Sohag Journal of Mathematics
An International Journal

http://dx.doi.org/10.12785/sjm/020104

Coupled Coincidence Point Results on Partial Metric Spaces

Reza Arab[*]

Department of Mathematics, Sari Branch, Islamic Azad University, Sari, Iran

Received: 7 Jun. 2012, Revised: 21 Sep. 2012, Accepted: 23 Sep. 2012
Published online: 1 Jan. 2015

Abstract: In this paper, we consider a new class of pairs of generalized contractive type mappings defined in partial metric spaces. Some coincidence and common fixed point results for these mapping are presented.

Keywords: Partial metric space , coincidence point, coupled fixed point, common coupled fixed point, Generalized contraction principle.

1 Introduction and Preliminaries

In 1992, Matthews [16,17] introduced the notion of a partial metric space which is a generalized metric space in which each object does not necessarily have to have a zero distance from itself.
First, we start with some preliminaries definitions on the partial metric spaces [1,2,3,4,5,6,8,10,11,12,13,14,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32].

Definition 1.*[16,17] A partial metric on a nonempty set X is a function $p : X\times X \longrightarrow R^+$ such that for all $x,y,z\in X$:*

$(P_1)\, x=y \Leftrightarrow p(x,x)=p(x,y)=p(y,y)$,
$(P_2)\, p(x,x)\leq p(x,y)$,
$(P_3)\, p(x,y)=p(y,x)$,
$(P_4)\, p(x,y)\leq p(x,z)+p(z,y)-p(z,z)$.

A partial metric space is a pair (X,p) such that X is a nonempty set and p is a partial metric on X.

*Remark.*It is clear that, if $p(x,y)=0$, then from (P_1) and (P_2) $x=y$. But if $x=y$, $p(x,y)$ may not be 0.

*Example 1.*Let a function $p : R^+\times R^+ \longrightarrow R^+$ be defined by $p(x,y)=max\{x,y\}$ for any $x,y\in R^+$. Then, (R^+,p) is a partial metric space.

*Example 2.*If $X=\{[a,b] : a,b\in R, a\leq b\}$, then $p : X\times X \longrightarrow R^+$ defined by $p([a,b],[c,d])=max\{b,d\}-min\{a,c\}$ defines a partial metric on X.

Each partial metric p on X generates a T_0 topology τ_p on X which has as a base the family open p-balls $\{B_p(x,\varepsilon) : x\in X, \varepsilon>0\}$, where $B_p(x,\varepsilon)=\{y\in X : p(x,y)<p(x,x)+\varepsilon\}$ for all $x\in X$ and $\varepsilon>0$.
If p is a partial metric on X, then the function $p^s : X\times X \longrightarrow R^+$ given by

$$p^s(x,y)=2p(x,y)-p(x,x)-p(y,y) \tag{1}$$

is a metric on X.

Definition 2.*[16,17]*

(i)*A sequence $\{x_n\}$ in a partial metric space (X,p) converges to a point $x\in X$ if*

$$p(x,x)=\lim_{n\longrightarrow\infty} p(x,x_n),$$

(ii)*a sequence $\{x_n\}$ in a partial metric space (X,p) is called a Cauchy sequence if there exists (and is finite)*

$$\lim_{n,m\longrightarrow\infty} p(x_m,x_n),$$

(iii)*a partial metric space (X,p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges, with respect to τ_p, to a point $x\in X$ such that*

$$p(x,x)=\lim_{n,m\longrightarrow\infty} p(x_m,x_n).$$

*Remark.*It is easy to see that, every closed subset of a complete partial metric space is complete.

[*] Corresponding author e-mail: mathreza.arab@iausari.ac.ir

© 2015 NSP
Natural Sciences Publishing Cor.

Lemma 101[16,17] *Let (X,p) be a partial metric space. Then*

(a) $\{x_n\}$ *is a Cauchy sequence in (X,p) if and only if it is a Cauchy sequence in the metric space (X,p^s).*
(b) *A partial metric space (X,p) is complete if and only if the metric space (X,p^s) is complete. Furthermore,*

$$\lim_{n\longrightarrow\infty} p^s(x_n,x)=0$$

if and only if

$$p(x,x)=\lim_{n\longrightarrow\infty} p(x,x_n)=\lim_{n,m\longrightarrow\infty} p(x_m,x_n).$$

Lemma 102[4] *A mapping $f: X\longrightarrow X$ is said to be continuous at $a\in X$, if for every $\varepsilon>0$, there exists $\delta>0$ such that $f(B(a,\delta))\subset B(f(a),\varepsilon)$.*

The following result is easy to check.

Lemma 103 *Let (X,p) be a partial metric space and $T: X\longrightarrow X$ be a given mapping. Suppose that T is continuous at x_0. Then, for all sequence $\{x_n\}\subseteq X$, if $\{x_n\}$ converges to x_0 in (X,p) implies $\{Tx_n\}$ converges to Tx_0 in (X,p).*

Definition 3.[9] *An element $(x,y)\in X\times X$ is said to be a coupled fixed point of the mapping $T: X\times X\to X$ if*

$$T(x,y)=x \text{ and } T(y,x)=y.$$

Definition 4.[15] *An element $(x,y)\in X\times X$ is called a coupled coincidence point of a mapping $T: X\times X\to X$ and $g: X\longrightarrow X$ if*

$$T(x,y)=gx \text{ and } T(y,x)=gy.$$

Definition 5.[15] *Let X be a non-empty set and $T: X\times X\to X$ and $g: X\longrightarrow X$. We say T and g are commutative if for all $x,y\in X$,*

$$g(T(x,y))=T(gx,gy).$$

H. Aydi[7] obtained the following.

Theorem 1. *Let (X,p) be a complete partial metric space. Suppose that the mapping $T: X\times X\longrightarrow X$ satisfies the following contractive condition for all $x,y,u,v\in X$*

$$p(T(x,y),T(u,v))\le kp(x,u)+lp(y,v), \tag{2}$$

where k and l are nonnegative constants with $k+l<1$. Then, T has a unique coupled fixed point.

The main purpose of this article is to present a generalization of Theorem 1.

2 Existence and uniqueness of coupled coincidence points

In this section, we will prove the existence and uniqueness of the coupled coincidence point. Our first main result is the following:

Theorem 2. *Let (X,p) be a complete partial metric space. Assume there exist $a_1,a_2,a_3\ge 0$ with $2a_1+3a_2+3a_3<2$ and also suppose $T: X\times X\longrightarrow X$ and $g: X\longrightarrow X$ are such that*

$$\begin{aligned} &p(T(x,y),T(u,v))\\ &\le a_1\frac{p(gx,gu)+p(gy,gv)}{2}\\ &+a_2\frac{p(gx,T(x,y))+p(gu,T(u,v))+p(gy,gv)}{2}\\ &+a_3\frac{p(gx,T(u,v))+p(gu,T(x,y))+p(gy,gv)}{2}, \end{aligned} \tag{3}$$

for all $x,y,u,v\in X$. Also Suppose $T(X\times X)\subseteq g(X)$, g is continuous and commutes with T. Then there exist $x,y\in X$ such that

$$gx=T(x,y) \text{ and } gy=T(y,x),$$

that is, T and g have a unique coupled coincidence point.

Proof. Let x_0,y_0 be two arbitrary elements in X. Since $T(X\times X)\subseteq g(X)$, we can choose $x_0,y_0\in X$ such that $gx_1=T(x_0,y_0)$ and $gy_1=T(y_0,x_0)$. Again from $T(X\times X)\subseteq g(X)$ we can choose $x_1,y_1\in X$ such that $gx_2=T(x_1,y_1)$ and $gy_2=T(y_1,x_1)$. Continuing this process, we can construct two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$gx_{n+1}=T(x_n,y_n) \text{ and } gy_{n+1}=T(y_n,x_n) \text{ for all } n\ge 0 \tag{4}$$

Now, let $a=\frac{p(gx_0,gx_1)+p(gy_0,gy_1)}{2}$ and $\lambda=\frac{2(a_1+a_2+a_3)}{2-a_2-a_3}$. Then, by (3), we have

$$\begin{aligned} &p(gx_1,gx_2)=p(T(x_0,y_0),T(x_1,y_1))\\ &\le a_1\frac{p(gx_0,gx_1)+p(gy_0,gy_1)}{2}\\ &\quad+a_2\frac{p(gx_0,T(x_0,y_0)+p(gx_1,T(x_1,y_1))+p(gy_0,gy_1)}{2}\\ &\quad+a_3\frac{p(gx_0,T(x_1,y_1)+p(gx_1,T(x_0,y_0))+p(gy_0,gy_1)}{2}\\ &=a_1\frac{p(gx_0,gx_1)+p(gy_0,gy_1)}{2}\\ &\quad+a_2\frac{p(gx_0,gx_1)+p(gx_1,gx_2)+p(gy_0,gy_1)}{2}\\ &\quad+a_3\frac{p(gx_0,gx_2)+p(gx_1,gx_1)+p(gy_0,gy_1)}{2}\\ &\le a_1\frac{p(gx_0,gx_1)+p(gy_0,gy_1)}{2}\\ &\quad+a_2\frac{p(gx_0,gx_1)+p(gx_1,gx_2)+p(gy_0,gy_1)}{2}\\ &\quad+a_3\frac{p(gx_0,gx_1)+p(gx_1,gx_2)+p(gy_0,gy_1)}{2}. \end{aligned}$$

Thus, we obtain

$$\begin{aligned} p(gx_1,gx_2)&\le\frac{2(a_1+a_2+a_3)}{2-a_2-a_3}.\frac{p(gx_0,gx_1)+p(gy_0,gy_1)}{2}\\ &=\lambda a. \end{aligned}$$

© 2015 NSP
Natural Sciences Publishing Cor.

Also, one can get

$$\begin{aligned}
&p(gy_1,gy_2) = p(T(y_0,x_0),T(y_1,x_1))\\
&\leq a_1\frac{p(gy_0,gy_1)+p(gx_0,gx_1)}{2}\\
&\quad + a_2\frac{p(gy_0,T(y_0,x_0)+p(gy_1,T(y_1,x_1))+p(gx_0,gx_1)}{2}\\
&\quad + a_3\frac{p(gy_0,T(y_1,x_1)+p(gy_1,T(y_0,x_0))+p(gx_0,gx_1)}{2}\\
&= a_1\frac{p(gy_0,gy_1)+p(gx_0,gx_1)}{2}\\
&\quad + a_2\frac{p(gy_0,gy_1)+p(gy_1,gy_2)+p(gx_0,gx_1)}{2}\\
&\quad + a_3\frac{p(gy_0,gy_2)+p(gy_1,gy_1)+p(gx_0,gx_1)}{2}\\
&\leq a_1\frac{p(gy_0,gy_1)+p(gx_0,gx_1)}{2}\\
&\quad + a_2\frac{p(gy_0,gy_1)+p(gy_1,gy_2)+p(gx_0,gx_1)}{2}\\
&\quad + a_3\frac{p(gy_0,gy_1)+p(gy_1,gy_2)+p(gx_0,gx_1)}{2}.
\end{aligned}$$

Thus, we obtain

$$\begin{aligned}
&p(gy_1,gy_2)\\
&\leq \frac{2(a_1+a_2+a_3)}{2-a_2-a_3}.\frac{p(gx_0,gx_1)+p(gy_0,gy_1)}{2} = \lambda a.
\end{aligned}$$

Similar to the above proof, one can show that

$$\begin{aligned}
&p(gx_n,gx_{n+1})\\
&\leq \frac{2(a_1+a_2+a_3)}{2-a_2-a_3}.\frac{p(gx_{n-1},gx_n)+p(gy_{n-1},gy_n)}{2}\\
&= \lambda\frac{p(gx_{n-1},gx_n)+p(gy_{n-1},gy_n)}{2}\\
&\leq \lambda^2\frac{p(gx_{n-2},gx_{n-1})+p(gy_{n-2},gy_{n-1})}{2}\\
&\quad \vdots\\
&\leq \lambda^n\frac{p(gx_0,gx_1)+p(gy_0,gy_1)}{2}\\
&= \lambda^n a,
\end{aligned}$$

and

$$\begin{aligned}
&p(gy_n,gy_{n+1})\\
&\leq \frac{2(a_1+a_2+a_3)}{2-a_2-a_3}.\frac{p(gx_{n-1},gx_n)+p(gy_{n-1},gy_n)}{2}\\
&= \lambda\frac{p(gx_{n-1},gx_n)+p(gy_{n-1},gy_n)}{2}\\
&\leq \lambda^2\frac{p(gx_{n-2},gx_{n-1})+p(gy_{n-2},gy_{n-1})}{2}\\
&\quad \vdots\\
&\leq \lambda^n\frac{p(gx_0,gx_1)+p(gy_0,gy_1)}{2}\\
&= \lambda^n a.
\end{aligned}$$

If $p(gx_0,gx_1)+p(gy_0,gy_1)=0$, then from remark 1, we get $gx_0=gx_1=T(gx_0,gy_0)$ and $gy_0=gy_1=T(gy_0,gx_0)$, meaning that (x_0,y_0) is a coupled coincidence point of T and g. Now, let $p(gx_0,gx_1)+p(gy_0,gy_1)>0$. For each $m\geq n$ we have in view of the condition (p_4)

$$\begin{aligned}
&p(gx_m,gx_n)\\
&\leq p(gx_m,gx_{m-1})+p(gx_{m-1},gx_{m-2})-p(gx_{m-1},gx_{m-1})\\
&\quad +p(gx_{m-2},gx_{m-3})+p(gx_{m-3},gx_{m-4})-p(gx_{m-3},gx_{m-3})\\
&\quad +\ldots+p(gx_{n+2},gx_{n+1})+p(gx_{n+1},gx_n)-p(gx_{n+1},gx_{n+1})\\
&\leq p(gx_m,gx_{m-1})+p(gx_{m-1},gx_{m-2})+\ldots+p(gx_{n+1},gx_n)\\
&\leq (\lambda^{m-1}+\lambda^{m-2}+\ldots+\lambda^n)a\\
&\leq \frac{\lambda^n}{1-\lambda}a.
\end{aligned}$$

Similarly, we have

$$\begin{aligned}
p(gy_m,gy_n) &\leq (\lambda^{m-1}+\lambda^{m-2}+\ldots+\lambda^n)a\\
&\leq \frac{\lambda^n}{1-\lambda}a.
\end{aligned}$$

Then

$$\lim_{n,m\longrightarrow\infty} p(gx_m,gx_n)=0 \ \ and \ \ \lim_{n,m\longrightarrow\infty} p(gy_m,gy_n)=0. \quad (5)$$

By (1), we have $p^s(x,y)\leq 2p(x,y)$, so for any $m\geq n$

$$p^s(gx_m,gx_n)\leq 2p(gx_m,gx_n)\leq 2\frac{\lambda^n}{1-\lambda}a,$$

$$p^s(gy_m,gy_n)\leq 2p(gy_m,gy_n)\leq 2\frac{\lambda^n}{1-\lambda}a.$$

So,

$$\lim_{n,m\longrightarrow\infty} p^s(gx_m,gx_n)=0 \ \ and \ \ \lim_{n,m\longrightarrow\infty} p^s(gy_m,gy_n)=0.(6)$$

Then $\{gx_n\}$ and $\{gy_n\}$ are Cauchy sequences in (X,p^s). Since the partial metric space (X,p) is complete hence thanks to Lemma 101, the metric (X,p^s) is complete, so there exist $x,y\in X$ such that

$$\lim_{n\longrightarrow\infty} p^s(gx_n,x)=0 \ \ and \ \ \lim_{n\longrightarrow\infty} p^s(gy_n,y)=0. \quad (7)$$

On the other hand, we have

$$p^s(gx_n,x)=2p(gx_n,x)-p(gx_n,gx_n)-p(x,x).$$

Letting $n\longrightarrow\infty$ in the above equation, we get

$$\lim_{n\longrightarrow\infty} p(gx_n,x)=\frac{1}{2}p(x,x). \quad (8)$$

On the other hand, we have $p(x,x)\leq p(gx_n,x)$ for all $n\in N$.
On letting $n\longrightarrow\infty$. We get that

$$p(x,x)\leq \lim_{n\longrightarrow\infty} p(gx_n,x). \quad (9)$$

Using (8) and (9), we get that

$$p(x,x)=\lim_{n\longrightarrow\infty} p(gx_n,x)=0.$$

© 2015 NSP
Natural Sciences Publishing Cor.

Similarly, one can show that

$$p(y,y)=\lim_{n\longrightarrow\infty}p(gy_n,y)=0.$$

Thus, we have

$$p(x,x)=\lim_{n\longrightarrow\infty}p(gx_n,x)=0 \quad \textit{and} \quad p(y,y)=\lim_{n\longrightarrow\infty}p(gy_n,y)=0. \tag{10}$$

From (10) and continuity of g,

$$p(gx,gx)=\lim_{n\longrightarrow\infty}p(ggx_n,gx)=0 \quad \textit{and} \quad p(gy,gy)=\lim_{n\longrightarrow\infty}p(ggy_n,gy)=0. \tag{11}$$

From (4) and commutativity of T and g,

$$ggx_{n+1}=g(T(x_n,y_n))=T(gx_n,gy_n) \quad \textit{and} \quad ggy_{n+1}=g(T(y_n,x_n))=T(gy_n,gx_n). \tag{12}$$

We now show that $gx=T(x,y)$ and $gy=T(y,x)$.

$$\begin{aligned}
&p(gx,T(x,y))\\
&\le p(gx,g(gx_{n+1}))+p(g(gx_{n+1}),T(x,y))\\
&\quad -p(g(gx_{n+1}),g(gx_{n+1}))\\
&\le p(gx,g(gx_{n+1}))+p(g(T(x_n,y_n)),T(x,y))\\
&\le p(gx,g(gx_{n+1}))+p(T(gx_n,gy_n),T(x,y))\\
&\le p(gx,g(gx_{n+1}))+a_1\frac{p(gx,ggx_n)+p(gy,ggy_n)}{2}\\
&\quad +a_2\frac{p(gx,T(x,y))+p(ggx_n,T(gx_n,gy_n))+p(gy,ggy_n)}{2}\\
&\quad +a_3\frac{p(gx,T(gx_n,gy_n))+p(ggx_n,T(x,y))+p(gy,ggy_n)}{2}\\
&\le p(gx,g(gx_{n+1}))+a_1\frac{p(gx,ggx_n)+p(gy,ggy_n)}{2}\\
&\quad +a_2\frac{p(gx,T(x,y))+p(ggx_n,T(gx_n,gy_n))+p(gy,ggy_n)}{2}\\
&\quad +a_3\Big[\frac{p(gx,T(gx_n,gy_n))+p(ggx_n,gx)}{2}\\
&\quad +\frac{p(gx,T(x,y))+p(gy,ggy_n)}{2}\Big].
\end{aligned}$$

Taking the limit as $n\longrightarrow\infty$ in above inequality, (11) and (12) we get

$$p(gx,T(x,y))\le\frac{a_2+a_3}{2}p(gx,T(x,y))<p(gx,T(x,y)),$$

which is a contradiction. Thus, we have $p(gx,T(x,y))=0$, which implies that $gx=T(x,y)$. Similarly one can show that $gy=T(y,x)$. Thus we proved that T and g have a coupled coincidence point.
Suppose that (x,y) and (z,t) are coupled coincidence points of T and g, that is

$$gx=T(x,y),\ gy=T(y,x),\ gz=T(z,t) \text{ and } gt=T(t,z).$$

We are going to show that $gx=gz$ and $gy=gt$. From condition (3) we have

$$\begin{aligned}
&p(gx,gz)=p(T(x,y),T(z,t))\\
&\le a_1\frac{p(gx,gz)+p(gy,gt)}{2}\\
&\quad +a_2\frac{p(gx,T(x,y))+p(gz,T(z,t))+p(gy,gt)}{2}\\
&\quad +a_3\frac{p(gx,T(z,t))+p(gz,T(x,y))+p(gy,gt)}{2}\\
&=a_1\frac{p(gx,gz)+p(gy,gt)}{2}\\
&\quad +a_2\frac{p(gx,gx)+p(gz,gz)+p(gy,gt)}{2}\\
&\quad +a_3\frac{p(gx,gz)+p(gz,gx)+p(gy,gt)}{2}\\
&=(\frac{a_1}{2}+a_3)p(gx,gz)+(\frac{a_1}{2}+\frac{a_2}{2}+\frac{a_3}{2})p(gy,gt)\\
&\quad +\frac{a_2}{2}(p(gx,gx)+p(gz,gz))\\
&\le(\frac{a_1}{2}+a_3)p(gx,gz)+(\frac{a_1}{2}+\frac{a_2}{2}+\frac{a_3}{2})p(gy,gt)\\
&\quad +\frac{a_2}{2}[p(gx,gz)+p(gx,gz)]\\
&=(\frac{a_1}{2}+a_2+a_3)p(gx,gz)+(\frac{a_1}{2}+\frac{a_2}{2}+\frac{a_3}{2})p(gy,gt).
\end{aligned}$$

Similarly

$$\begin{aligned}
p(gy,gt)&\le(\frac{a_1}{2}+a_2+a_3)p(gy,gt)\\
&\quad +(\frac{a_1}{2}+\frac{a_2}{2}+\frac{a_3}{2})p(gx,gz).
\end{aligned}$$

Then above inequality and the property (p_2), we have

$$\begin{aligned}
&p(gx,gz)+p(gy,gt)\\
&\le(a_1+\frac{3a_2}{2}+\frac{3a_3}{2})[p(gx,gz)+p(gy,gt)]\\
&<p(gx,gz)+p(gy,gt),
\end{aligned}$$

which is a contradiction. Thus $p(gx,gz)+p(gy,gt)=0$. It implies that $p(gx,gz)=0$ and $p(gy,gt)=0$.

An immediate consequence of Theorem 1 are the following results.

Corollary 201*Let (X,p) be a complete partial metric space. Assume there exist $0\le k<1$ and $T:X\times X\longrightarrow X$ and $g:X\longrightarrow X$ are such that*

$$p(T(x,y),T(u,v))\le\frac{k}{2}[p(gx,gu)+p(gy,gv)], \tag{13}$$

for all $x,y,u,v\in X$. Also Suppose $T(X\times X)\subseteq g(X)$, g is continuous and commutes with T. Then there exist $x,y\in X$ such that

$$gx=T(x,y) \ \textit{ and } \ gy=T(y,x),$$

that is, T and g have a unique coupled coincidence point.

© 2015 NSP
Natural Sciences Publishing Cor.

*Proof.*If T and g satisfies (13), then T and g satisfies (3) with $a_1 = k$ and $a_2 = a_3 = 0$.

Then, the result follows from Theorem 1.

Corollary 202*Let (X,p) be a complete partial metric space. Assume there exist $a_1,a_2,a_3 \geq 0$ with $2a_1 + 3a_2 + 3a_3 < 2$ and also suppose that $T : X\times X \longrightarrow X$ is such that*

$$\begin{aligned} &p(T(x,y),T(u,v)) \\ &\leq a_1 \frac{p(x,u)+p(y,v)}{2} \\ &\quad + a_2 \frac{p(x,T(x,y))+p(u,T(u,v))+p(y,v)}{2} \\ &\quad + a_3 \frac{p(x,T(u,v))+p(u,T(x,y))+p(y,v)}{2}, \end{aligned} \tag{14}$$

for all $x,y,u,v \in X$. Then there exist $x,y \in X$ such that

$x = T(x,y)$ *and* $y = T(y,x)$,

that is, T have a unique coupled fixed point.

*Proof.*Putting $g = I$ (I the identity mapping) in Theorem 1, we obtain corollary 202.

Corollary 203*Let (X,p) be a complete partial metric space. Assume there exist $0 \leq k < 1$ and $T : X\times X \longrightarrow X$ and $g : X \longrightarrow X$ are such that*

$$p(T(x,y),T(u,v)) \leq \frac{k}{2}[p(x,u)+p(y,v)], \tag{15}$$

for all $x,y,u,v \in X$. Then T has a unique coupled fixed point.

Corollary 204*Let (X,p) be a complete partial metric space. Assume there exist $0 \leq k < 1$ and also suppose $T : X\times X \longrightarrow X$ and $g : X \longrightarrow X$ are such that*

$$\begin{aligned} &p(T(x,y),T(u,v)) \\ &\leq \frac{k}{2}[p(gx,T(u,v))+p(gu,T(x,y))+p(gy,gv)], \end{aligned} \tag{16}$$

for all $x,y,u,v \in X$. Also Suppose $T(X\times X) \subseteq g(X)$, g is continuous and commutes with T. Then there exist $x,y \in X$ such that

$gx = T(x,y)$ *and* $gy = T(y,x)$,

that is, T and g have a unique coupled coincidence point.

*Example 3.*Let $X = [0,1]$ endowed with the usual partial metric p defined by $p(x,y) = max\{x,y\}$. Since

$$\begin{aligned} p^s(x,y) &= 2p(x,y)-p(x,x)-p(y,y) \\ &= \max\{x,y\}-x-y \\ &= |x-y|, \end{aligned}$$

is Euclidean metric, then (X,p^s) is complete. So it is clear that (X,p) is a complete partial metric space. Define $T : X\times X \longrightarrow X$ as $T(x,y) = \dfrac{x+y}{16}$ for all $x,y \in X$ and $g : X \longrightarrow X$ be defined by $gx = \frac{1}{2}x$. We show that condition (3) is satisfied.
If $x,y \in X$, then we have

$$\begin{aligned} p(T(x,y),T(u,v)) &= \max\{\frac{x+y}{16},\frac{u+v}{16}\} \\ &\leq \frac{1}{16}[\max\{x,u\}+\max\{y,v\}] \\ &\leq \frac{1}{4}\times\frac{\max\{gx,gu\}+\max\{gy,gv\}}{2} \\ &\leq a_1\frac{p(gx,gu)+p(gy,gv)}{2} \\ &\leq a_1\frac{p(gx,gu)+p(gy,gv)}{2} \\ &\quad + a_2\frac{p(gx,T(x,y))+p(gu,T(u,v))+p(gy,gv)}{2} \\ &\quad + a_3\frac{p(gx,T(u,v))+p(gu,T(x,y))+p(gy,gv)}{2}. \end{aligned}$$

Thus all the conditions of theorem 1 are satisfied. Moreover, $(0,0)$ is the unique coupled coincidence point of T and g.

References

[1] T. Abdeljawad, E. Karapinar, K. Tas, Existence and uniqueness of a common fixed point on partial metric spaces, Applied Mathematics Letters. 2011, 24(11):1900-1904.

[2] A. Aghajani, R. Arab,Some fixed point results for generalized contractions on partial metric spaces, European Journal of Scientific Research, Volume 107 Issue 1,(2013).

[3] Ya.I. Alber, S. Guerre-Delabrere, Principle of weakly contractive maps in Hilbert spaces, in: I. Gohberg, Yu. Lyubich (Eds.), New Results in Operator Theory, in: Advances and Appl., vol. 98, Birkhuser Verlag, Basel, 1997, pp. 722.

[4] I. Altun, A. Erduran, Fixed point theorems for monotone mappings on partial metric spaces. Fixed Point Theory Appl 2011, 10.

[5] I. Altun, F. Sola, H. Simsek, Generalized contractions on Partial metric spaces, Topology and its Applications.157(2010) 2778-2785.

[6] H. Aydi, Some fixed point results in ordered partial metric spaces, Accepted in J. Nonlinear Sci. Appl, (2011).

[7] H. Aydi, some coupled fixed point results on partial metric spaces, International Journal of Mathematics and Mathematical Sciences, Article ID 647091, 11 pages, 2011.

[8] M. Bukatin, R. Kopperman, S. Matthews, H. Pajoohesh, Partial metric spaces. Am. Math. Mon. 116, 708-718(2009).

[9] T. Gnana Bhaskar, V. Lakshmikantham, Fixed point theorems in partially ordered metric spaces and applications, Nonlinear Analysis. 65 (2006) 1379-1393.

[10] S. Gulyaz, E. Karapinar, A Coupled Fixed Point Result in Partially Ordered Partial Metric Spaces Through Implicit Function, Hacettepe Journal of Mathematics and Statistics, Volume 42(4) (2013), 347 357.

[11] R. Heckmann, Approximation of metric spaces by partial metric spaces, Appl. Categ. Structures 7 (1999) 7183.

© 2015 NSP
Natural Sciences Publishing Cor.

[12] E. Karapinar, M. E. Inci, Fixed point theorems for operators on partial metric spaces. Appl Math Lett 2011, 24(11):1894-1899.
[13] Z. Kadelburg, S. Radenović, Fixed points under $\psi-\alpha-\beta-$ conditions in ordered partial metric spaces, International Journal of Analysis and Applications, Volume 5, Number 1 (2014), 91-101.
[14] Z. Kadelburg, H.K. Nashine, and S. Radenović, Some new coupled fixed point results in 0-complete ordered partial metric spaces, J. Adv. Math. Studies Vol. 6 (2013), No. 1, 159-172.
[15] V. Lakshmikantham, Lj. Ćirić, Coupled fixed point theorems for nonlinear contrac- tions in partially ordered metric spaces, Nonlinear Analysis 70 (2009) 4341-4349.
[16] S. G. Matthews, Partial metric topology. Research Report 212. Department of Computer Science, University of Warwick 1992.
[17] S. G. Matthews, Partial metric topology, in: Proc. 8th Summer Conference on General Topology and Applications, in: Ann. New York Acad. Sci., vol. 728,1994, pp. 183-197.
[18] S. Oltra, O. Valero, Banachs fixed point theorem for partial metric spaces, Rend. Istit. Mat. Univ. Trieste 36 (2004) 17-26.
[19] S. J. ONeill, Two topologies are better than one, Tech. report, University of Warwick, Coven-try, UK, http : //www.dcs.warwick.ac.uk/reports/283.html,1995.
[20] S. J. ONeill, Partial metrics, valuations and domain theory, in: Proc. 11th Summer Conference on General Topology and Applications, in: Ann. New York Acad. Sci., vol. 806, 1996, pp. 304-315.
[21] S. Radenović, Remarks on some coupled fixed point results in partial metric spaces, Nonlinear Funct. Anal. and Appl. Vo;l.18, No. 1 (2013), 39-50.
[22] S. Radenović, Coincidence point results for nonlinear contraction in ordered partial metric spaces, Journal of Indian Math. soc. Vol. 81, Nos. X-X, (2014), XX-YY.
[23] V. Ć. Rajić, S. Radenovi, W. Shatanawi, N. Tahat, Common fixed point results for weakly isotone increasing mappings in partially ordered partial metric spaces, Le Matematiche Vol. LXVIII (2013)-Fasc. II, pp. 191-204.
[24] S. Romaguera, A Kirk type characterization of completeness for partial metric spaces. Fixed Point Theory Appl. 2010(Article ID 493298), 6 (2010).
[25] S. Romaguera, M. Schellekens, Partial metric monoids and semivaluation spaces, Topology Appl. 153 (5-6) (2005), 948-962.
[26] S. Romaguera, O. Valero, A quantitative computational model for complete partialmetric spaces via formal balls, Math. Structures Comput. Sci. 19 (3) (2009), 541-563.
[27] B. Samet, M. Rajovic, R. Lazovi, R. Stoiljkovic, Common Fixed Point Results For Nonlinear Contractions in Ordered Partial Metric Spaces.Fixed Point Theory Appl. 2011.doi:10.1186/1687-1812-2011-71.
[28] M. P. Schellekens, The correspondence between partial metrics and semivaluations, Theoret. Comput. Sci. 315 (2004), 135-149.
[29] H. Sheng Ding, L. Li, coupled fixed point theorems in partialy orderd con metric spaces, Faculty of Sciences and Mathematics, University of Niš, Serbia, (2011), 137-149.
[30] S. Shukla, S. Radenović, C. Vetro, Set-valued Hardy-Rogers type contraction in 0-complete partial metric spaces, International Journal of Mathematics and Mathematical Sciences, Volume 2014, Article ID 652925, 9 pages.
[31] S. Shukla, S. Radenović, Some common fixed point theorems for F-contraction type mappings in 0-complete partial metric spaces, Journal of Mathemnatics, Volume 2013, Article ID 878730, 7 pages.
[32] O. Valero, On Banach fixed point theorems for partial metric spaces, Appl. Gen. Topol, 6 (2) (2005) 229-240.

Reza Arab, He has a great experience in teaching Mathematics as a lecturer in Islamic Azad university, Sari branch, since 1999 and now He is a head of Department of Mathematics. His favorite field of study is Mathematical Analysis and He is really interested in Fixed point theorems and Measure of non-compactness. His PhD thesis entitled Fixed and coupled fixed point theorems on complete metric spaced and applications last year.

© 2015 NSP
Natural Sciences Publishing Cor.