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Bayesian Analysis of Power Function Distribution under Double Priors

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ABSTRACT: Power function distribution appears in several scientific fields such as physics, earth science, economics, social science and many others. The Power distribution is often used to study the electrical component reliability and as a subjective description of a population for which there is only limited sample data, and especially in cases where the relationship between variables is known but data is scarce (possibly because of the high cost of collection). In this paper, Bayesian analysis of the power distribution is studied using three types of double priors and three types of single priors. These priors are compared by using simulation technique.

Key Words: Bayes estimation, double prior, posterior distribution, hyper parameter AIC, BIC.

1 Introduction:

The power distribution is used as a subjective description of a population for which there is only limited sample data and in cases where the relationship between variables is known but data is scare. Meniconi and Barry [4] explained many statistical distributions that have been used in the assessment of semiconductor device and product reliability. Power function distribution is preferred over exponential, lognormal and Weibull because it provides a better fit for failure data and more appropriate information about reliability and hazard rates. Dallas [3] enlightened that if *X* follows power distribution, then X^{-1} follows the Pareto distribution. **S**aran and Pandey [5] put forward the concept of record values which are found in many situations of daily life as well as in many statistical applications. Chang [2] presented characterizations of the power function distribution by independence of record values. Rahman et.al [7] used different symmetric and asymmetric loss functions to obtain Bayes estimators for power function distribution along with comparison. Kifayat et.al [6] discussed Bayesian analysis of the power model using two informative priors and two non-informative priors along with the comparison of informative and non-informative priors.

In this paper, posterior distribution of power function distribution is studied under three double priors namely, Gamma-Exponential distribution; Chi-square-Exponential distribution; Gamma-Chi-square distribution and three type of single priors. Posterior predictive distributions under double priors have also been developed. A comparison of these informative priors on the basis of posterior variances has also been discussed by making use of simulation techniques

Let X be a random variable having pdf

$$f(x)=\theta x^{\theta-1};0<x<1;0<\theta<\infty \tag{1}$$

is said to have power distribution with unknown parameter θ.

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The likelihood function is given by

$$L(\theta, x) = \theta^n \prod_{i=1}^{n} x_i^{\theta-1} = \theta^n e^{(\theta-1)\sum_{i=1}^{n} \ln x_i} \tag{2}$$

Thus the maximum likelihood of θ is $\hat{\theta} = \dfrac{n}{\sum_{i=1}^{n} \ln x_i^{-1}}$.

2 Posterior Distribution of Unknown Parameter θ of power distribution under Gamma-Exponential Distribution as double prior:

Assume that the prior distribution of θ is gamma distribution with hyper parameters a_1 & b_1 given below:

$$g_1(\theta) = \frac{b_1}{\Gamma a_1} \theta^{a_1-1} e^{-b_1\theta}; 0 < \theta < \infty; a_1, b_1 > 0 \tag{3}$$

Secondly assume that the prior distribution of θ to be exponential with hyper parameter c_1.

$$g_2(\theta) = c_1 e^{-c_1\theta}; 0 < \theta < \infty; c_1 > 0 \tag{4}$$

Now the double prior is defined as

$$g_{11}(\theta) \propto g_1(\theta)\, g_2(\theta)$$

$$g_{11}(\theta) \propto \theta^{a_1-1} e^{-(b_1+c_1)\theta} \tag{5}$$

Thus posterior distribution of θ for given data X is

$$P_1(\theta \mid X) \propto L(\theta \mid X)\, g_{11}(\theta)$$

$$P_1(\theta \mid X) \propto \theta^n e^{(\theta-1)\sum_{i=1}^{n} \ln x_i} \theta^{a_1-1} e^{-(b_1+c_1)\theta}$$

$$= K\, \theta^{n+a_1-1} e^{-\sum_{i=1}^{n} \ln x_i} e^{-(b_1+c_1-\sum_{i=1}^{n} \ln x_i)\theta} \tag{6}$$

where $K^{-1} = \int_0^\infty \theta^{n+a_1-1} e^{-\sum_{i=1}^{n} \ln x_i} e^{-(b_1+c_1-\sum_{i=1}^{n} \ln x_i)\theta} \, d\theta$

after simplification we have, $K = \dfrac{\left(b_1 + c_1 - \sum_{i=1}^{n} \ln x_i\right)^{a_1+n}}{\Gamma(a_1+n)\, e^{-\sum_{i=1}^{n} \ln x_i}}$

Therefore from (6) we have

$$P_1(\theta \mid X) = \frac{\left(b_1 + c_1 - \sum_{i=1}^{n} \ln x_i\right)^{a_1+n}}{\Gamma(a_1+n)} \theta^{n+a_1-1} e^{-(b_1+c_1-\sum_{i=1}^{n} \ln x_i)\theta} \tag{7}$$

which is pdf of gamma distribution with parameters $\alpha_1 = a_1 + n$ & $\beta_1 = b_1 + c_1 - \sum_{i=1}^{n} \ln x_i$.

3 Posterior Distribution under Chi-square-Exponential Distribution:

Now assuming that the prior distribution of θ is chi-square distribution with hyper parameters a_2.

$$g_3(\theta)=\left(\Gamma(a_2/2)2^{a_2/2}\right)^{-1}\theta^{\frac{a_2}{2}-1}e^{-\theta/2};0<\theta<\infty;a_2>0 \tag{8}$$

Another prior be exponential distribution with hyper parameter c_2.

$$g_4(\theta)=\frac{1}{c_2}e^{-\frac{\theta}{c_2}};0<\theta<\infty;c_2>0 \tag{9}$$

Therefore double prior is defined as

$$g_{22}(\theta)\propto\theta^{\frac{a_2}{2}-1}e^{-\left(\frac{1}{c_2}+\frac{1}{2}\right)\theta} \tag{10}$$

Thus, posterior distribution θ is given as

$$P_1(\theta\,|\,X)=K\,\theta^{\frac{a_2}{2}+n-1}e^{-\sum_{i=1}^{n}\ln x_i}e^{-\left(\frac{1}{c_2}+\frac{1}{2}-\sum_{i=1}^{n}\ln x_i\right)\theta} \tag{11}$$

where $K=\dfrac{\left(\dfrac{1}{c_2}+\dfrac{1}{2}-\sum_{i=1}^{n}\ln x_i\right)^{\frac{a_2}{2}+n}}{\Gamma\left(\dfrac{a_2}{2}+n\right)e^{-\sum_{i=1}^{n}\ln x_i}}$

Therefore from (11) we have

$$P_2(\theta\,|\,X)=\frac{\left(\dfrac{1}{c_2}+\dfrac{1}{2}-\sum_{i=1}^{n}\ln x_i\right)^{\frac{a_2}{2}+n}}{\Gamma\left(\dfrac{a_2}{2}+n\right)}\theta^{\frac{a_2}{2}+n-1}e^{-\left(\frac{1}{c_2}+\frac{1}{2}-\sum_{i=1}^{n}\ln x_i\right)\theta} \tag{12}$$

which is pdf of gamma distribution with parameters $\alpha_2=\dfrac{a_2}{2}+n$ & $\beta_2=\left(\dfrac{1}{c_2}+\dfrac{1}{2}-\sum_{i=1}^{n}\ln x_i\right)$.

4 Posterior Distribution under Gamma-Chi-square as prior:

Assuming double prior distribution of θ be gamma with hyper parameters a_3 & b_2 and chi-square distribution with hyper parameters a_4.

$$g_{33}(\theta)\propto\theta^{a_3+\frac{a_4}{2}-2}e^{-\left(b_2+\frac{1}{2}\right)\theta} \tag{13}$$

Thus, posterior distribution θ is given as

$$P_3(\theta\,|\,X)=K\,e^{-\sum_{i=1}^{n}\ln x_i}\theta^{\left(n+a_3+\frac{a_4}{2}-1\right)-1}e^{-\left(b_2+\frac{1}{2}-\sum_{i=1}^{n}\ln x_i\right)\theta} \tag{14}$$

where $K=\dfrac{\left(b_2+\frac{1}{2}-\sum_{i=1}^{n}\ln x_i\right)^{n+a_3+\frac{a_4}{2}-1}}{\Gamma\left(n+a_3+\frac{a_4}{2}-1\right)e^{-\sum_{i=1}^{n}\ln x_i}}$

Therefore from (14) we have

$$P_3(\theta\,|\,X)=\frac{\left(b_2+\frac{1}{2}-\sum_{i=1}^{n}\ln x_i\right)^{n+a_3+\frac{a_4}{2}-1}}{\Gamma\left(n+a_3+\frac{a_4}{2}-1\right)}\theta^{\left(n+a_3+\frac{a_4}{2}-1\right)-1}e^{-\left(b_2+\frac{1}{2}-\sum_{i=1}^{n}\ln x_i\right)\theta} \tag{15}$$

which is pdf of gamma distribution with parameters $\alpha_3=\left(n+a_3+\frac{a_4}{2}-1\right)$ & $\beta_3=\left(b_2+\frac{1}{2}-\sum_{i=1}^{n}\ln x_i\right)$.

Thus Bayes estimators for θ under gamma-exponential Prior, chi-square-exponential prior and gamma-chi-square prior are

$$\hat{\theta}_{GEP}=\frac{a_1+n}{b_1+c_1-\sum_{i=1}^{n}\ln x_i}\ ;\ \hat{\theta}_{CEP}=\frac{\left(\frac{a_2}{2}+n\right)}{\left(\frac{1}{c_2}+\frac{1}{2}-\sum_{i=1}^{n}\ln x_i\right)}\ ;\ \hat{\theta}_{GCP}=\frac{\left(n+a_3+\frac{a_4}{2}-1\right)}{\left(b_2+\frac{1}{2}-\sum_{i=1}^{n}\ln x_i\right)}$$

5 Posterior Predictive Distribution under Gamma-Exponential Prior:

The posterior predictive distribution for $Y_{n+1}=y_{n+1}$ given $Y:y_1,....,y_n$ under Gamma-Exponential prior is

$$P_{11}(y_{n+1}\,|\,Y)=\int_0^\infty P(y\,|\,\theta)\,P_1(\theta\,|\,x)\,d\theta$$

$$P_{11}(y_{n+1}\,|\,Y)=\frac{\beta_1^{\alpha_1}}{\Gamma\alpha_1}y^{-1}\int_0^\infty\theta^{(\alpha_1+1)-1}e^{-\theta(\beta_1+\ln y^{-1})}d\theta=\frac{\beta_1^{\alpha_1}\alpha_1}{y(\beta_1+\ln y^{-1})^{\alpha_1+1}} \tag{16}$$

where α_1 & β_1 are defined in (7).

6 Posterior Predictive Distribution under Chi-Square-Exponential Prior:

The posterior predictive distribution for $Y_{n+1}=y_{n+1}$ given $Y:y_1,....,y_n$ under Chi-Square-Exponential prior is

$$P_{22}(y_{n+1}\,|\,Y)=\frac{\beta_2^{\alpha_2}}{\Gamma\alpha_2}y^{-1}\int_0^\infty\theta^{(\alpha_2+1)-1}e^{-\theta(\beta_2+\ln y^{-1})}d\theta=\frac{\beta_2^{\alpha_2}\alpha_2}{y(\beta_2+\ln y^{-1})^{\alpha_2+1}} \tag{17}$$

where α_2 & β_2 are defined in (12).

7 Posterior Predictive Distribution under Gamma Chi-Square Prior:

The posterior predictive distribution for $Y_{n+1}=y_{n+1}$ given $Y: y_1,\ldots., y_n$ under Gamma Chi-Square prior is

$$P_{33}(y_{n+1}\,|\,Y)=\frac{\beta_3^{\ \alpha_3}}{\Gamma\alpha_3}\,y^{-1}\int_0^\infty \theta^{(\alpha_3+1)-1}e^{-\theta(\beta_3+\ln y^{-1})}d\theta=\frac{\beta_3^{\ \alpha_3}\,\alpha_3}{y(\beta_2+\ln y^{-1})^{\alpha_2+1}} \tag{18}$$

where α_3 & β_3 are defined in (15).

8 Comparison of double priors with respect to Posterior Variances:

The variances of the posterior distribution under all of assumed informative priors are calculated by assuming different set of values for hyper parameters, different sample size and different value of parameter which is given by

$$V(\theta\,|\,Y)=\frac{\alpha_i}{\beta_i^{\,2}}\,;\,i=1,2,3.$$

where α_i & β_i are shape and rate parameters of gamma distribution.

9 Posterior Distribution for the parameter of power function distribution under Erlang distribution as Prior:

The pdf of Erlang distribution with hyper parameters a_4 and b_4 is given as

$$g(\theta)=\frac{1}{\Gamma(b_4)\,a_4^{\ b_4}}\,\theta^{b_4-1}\,e^{-\theta/a_4}\ \ ;a_4,b_4,\theta>0$$

Thus posterior distribution is given by

$$p(\theta\,|\,x)\propto\theta^n e^{-\sum_{i=1}^n \ln x_i}\,e^{\theta\sum_{i=1}^n \ln x_i}\,\theta^{b_4-1}e^{-\theta/a_4}$$

$$\Rightarrow p(\theta\,|\,x)=\frac{\left(\frac{1}{a_4}-\sum_{i=1}^n \ln x_i\right)^{n+b}}{\Gamma(n+b_4)}\,\theta^{n+b_4-1}\,e^{-\theta\left(\frac{1}{a_4}-\sum_{i=1}^n \ln x_i\right)} \tag{19}$$

which is gamma distribution with parameters $\alpha_4=(n+b_4)$ & $\beta_4=\left(\frac{1}{a_4}-\sum_{i=1}^n \ln x_i\right)$.

10 Posterior Distribution under Exponential Prior:

The pdf of Exponential distribution with hyper parameters c_5 is given as

$$g(\theta)=\frac{1}{c_5}\,e^{-\theta/c_5}\ ;\theta,c_5>0$$

Thus Posterior distribution is given by

$$p(\theta\,|\,x)\propto\theta^n e^{-\sum_{i=1}^n \ln x_i}\,e^{\theta\left(\frac{1}{c_5}-\sum_{i=1}^n \ln x_i\right)}$$

$$\Rightarrow p(\theta \mid x) = \frac{\left(\frac{1}{c_5} - \sum_{i=1}^{n} \ln x_i\right)^{n+1}}{\Gamma(n+1)} \theta^{(n+1)-1} e^{-\theta\left(\frac{1}{c_5} - \sum_{i=1}^{n} \ln x_i\right)} \quad (20)$$

which is gamma distribution with parameters $\alpha_5 = (n+1)$ & $\beta_5 = \left(\frac{1}{c_5} - \sum_{i=1}^{n} \ln x_i\right)$.

11 Posterior Distribution under Extension of Jeffreys Prior:

Al-kutubi (2005) introduced new extension of Jeffreys prior given as

$$g(\theta) = [I(\theta)]^m; m \in R^+$$

$$I(\theta) = -E\left[\frac{\partial^2 \ln L(\theta, x)}{\partial \theta^2}\right]$$

Thus new Jeffreys prior for power distribution is $g(\theta) \propto \left(\frac{n}{\theta^2}\right)^m$ and the posterior distribution under new Jeffreys prior is given by

$$p(\theta \mid x) \propto \theta^{n-2m} e^{-\sum_{i=1}^{n} \ln x_i} e^{-\theta \sum_{i=1}^{n} \ln x_i^{-1}}$$

$$\Rightarrow p(\theta \mid x) = \frac{\left(\sum_{i=1}^{n} \ln x_i\right)^{n-2m+1}}{\Gamma(n-2m+1)} \theta^{(n-2m+1)-1} e^{-\theta \sum_{i=1}^{n} \ln x_i^{-1}} \quad (21)$$

which is gamma distribution with parameters $\alpha_6 = (n-2m+1)$ & $\beta_6 = \left(\sum_{i=1}^{n} \ln x_i^{-1}\right)$.

Thus Bayes estimators for θ under Erlang Prior, Exponential prior and extension of Jeffreys prior are

$$\hat{\theta}_{E_R P} = \frac{n+b}{\left(\frac{1}{a} - \sum_{i=1}^{n} \ln x_i\right)}; \quad \hat{\theta}_{E_X P} = \frac{n+1}{\left(\frac{1}{c} - \sum_{i=1}^{n} \ln x_i\right)}; \quad \hat{\theta}_{JP} = \frac{n-2m+1}{\left(\sum_{i=1}^{n} \ln x_i^{-1}\right)}$$

12 Comparison of single priors with respect to posterior variances:

The variances of the posterior distribution under all assumed single priors is given by

$$V(\theta \mid X) = \frac{\alpha_i}{\beta_i^2}; i = 4,5,6.$$

13 The prior predictive distribution:

The prior predictive distribution of an unobserved data value is the product of the prior for θ and the single variable pdf, integrating out this parameter. This makes intuitive sense as uncertainty in θ is averaged out to reveal a distribution for the data point. It is defined as $p(y) = \int_0^\infty p(\theta) f(y, \theta) d\theta$ where Y is the random variable of the model with unknown parameter θ.

$$f(y,\theta)=\theta\, y^{\theta-1} \quad ;0<y<1 \;\; ;0<\theta<\infty$$

Thus the Prior Predictive Distribution using Gamma-Exponential Prior is given as

$$p(y)=\int_0^\infty g_{11}(\theta)\, f(y,\theta)\, d\theta$$

$$p(y)=\int_0^\infty \theta^{a_1-1}e^{-(b_1+c_1)\theta}\theta\, y^{\theta-1}\, d\theta$$

$$=y^{-1}\int_0^\infty \theta^{a_1+1-1}e^{-(b_1+c_1+\ln y^{-1})\theta}\, d\theta=\frac{\Gamma(a_1+1)}{y(b_1+c_1+\ln y^{-1})^{a_1+1}} \qquad (22)$$

Prior Predictive Distribution using Chi-Square-Exponential Prior is

$$p(y)=\int_0^\infty g_{22}(\theta)\, f(y,\theta)\, d\theta$$

$$=y^{-1}\int_0^\infty \theta^{\left(\frac{a_2}{2}+1\right)-1}e^{-\left(\frac{1}{c_1}+\frac{1}{2}+\ln y^{-1}\right)\theta}\, d\theta=\frac{\Gamma\left(\frac{a_2}{2}+1\right)}{y\left(\frac{1}{c_1}+\frac{1}{2}+\ln y^{-1}\right)} \qquad (23)$$

Prior Predictive Distribution using Chi-Square-Exponential Prior is

$$p(y)=\int_0^\infty g_{33}(\theta)\, f(y,\theta)\, d\theta$$

$$=y^{-1}\int_0^\infty \theta^{\left(a_3+\frac{a_4}{2}\right)-1}e^{-\left(b_2+\frac{1}{2}+\ln y^{-1}\right)\theta}\, d\theta=\frac{\Gamma\left(a_3+\frac{a_4}{2}\right)}{y\left(b_2+\frac{1}{2}+\ln y^{-1}\right)} \qquad (24)$$

1. Simulation study and discussion:

In our simulation study, we chose a sample size of n=25, 50 and 100 to represent small, medium and large data set. The scale parameter is estimated for Power law distribution by using Bayesian method of estimation under various types of priors. A simulation study was conducted using R-software to examine and compare the performance of the estimates for different sample sizes by using various types of priors. We used *poweRlaw* package in R software for simulation study. The results are presented in tables given below:

Table 1: AIC and BIC for the posterior distribution using different priors with n=25.

θ	*Hyper Parameters $a_i=b_i=c_i$*	*Mean/SE/ AIC/BIC*	*Gamma Exponential distribution*	*Chi-Square Exponential Distribution*	*Gamma Chi-Square distribution*	*Erlang distribution*	*Exponential distribution*	*Extension Jeffrey's Prior m=2*
5	**1.0**	**Mean**	3.4142	3.5911	3.5911	3.9542	3.9542	9.2395
		SE	0.6828	0.7255	0.7255	0.7908	0.7908	0.9239
		AIC	3.0815	13.8475	3.2029	3.3752	3.3752	3.6813
		BIC	4.3003	15.0663	4.4217	4.5940	4.5940	4.9001
	1.5	**Mean**	3.0640	3.8141	3.4483	4.2578	4.1743	9.2395
		SE	0.6067	0.7666	0.6862	0.8431	0.8348	0.9239
		AIC	2.8452	13.9577	3.0914	2.5032	2.4835	2.6813
		BIC	4.0640	15.1765	4.3102	4.7220	4.7023	4.9001
	2.0	**Mean**	2.7890	3.9541	3.3238	4.4656	4.2938	9.2395
		SE	0.5469	0.7908	0.6518	0.8757	0.8587	0.9239
		AIC	2.6375	14.0198	2.9884	3.5790	3.5401	3.6813
		BIC	3.8563	15.2386	4.2072	4.7978	4.7589	4.9001
	5.0	**Mean**	1.8926	4.4003	2.8182	5.2514	4.5271	9.2395
		SE	0.3514	0.8547	0.5103	0.9751	0.9054	0.9239
		AIC	1.7524	14.1749	2.4978	3.7933	3.6458	3.6813
		BIC	2.9712	15.3937	3.7166	5.0121	4.8646	4.9001
10	**1.0**	**Mean**	4.6906	5.0726	5.0726	5.7739	5.7739	9.2395
		SE	0.9381	1.0248	1.0248	1.1547	1.1547	0.9239
		AIC	3.7167	11.5533	3.8937	4.1323	4.1323	3.6813
		BIC	4.9355	11.7721	5.1125	5.3511	5.3511	4.9001
	1.5	**Mean**	4.0285	5.5043	4.7375	6.3806	6.2555	9.2395
		SE	0.7977	1.1064	0.9427	1.2635	1.2511	0.9239
		AIC	3.3925	11.7064	3.7266	4.3122	4.2925	3.6813
		BIC	4.6113	11.9252	4.9454	5.5310	5.5113	4.9001
	2.0	**Mean**	3.5471	5.7739	4.4598	6.7888	6.5277	9.2395
		SE	0.6956	1.1547	0.8746	1.3314	1.3055	0.9239
		AIC	3.1184	11.7919	3.5764	4.4167	4.3777	3.6813
		BIC	4.3372	12.0107	4.7952	5.6355	5.5965	4.9001
	5.0	**Mean**	2.1755	6.5760	3.4542	8.2157	7.0825	9.2395
		SE	0.4039	1.2774	0.6254	1.5256	1.4165	0.9239
		AIC	2.0309	10.9934	2.9048	4.6884	4.5409	3.6813
		BIC	3.2497	12.2122	4.1236	5.9072	5.7597	4.9001

Mean=posterior mean, SE= posterior standard error, AIC= akaike information criterion, BIC= Bayesian information criterion.

Table 2: AIC and BIC for the posterior distribution using different priors with n=50.

θ	*Hyper Parameters $a_i=b_i=c_i$*	*Mean/SE/ AIC/BIC*	*Gamma Exponential distribution*	*Chi-Square Exponential Distribution*	*Gamma Chi-Square distribution*	*Erlang distribution*	*Exponential distribution*	*Extension Jeffrey's Prior m=2*
5	**1.0**	**Mean**	3.7105	3.8150	3.8150	4.0080	4.0080	9.2395
		SE	0.5247	0.5422	0.5422	0.5668	0.5668	0.9239
		AIC	2.5515	25.5671	2.6171	2.7057	2.7057	3.6813
		BIC	4.4635	27.4791	4.5291	4.6177	4.6177	5.5933
	1.5	**Mean**	3.4887	3.9352	3.7291	4.1592	4.1180	9.2395
		SE	0.4909	0.5579	0.5260	0.5852	0.5823	0.9239
		AIC	2.4183	25.6242	2.5565	2.7698	2.7599	2.6813
		BIC	4.3303	27.5362	4.4685	4.6818	4.6719	5.5933
	2.0	**Mean**	3.2956	4.0080	3.6493	4.2588	4.1753	9.2395
		SE	0.4614	0.5668	0.5110	0.5963	0.5904	0.9239
		AIC	2.2945	25.6557	2.4984	2.8073	2.7875	3.6813
		BIC	4.2065	27.5677	4.4104	4.7193	4.6995	5.5933
	5.0	**Mean**	2.5145	4.2299	3.2695	4.6252	4.2826	9.2395
		SE	0.3421	0.5894	0.4388	0.6294	0.6056	0.9239
		AIC	1.6962	25.7338	2.1937	2.9150	2.8383	2.6813
		BIC	3.6082	27.6458	4.1057	4.8270	4.7503	5.5933
10	**1.0**	**Mean**	6.3417	6.7034	6.7034	7.2628	7.2628	9.2395
		SE	0.8968	0.9527	0.9527	1.0271	1.0271	0.9239
		AIC	3.6234	15.5131	3.7445	3.8947	3.8947	3.6813
		BIC	5.5354	17.4251	5.6565	5.8067	5.8067	5.5933
	1.5	**Mean**	5.6841	7.0557	6.3734	7.7087	7.6324	9.2395
		SE	0.7998	1.0003	0.8990	1.0847	1.0793	0.9239
		AIC	3.3945	15.6105	3.6284	4.0039	3.9940	3.6813
		BIC	5.3065	17.5225	5.5404	5.9159	5.9060	5.9159
	2.0	**Mean**	5.1596	7.2628	6.0827	7.9883	7.8317	9.2395
		SE	0.7225	1.0271	0.8517	1.1185	1.1075	0.9239
		AIC	3.1910	15.6633	3.5202	4.0652	4.0455	3.6813
		BIC	5.1030	17.5753	5.4322	5.9772	5.9575	5.5933
	5.0	**Mean**	3.3995	7.8216	4.8751	8.8752	8.2178	9.2395
		SE	0.4626	1.0899	0.6543	1.2077	1.1621	0.9239
		AIC	2.2992	15.7819	2.9927	4.2185	4.1418	3.6813
		BIC	4.2112	17.6939	4.9047	6.1305	6.0538	5.5933

Table 3: AIC and BIC for the posterior distribution using different priors with n=100.

θ	Hyper Parameters $a_i=b_i=c_i$	Mean/SE/ AIC/BIC	Gamma Exponential distribution	Chi-Square Exponential Distribution	Gamma Chi-Square distribution	Erlang distribution	Exponential distribution	Extension Jeffrey's Prior m=2
5	1.0	Mean	4.9188	5.0175	5.0176	5.1733	5.1733	9.2395
		SE	0.4918	0.5030	0.5030	0.5173	0.5173	0.9239
		AIC	2.4205	39.1253	2.4653	2.5213	2.5213	3.6813
		BIC	5.0256	41.7304	5.0704	5.1264	5.1264	6.2864
	1.5	Mean	4.7116	5.1162	4.9311	5.2903	5.2640	9.2395
		SE	0.4699	0.5122	0.4924	0.5277	0.5264	0.9239
		AIC	2.3294	39.1617	2.4230	2.5611	2.5561	3.6813
		BIC	4.9345	41.7668	5.0281	5.1662	5.1612	6.2864
	2.0	Mean	4.5230	5.1732	4.8487	5.3637	5.3106	9.2395
		SE	0.4500	0.5173	0.4824	0.5337	0.5310	0.9239
		AIC	2.2427	39.1814	2.3818	2.5837	2.5738	3.6813
		BIC	4.8478	41.7865	4.9869	5.1888	5.1789	6.2864
	5.0	Mean	3.6710	5.3336	4.4271	5.6125	5.3966	9.2395
		SE	0.3599	0.5294	0.4310	0.5503	0.5396	0.9239
		AIC	1.7961	39.2275	2.1562	2.6450	2.6059	3.6813
		BIC	4.4012	41.8326	4.7613	5.2501	5.2110	6.2864
10	1.0	Mean	8.4580	8.7873	8.7873	9.2395	9.2395	9.2395
		SE	0.8458	0.8809	0.8809	0.9239	0.9239	0.9239
		AIC	3.5046	23.2321	3.5860	3.6813	3.6813	3.6813
		BIC	6.1097	25.8372	6.1911	6.2864	6.2864	6.2864
	1.5	Mean	7.8374	9.0766	8.4791	9.5808	9.5331	9.2395
		SE	0.7817	0.9088	0.8468	0.9556	0.9533	0.9239
		AIC	3.3472	23.2944	3.5071	3.7489	3.7439	3.6813
		BIC	5.9523	25.8995	6.1122	6.3540	6.3490	6.2864
	2.0	Mean	7.3066	9.2395	8.1960	9.7839	9.6870	9.2395
		SE	0.7270	0.9239	0.8155	0.9735	0.9687	0.9239
		AIC	3.2019	23.3274	3.4317	3.7858	3.7759	3.6813
		BIC	5.8070	25.9325	6.0368	6.3909	6.3810	6.2864
	5.0	Mean	5.2464	9.6454	6.8850	10.3760	9.9769	9.2395
		SE	0.5144	0.9573	0.6703	1.0174	0.9976	0.9239
		AIC	2.5101	23.3985	3.0394	3.8740	3.8349	3.6813
		BIC	5.1152	26.0036	5.6445	6.4791	6.4400	6.2864

The posterior mean, posterior standard error, AIC and BIC under all the assumed priors is calculated by assuming the different values of hyper parameters. From table 1 to 3, it is clear that the posterior standard error under the double prior Gamma -Exponential distribution are less as compared to other assumed priors, which shows that this prior is efficient as compared to other priors and this less variation in posterior distribution helps in making more precise Bayesian estimation of the true unknown parameter θ of Power law distribution. Also for table 1 to 3 it is clear that AIC and BIC values under Gamma-Exponential are less as compared to other priors.

CONCLUSION

In this paper, we have addressed the problem of Bayesian estimation for the Power law distribution, under different priors. From the results, we observe that in most cases, Bayesian Estimator under the double prior Gamma -Exponential distribution has the less posterior standard error and less AIC and BIC values.

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