similar behaviors for the doubly polarized total cross sections and spin asymmetry. The results using various models for the deuteron wave function are different, specially at the peak position. We find that the deuteron wave function of the Paris potential leads to an overall strong reduction of the doubly polarized total cross sections σ^P and σ^A, spin asymmetry $\sigma^P - \sigma^A$ as well as the GDH integral. This means that these observables are also sensitive to the choice of the potential model used for the deuteron wave function. Thus, the process $\vec{\gamma}\vec{d} \to \pi^0 d$ can be used as a test not only of the elementary operator employed but also of the potential model used for the deuteron wave function.

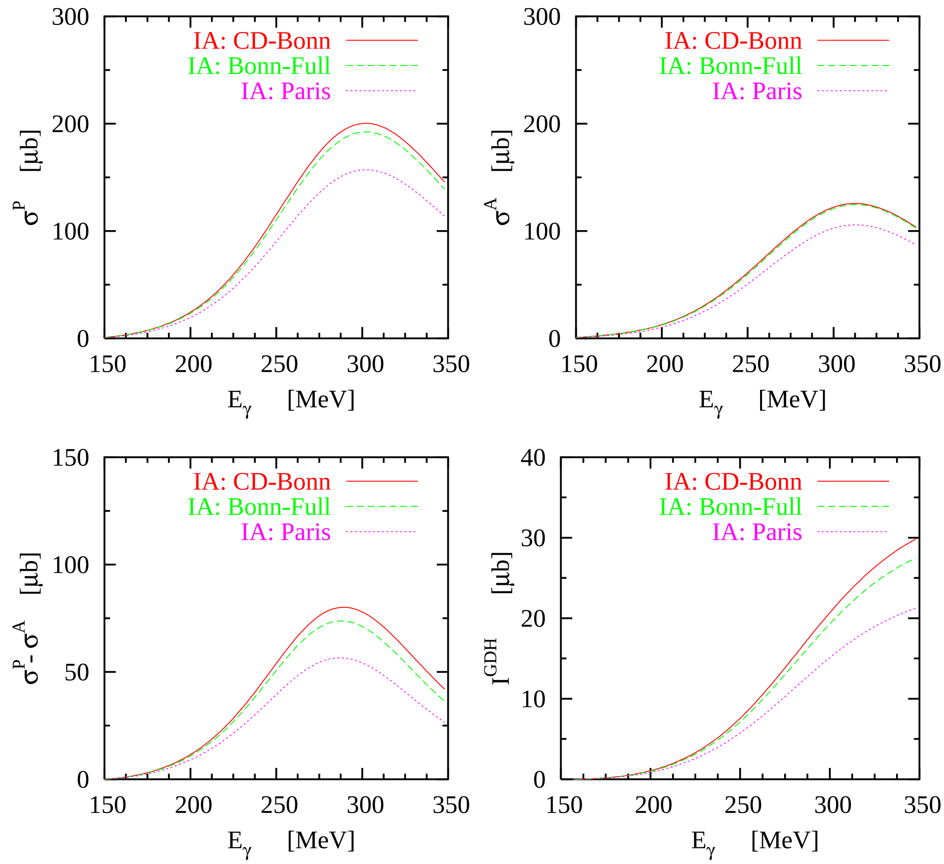

Figure 3.4: (Color online) Contribution of coherent neutral-pion photoproduction to the spin asymmetry and the GDH integral for the deuteron using the bare electromagnetic multipoles of ELA [5] and different realistic models for the deuteron wave function. Curve conventions: solid, results using the CD-Bonn potential [13] for the deuteron wave function; dashed, using the Bonn potential (full model) [14]; dotted, using the Paris potential [15].

The contribution of coherent neutral-pion photoproduction channel to the finite GDH integral for the deuteron up to 350 MeV using different elementary pion photoproduction operators and various potential models for the deuteron wave function are summarized in Table 3.1. Compared the values using MAID to the values using ELA (with bare and dressed multipoles) of a particular potential model for the deuteron wave function, we find a small difference between the computations with the MAID and the bare and dressed multipoles of ELA. On the other hand, a big difference is found between the results with the deuteron wave function from the Bonn and Paris potentials are used. This means that the GDH integral is found to be more sensitive to the deuteron wave function than the

Table 3.1: Contribution of coherent neutral-pion photoproduction from the deuteron to the finite GDH integral using different elementary pion photoproduction operators and various models for the deuteron wave function explicitly integrated up to a photon lab-energy of $E_\gamma = 350$ MeV in μb.

Contribution	CD-Bonn [13]	Bonn [14]	Paris [15]
IA using bare multipoles of ELA [5]	30.09	27.69	21.40
IA* using dressed multipoles of ELA [5]	32.23	29.46	22.85
IA* using MAID-2003 [6]	31.82	29.03	22.48

elementary pion photoproduction amplitude.

A theoretical calculation of the deuteron GDH integral has been performed in [3] by explicit integration up to an energy of 2.2 GeV. This calculation includes deuteron photodisintegration (up to 0.8 GeV), coherent and quasifree single-pion and eta photoproduction (up to 1.5 GeV), and double-pion photoproduction (up to 2.2 GeV). Contribution of various channels to the finite GDH integral was given in said work. For the coherent pion photoproduction channel, the reaction of our interest in the present work, a value of about 90 μb was found [3] by explicit integration up to 350 MeV. Our present model using only the impulse approximation gives about 31 μb for the contribution of the deuteron GDH integral of this channel. The rest should be due to the neglected contributions from pion rescattering and two-body effects in our model. Our goal in the present work is to study the influence of the GDH integral on both the deuteron wave function and the elementary operator on the free nucleon. To the best of our knowledge, this influence was not studied before in the literature.

From the preceding discussion it is apparent that the choice of the elementary pion photoproduction operator and the potential model used for the deuteron wave function have a visible effect on the doubly polarized total cross sections, spin asymmetry and the GDH integral for the deuteron. Summarizing, we can say that the MAID model provides different predictions for polarization observables than the ELA model and that the GDH integral provides an excellent observable to test not only different pion production operators but also various parameterizations of the deuteron wave functions.

4 Conclusions

The main topic of this paper was the investigation of the helicity structure of the partial total cross sections and their contribution to the spin asymmetry and the GDH integral for the deuteron. Contribution from coherent neutral-pion photoproduction channel from the deuteron has been explicitly evaluated in the energy region from near threshold up to photon lab-energy of $E_\gamma = 350$ MeV. For the elementary operator, a realistic effective Lagrangian approach has been used which displays chiral symmetry, gauge invariance, and crossing symmetry, as well as a consistent treatment of the spin-3/2 interaction. The sensitivity to the elementary $\gamma N \to \pi^0 N$ operator and the deuteron wave function of the results has also been investigated.

Within our model, we have found that the doubly polarized total cross sections, spin asymmetry, and GDH integral are more sensitive to the deuteron wave function than the elementary operator. In many cases, the deviation among results obtained using different deuteron wave functions is very large. In view of these results, we conclude that the process $d(\gamma, \pi^0)d$ can serve as a filter for different elementary operators and deuteron wave functions since their predictions provide very different values for observables. We have also evaluated the contribution of $\vec{\gamma}\vec{d} \to \pi^0 d$ to the GDH integral by explicit integration up to 350 MeV. The value $I^{GDH}_{\gamma d \to \pi^0 d}(350 \text{ MeV}) = 30.09$ μb has been computed using the bare electromagnetic multipoles of the ELA model and the deuteron wave function from the CD-Bonn potential. The GDH integral is found to be an excellent observable to discriminate among different deuteron wave functions. We obtain quite different values for the GDH integral for the Paris and the Bonn potentials.

Finally, we would like to point out that future improvements of the present model can be achieved by including pion rescattering and two-body effects. Polarization observables in general constitute more stringent tests for theoretical models due to their sensitivity to small amplitudes. At this point, a much needed measurement on the deuteron spin asymmetries will certainly provide us with an important observable to test our knowledge of the pion photoproduction on the neutron process and, hence, to provide us with valuable information on the neutron spin asymmetry in an indirect way. An independent test within the framework of effective field theory will be also of great interest.

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