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Generalized Fractal Dimensions in Image Thresholding Technique

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Abstract: A digital grayscale image can be described by intensity or pixel values. The gray levels are spread over the images as irregular or inhomogeneous fashion. A number of proposed methods for calculating the optimal thresholding value for image segmentation, but the fractal analysis is an expeditious and significant mathematical approach that distributes with irregular geometric objects. In the recent years, fractal analysis is widely used in the image processing. In this study we use more general approach, multifractal analysis, for evaluating the optimal thresholding value for image segmentation, since the generalized fractal dimensions measures the irregularity or chaos object as exactly.

Keywords: Image segmentation, thresholding, Generalized Fractal Dimensions, Otsu method, Region nonuniformity.

1 Introduction

Image segmentation is a tremendously democratic image processing procedure, as of all image processing strategies involve some sort of operation of the pixels into dissimilar classes. At present thresholding is a noticeable and glorious technique, and it is enormously used in the field of image processing. There are two types of thresholding methods, are global thresholding and local thresholding. Broadly speaking, a local thresholding method better suits, poor and unevenly illuminated images [1,2]. However, a global thresholding approach is a more appropriate choice for images in which the object and background can be separated with an optimal threshold. Fundamentally, it is a technique of partitioning the original image into distinguishable regions, such as the background and region of interest or foreground region. In order to find out thresholds for segmentation, majority techniques study the histogram of the image. The optimal thresholds are those values of intensity that can be independent antithetic objects from each other or from the background to such an extent that decisiveness can be attained without further processing [3,4] and these are frequently regenerate by either minimizing or maximizing an objective function in the sense of the threshold value. It is generally unproblematic and streamlined assessment and based on the hypothesis that the objects can be spotted through their gray levels. The automatically electing of these thresholds from the image with irregular gray level distribution is one of the outstanding challenges in image segmentation.

In this study, we shall concentrate on global thresholding methods. Global thresholding methods compare each pixel in a gray level image with an estimated threshold value. So, how to find an optimal threshold becomes a classical challenge in global thresholding segmentation [5]. As far as there are several methods for image segmentation [3]-[12]. After reviewing various methods for gray level image segmentation, Pal and Pal [13] state that image thresholding is a popular segmentation method because of its simplicity and ease of implementation. Basically, thresholding is habituated to perceive and infused a target region from its background or texture in image foreground region in the sense of dispersion of gray levels. Identifying or analysis of any image wants the image to be precisely segmented into significant regions. Numerous methods have been proposed to determine the optimal threshold for the past several authors. An early review of thresholding approaches was reported in [6]. A comparative performance study of global thresholding techniques was reported by Lee et al. [14]. Another comparative analysis of the performance of eleven histogram based thresholding methods was carried out by

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Glasbey [15]. Among these global thresholding approaches, one of the most popular thresholding method is an Otsu method . This method is widely used in practice and highly cited in scientific publications. Otsu method utilizes the discriminant analysis to find the maximum separability of two classes. For every possible gray level, this method evaluates the goodness of this value if it is used as a threshold. Recently, several interesting views of Otsu method have been reported, which enhances understanding of the properties and thresholding performance of Otsu method [16].

In this circumstance we have used the multifactal analysis. The Euclidean geometry handles regular objects with integer dimension, while the fractal geometry addresses irregular objects with non-integer dimension. Professor Mandelbrot defined fractal as a set with Hausdorff dimension is strictly greater than its topological dimension [17]-[21]. Fractal dimension analyses the irregularity of the given object with homogeneous scaling properties. The concept of fractal dimension can be practicable in the measurement and categorization of shape and texture. Numerous research works have been described in medical image analysis employing fractal analysis [22]-[27]. Fractal dimension is insufficient to characterizing the object having complex and inhomogeneous scaling properties. Therefore, we used the generalized fractal dimension to elect an optimal threshold value for image segmentation.

The interpretation of medical images is a multilevel process where the ultimate goal is the identifying the irregularities of tissues. This goal is perfectly detected by clinician, when he incorporates the image observation to realize image practices as unique and the identification or analysis of the resemblance between perceived practices and practical diagnoses. One of the precise characters in this process is segmentation of the medical images. The multifractal analysis is justified by the irregular self-similarity of medical images with finite resolution. In reality, the images are not only spiritual complex or inhomogeneous, but they often demonstrate certain similarities at irregular spatial scales. This situation induces that spatially complex practices of medical image could be segmented by multifractal analysis.

Thresholding based image segmentation receives troubles whenever the foreground region establishes an unequal or irregular region of the panorama in whole or part of the image, or while both foreground region and background region gray levels are dense, even ensuing in an unimodal distribution. Moreover, the optimal threshold value estimation causes histogram as noisy if it attains from humble sample size, or it may have a ransack structure because of histogram stretching processes. Accordingly, relegate pixels and shape contortions of the object may discriminatorily regard the outstanding testing process. Then again, noise pixels corrupting the original quality bitmaps of given grayscale image. Therefore, the standards to evaluate thresholding based segmentation algorithms must acquire into thoughtfulness both the noisiness of the segmentation function in addition to the shape distortion of the eccentrics, especially in the medical recognition process. For these circumstances, Jaccad coefficient [28] and region nonunifomity [29,30] have been used to analyses the performance between our proposed method and Otsu method [4,6].

Presently there are various thresholding measure precisely applied to compare the performance and accurse of the resultant images, evidently in this study we have been used Otsu (or global thresholding) method [4] for performance evaluation both quantitatively and qualitatively, since Otsu method is broadly utilised as a pre-processing measure to segment images for advance processing, it is significant to attain a eminent accuracy.

2 Multifractal Analysis

The Renyi Entropies [31,32] are important in Non-linear Analysis and Statistics as indices of uncertainty or randomness. They also lead to a spectrum of indices of Fractal Dimension (Renyi Fractal Dimensions or Generalized Fractal Dimensions) systematically developed the multifractal theory, which is based upon Generalized Fractal Dimensions (GFD). For the ancient surveys [33]-[37] showed that the elaborate information about multifractal analysis and generalized fractal dimensions, hence we present here only a formal definition of generalized fractal dimensions.

The *Renyi Fractal Dimensions or Generalized Fractal Dimensions (GFD)* of order $q \in (-\infty, \infty)$ defined in terms of generalized Renyi Entropy as

$$D_q = \lim_{r \to 0} \frac{1}{q-1} \frac{log_2 \left(\sum_{i=1}^{N} p_i^q\right)}{log_2 r} \tag{1}$$

Where p_i is probability distribution. As $q \longrightarrow 1$, D_q converges to D_1, which is given by

$$D_1 = \lim_{r \to 0} \frac{\sum_{i=1}^{N} p_i log_2 p_i}{log_2 r}.$$

D_1 is called information dimension. D_q is monotonically decreasing function of q. $D_0 \geq D_1 \geq D_2$ and they represent the Fractal Dimension, Information Dimension and Correlation Dimension respectively.

3 Algorithm

After sampling and quantization process, image can be represented as matrix form. Let M, N be the finite subsets of natural numbers and $G = \{0, 1, 2, ..., k-1\}$ be a set of positive integers denote the graylevels of k-bit. Then, an $M \times N$ dimensional image can be defined as a function $f : M \times N \longrightarrow G$ by $f(x,y) = i, i \in G$. Assume that $t \in G$ is a optimal thresholding value and $B = \{0, 1\}$ be a binary gray levels in G. Then the thresholding image can be defined as a mapping $T : M \times N \longrightarrow B$, such that

$$T(x,y) = \begin{cases} 0 \ if \ \ f(x,y) \leq T \\ 1 \ if \ \ f(x,y) \ > T \end{cases}$$

A grayscale image can be described in term of a mass distribution. We assume that the total intensity as a finite mass scattered to whole image so that white areas have high density and black areas have low density. we have to determine threshold (t) value as alone from the gray level of each pixel through following steps.

1. Read the input MRI.
2. Let N be the number of boxes to cover the image with box size r.
3. The probability p_i for i^{th} box of size r in the image is defined as,
$$p_i = \frac{X_i}{X}$$
where X_i is the intensity value of the image in the corresponding i^{th} box of size r and X is the total intensity value image.
4. Estimate the value of N.
5. Fix q, calculate D_q as defined in equation (1) for various $r \to 0$.
6. Repeat setp 5 for various $q \in (-150, 150)$, we found D_q for each intensity level of the given input image using equation (1).
7. Find the median value of D_q's and fix corresponding intensity level as optiml threshold, $t = med(D_q)$
8. Based on threshold value, image is partitioning as foregroud and backgroud region. Binary image B(x,y) generated from the original image $f(x,y)$ as
$$B(x,y) = \begin{cases} 0 \ if \ \ f(x,y) \leq t \\ 1 \ if \ \ f(x,y) \ > t \end{cases}$$
9. Mask the input image by generated binary image.

4 Evaluation metric

The choice of a suitable distance function is essential when analyzing the quality of segmentation. If we choose the unsuitable metric for the qualitative analysis, an extreme problem of this approach is that the metric can only be applied to the output image of the corresponding algorithm, and not on the original input image. Hence, it is not possible to account for differences between the original input image and their corresponding image. The evaluation may therefore favor models learned from image, which represent the original segmentation poorly. This problem was first addressed by Heimann et al. [41], who proposed to use the Jaccard coefficient as a similarity measure, which was later also employed in [42]. This metric measure the similarity between our segmentation and the expected segmentation output. This naturally means that we will need a ground truth for comparison.

Table 4: Jaccard coefficient of Otsu method and Proposed method

General Images	Otsu Method (J)	Proposed Method (J)
G_1	0.9905	0.9925
G_2	0.7551	0.7696
G_3	0.9848	0.9867
G_4	0.9770	0.9794
G_5	0.9913	0.9973
G_6	0.9573	0.9613
G_7	0.9820	0.9854
G_8	0.9634	0.9677
G_9	0.9956	0.9978
G_{10}	0.9509	0.9589

Among a number of region-based coefficients based upon the measure of spatial overlap, the Jaccard [28] coefficients have been extensively used for the performance evaluation of segmentation methods in image processing technique. The Jaccard coefficient J measures the ratio of the intersection area of two sets (A and B) divided by the area of their union,

$$J = \frac{|A \cap B|}{|A \cup B|}.$$

Jaccard coefficient vary between 0 and 1, with 1 corresponding to perfectly matched classifications.

The region nonunifomity measure is pronounces the inherent quality of the segmented region. Assume that $f(x,y)$ is given grayscale image then the region nonuniformity measure RNU is defined as

$$RNU = \frac{|FG| \times Var(FG)}{|FG + BG| \times Var(f)},$$

where FG is foreground image pixels, BG is background image pixels, Variance of whole image denoted by $Var(f)$ and $Var(FG)$ represent the variance of foreground region of given image $f(x,y)$, $|.|$ cardinality of the given object. The good segmented image have a RNU value close to 0.

5 Performance Analysis

Our proposed algorithm has been applied on general images, standard images and medical image samples and results are compared with Otsu method. we have used MATLAB R2008a software for algorithm implementation.

For the explanation of Professor Mandelbrot, Fractal is a set with non-integral Hausdorff dimension, which exceeds its topological dimension. As discussed earlier about generalized fractal dimensions, if $q = 0$, then D_q generates the fractal dimensions. Table 1, 2, 3indicates that all the tested images have fractal dimension D_0 values in between 0 and 1, as well sample images are subset of Euclidean plane, therefore it has the topological dimension as 1. Hence fractal dimension of image samples are strictly greater than its topological

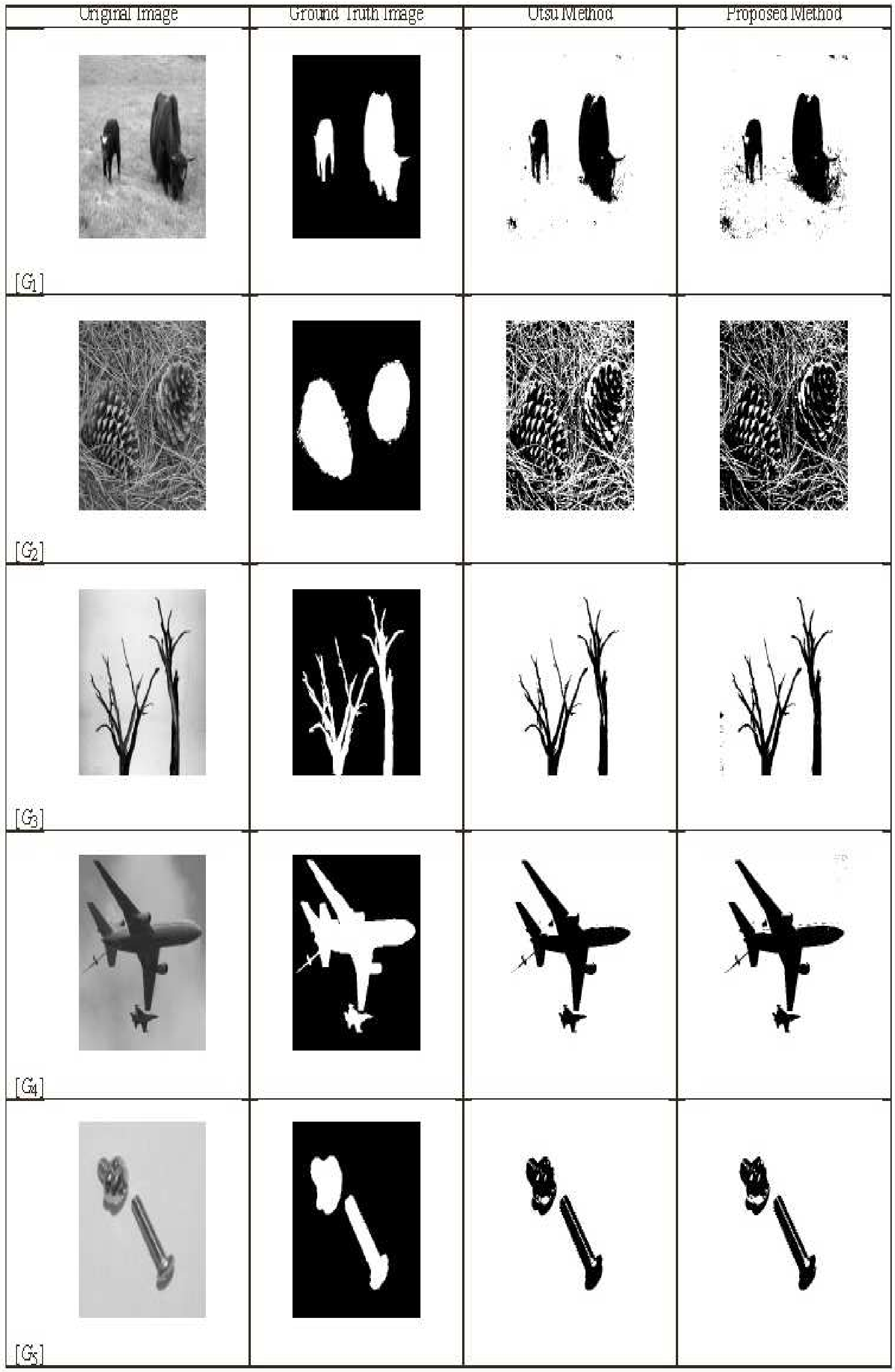

Fig. 1: Qualitative analysis of proposed algorithm with ground truth images and Otsu method resultant images

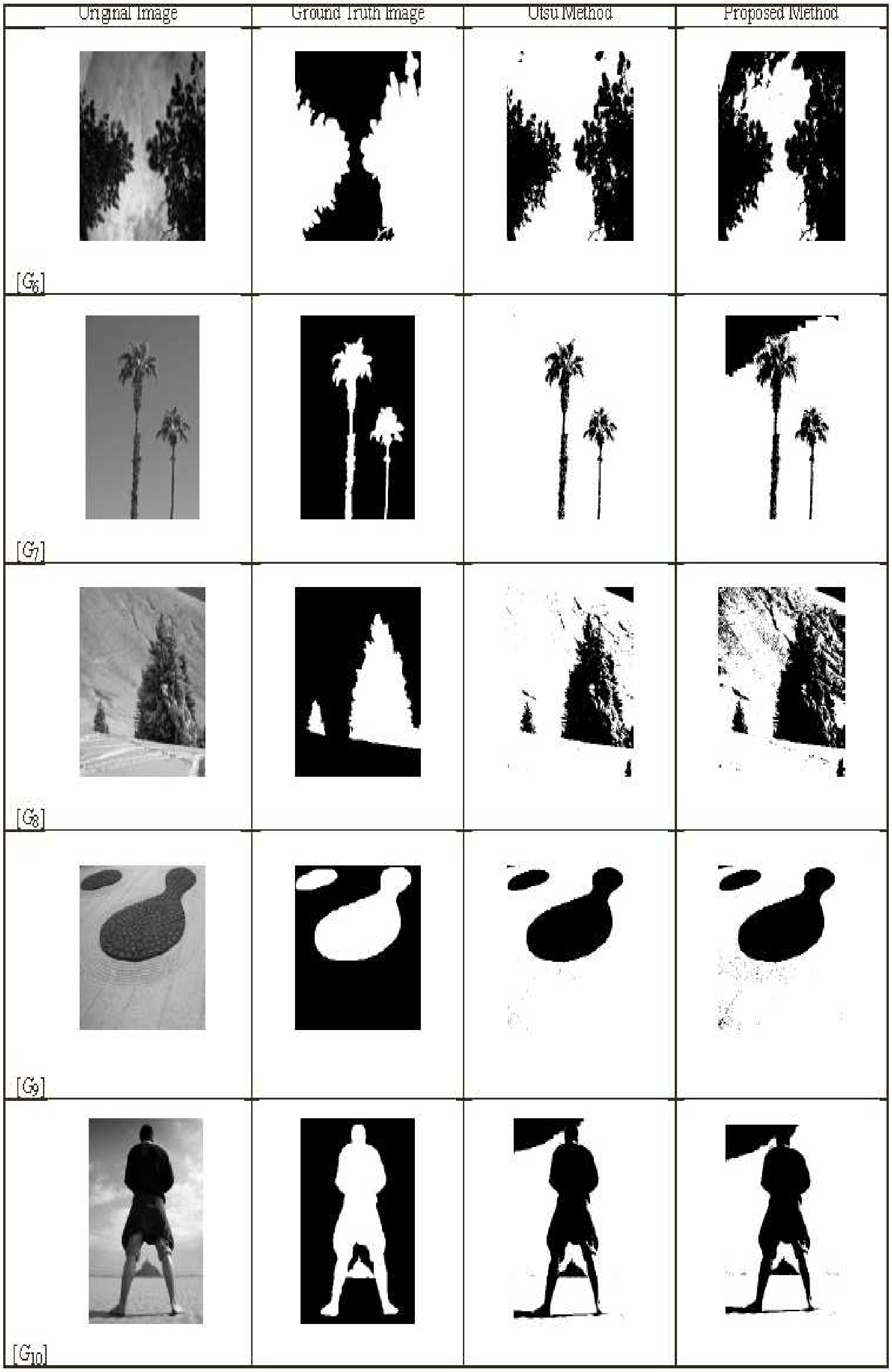

Fig. 2: Qualitative analysis of proposed algorithm with ground truth images and Otsu method resultant images

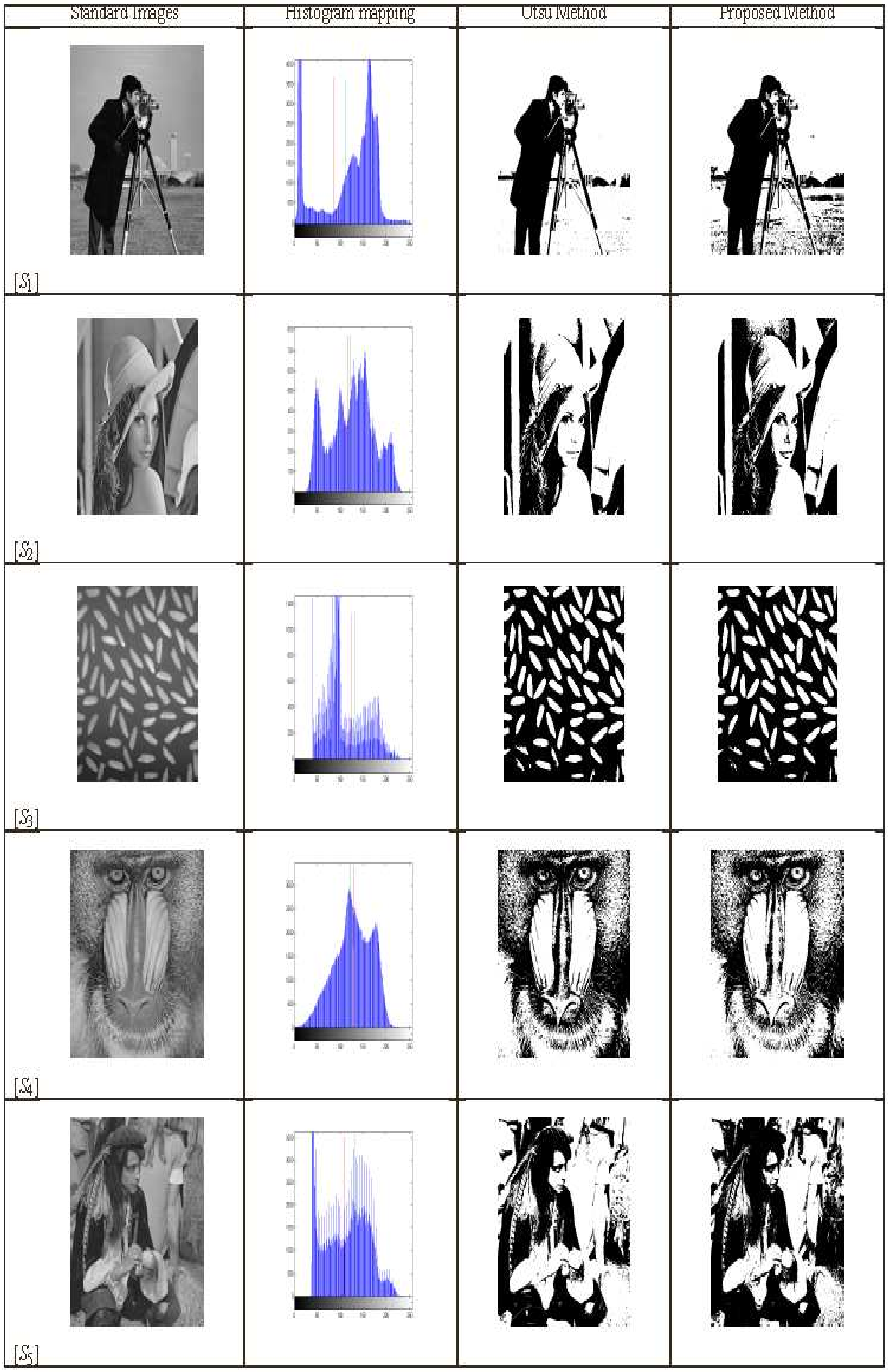

Fig. 3: Qualitative analysis of proposed algorithm with Otsu method resultant images

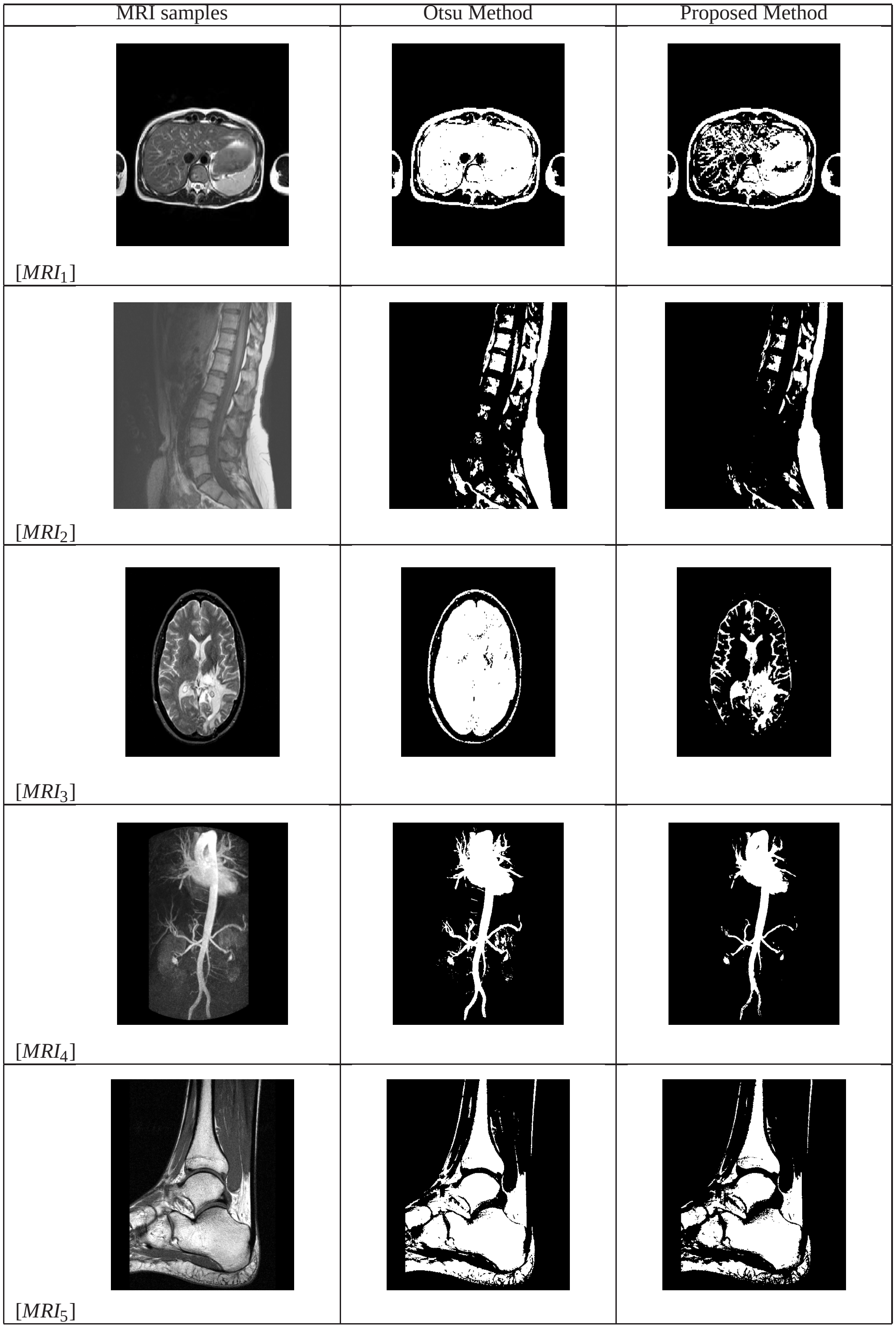

Fig. 4: Qualitative analysis of proposed algorithm with Otsu method resultant images

Table 1: Threshold value and region nonuniformity of Otsu method and Proposed method comparison of general image samples

General Images	Otsu Method		Proposed Method		Fractal Dimension D_0
	Threshold value	Region nonunifomity	Threshold value	Region nonunifomity	
G_1	113	0.0254	135	0.0234	1.3014
G_2	106	0.2029	128	0.1608	1.2935
G_3	126	0.0116	148	0.0108	1.2982
G_4	110	0.0328	121	0.0321	1.2869
G_5	158	0.0042	143	0.0073	1.2853
G_6	84	0.1430	119	0.1414	1.2935
G_7	92	0.0358	127	0.0813	1.2974
G_8	114	0.0658	140	0.0789	1.2938
G_9	121	0.0318	127	0.0309	1.3242
G_{10}	112	0.0567	144	0.0456	1.2872

Table 2: Threshold value and region nonuniformity of Otsu method and Proposed method comparison of Standard Images

Standard Images	Otsu Method		Proposed Method		Fractal Dimension D_0
	Threshold value	Region nonunifomity	Threshold value	Region nonunifomity	
S_1	86	0.0466	112	0.0411	1.2500
S_2	117	0.1374	122	0.1357	1.2857
S_3	125	0.2170	130	0.1916	1.2875
S_4	127	0.1374	122	0.1338	1.2567
S_5	107	0.1267	129	0.1115	1.2506

Table 3: Threshold value and region nonuniformity of Otsu method and Proposed method comparison of Medical Images

MRI Samples	Otsu Method		Proposed Method		Fractal Dimension D_0
	Threshold value	Region nonunifomity	Threshold value	Region nonunifomity	
MRI_1	65	0.4703	130	0.3381	1.2684
MRI_2	132	0.6744	148	0.4817	1.2633
MRI_3	61	0.4139	120	0.3424	1.2857
MRI_4	99	0.5727	134	0.3327	1.2500
MRI_5	102	0.1912	136	0.1193	1.2579

dimension. It is evident that these sample images are a fractal object, that is gray levels of the each image samples are distributed irregularly. Therefore optimal thresholding value estimated through generalized fractal dimensions.

The general grayscale image and its ground truth datasets were stochastically obtained from the source [38]. In order to compare proposed method with Otsu method and manually segmented image samples shows in Figure.1 our method presents good results. Moreover, the region nonunifomity measure of proposed method and Otsu method are presented in table 1. We take the average of each methods, Otsu method have average value 0.04105 and multifractal analysis method receive the average value 0.0345. Although the Jaccard coefficient provided differences to characterize the performance of segmentation. Table 4 gives the Jaccard coefficient of proposed method and Otsu method at different general image samples. The table 4 give the average J values of the proposed method is greater than the Otsu method. Proposed method obtained the best Jaccard coefficient for almost all general image samples.

The Standard images, because most of the literature or image processing technique process these images, are randomly selected from the image processing website [39]. These images were used to test the efficiency of proposed methods. In Figure.3 column 2 shows that, how the thresholding values are fragmented the foreground region and back ground region from histogram of given image. Red line indicates the threshold value of Otsu method as well as green line delivers the optimal threshold value of proposed method. In order to qualitative analysis between our method and Otsu method from Figure.3 and estimating the average region nonunifomity values obtaind from table 2, generalized fractal dimensions based thresholding method provides the better results. Since Otsu method having average region nonunifomity 0.08665, while multifractal method have 0.076.

The robustness of the algorithm to medical images was checked using anatomic magnetic resonance images received from data source [40]. We have processed our algorithm for most of the T1- and T2- weighted images provided by the resource [40] and likened with Otsu method, while here we present only the 5 MRI samples for presentation. The performance evaluated from the Figure 4 and table 3 proposed algorithm gives significant results compared with other method. Because medical images have more irregular gray level distribution, therefore multifractal analysis is efficient to measure the irregularity of gray level diffusion in magnetic resonance images.

6 Conclusion

For novelty, we have used multifractal analysis for evaluating the optimal threshold value in favour of image segmentation. The method coalescing generalized fractal dimensions for robustness and exactness. From this work, we justified that for the above mentioned data sets proposed, generalized fractal dimensions based thresholding, technique reasonably good thresholding method for digital grayscale images having irregular or nonuniform gray level distribution.

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