Sohag Journal of Mathematics
An International Journal

http://dx.doi.org/10.12785/sjm/020203

Generalized (1,Θ)-Derivations of Noncommutative Prime Rings

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Received: 16 Jun. 2014, Revised: 27 Dec. 2014, Accepted: 23 Jan. 2015
Published online: 1 May 2015

Abstract: In the present paper ,we study when a noncommutative prime ring and semiprime ring R admitting a generalized (1,Θ)-derivation F satisfying any one of the properties: (i) $F(x)F(y) \mp xy \in Z(R)$ for all x,y∈ I. (ii) $F(x)F(y) \mp yx \in Z(R)$ for all x,y∈ I. (iii) $F(xy) - xoy \in Z(R)$ for all x,y∈ I.

Keywords: Derivations,(Φ,Θ)-derivations, Generalized derivations, Generalized (Φ,Θ)-derivations, Prime ring,Commuting mapping

1 Introduction

Over last few decades, several authors have investigate the relationship between the commutativity of the ring R and certain specific types of derivations of R. The first result in this direction is due to Posner [1] who proved that if R is a prime ring and D a nonzero derivation on R such that $[D(x),x] \in Z(R)$ for all x $\in R$, then R is commutative. In [2] M.N Daif, proved that, let R be a semiprime ring and d a derivation of R with $d^3 \neq 0$.If $[d(x),d(y)] = 0$for all $x,y \in R$,then R contains a non-zero central ideal.H.E.Bell and W.S.Martindal III [3] proved that the center of semiprime ring contains no non-zero nilpotent elements.M.N.Daif and H.E.Bell [4] proved that,let R be a semiprime ring admitting a derivation d for which either $xy + d(xy) = yx + d(yx)$ for all x,y ∈ R or $xy - d(xy) = yx - d(yx)$ for all x,y∈ R,then R is commutative.V.DeFilippis [5] proved that,when R be a prime ring let d a non-zero derivation of R ,U≠(0) a two-sided ideal of R, such that $d([x,y]) = [x,y]$ for all x,y ∈ U ,then R is commutative.A.H.Majeed and Mehsin Jabel [6],them gave some results as,let R be a 2-torsion free semiprime ring and U a non-zero ideal of R.R admitting a non-zero derivation d satisfying $([d(x),d(y)]) = [x,y]$ for all x,y∈ U .If d acts as a homomorphism, then R contains a non-zero central ideal. Mehsin Jabel [7] proved ,let$R$ be a semiprime ring and U be a non-zero ideal of R.If R admits a generalized derivation D associated with a non-zero derivation d such that $D(xy) - xy \in$Z(R) for all x,y∈ U , then R contains a non-zero central ideal,L.Oukhtite and S.Salhi [8] proved, let R be a 2-torsion free σ -prime ring and let d be a non-zero derivation.If $[d(x),x] \in$ Z(R) for all x ∈ R,then R is commutative , where if a prime ring R has an involution σ ,then R is said to be σ-prime if

$aRb = aR\sigma(b) = 0$ implies $a = 0$ or $b = 0$.Obviously,every prime ring equipped with an involution σ is σ-prime,the converse need not be true in general.M.A.Chaudhry and Allah-Bakhsh Thaheem [9] proved,let α,β be epimorphisms of a semiprime ring R such that β is centralizing .If d is a commuting (α,β)-derivation of R ,then$[x,y]d(u) = 0 = d(u)[x,y]$ for all x,y,u∈ R; in particular, d maps R into its center,where let α,β be mappings of R into itself. An additive mapping d of R into itself is called an (α,β) -derivation if $d(xy) = \alpha(x)(y) + d(x)\beta(y)$ for all x,y∈ R.A number of authors have studied the commutativity theorems in prime and semiprime rings admitting derivation and generalized derivation the notion of a generalized derivation of a ring was introduced by Brešar [10] and Hvala [11]. They have studied some properties of such derivations. An additive mapping g of R into itself is called a generalized derivation of R, with associated derivation δ , if there is a derivation δ of R such that $g(xy) = g(x)y + x\delta(y)$ for all x,y∈ R.Chang [12] introduced the notion of a generalized (α,β)-derivation of a ring R and investigated some properties of such derivations. Letα, β be mappings of R into itself. An additive mapping g of R into itself is called a generalized (α,β) -derivation of R, with associated (α,β) -derivation δ, if there exists an (α,β) -derivation

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δ of R such that $g(xy) = g(x)\alpha(y) + \beta(x)\delta(y)$ for all x,y $\in$ R. Obviously this notion covers the notion of a generalized derivation (in case $\alpha = \beta = 1$), notion of a derivation (in case$g = \delta, \alpha = \beta = 1$), notion of a left centralizer (in case $\delta = 0$, $\alpha = 1$), notion of (α,β)-derivation (in case $g = \delta$) and the notion of left α-centralizer (in case $\delta = 0$). Thus it is interesting to investigate properties of this general notion. Recently, Mehsin Jabel [13] proved some results concerning generalized derivations on prime and semiprime rings.In this paper we shall study and investigate some results concerning a generalized $(1,\Theta)$ -derivation on noncommutative prime ring R , where 1 is an identity automorphism of R ,we give some results about that.

2 Preliminaries

Let R be an associative ring with identity and center $Z(R)$.A ring R is said to be prime (resp.semiprime)if $aRb = 0$ implies that either $a = 0$ or $b = 0$ (resp.$aRa = 0$implies that $a = 0$).A prime ring is semiprime but the converse is not true in general.For any $x,y \in$R we shall write $[x,y] = xy - yx$and $xoy = xy + yx$.An additive mapping $d : R \to R$ is called a derivation if $d(xy) = d(x)y + xd(y)$ for all $x,y \in R$,and is said to be n-centralizing on U (resp.n-commuting on U),if $[x^n, d(x)] \in Z(R)$ holds for all x $\in U$(resp.$[x^n, d(x)] = 0$ holds for all x $\in$U , where n be a positive integer. Also,an additive mapping $d : R \to R$ is called a left (right) centralizer if $d(xy) = d(x)y(d(xy) = xd(y))$ for all $x,y \in R$.Let Φ,Θ be endomorphisms of$R$.An additive mapping $d : R \to R$ is called a (Φ,Θ)-derivation if $d(xy) = d(x)\Phi(y) + \Theta(x)d(y)$ for all $x,y \in R$ .An additive mapping $F : R \to R$ is called a generalized (Φ,Θ)-derivation on R if there exists a (Φ,Θ)-derivation $d : R \to R$ such that $F(xy) = F(x)\Phi(y) + \Theta(x)d(y)$ for all $x,y \in R$.We shall call a generalized $(\Phi,1)$-derivation a generalized Φ-derivation, where 1 is the identity automorphism of R.Similarly a generalized $(1,\Theta)$-derivation will be called a generalized Θ-derivation. The following lemmas are necessary for this paper.

Lemma 2.1:[14:Lemma3.1] Let R be a semiprime ring and a R some fixed element.If $a[x,y] = 0 forall x,y \in R$,then there exists an ideal U of R such that a $\in U \subset Z(R)$ holds.

Lemma 2.2:[15,Lemma3] If a prime ring R contains a nonzero commutative right ideal I,then R is commutative.

3 The main results

Theorem 3.1: Let$R$ be a noncommutative prime ring and I a nonzero ideal of R.Suppose that Θ is an automorphism of$R$.If R admits a generalized Θ-derivation F with associated Θ-derivation d such that $F(xy) - xy \in Z(R)$ for all $x,y \in I$,then F is commuting on I.

Proof: By assumption,we have

$F(xy) - xy \in Z(R)$ for all $x,y \in I$. This can be written as $F(x)y + \Theta(x)d(y) - xy \in Z(R)$. Replacing y by yz,we obtain

$$F(x)yz + \Theta(x)d(y)z + \Theta(x)\Theta(y)d(z) - xyz \in Z(R), \quad (3.1)$$

. for all x,y,z $\in I$ Thus,in particular

$$[(F(x)y + \Theta(x)d(y)\Theta(xy)z + \Theta(x)\Theta(y)d(z), z] = 0 \quad (3.2)$$

for all $x,y,z \in I$ Using (3.1) and (3.2),we get

$$[\Theta(x)\Theta(y)d(z), z] = 0 \quad (3.3)$$

for all $x,y,z \in I$ Replacing x by rx in the above expression we obtain $[\Theta(r),z]\Theta(x)\Theta(y)d(z) = 0$ for all $x,y,z \in I, r \in R$. Now replace y by ty, to get $[\Theta(r),z]\Theta(x)\Theta(ty)\ d(z) = 0$ for all $x,y,z \in I, t \in R$. Since Θ is an automorphism of R ,then $[\Theta(r),z]\Theta(x)\Theta(t)\Theta(y)(d(z)) = 0$ for all $x,y,z \in I, t \in R$. That is, $[\Theta(r),z]\Theta(x)t\Theta(y)(d(z)) = 0$ for all $x,y,z \in I, t \in R$. $[\Theta(r),z]\Theta(x)R\Theta(y)(d(z)) = (0)$ for all $x,y,z \in I$. Thus,the primeness of R yields that for each $z \in I$, either $[\Theta(r),z]\ \Theta(x) = 0$ or $\Theta(y)d(z) = 0$. Let $I_1 = z \in I|[\Theta(r),z]\Theta(x) = 0$, for all $x \in I$ and $r \in R$ and $I_2 = z \in I|\Theta(y)d(z) = 0$,for all $x \in I$. Then I_1 and I_2 are two additive subgroups of I whose union is I. Therefore either $I_1 = I$ or $I_2 = I$. If $I_2 = I$ then $\Theta(y)d(z) = 0$ for all $y,z \in I$. Replace y by [y,q] to get $[\Theta(y),\Theta(q)]d(z) = 0$ for all $y,z \in I, q \in R$. Now replacing q by sq ,to get $[\Theta(y),\Theta(sq)]\Theta(r)d(z) = 0$ for all $y,z \in I$,q, $s \in R$. Since Θ is an automorphism of R ,then $[\Theta(y),\Theta(s)\Theta(q)]\Theta(r)d(z) = 0$ for all $y,z \in I, q,s \in R$. $[\Theta(y),\Theta(s)]\Theta(q)\Theta(r)d(z) = 0$ for all $y,z \in I, q,s \in R$. Again since Θ is an automorphism of R,then $[\Theta(y),\Theta(s)]q\Theta(r)d(z) = 0$ for all $y,z \in I, q,s \in R$. i.e.,$[\Theta(y),\Theta(s)]Rd(z) = (0)$,for all $y,z \in I, s \in R$. Again the primeness of R gives that either $[\Theta(y),\Theta(s)] = 0$ or $d(z) = 0$ for all $y \in I, s \in R$. But according to our hypothesis R is noncommutative.So,in the our hand $d(z) = 0$ for all $z \in I$.,this implies that $d = 0$ on R. Then from the main relation,we have $(F(x) - x)y \in Z(R)$,which leads to $[F(x),v] + [x,v] + [y,v] = 0$ for all $x,y \in I, v \in R$. Replacing v and y by x, gives $[F(x),x] = 0$ for all $x \in I$. Thus F is commuting on I,hence we get the required result. One can note that if R admits a generalized Θ-derivation F satisfying $F(xy) + xy \in Z(R)$ for all $x,y \in I$,then the generalized Θ-derivation $(-F)$ also satisfies $(-F)(xy) - xy \in Z(R)$ for all $x,y \in I$. Hence in view of Theorem 3.1 we conclude the following.

Corollary 3.2: Let R be a noncommutative prime ring and I a nonzero ideal of R. Suppose Θ is an automorphism of R. If R admits a generalized Θ -derivation F with associated Θ-derivation d such that $F(xy) + xy \in Z(R)$ for all $x,y \in I$,then F is commuting on I.

Theorem 3.3: Let R be a noncommutative prime ring and I a nonzero ideal of R. SupposeΘ is an automorphism of

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R. If F is a generalized Θ-derivation with associated Θ-derivation d such that $F(xy)-yx \in Z(R)$ for all $x,y \in I$,then

$$(i)d(R) = 0. \quad (1)$$

(ii) F is left centralizer of I.

Proof: (i)For any $x,y \in I$, we have $F(xy)-yx \in Z(R)$. This can be written as $F(x)y+\Theta(x)d(y)-yx \in Z(R)$ for all $x,y \in I$. Substituting xy for x,we obtain

$$F(x)yy+\Theta(x)d(y)y+\Theta(x)\Theta(y)d(y)-yxy \in Z(R), \quad (3.4)$$

for all x,y $\in I$. In particular

$$[(F(x)y+\Theta(x)d(y)\Theta(yx)y+\Theta(x)\Theta(y)d(y),y] = 0, \quad (3.5)$$

for all $x,y \in I$. An application of (3.4) and (3.5) gives

$$[\Theta(x)\Theta(y)d(y),y] = 0 \quad (2)$$

for all $x,y \in I$,i.e.

$$\Theta(x)\Theta(y)[d(y),y]+\Theta(x)[\Theta(y),y]d(y)+[\Theta(x),y]\Theta(y)d(y) = 0 \quad (3)$$

for all $x,y \in I$. Replacing x by z x in (3.6) and using (3.6),we find that

$$[\Theta(z),y]\Theta(x)\Theta(y)d(y) = 0, \quad (3.7)$$

for all $x,y,z \in I$. Replacing x by x r in (3.7), we get

$[\Theta(z),y]\Theta(x)\Theta(r)\Theta(y)d(y) = 0$ for all $x,y,z \in I, r \in R$, i.e. $[\Theta(z),y]\Theta(x)R\Theta(y)d(y) = (0)$ for all $x,y,z \in I$. Thus the primeness of R gives that for each $y \in I$, either

$[\Theta(z),y]\Theta(x) = 0$ or $\Theta(y)d(y) = 0$ for all $y \in I$. The sets y $\in I$ for which these two properties hold, are additive subgroups of I whose union is I. Then either $[\Theta(z),y]\Theta(x) = 0$ or $\Theta(y)d(y) = 0$, for all $x,y,z \in I$. If $\Theta(y)d(y) = 0$, for all $y \in I$, then linearization gives

$$\Theta(x)d(y)+\Theta(y)d(x) = 0, \quad (3.8)$$

for all $x,y \in I$. Replace y by zy to get

$$\Theta(x)d(z)y+\Theta(x)\Theta(z)d(y)+\Theta(z)\Theta(y)d(x) = 0 \quad (3.9)$$

for all $x,y \in I$. Comparing (3.8) and (3.9),we get

$\Theta(x)d(z)y+\Theta(x)\Theta(z)d(y)-\Theta(z)\Theta(x)d(y) = 0$ for all $x,y,z \in I$. That is,

$$\Theta(x)d(z)yr+[\Theta(x),\Theta(z)]d(y)r+[\Theta(x),\Theta(z)]\Theta(y)d(r) = 0 \quad (3.10)$$

for all $x,y,z \in I$, $r \in R$ An application of (3.9) in(3.10) yields that

$$\Theta(x),\Theta(z)]\Theta(y)d(r) = 0 \quad (3.11)$$

for all $x,y,z \in I, r \in R$

Now replace y by y s to get

$[\Theta(x),\Theta(z)]\Theta(y)\Theta(s)d(r) = 0$ for all $x,y,z \in I,r,s \in R$, i.e. $[\Theta(x),\Theta(z)]RIRd(r) = (0)$ for all $x,z \in I,r \in R$.

Thus the primeness of R with noncommutative, lead to $RIRd(r) = (0)$ for all $r \in R$. By using the priemness of R and I is nonzero ideal, we obtain $d(r) = 0$ for all $r \in R$. Thus,we have $d(R) = 0$. (ii)Since F is a generalized Θ-derivation with associated Θ-derivation d,then by using the fact Θ is an automorphism of R and result in (i) , we get the required result.

Arguing as above we can prove the following.

Theorem 3.4: Let R be a noncommutative prime ring and I a nonzero ideal of R.SupposeΘ is an automorphism of R. If F is a generalized Θ-derivation with associated Θ-derivation d is such that $F(xy)+yx \in Z(R)$ for all $x,y \in I$,then

(i)$d(R) = 0$ (ii) F is left centralizer of I.

Theorem 3.5:

Let R be a noncommutative prime ring and I a nonzero ideal of R. SupposeΘ is an automorphism of$R$. If R admits a generalized Θ-derivation F with associated nonzero Θ-derivation d such that $F(x)F(y)-xy \in Z(R)$ for all $x,y \in I$, then either $d(R) = 0$ or F is commuting of I.

Proof: By assumption we have $F(x)F(y)-xy \in Z(R)$ for all $x,y \in I$. Replacing y by yr, we find that $(F(x)F(y)-xy)r+F(x)\theta(y)d(r) \in Z(R)$,for all $x,y \in I, r \in R$. (3.12) This implies that

$$[F(x)\Theta(y)d(r),r] = 0, \quad (3.13)$$

for all $x,y \in I,r \in R$. This can be rewritten as

$$F(x)[\Theta(y)d(r),r]+[F(x),r]\Theta(y)d(r) = 0 \quad (3.14)$$

for all $x,y \in I,r \in R$. Substituting $(\Theta^{-1}(F(x)))$ y for y in (3.14) and using (3.13), we find that

$$[F(x),r]F(x)\Theta(y)d(r) = 0, \quad (3.15)$$

for all $x,y \in I,r \in R$. That is, $[F(x),r]F(x)R\Theta(y)d(r) = (0)$. Thus for each $r \in R$ the primeness of R forces that either

$[F(x),r]F(x) = 0$ or $\Theta(y)d(r) = 0$. The sets of all $r \in R$ for which these two properties hold form additive subgroups of R whose union is I. Hence either $[F(x),r]F(x) = 0$ or $\Theta(y)d(r) = 0$ for all $x,y \in I$ and $r \in R$.

If $\Theta(y)d(r) = 0$ then replace y by y s,to obtain $\Theta(y)\Theta(s)d(r) = 0$ for all $y \in I$ and $r,s \in R$, i.e. $\Theta(y)$Rd(r)=(0) for all $r \in R$ and $y \in I$. Since I is a nonzero ideal of R and R is prime, the above relation yields that $d(r) = 0$ for all $r \in R$. Thus, we get $d(R) = 0$. If $[F(x),r]F(x) = 0$ for all $x \in I$ and $r \in R$. Substituting r by s r and using this we find that $[F(x),r]RF(x) = (0)$ for all $x \in I$ and $r \in R$. The primeness of R implies that for each $x \in I$, $F(x) = 0$, which gives

$[F(x),x] = 0$ for all $x \in I$. i.e. F is commuting on I. or $[F(x),r] = 0$ for all $x \in I,r \in R$. This meaning, we get the required result. Using the same arguments we can prove the following.

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Theorem 3.6 : Let R be a noncommutative prime ring and I a nonzero ideal of R.Suppose Θ is an automorphism of R.If R admits a generalized Θ-derivation F with associated Θ-derivation d such that $F(x)F(y)+xy \in Z(R)$ for all $x,y \in I$, then $d(R)=0$ or F is commuting of I.

Theorem 3.7: Let R be a noncommutative semiprime ring and I a nonzero ideal of R. Suppose thatΘ is an automorphism of R. If R admits a generalized Θ-derivation F with associated Θ-derivation d such that $F(xy) - xoy \in Z(R)$ for all $x,y \in I$,then R contains nonzero central ideal.

Proof: By assumption, we have $F(xy) - xoy \in Z(R)$ for all $x,y \in I$. This can be written as $F(x)y + \Theta(x)d(y) - xoy \in Z(R)$. Replacing y by y z, we obtain

$$F(x)yz + \Theta(x)d(y)z + \Theta(x)\Theta(y)d(z) - xoyz \in Z(R), \tag{3.16}$$

for all $x,y,z \in I$. Replacing x by xy in (3.16),we obtain

$$F(xy)yz + \Theta(xy)d(y)z + \Theta(xy)\Theta(y)d(z) - xyoyz \in Z(R) \tag{3.17}$$

, for all $x,y,z \in I$.

Then form (3.17),we get $(F(xy) - xy)yz + \Theta(xy)d(y)z + \Theta(xy)\Theta(y)d(z) - yzxy \in Z(R)$,for all $x,y,z \in I$. Then $(F(xy) - xy - xy + xy)yz + \Theta(xy)d(y)z + \Theta(xy)\Theta(y)d(z) - yzxy \in Z(R)$, for all $x,y,z \in I$.

$(F(xy) - (xoy) + xy)yz + \Theta(xy)d(y)z + \Theta(xy)\Theta(y)d(z) - yzxy \in Z(R)$, for all $x,y,z \in I$.

$$(F(xy) - (xoy))yz + xyyz + \Theta(xy)d(y)z + \Theta(xy)\Theta(y)d(z) - yzxy \in Z(R), \tag{3.18}$$

for all $x,y,z \in I$. Since $F(xy) - xoy \in Z(R)$ for all $x,y \in I$, then from above equation(3.18) ,we arrived at

$$yz(F(xy) - (xoy)) + xyyz + \Theta(xy)d(y)z + \Theta(xy)\Theta(y)d(z) - yzxy \in Z(R), \tag{3.19}$$

for all $x,y,z \in I$. Subtracting (3.19) and(3.18),we obtain $[F(xy) - (xoy), yz] = 0$ for all $x,y,z \in I$. Then

$[F(xy), yz] - [(xoy), yz] = 0$ for all $x,y,z \in I$. $[F(x)y + \Theta(x)d(y), yz] - [(xoy), yz] = 0$ for all $x,y,z \in I$. Replacing y and z by x,we get $[F(x)x + \Theta(x)d(x), x^2] = 0$ for all $x,y,z \in I$.

$$[F(x), x^2]x + \Theta(x)[d(x), x^2] + [\Theta(x), x^2]d(x) = 0 \tag{3.20}$$

for all $x \in I$.

From the relation $F(xy) - xoy \in Z(R)$ for all $x,y \in I$, after replacing r and y by x,we get, $[F(x^2), x^2] = 0$ for all $x \in I$. Replacing x by x^2 in above equation (3.20),with using the relation $[F(x^2), x^2] = 0$ for all $x \in I$, we obtain

$$x^2[d(x^2), x^4] = 0 \tag{3.21}$$

for all $x \in I$.

Right-multiplying (3.21) by [s,t],we arrived to $x^2[d(x^2), x^4][s,t] = 0$ for all $x \in I, s,t \in R$. We set $a = x^2[d(x^2), x^4], a \in R$,then $a[s,t] = 0$ for all $y,z \in R$. Apply Lemma 2.1,we get$R$ contains nonzero central ideal. This meaning, we get the required result. By using the result in Theorem(3.7) with apply Lemma(2.2),we can prove the following theorem.

Theorem 3.8: Let R be a noncommutative prime ring and I a nonzero ideal of$R$. Suppose that Θ is an automorphism of R.If R admits a generalized Θ-derivation F with associated Θ-derivation d such that $F(xy) - xoy \in Z(R)$for all $x,y \in I$, then R is commutative ring.

Acknowledgments

The author would like to thank the referee for her/his useful comments.

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