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Phase Properties of a Multi-Photon Jaynes-Cummings Model

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Abstract: Phase properties are important in understanding quantum entanglement. Several studies have been carried out on the cases for one and two photon transitions. The effect of multi-photon transition on phase properties in Jaynes-Cummings model coupled with Kerr-medium is studied to further understand phase properties in larger system. This paper develops a model as well as compares the results for different photon transition numbers from one to five photon transition using Pegg-Barnett formalism based measurement. From the developed model, it has been found that the phase state of the quantum system increases during both collapse and revival of photon transition when the Kerr effect increases. This finding shows the behavior of phase properties at higher photon transitions which can benefit further studies in quantum entanglement.

Keywords: Jaynes-Cumming model, Kerr-like medium, phase properties

1 Introduction

Jaynes-Cummings model explains the interaction between the field and the atom. This atom is explained in two levels such as excitation level and ground level. Jaynes-Cummings is a commonly used model for its simplicity and ability to give rise to commensurable Rabi frequencies [3]. Rabi frequencies are used to represent the strength of coupling between light and atomic transition which is proportional to Jaynes-Cummings model.

There have been studies on the phase properties using Jaynes-Cummings model [6]. Further studies have also been conducted to accommodate more factors in the study of phase properties of one and two photons such as with Kerr medium [1] and with Stark shift, and Kerr medium [3]. Kerr medium is the change of refractive index of a material using electric field which is one of the factors affecting the change of phase properties in Jaynes-Cummings model. Besides, Jaynes-Cummings model also provides quantum features which are useful in further understanding of quantum behavior such as squeezing, quantum entanglement, and coherent.

This paper will focus on the multi-photon transition of Jaynes-Cummings model under the influence of Kerr-medium. The phase properties will be measured using Pegg-Barnett Hermitian phase operator formalism. Pegg-Barnett Hermitian phase operator formalism is the mathematical representation of the concept of quantum observable using normalized positive operator to measure the phase shift [2]. Since previous studies have been conducted on one and two-photon transitions, this work moves forward to study on multi-photon transition of Jaynes-Cummings model. Models for one, two, three four and five-photon transitions will be analyzed and compared to see the differences along with the Kerr-medium effect when there is an increase in photon number transition. The organization of this paper includes the introduction section, a section on developing Jaynes-Cummings model for multi-photon transition, a section on the results which include phase probability distribution under the effect of Kerr-medium and finally the conclusion.

2 Model Development

In this paper, Hamiltonian with rotating-wave approximation for Jaynes-Cummings model with Kerr-like medium is considered [4]. This model describes the interaction between the two level atom and the field with the effect of Kerr-medium as below,

$$H=\omega a^t a+\frac{\omega_0}{2}S_z+g(S_+a^m+S_-a^{tm})+\chi a^t 2a^2 \qquad (1)$$

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where ω_0 is the atomic transition frequency and ω cavity field frequency which will have the exact resonance $\Delta = \omega - k\omega_0 = 0$ .S_z, S_+ and S_- are the atomic pseudo spin operator,g is atom field coupling constant and χ is the Kerr-medium. Besides that, a^t and a is the annihilation and creation operators and m represent the number of photon transition.The Hamiltonian model consists of two parts as follows,

$$H = H_0 + H_1 \qquad [H_0, H_1] = 0 \tag{2}$$

where,

$$H_0 = \omega_0(a^t a) + S_z \tag{3}$$
$$H_1 = \Delta S_z + g(S_+ a^m + S_- a^(tm)) + \chi a^t 2a^2 \tag{4}$$

After the factorization of $exp(-iH_0 t)$ from the evolution operator, the quantum state of the system will be as follows,

$$|\Psi(t)\rangle = exp(-iH_1)|\Psi(0)\rangle \tag{5}$$

where, $|\Psi(0)\rangle$ as the initial quantum state.

In this case, the atom is initially in an excited state and the cavity field is in a coherent state such that

$$|\alpha\rangle = \sum_{n=0}^{\infty} Q_n exp(in\varphi \| n\rangle \tag{6}$$

$$Q_n = exp(\frac{-\bar{n}}{2})\frac{\bar{n}^n/2}{\sqrt{n!}} \tag{7}$$

where, $\bar{n} = |\alpha|^2$ is the average photon number of the initial coherent field and φ is the phase angle $\alpha = |\alpha|exp(i\varphi)$. The initial state of the system is in (5). With the combination of (5) and (6), the resulting state will become as follows,

$$|\Psi(0)\rangle = |e\rangle \otimes |\alpha\rangle = \sum_n Q_n exp(in\varphi)|e, n\rangle \tag{8}$$

$$|\Psi(t)\rangle = \sum_{n=0}^{\infty} Q_n e^i n\varphi exp\{-i\chi t[n^2 + (k-1)(n + \frac{k}{2})]\}$$
$$[C_n(t)|g, n+k\rangle + D_n(t)|e, n\rangle] \tag{9}$$
$$C_n(t) = -i \sin 2\beta_n \sin \Omega_n t \tag{10}$$
$$D_n(t) = cos\Omega_n t - icos2\beta_n cos2\Omega_n \tag{11}$$

$$\sin 2\beta_n = \frac{g\sqrt{\frac{(n+k)!}{n!}}}{\Omega_n} \tag{12}$$

$$\cos 2\beta_n = \frac{\Delta/2 + k\chi/2(1-2n-k)}{\Omega_n} \tag{13}$$

$$\Omega_n^2 = [frac\Delta 2 + k\chi/2(1-2n-k)^2 + g^2 \frac{(n+k)!}{n!} \tag{14}$$

where, k is the number of photon transition. The following section illustrates the model for one, two, three, four and five- photon transitions.

3 Phase Properties

Pegg-Barnett Hermitian phase operator formalism is used to evaluate the phase properties in Jaynes-Cummings model. This formalism will be introducing a finite $(s+1)$-dimensional space ψ spanned by the number states $|0\rangle, |1\rangle, |2\rangle, ..., |s\rangle$ [1] . Expectation value in ψ will be calculated and the value of s will tend to infinity. (15) showed the complete orthonormal basis of$(s+1)$ states such that

$$|\theta_m\rangle \equiv \frac{1}{\sqrt{s+1}} \sum_{n=0}^{s} exp(in\theta_m)(|n\rangle \tag{15}$$

where,

$$\theta_m \equiv \theta_0 + \frac{2\pi m}{s+1} \qquad m = 0, 1, , s \tag{16}$$

The value of θ_0 is arbitrary for a particular basis set of $(s+1)$ which is a mutual orthogonal phase states. The Hermitian phase operator is shown in (17) below.

$$\bar{\phi}_\theta \equiv \sum_{m=0}^{s} \theta_m |\theta_m\rangle\langle\theta_m| \tag{17}$$

θ is the dependence on the choice of θ_0. (15) is the eigenstates for phase operator in (17) where the eigenvalues lie betweenθ_0 and $\theta_0 + 2\pi$.Hence, the expectation value will be in a state described by the density operator as shown below.

$$\langle\hat{\phi}_\theta^k\rangle = Tr\{\rho\hat{\phi}_\theta^k\} = \sum_{m=0}^{s} \theta_m^k \langle\theta_m|\rho|\theta_m\rangle \tag{18}$$

$\langle\theta_m|\rho|\theta_m\rangle$ is probability in phase state $|\theta_m\rangle$.Based on (18), as s tends to infinity with density phase states of $s+1/2\pi$, the equation can be written as,

$$\langle\hat{\phi}_\theta^k\rangle = \int_{\theta_0}^{\theta_0+2\pi} \theta^k P(\theta) d\theta \tag{19}$$

and continuum phase distribution can be introduced as below.

$$P(\theta) = \lim_{s\to\infty} \frac{s+1}{2\pi} \langle\theta_m|\rho|\theta_m\rangle$$
$$= \frac{1}{2\pi} \sum_{n,n'=0}^{\infty} \rho(n, n') \exp[-i(n-n')\theta] \tag{20}$$

The continuous phase variable, θ replaces θ_m and $\rho(n, n')$ are the matrix elements of density operator in number state basis. Using the phase state in (9), it can be written in the following form.

$$|\Psi(t)\rangle = |\Psi_g(t)\rangle|g\rangle + |\Psi_e(t)\rangle|e\rangle \tag{21}$$

where,

$$|\Psi(t)_g\rangle = -i\sum_{n=0} Q_n e^i n\varphi \exp\{-i\chi t[n^2+(k+1)(n+\tfrac{k}{2})]\} \sin 2\beta_n \sin \Omega_n t|n+k\rangle \quad (22)$$

$$|\Psi(t)_e\rangle = \sum_{n=0} Q_n e^i n\varphi \exp\{-i\chi t[n^2+(k-1)(n+\tfrac{k}{2})]\} \sin 2\beta_n \sin \Omega_n t|n+k-1\rangle \quad (23)$$

By tracing over the atomic variables the reduced density operator will be

$$\rho(t) = Tr_A|\Psi(t)\rangle\langle\Psi(t)| = |\Psi(t)_g\rangle\langle\Psi_g(t)| + |\Psi_e(t)\rangle\langle\Psi_e(t)| \quad (24)$$

Base on Eq (24), the phase probability distribution is written as

$$P(\theta,t) = \frac{1}{2\pi}(1+2\sum_{m>n} Q_nQ_mA_mn(t)\cos[(m-n)\theta+\chi t(\gamma_m n)] + 2\sum_{m>n} Q_nQ_mB_mn(t)\sin[(m-n)\theta+\chi t(\gamma_m n)]) \quad (25)$$

$$A_mn(t) = \cos(\Omega_m t)\cos(\Omega_n t) + \cos[2(\beta_m-\beta_n)] \sin(\Omega_m t)\sin(\Omega_n t) \quad (26)$$

$$B_mn(t) = \cos(2\beta_n)\cos(\Omega_m t)\sin(\Omega_n t) - \cos(2\beta_m) \cos(\Omega_n t)\sin(\Omega_m t) \quad (27)$$

$$\gamma_{mn} = [m^2+(k-1)(m+\frac{k}{2})] - [n^2+(k-1)(n+\frac{k}{2})] \quad (28)$$

$$\Omega_n^2 = [\frac{\Delta}{2} - \chi(1-2n-k)]^2 + g^2\frac{(n+k)!}{n!} \quad (29)$$

$$\cos 2\beta_n = \frac{\Delta/2 - \chi(1-2n-k)}{\Omega_n} \quad (30)$$

$$\sin 2\beta_n = \frac{g\sqrt{\frac{(n+k)!}{n!}}}{\Omega_n} \quad (31)$$

The following equations show γ for different photon transitions. Different photon transition numbers are considered as follows,

One photon transition

$$\gamma_{mn} = m^2 - n^2 \quad (32)$$

Two photon transition

$$\gamma_{mn} = (m^2+m+1) - (n^2+n+1) \quad (33)$$

Three photon transition

$$\gamma_{mn} = (m^2+2m+3) - (n^2+2n+3) \quad (34)$$

Four photon transition

$$\gamma_{mn} = (m^2+4m+10) - (n^2+4n+10) \quad (35)$$

Five photon transition

$$\gamma_{mn} = (m^2+m4+10) - (n^2+4n+10) \quad (36)$$

4 Analysis

Based on (25), the phase properties are analyzed for one, two, three, four and five photon transition scenarios. Here, it is considered that $|\alpha^2| = 10$ and $\Delta/g = 0$. The variable will be χ/g to see the changes on the phase properties when there is an increase in Kerr effect. Figures 1, 2, 3, 4, and 5 illustrate the phase properties for one, two, three, four and five photon transitions, respectively.

Figure 1 shows that with the increase in Kerr effect, the collapse and revival have been increased and the amplitude of phase probability has been reduced [3]. As a comparison,Figure 2 shows that when the photon transition has increase to two photon, the collapse and revival have been increased with the increase in amplitude of phase probability when the photon transition increases to two photons. It is shown that in Figure 2a that wave of the phase probability had increased compare to Figure 1a. Figure 2 also shown that the collapse and revival is also increase with amplitude compare to Figure 1 when coupling strength of Kerr medium is increase as shown in figure 2b, 2c which compare to figure 1b, 1c [1].

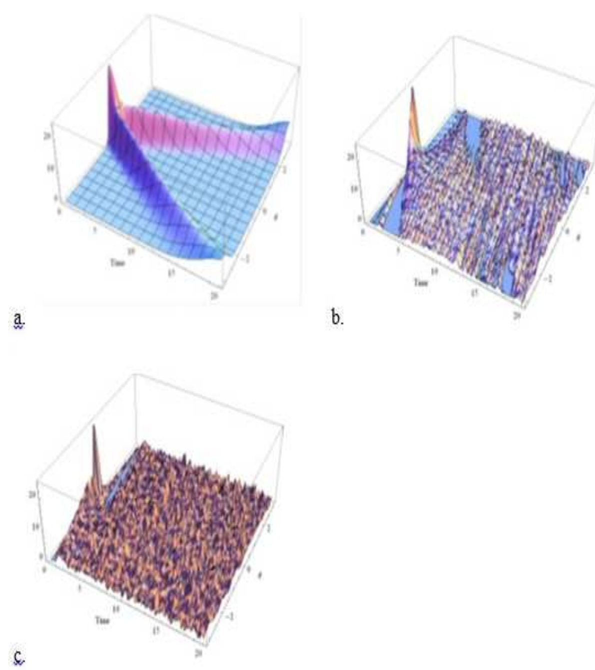

Fig. 1: Phase probability distribution for $k = 1$, $|\alpha^2| = 10$ and $\Delta/g = 0$. The parameter of Kerr affect will be a. $\chi/g = 0$ b. $\chi/g = 0.01$ c. $= \chi/g = 0.1$.

Figure 3 shows the results for the case of three photon transition. Here, the graph shows that initially the peak spreads across the phase angle, but eventually the spread reduces when the quantum system is entangled with the

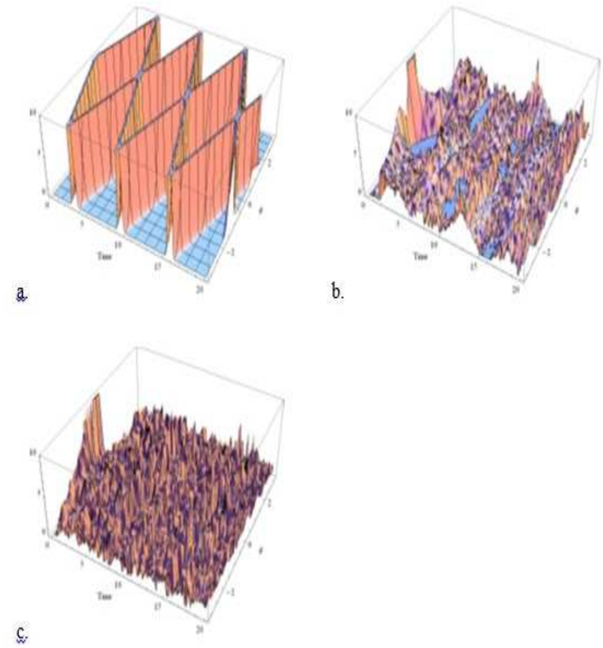

Fig. 2: Phase probability distribution for$k = 2$, $|\alpha^2| = 10$ and $\Delta/g = 0$. The parameter of Kerr affect will be a. $\chi/g = 0$ b. $\chi/g = 0.01$ c. = $\chi/g = 0.1$.

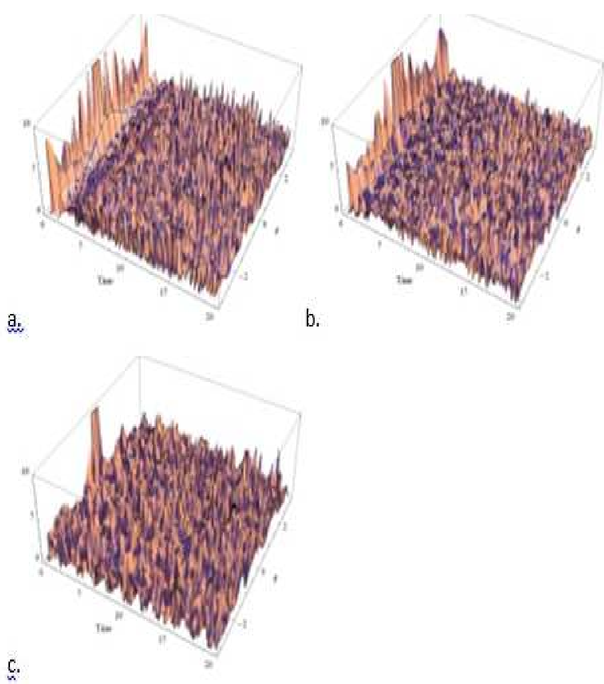

Fig. 3: Phase probability distribution for $k = 3$, $|\alpha^2| = 10$ and $\Delta/g = 0$. The parameter of Kerr affect will be a. $\chi/g = 0$ b. $\chi/g = 0.01$ c. = $\chi/g = 0.1$.

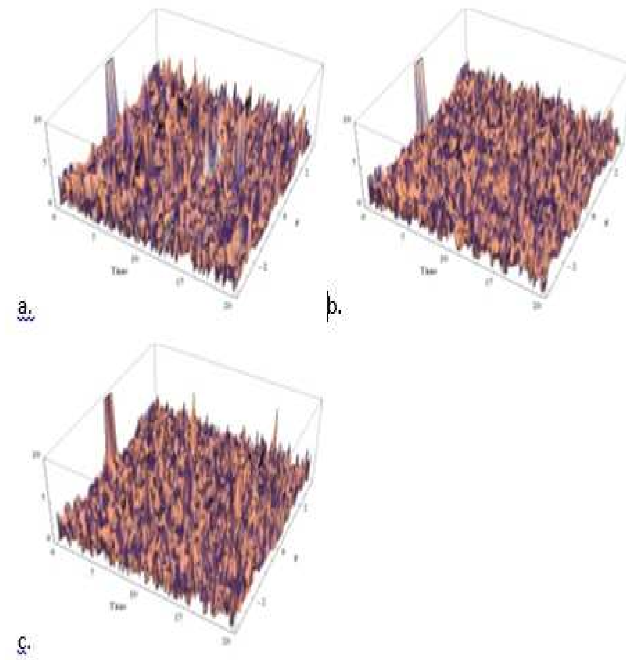

Fig. 4: Phase probability distribution for $k = 4$, $|\alpha^2| = 10$ and $\Delta/g = 0$. The parameter of Kerr affect will be a. $\chi/g = 0$ b. $\chi/g = 0.01$ c. = $\chi/g = 0.1$.

Kerr effect. Besides that, the phase properties also show a wave like pattern from the the time when the Kerr effect starts with a small value . When the Kerr effect increases to a higher value, the wave disappears and is substituted by the collapse revival which manifests a lot of small peaks in Figure 3c.

Figure 4 and 5 show the results when the photon transition increases to four and five, respectively. At these higher photon transitions, the initial spread and the wave like pattern disappear that are found in photon transitions of one, two, and three. The four photon transition case in Figure 4a shows that there are some large peaks at a certain time when there is no Kerr effect. However, the large peak disappears with the addition of a small Kerr effect as shown in Figure 4b. The large peaks appear back with further increase in Kerr effect, but to a lesser extent in compared to the case when there is no Kerr effect as shown in Figure 4c. Similar phenomenon is observed for five photon transition where higher peaks of collapse revivals appear. Although five photon transition shows higher peak, the phase probability amplitude is lesser than that found in other number of photon transition.

5 Conclusion

This paper develops a model for multi-k photon transition and applies Pegg-Barnett Hermitian phase operator

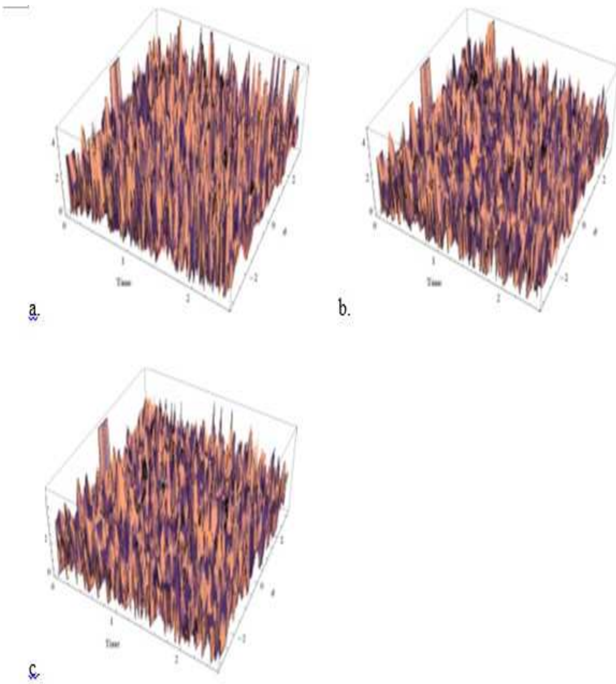

Fig. 5: Phase probability distribution for $k = 5$, $|\alpha^2| = 10$ and $\Delta/g = 0$. The parameter of Kerr affect will be a. $\chi/g = 0$ b. $\chi/g = 0.01$ c. = $\chi/g = 0.1$.

formalism to measure the quantum phase properties. Using phase probability distribution, it is found that phase probability distribution amplitude decreases, whereas the photon transition increases with the Kerr effect. Besides, there exists a wave shaped pattern for one and two photon transition cases. However, such pattern disappears at higher photon transitions resulting in a more frequent fluctuation on the collapse and revival. Hence, the Kerr effect is decreased when the photon transition increases. This result can be useful in further studies for quantum entanglement as this demonstrates the behavior of the quantum state for the two level atom when there is an increase in photon transition.

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