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Bayesian Reliability Estimation of Inverted Exponential Distribution under Progressive Type-II Censored Data

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Abstract: The present study deals with the estimation procedure for the parameter, reliability and hazard functions of the inverted exponential distribution under progressive Type-II censored data. For estimation purpose, we have considered both classical and Bayesian method of estimation. The Bayes estimates of the parameter, reliability and hazard functions are calculated under symmetric and asymmetric loss functions. We have also computed the 95% asymptotic and highest posterior density (HPD) intervals of the parameter. Further, Monte Carlo simulation technique has been used to compare the performances of the Bayes estimators with corresponding maximum likelihood estimators under both the loss functions. Finally, we analysed one real data set for illustrative purposes of the study.

Keywords: Inverted exponential distribution, Progressive censoring, Bayesian inferences, Importance sampling, Metropolis-Hastings algorithm.

1 Introduction

The exponential distribution is the most popular distribution for lifetime data analysis because of its simplicity and mathematical feasibility. The applicability of this distribution is restricted due to its constant failure rate but in realistic phenomenon, it seems to be meaningless when failure rate is non-constant. Therefore, a number of lifetime models such as Weibull, gamma, generalized exponential etc. have been proposed as a life time models which passes the various type of failure rate i.e. monotonically increasing or decreasing failure rate function and it reduces to exponential distribution for particular choice of shape parameter. However, non-monotonicity of the hazard rate has also been observed in many situation. For example, in the course of the study of mortality associated with some of the diseases, the hazard rate initially increases with time and reaches a peak after some finite period of time and then decline slowly, see Singh et al. (2013). Thus, need to analyse such a data whose hazard rate was non-monotonically realized. Then, in view of this, Inverted exponential distribution (IED) has been proposed as a lifetime model by Lin et al. (1989). Lin et al. (1989), Prakash (2009), Singh et al. (2013) have advocated the use of IED as an appropriate model for this satiation. Prakash (2009) have obtained Maximum Likelihood estimates (MLEs), confidence limits and uniformly minimum variance unbiased estimators for the parameters and reliability function of IED with complete sample. Dey (2007) provides the estimation of the parameter of IED by assuming the parameter involved in the model as a random variable.

G. Prakash (2009) has discussed the properties of the various estimators for the IED and also presented the estimation of the parameter, based on lower record values. Recently, Singh et al. (2012) propose Bayes estimators of the parameter and reliability function for the same under the general entropy loss function for complete, Type-I and Type-II censored samples. The probability density function (pdf) and cumulative distribution function (cdf) of inverted exponential distribution (IED) are given as;

$$f(x,\alpha)=\frac{\alpha}{x^2}e^{-\alpha/x} \quad ;x\geq 0,\alpha>0, \tag{1}$$

$$F(x,\alpha)=e^{-\alpha/x} \quad ;x\geq 0,\alpha>0 \tag{2}$$

respectively.

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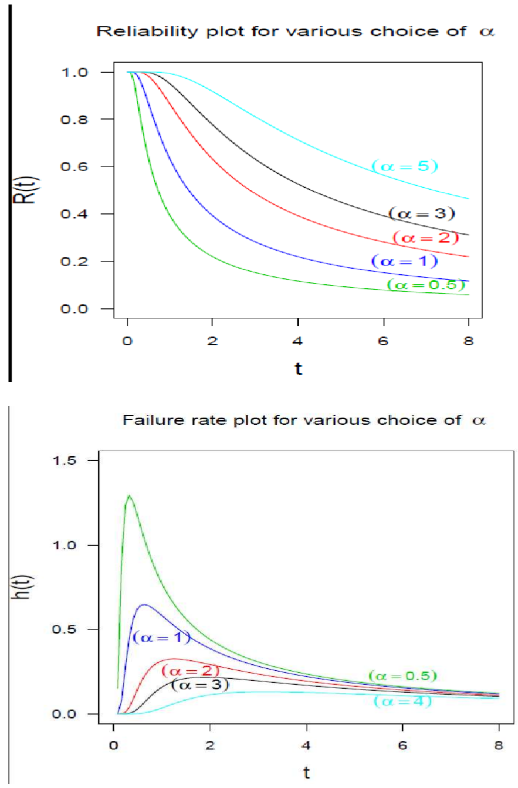

Fig. 1: Figure represents the shape of reliability and hazard function for different choices of the shape and scale parameters.

1.1 Reliability Function and Hazard Function

The reliability function and hazard function of the IED for specified time t are given by the following equations;

$$R(t) = 1 - e^{-\alpha/t} \;\; ;\alpha, t > 0, \tag{3}$$

$$H(t) = \frac{\alpha e^{-\alpha/t}}{t^2(1 - e^{-\alpha/t})};\; \alpha, t > 0 \tag{4}$$

respectively. The plots of the reliability and hazard function are presented below. From graph we see that the hazard function has a non-monotonic shape which increases initially, reaches a maximum and drops slowly. The other advantages of this model is that, it is the special case of the inverted gamma distribution with known shape parameter.

1.2 Progressive Censoring and Likelihood Function

The problem of censoring in life time data analysis is obvious because of time and other circumstances. The most common censoring schemes are Type-I and Type-II censoring schemes. In Type-I censoring scheme, the life testing experiment will be terminated at a prefixed time T, and in Type-II censoring, the life testing experiment will be terminated after observing the $r^t h$ failure. Therefore, Type-I and Type-II censoring schemes are also called as time and failure censoring schemes respectively. However, the conventional Type-I and Type-II censoring schemes do not have the flexibility of allowing removal of the units during the experiment. Due to this reason, a more general censoring scheme called progressive Type-II (PT-II) censoring scheme has been introduced by Cohen (1963).

PT-II censoring scheme can be abbreviated as follows; Suppose n units are placed on experiment and m failures are going to be observed. When the first failure is observed, R_1 of the surviving units are randomly selected and removed. At the second observed failure, R_2 of the surviving units are randomly selected and removed. This experiment terminates at the time when the m^th failure is observed and the remaining $R_m = n - R_1 - R_2 - \cdots - m$ surviving units are all removed. The statistical inference on the parameters of life time distributions under progressive Type-II censoring has been studied by several authors such as Cohen (1963), Viveros and Balakrishnan (1994), Aggarwala (2001) and Krishna and Kumar (2011).

Thus, the likelihood function in the case of PT-II censoring scheme is given by Balakrishnan and Sandhu (1995) as follows;

$$L(\underline{x}|\alpha) = C\prod_{i=1}^{m} f(x_i,\alpha)\left\{1 - F(x_i)\right\}^{R_i}, \tag{5}$$

where C is the constant given as follows;

$$C = n(n - R_1 - 1), \cdots, (n - R_1 - R_2 - \cdots - R_{m-1} - m + 1).$$

Based on such a progressive Type-II right censored sample, the likelihood function using (1) and (2) can be expressed as;

$$L^*(\underline{x}|\alpha) \propto \prod_{i=1}^{m} \frac{\alpha}{x_i^2} e^{-\alpha/x_i}\left\{1 - e^{-\alpha/x_i}\right\}^{R_i}. \tag{6}$$

Then, the log-likelihood function can be written as;

$$L = \ln L^*(\underline{x}|\alpha) \propto m\ln\alpha - \sum_{i=1}^{m}\frac{\alpha}{x_i} - 2\sum_{i=1}^{m}\ln(x_i) + \sum_{i=1}^{m} R_i \ln(1 - e^{-\alpha/x_i}). \tag{7}$$

It is important to mention here that, the most of the authors have discussed the various estimation procedures based on Type-I or Type censoring scheme but no one has paid attention about the estimation of the parameter under Progressive censoring, which is very popular and applicable censoring scheme now a days. Thus, our aim of this paper is to consider the estimation of the unknown parameters, reliability function and hazard function of IED using PT-II censoring scheme under squared error loss function (SELF) and general entropy loss function (GELF). It is observed that, the MLE of the unknown parameter cannot be obtained in nice closed form. Therefore, Newton- Raphson (N-R) method has been implemented to obtain the MLEs. It is also observed that the Bayes estimators are not in explicit form. Thus, among existing various approximation techniques, one of the most popular Markov Chain Monte Carlo (MCMC) technique has been used to obtain the Bayes estimators based on posterior samples. Monte Carlo simulations are conducted to compare the performances of the classical estimators with corresponding Bayes estimators obtained in both informative and non-informative set-up. Further, we have also constructed 95% approximate confidence intervals and highest posterior density (HPD) credible intervals for the parameters.

The rest of the article is organized as follows: Introductory part of the paper has been covered in section and subsection of 1. In Section 2, we derived the maximum likelihood estimators (MLEs) of the parameters, reliability and hazard functions and obtained Fisher information for constructing 95% approximate confidence intervals. In section 3, we obtained the expressions for Bayes estimators under the non-informative prior and two informative prior using SELF and GELF. Monte Carlo simulation results and the analysis of data sets are presented in section 4. A real data illustration has been discussed in section 5 and finally in Section 6, we conclude the paper.

2 Maximum Likelihood Estimation

Now in order to obtain the maximum likelihood estimators of the parameter, we have to maximize the above equation (7) w.r.t. the parameter α. Therefore, if we assume that α is unknown then the MLE $(\hat{\alpha}_M)$ of α can be obtained by solving the following equation.

$$\frac{m}{\alpha} - \sum_{i=1}^{m}\left(\frac{1}{x_i}\right) + \sum_{i=1}^{m}\frac{R_i e^{-\alpha/x_i}}{x_i(1 - e^{-\alpha/x_i})} = 0. \tag{8}$$

From the above equation, we observed that the MLE of α can not be obtained analytically. Therefore, we have used Newton-Raphson method to obtain MLE of the parameter. Now for specified value of t the MLE of the reliability function $(\hat{R}_M)$ and hazard function $(\hat{H}_M)$ have been obtained by using invariance property. i.e.

$$\hat{R}_M = 1 - e^{-\hat{\alpha}_M/t},$$

$$\hat{H}_M = \frac{\hat{\alpha}_M e^{-\hat{\alpha}_M/t}}{t^2(1 - e^{-\hat{\alpha}_M/t})}$$

2.1 Asymptotic Confidence Interval

To construct asymptotic confidence intervals we need the observed Fisher information. Therefore, the observed Fisher information is given by;

$$I(\hat{\alpha}) = -E\left(\frac{\partial^2 L}{\partial \alpha^2}\right)_{\alpha=\hat{\alpha}}$$

also the asymptotic variance of α is given by

$$Var(\hat{\alpha}) = \frac{1}{I(\hat{\alpha})}.$$

But, the exact mathematical expressions for the above expectation is not exist. Therefore, by using the concept of large sample theory the $100(1-\lambda)\%$ confidence interval of α is given by;

$$[\hat{\alpha}_L, \hat{\alpha}_U] = \hat{\alpha}_M \mp z_{\lambda/2}\sqrt{Var(\hat{\alpha}_M)}.$$

3 Bayesian Estimation

In this section, we have obtained the Bayes estimates for unknown parameter α, reliability function R(t) and the hazard rate function h(t). For estimating these quantities, two loss functions have been taken into consideration, which are defined as;
Squared error loss function (SELF): $L_S(\alpha,\hat{\alpha}) = (\hat{\alpha}-\alpha)^2$,
General entropy loss function (GELF):$L_G \propto \left(\frac{\hat{\alpha}}{\alpha}\right)^{\delta} - \delta \ln\left(\frac{\hat{\alpha}}{\alpha}\right) - 1 \;\; ; \delta \neq 0.$
In each case, $\hat{\alpha}$ represents the estimate of unknown parameter α and δ is the shape parameters of GELF which reflect the departure from symmetry. The Bayes estimate of α with respect to the L_S and L_G loss function is obtained from its posterior distribution as;

$$\hat{\alpha}_S = \{E_\alpha(\alpha|\underline{x})\}$$

$$\hat{\alpha}_G = \left\{E_\alpha(\alpha^{-\delta}|\underline{x})\right\}^{-1/\delta}$$

provided the expectation of the above quantity must exist.

3.1 Bayes Estimators of the Parameter, Reliability Function and Hazard Function under Gamma Prior (Prior 1)

In this subsection, we consider the Bayes procedure to derive the point estimates of the parameters α, reliability function $R(t)$ and hazard function $H(t)$ based on PT-II censored data. In Bayesian analysis, the parameter of interest is to be considered as a random variable and follows some prior distribution. Here, we assume that, parameter α having $gamma(a,b)$ density i.e.

$$\pi_1(\alpha) \propto \alpha^{a-1}e^{-b\alpha}; \alpha > 0$$

where, a, b are the hyper parameters assume to be known. The above considered prior is more applicable in the sense that, it is more flexible and having different variety of prior distribution which may be the reason behind its popularity. Therefore, based on the above prior, the posterior distribution of α is given as;

$$p_1(\alpha|\underline{x}) \propto \alpha^{m+a-1}e^{-\sum_{i=1}^{m}\frac{\alpha}{x_i}-b\alpha}\prod_{i=1}^{m}U(\alpha,x_i,R_i) \propto Gamma\left[m+a, b+\sum_{i=1}^{m}\left(\frac{1}{x_i}\right)\right]\prod_{i=1}^{m}U(\alpha,x_i,R_i), \tag{9}$$

where $U(\alpha,x_i,R_i) = \frac{1}{x_i^2}\left\{1-e^{-\alpha/x_i}\right\}^{R_i}$.

Now, the Bayes estimators of the parameter, reliability function and hazard function under SELF and GELF are given as;

$$\hat{\alpha}_{BS1} = \frac{\int\limits_{\alpha=0}^{\infty} \alpha^{m+a} e^{-\sum_{i=1}^{m}\frac{\alpha}{x_i}-b\alpha} \prod\limits_{i=1}^{m} U(\alpha,x_i,R_i)d\alpha}{\int\limits_{\alpha=0}^{\infty} \alpha^{m+a-1} e^{-\sum_{i=1}^{m}\frac{\alpha}{x_i}-b\alpha} \prod\limits_{i=1}^{m} U(\alpha,x_i,R_i)d\alpha} \tag{3.11}$$

$$\hat{R}_{BS1} = \frac{\int\limits_{\alpha=0}^{\infty} \left(1-e^{-\alpha/t}\right) \alpha^{m+a-1} e^{-\sum\limits_{i=1}^{m}\frac{\alpha}{x_i}-b\alpha} \prod\limits_{i=1}^{m} U(\alpha,x_i,R_i)d\alpha}{\int\limits_{\alpha=0}^{\infty} \alpha^{m+a-1} e^{-\sum\limits_{i=1}^{m}\frac{\alpha}{x_i}-b\alpha} \prod\limits_{i=1}^{m} U(\alpha,x_i,R_i)d\alpha} \tag{3.12}$$

$$\hat{H}_{BS1} = \frac{\int\limits_{\alpha=0}^{\infty} \left(\frac{\alpha e^{-\alpha/t}}{t^2(1-e^{-\alpha/t})}\right) \alpha^{m+a-1} e^{-\sum\limits_{i=1}^{m}\frac{\alpha}{x_i}-b\alpha} \prod\limits_{i=1}^{m} U(\alpha,x_i,R_i)d\alpha}{\int\limits_{\alpha=0}^{\infty} \alpha^{m+a-1} e^{-\sum\limits_{i=1}^{m}\frac{\alpha}{x_i}-b\alpha} \prod\limits_{i=1}^{m} U(\alpha,x_i,R_i)d\alpha} \tag{3.13}$$

$$\hat{\alpha}_{BG1} = \left[\frac{\int\limits_{\alpha=0}^{\infty} \alpha^{m+a-\delta-1} e^{-\sum\limits_{i=1}^{m}\frac{\alpha}{x_i}-b\alpha} \prod\limits_{i=1}^{m} U(\alpha,x_i,R_i)d\alpha}{\int\limits_{\alpha=0}^{\infty} \alpha^{m+a-1} e^{-\sum\limits_{i=1}^{m}\frac{\alpha}{x_i}-b\alpha} \prod\limits_{i=1}^{m} U(\alpha,x_i,R_i)d\alpha}\right]^{(-1/\delta)} \tag{3.14}$$

$$\hat{R}_{BG1} = \left[\frac{\int\limits_{\alpha=0}^{\infty} \left\{1-e^{-\alpha/t}\right\}^{-\delta} \alpha^{m+a-1} e^{-\sum\limits_{i=1}^{m}\frac{\alpha}{x_i}-b\alpha} \prod\limits_{i=1}^{m} U(\alpha,x_i,R_i)d\alpha}{\int\limits_{\alpha=0}^{\infty} \alpha^{m+a-1} e^{-\sum\limits_{i=1}^{m}\frac{\alpha}{x_i}-b\alpha} \prod\limits_{i=1}^{m} U(\alpha,x_i,R_i)d\alpha}\right]^{(-1/\delta)} \tag{3.15}$$

$$\hat{H}_{BG1} = \left[\frac{\int\limits_{\alpha=0}^{\infty} \left\{\frac{\alpha e^{-\alpha/t}}{t^2(1-e^{-\alpha/t})}\right\}^{-\delta} \alpha^{m+a-1} e^{-\sum\limits_{i=1}^{m}\frac{\alpha}{x_i}-b\alpha} \prod\limits_{i=1}^{m} U(\alpha,x_i,R_i)d\alpha}{\int\limits_{\alpha=0}^{\infty} \alpha^{m+a-1} e^{-\sum\limits_{i=1}^{m}\frac{\alpha}{x_i}-b\alpha} \prod\limits_{i=1}^{m} U(\alpha,x_i,R_i)d\alpha}\right]^{(-1/\delta)} \tag{3.16}$$

3.2 Bayes Estimators of the Parameter, Reliability Function and Hazard Function under Inverted Gamma Prior (Prior 2)

In this subsection, we have obtained the Bayes estimates under the consideration of inverted gamma prior $IG(c,d)$ based on PT-II censored data. Inverted gamma prior is also a flexible and conjugate prior for this distribution in complete sample. Keeping these points in mind we have considered it. It was also considered by several authors see Prakash (2009) and Singh et al. (2011). Therefore, here we assume that, parameter α has inverted gamma density. i.e.

$$\pi_2(\alpha) \propto \alpha^{-c-1} e^{-d/\alpha}; \alpha > 0, c, d > 0,$$

where c, d are the hyper parameters assume to be known. Therefore, based on the above prior, the posterior distribution of α will be;

$$p_2(\alpha|\underline{x}) \propto \alpha^{m-c-1} e^{-\sum\limits_{i=1}^{m}\frac{\alpha}{x_i}-d/\alpha} \prod_{i=1}^{m} U(\alpha,x_i,R_i). \tag{10}$$

Therefore, the Bayes estimators of the parameter, reliability function and hazard function under SELF and GELF are given as;

$$\hat{\alpha}_{BS2} = \frac{\int\limits_{\alpha=0}^{\infty} \alpha^{m-c} e^{-\sum\limits_{i=1}^{m}\frac{\alpha}{x_i}-d\alpha} \prod\limits_{i=1}^{m} U(\alpha,x_i,R_i)d\alpha}{\int\limits_{\alpha=0}^{\infty} \alpha^{m-c-1} e^{-\sum\limits_{i=1}^{m}\frac{\alpha}{x_i}-d\alpha} \prod\limits_{i=1}^{m} U(\alpha,x_i,R_i)d\alpha} \tag{3.2.1}$$

$$\hat{R}_{BS2}=\frac{\int\limits_{\alpha=0}^{\infty}\left(1-e^{-\alpha/t}\right)\alpha^{m-c}e^{-\sum\limits_{i=1}^{m}\frac{\alpha}{x_i}-d/\alpha}\prod\limits_{i=1}^{m}U(\alpha,x_i,R_i)d\alpha}{\int\limits_{\alpha=0}^{\infty}\alpha^{m-c-1}e^{-\sum\limits_{i=1}^{m}\frac{\alpha}{x_i}-d/\alpha}\prod\limits_{i=1}^{m}U(\alpha,x_i,R_i)d\alpha} \tag{3.2.2}$$

$$\hat{H}_{BS2}=\frac{\int\limits_{\alpha=0}^{\infty}\left(\frac{\alpha e^{-\alpha/t}}{t^2(1-e^{-\alpha/t})}\right)\alpha^{m-c-1}e^{-\sum\limits_{i=1}^{m}\frac{\alpha}{x_i}-d/\alpha}\prod\limits_{i=1}^{m}U(\alpha,x_i,R_i)d\alpha}{\int\limits_{\alpha=0}^{\infty}\alpha^{m-c-1}e^{-\sum\limits_{i=1}^{m}\frac{\alpha}{x_i}-d/\alpha}\prod\limits_{i=1}^{m}U(\alpha,x_i,R_i)d\alpha} \tag{3.2.3}$$

$$\hat{\alpha}_{BG2}=\left[\frac{\int\limits_{\alpha=0}^{\infty}\alpha^{m-c-\delta-1}e^{-\sum\limits_{i=1}^{m}\frac{\alpha}{x_i}-d\alpha}\prod\limits_{i=1}^{m}U(\alpha,x_i,R_i)d\alpha}{\int\limits_{\alpha=0}^{\infty}\alpha^{m-c-1}e^{-\sum\limits_{i=1}^{m}\frac{\alpha}{x_i}-d\alpha}\prod\limits_{i=1}^{m}U(\alpha,x_i,R_i)d\alpha}\right]^{(-1/\delta)} \tag{3.2.4}$$

$$\hat{R}_{BG2}=\left[\frac{\int\limits_{\alpha=0}^{\infty}\left\{1-e^{-\alpha/t}\right\}^{-\delta}\alpha^{m-c-1}e^{-\sum\limits_{i=1}^{m}\frac{\alpha}{x_i}-d/\alpha}\prod\limits_{i=1}^{m}U(\alpha,x_i,R_i)d\alpha}{\int\limits_{\alpha=0}^{\infty}\alpha^{m-c-1}e^{-\sum\limits_{i=1}^{m}\frac{\alpha}{x_i}-d\alpha}\prod\limits_{i=1}^{m}U(\alpha,x_i,R_i)d\alpha}\right]^{(-1/\delta)} \tag{3.2.5}$$

$$\hat{H}_{BG2}=\left[\frac{\int\limits_{\alpha=0}^{\infty}\left\{\frac{\alpha e^{-\alpha/t}}{t^2(1-e^{-\alpha/t})}\right\}^{-\delta}\alpha^{m-c-1}e^{-\sum\limits_{i=1}^{m}\frac{\alpha}{x_i}-d/\alpha}\prod\limits_{i=1}^{m}U(\alpha,x_i,R_i)d\alpha}{\int\limits_{\alpha=0}^{\infty}\alpha^{m-c-1}e^{-\sum\limits_{i=1}^{m}\frac{\alpha}{x_i}-d/\alpha}\prod\limits_{i=1}^{m}U(\alpha,x_i,R_i)d\alpha}\right]^{(-1/\delta)} \tag{3.2.6}$$

3.3 Bayes Estimators of the Parameter, Reliability Function and Hazard Function under Non-Informative Prior (Prior 0)

The selection of prior distribution is often based on the type of prior information available to us. When we have little or no information about the parameter, a non-informative prior should be used. Jeffreys prior is one of the general class of non-informative prior. The important feature of this prior is that it is not affected by the restriction of the parameter space. Several authors have given a general justification for using Jeffreys prior for an exponential family by showing that a proper posterior is produced. It motivated us to consider non-informative prior for the parameter. The prior of α may be taken as;

$$\pi_3(\alpha)\propto\frac{1}{\alpha};\alpha>0$$

Therefore, based on the above prior, the posterior distribution of α will be;

$$p_3(\alpha|\underline{x})\propto\alpha^{m-1}e^{-\sum\limits_{i=1}^{m}\frac{\alpha}{x_i}}\prod_{i=1}^{m}U(\alpha,x_i,R_i)$$

Therefore, the Bayes estimator of the parameter, reliability function and hazard function under above two loss functions are expressed as;

$$\hat{\alpha}_{BS0}=\frac{\int\limits_{\alpha=0}^{\infty}\alpha^{m}e^{-\sum\limits_{i=1}^{m}\frac{\alpha}{x_i}}\prod\limits_{i=1}^{m}U(\alpha,x_i,R_i)d\alpha}{\int\limits_{\alpha=0}^{\infty}\alpha^{m-1}e^{-\sum\limits_{i=1}^{m}\frac{\alpha}{x_i}}\prod\limits_{i=1}^{m}U(\alpha,x_i,R_i)d\alpha} \tag{3.3.1}$$

$$\hat{R}_{BS0} = \frac{\int\limits_{\alpha=0}^{\infty} \left\{1-e^{-\alpha/t}\right\} \alpha^{m-1} e^{-\sum\limits_{i=1}^{m} \frac{\alpha}{x_i}} \prod\limits_{i=1}^{m} U(\alpha, x_i, R_i) d\alpha}{\int\limits_{\alpha=0}^{\infty} \alpha^{m-1} e^{-\sum\limits_{i=1}^{m} \frac{\alpha}{x_i}} \prod\limits_{i=1}^{m} U(\alpha, x_i, R_i) d\alpha} \tag{3.3.2}$$

$$\hat{H}_{BS0} = \frac{\int\limits_{\alpha=0}^{\infty} \left\{\frac{\alpha e^{-\alpha/t}}{t^2(1-e^{-\alpha/t})}\right\} \alpha^{m-1} e^{-\sum\limits_{i=1}^{m} \frac{\alpha}{x_i}} \prod\limits_{i=1}^{m} U(\alpha, x_i, R_i) d\alpha}{\int\limits_{\alpha=0}^{\infty} \alpha^{m-1} e^{-\sum\limits_{i=1}^{m} \frac{\alpha}{x_i}} \prod\limits_{i=1}^{m} U(\alpha, x_i, R_i) d\alpha} \tag{3.3.3}$$

$$\hat{\alpha}_{BG0} = \left[\frac{\int\limits_{\alpha=0}^{\infty} \alpha^{m-\delta-1} e^{-\sum\limits_{i=1}^{m} \frac{\alpha}{x_i}} \prod\limits_{i=1}^{m} U(\alpha, x_i, R_i) d\alpha}{\int\limits_{\alpha=0}^{\infty} \alpha^{m-1} e^{-\sum\limits_{i=1}^{m} \frac{\alpha}{x_i}} \prod\limits_{i=1}^{m} U(\alpha, x_i, R_i) d\alpha}\right]^{(-1/\delta)} \tag{3.3.4}$$

$$\hat{R}_{BG0} = \left[\frac{\int\limits_{\alpha=0}^{\infty} \left\{1-e^{-\alpha/t}\right\}^{-\delta} \alpha^{m-1} e^{-\sum\limits_{i=1}^{m} \frac{\alpha}{x_i}} \prod\limits_{i=1}^{m} U(\alpha, x_i, R_i) d\alpha}{\int\limits_{\alpha=0}^{\infty} \alpha^{m-1} e^{-\sum\limits_{i=1}^{m} \frac{\alpha}{x_i}} \prod\limits_{i=1}^{m} U(\alpha, x_i, R_i) d\alpha}\right]^{(-1/\delta)} \tag{3.3.5}$$

$$\hat{H}_{BG0} = \left[\frac{\int\limits_{\alpha=0}^{\infty} \left\{\frac{\alpha e^{-\alpha/t}}{t^2(1-e^{-\alpha/t})}\right\}^{-\delta} \alpha^{m-1} e^{-\sum\limits_{i=1}^{m} \frac{\alpha}{x_i}} \prod\limits_{i=1}^{m} U(\alpha, x_i, R_i) d\alpha}{\int\limits_{\alpha=0}^{\infty} \alpha^{m-1} e^{-\sum\limits_{i=1}^{m} \frac{\alpha}{x_i}} \prod\limits_{i=1}^{m} U(\alpha, x_i, R_i) d\alpha}\right]^{(-1/\delta)} \tag{3.3.6}$$

3.4 Markov Chain Monte Carlo Method

From previous sections 3.1, 3.2 and 3.3 we observed that analytical solution of all considered estimators are not possible. Therefore, MCMC method is used to resolve such type of situations. MCMC method is the one of the best method for obtaining the approximate solution of the posterior expectations. After extracting or simulating the posterior samples, we may easily obtain the estimates of the parameter and reliability characteristics. To obtain the solutions of the posterior expectation, we have considered the Importance sampling method and Metropolis-Hastings algorithm of MCMC method under Prior 1 and Prior 2 respectively. For more detail about MCMC method, see, Smith and Robert (1993), Upadhyay et al. (2001) and Singh et al. (2013).

Importance Sampling Method:

To implement the importance sampling method under the consideration of Prior 1, the posterior distribution of α is given as;

$$p_1(\alpha|\underline{x}) \propto Gamma\left[m+a, b+\sum_{i=1}^{m}\left(\frac{1}{x_i}\right)\right] \prod_{i=1}^{m} U(\alpha, x_i, R_i)$$

Therefore, the following steps are taken to extract the sample from the above posterior distribution.

- generate α_1 from Gamma(.,.)
- repeat step 1 to generate $\alpha_1, \alpha_2, \cdots, \alpha_s$

Now, the Bayes estimates of $\tau(\alpha)$ with respect to the SELF and GELF will be;

$$\hat{\tau}(\alpha)_{BS1} = \frac{1}{s}\left\{\frac{\sum\limits_{i=1}^{s} \tau(\alpha) U(\alpha, x_i, R_i) d\alpha}{\sum\limits_{i=1}^{s} U(\alpha, x_i, R_i) d\alpha}\right\}$$

and

$$\hat{\tau}(\alpha)_{BG1} = \left[\frac{1}{s} \left\{ \frac{\sum_{i=1}^{s} \tau(\alpha)^{-\delta} U(\alpha, x_i, R_i) d\alpha}{\sum_{i=1}^{s} U(\alpha, x_i, R_i) d\alpha} \right\} \right]^{(-1/\delta)}$$

respectively.
The $100(1-\lambda)\%$ HPD intervals for the parameter α using the method of Chen and Shao (1990). One may also refer to Kundu and Pradhan (2009a) for a review on this method.

Metropolis-Hastings Algorithm:

In case of Prior 2 no any standard distribution is found for simulating the samples from its respective posterior distribution. Therefore most suitable method for extracting the posterior samples is Metropolis under Gibbs algorithms are taken into consideration. The M-H under Gibbs algorithm consist the following steps;

- set the initial values of α say α_0
- Set l=1
- Generate posterior sample for α from (10).
- Repeat step 2, for all $l = 1, 2, 3, \cdots s$ and obtained $\alpha_1, \alpha_2, \cdots, \alpha_s$

After obtaining the posterior samples, the Bayes estimates of the parameters, reliability function and hazard function under SELF are the mean of the posterior samples. Therefore, we have,

$$\hat{\alpha}_{BS2} = \frac{1}{s} \sum_{l=1}^{s} \alpha_l$$

$$\hat{R}_{BS2} = \frac{1}{s} \sum_{l=1}^{s} \left\{ 1 - e^{-\alpha_l/t} \right\}$$

$$\hat{H}_{BS2} = \frac{1}{s} \sum_{l=1}^{s} \frac{\alpha_l e^{-\alpha_l/t}}{t^2 (1 - e^{-\alpha_l/t})}$$

and Bayes estimators under GELF are given by;

$$\hat{\alpha}_{GS2} = \left\{ \frac{1}{s} \sum_{l=1}^{s} \alpha_l^{-\delta} \right\}^{(-1/\delta)}$$

$$\hat{R}_{GS2} = \left\{ \frac{1}{s} \sum_{l=1}^{s} \left\{ 1 - e^{-\alpha_l/t} \right\}^{-\delta} \right\}^{(-1/\delta)}$$

$$\hat{H}_{GS2} = \left\{ \frac{1}{s} \sum_{l=1}^{s} \left[\frac{\alpha_l e^{-\alpha_l/t}}{t^2 (1 - e^{-\alpha_l/t})} \right]^{-\delta} \right\}^{(-1/\delta)}$$

After extracting the posterior samples we can easily construct the HPD credible intervals for α. Therefore, for this purpose order $\alpha_1, \alpha_2, ..., \alpha_s$ as $\alpha_1 < \alpha_2 < \cdots < \alpha_s$. Then $100(1-\lambda)\%$ credible intervals of α is

$$(\alpha_1, \alpha_{[s(1-\lambda)+1]}), \cdots, (\alpha_{[s\lambda]}, \alpha_s)$$

. Here $[x]$ denotes the greatest integer less than or equal to x. Then, the HPD credible interval is that interval which has the shortest length. Further, the Bayes estimates under non-informative prior can be obtained by setting the values of hyper parameter is to be zero in any of the above algorithm.

4 Algorithm for Sample Generation under PT-II

Using the algorithm of Balakrishnan and Agrawala (1995), we have used the following steps to generate a PT-II censored sample from the IED.

* Specify the values of n, m and α.
* Generate m iid random numbers $u_1, u_2, \cdots, u_m$ from $U(0,1)$.
* Set $\xi_i = \ln(1-u_i)$, so that ξ_i's are iid standard exponential variates.
* For given censoring scheme $R = (R_1, R_2, \cdots, R_m)$, set $y_1 = \xi_1/m$ and for $i = 2, 3, \cdots, m$, $\phi_i = \phi_{i-1} + \dfrac{\xi_i}{n - \sum_{j=1}^{i-1} R_j - i + 1}$

  Now $(\phi_1, \phi_2, ..., \phi_m)$ are the progressive Type II censored sample from standard exponential distribution with censoring scheme $R = (R_1, R_2, \cdots, R_m)$.
* Set $\psi_i = 1 - exp(-\phi_i)$, so that ψ_is form a progressive Type II censored sample from U(0,1).
* Set $x_i = F^{-1}(\psi_i) = -\left\{\dfrac{\alpha}{\ln\psi}\right\}$.

Now, $(x_1, x_2, ..., x_m)$ are the PT-II right censored sample from IED(α) with a censoring scheme $R = (R_1, R_2, \cdots, R_m)$.

5 Simulation Study and Comparison

In this section, we investigate the performances of the Bayes estimators with corresponding ML estimators based on 1000 replications. In order to perform the comparison study following steps taken into account.

1. For specified values of n, m and α, generate m PT-II censored samples using the algorithm discussed in previous section.
2. For different censoring scheme n is taken as 30 and the values of m has been chosen in such a way that the observations are censored as $10\%, 20\%, 40\%, 50\%$ and 60% for fixed value of $\alpha = 2$.
3. The ML estimators of the parameter, reliability function and hazard function and corresponding asymptotic confidence intervals have been computed.
4. For Bayesian analysis, we have assumed that parameter has gamma and inverted gamma prior and Jeffreys prior. The values of hyper parameters are taken as (a=c=4 and b=d=2).
5. Importance sampling method and Metropolis-Hastings algorithm of MCMC technique have been used to extract posterior samples under Prior 1 and Prior 2 respectively.
6. On the basis of simulated posterior samples, we have obtained the Bayes estimators of the parameter, reliability and hazard functions under the assumption of the above prior using SELF and GELF.
7. In simulation study under GELF, only one choice of loss parameter δ is considered. $\delta = 0.5$ (for over estimation) and $\delta = -0.5$ (for under estimation).
8. We have also constructed 95% highest posterior density (HPD) interval for the parameter.
9. The performances of the estimators have been reported under SELF and GELF both, see, Tables [2-12].

In order to choose different censoring scheme $a*s$ represents that the number a is repeated s times, see Table 1. From this extensive study, we may conclude the followings;

i. The risks of the Bayes estimators is least as compared to the risk of the ML estimators.
ii. The risk of the estimators decreases as the percentage of m is increases.
iii. From the Tables 2 and 3, we observed that, the risk of the Bayes estimators of the parameter, reliability function and hazard function under Prior 1 is smaller than the risk of the estimators under Prior 2.
iv. From Table 3, we noticed that the length of HPD intervals is smaller than the length of asymptotic confidence interval. Further, we have also observed that the average confidence length in case of Prior 1 is minimum as compared Prior 2.
v. From the Table 4, 5 and 6, we see that, the Bayes estimators under GELF have smaller risk when $\delta = 0.5$ as compared to the $\delta = -0.5$.
vi. From the Table 7, we observed that, the HPD length and risk of the Bayes estimators under Prior 1 and Prior 2 is minimum as compared of Prior 0.
vii. From the Table 8, we observed that the risks of the estimators under GELF is minimum for $\delta = 0.5$ as compared to those of the estimators for $\delta = -0.5$. It means that under estimation is more serious then over estimation.

6 Real Data Analysis

In this section, we propose data analysis to illustrate our proposed methodology based on two data set . The considered data set has been used by several authors when life time follows inverted exponential distribution.
Data Set-I: We have generated a random sample of size 50 from IED with parameter $\alpha = 3$ and chooses different censoring schemes. The simulated data is presented as follows;
$0.4, 0.77, 0.78, 1.03, 1.11, 1.16, 1.82, 1.83, 1.86, 1.9, 1.96, 2.02, 2.79, 2.84, 2.85, 3.4, 3.71, 3.75, 3.82,$
$4.03, 4.2, 4.29, 4.29, 4.42, 4.57, 4.74, 4.79, 5.01, 5.03, 5.26, 5.61, 5.7, 6.44, 6.98, 8.09, 8.42, 8.82, 9.28,$
$9.49, 10.38, 11.49, 11.65, 14.86, 22.39, 24.29, 27.63, 36.28, 49.38, 96.77, 100.53$
Data Set-II:
$12, 15, 22, 24, 24, 32, 32, 33, 34, 38, 38, 43, 44, 48, 52, 53, 54, 54, 55, 56, 57, 58, 58, 59, 60, 60, 60, 60, 61,$
$62, 63, 65, 65, 67, 68, 70, 70, 72, 73, 75, 76, 76, 81, 83, 84, 85, 87, 91, 95, 96, 98, 99, 109, 110, 121, 127,$
$129, 131, 143, 146, 146, 175, 175, 211, 233, 258, 258, 263, 297, 341, 341, 376.$

The data set-II was initially proposed by Bjerkedal (1960) and used by Kundu and Howlader (2010) for Bayesian estimation and prediction of the inverse Weibull distribution under Type-II censored data. Bayes estimators of the parameter and reliability function of inverted exponential distribution under the general entropy loss function based on complete, Type-I and Type-II censored samples have discussed by Singh et al. (2012). Recently, Maheshwari et al. (2014) have considered this data set and derived the Bayes estimation procedure under hybrid censoring schemes. Here, we have also used this data set under progressive Type-II censoring scheme. For this purpose, we have taken different censoring schemes, see Table [8-11]. Based on these two data set, we have calculated the ML and Bayes estimators of the parameter, reliability function, hazard function and also provided interval estimates for different censoring scheme and loss parameter $\delta[(-1.5, 1.5), (-1.0, 1.0), (-0.5, 0.5)]$.

Table 1: Table represents the different censoring schemes which are considered for simulation.

n	m	r1	r2	r3	r4
30	12	3*6,0*6	0*5,6*3,0*4	0*2,2*9,0	0*6,3*6
	15	3*5,0*10	1*15	0*6,5*3,0*6	0*10,3*5
	18	2*6,0*12	0*8,4*3,0*7	0*3,1*12,0*3	0*12,2*6
	24	1*6,0*18	0*11,2*3,0*10	0*9,1*6,0*9	0*18,1*6
	27	3,0*26	0*12,1*3,0*12	0*13,3,0*13	0*26,3
	30	0*30			

7 Concluding Remarks

In this paper, we proposed Bayesian and maximum Likelihood estimation for the parameter, reliability function and hazard function under PT-II censoring scheme using different prior information and loss functions. Therefore, from this extensive study of the results of simulation, we observed that the Bayes estimators with an informative prior performs well in all considered cases.. Therefore, we conclude that the Bayes estimator with an informative prior may be used particularly when some a priori information about the parameter is known. However, if no a priori information about the parameter is available, MLEs may be recommended for their use.

Table 2: Risk of the parameter α and reliability function R under SELF using Prior 1 and Prior 2 for fixed value of n=30.

m	RI	$\hat{\alpha}_M$	$\hat{\alpha}_{BS1}$	$\hat{\alpha}_{BS2}$	$\hat{\alpha}_{BG1}$	$\hat{\alpha}_{BG2}$	$\hat{R}_M$	$\hat{R}_{BS1}$	$\hat{R}_{BS2}$	$\hat{R}_{BG1}$	$\hat{R}_{BG1}$
12	r1	0.3854	0.2651	0.3570	0.3794	0.3642	0.0147	0.0104	0.0137	0.0137	0.0137
	r2	0.6362	0.4551	0.5839	0.6061	0.5911	0.0254	0.0183	0.0232	0.0232	0.0232
	r3	0.6643	0.4765	0.6092	0.6315	0.6165	0.0265	0.0192	0.0242	0.0242	0.0242
	r4	0.7985	0.5829	0.7324	0.7532	0.7392	0.0326	0.0237	0.0297	0.0297	0.0297
15	r1	0.2382	0.1687	0.2222	0.2343	0.2260	0.0085	0.0063	0.0081	0.0081	0.0081
	r2	0.4495	0.3330	0.4261	0.4427	0.4316	0.0172	0.0130	0.0164	0.0164	0.0164
	r3	0.4293	0.3172	0.4080	0.4245	0.4134	0.0164	0.0124	0.0157	0.0157	0.0157
	r4	0.5980	0.4521	0.5651	0.5816	0.5705	0.0236	0.0180	0.0223	0.0223	0.0223
18	r1	0.1764	0.1282	0.1638	0.1704	0.1658	0.0060	0.0046	0.0058	0.0058	0.0058
	r2	0.3171	0.2405	0.3066	0.3183	0.3104	0.0117	0.0092	0.0114	0.0114	0.0114
	r3	0.3231	0.2465	0.3122	0.3236	0.3159	0.0120	0.0094	0.0117	0.0117	0.0117
	r4	0.4245	0.3292	0.4102	0.4227	0.4143	0.0162	0.0128	0.0157	0.0157	0.0157
24	r1	0.1460	0.1064	0.1212	0.1187	0.1203	0.0042	0.0031	0.0037	0.0037	0.0037
	r2	0.1602	0.1242	0.1516	0.1549	0.1526	0.0054	0.0043	0.0052	0.0052	0.0052
	r3	0.1682	0.1303	0.1583	0.1612	0.1592	0.0056	0.0045	0.0054	0.0054	0.0054
	r4	0.2009	0.1569	0.1912	0.1950	0.1924	0.0069	0.0056	0.0067	0.0067	0.0067
27	r1	0.1602	0.1161	0.1288	0.1234	0.1269	0.0043	0.0031	0.0036	0.0036	0.0036
	r2	0.1413	0.1069	0.1233	0.1220	0.1228	0.0042	0.0033	0.0038	0.0038	0.0038
	r3	0.1251	0.0954	0.1108	0.1101	0.1105	0.0038	0.0030	0.0035	0.0035	0.0035
	r4	0.1449	0.1119	0.1314	0.1314	0.1313	0.0045	0.0036	0.0042	0.0042	0.0042
30	r1	0.1532	0.1132	0.1243	0.1190	0.1225	0.0039	0.0029	0.0033	0.0033	0.0033

Table 3: Risk of the hazard function H under SELF and interval estimates of the parameter using Prior 1 and Prior 2.

m	RI	$\hat{H}_M$	$\hat{H}_{BS1}$	$\hat{H}_{BS2}$	$\hat{H}_{BG1}$	$\hat{H}_{BG2}$	Asypmtotic Interval			HPD (Prior 1)			HPD (Prior 2)		
							$\hat{\alpha}_L$	$\hat{\alpha}_U$	Length	$\hat{\alpha}_L$	$\hat{\alpha}_U$	Length	$\hat{\alpha}_L$	$\hat{\alpha}_U$	Length
12	r1	0.0505	0.0333	0.0463	0.0481	0.0469	0.6318	2.8789	2.2471	0.8430	2.3140	1.4709	1.0385	2.8943	1.8558
	r2	0.0784	0.0556	0.0722	0.0738	0.0727	0.5412	2.9518	2.4107	0.7457	2.0435	1.2978	0.9442	2.6110	1.6668
	r3	0.0817	0.0581	0.0752	0.0768	0.0757	0.5296	2.9101	2.3805	0.7320	2.0065	1.2745	0.9330	2.5747	1.6417
	r4	0.0954	0.0700	0.0882	0.0895	0.0886	0.4933	2.7794	2.2860	0.6886	2.0093	1.3207	0.8895	2.4598	1.5703
15	r1	0.0348	0.0237	0.0312	0.0317	0.0313	0.8195	2.4992	1.6797	0.9898	2.4845	1.4947	1.1889	2.4946	1.3057
	r2	0.0585	0.0422	0.0551	0.0563	0.0555	0.6860	2.2921	1.6060	0.8539	2.1448	1.2909	1.0603	2.6739	1.6136
	r3	0.0559	0.0401	0.0528	0.0541	0.0532	0.6956	2.2213	1.5257	0.8644	2.1719	1.3075	1.0705	2.5764	1.5059
	r4	0.0747	0.0559	0.0706	0.0718	0.0710	0.6253	2.5907	1.9654	0.7908	2.2098	1.4190	0.9950	2.5275	1.5324
18	r1	0.0279	0.0197	0.0246	0.0245	0.0245	0.9481	2.5765	1.6284	1.0899	2.5469	1.4571	1.2958	2.3728	1.0770
	r2	0.0435	0.0318	0.0414	0.0423	0.0417	0.5180	2.4229	1.9049	0.9599	2.2476	1.2878	1.1731	2.8612	1.6881
	r3	0.0438	0.0324	0.0418	0.0426	0.0421	0.6819	2.4260	1.7441	0.9606	2.2497	1.2891	1.1741	2.8640	1.6899
	r4	0.0553	0.0419	0.0532	0.0542	0.0535	0.7589	2.5623	1.8035	0.8993	2.1064	1.2072	1.1112	2.7176	1.6064
24	r1	0.0302	0.0230	0.0239	0.0220	0.0232	1.1761	2.7451	1.5690	1.2690	2.6856	1.4166	1.1002	2.5433	1.4430
	r2	0.0259	0.0198	0.0236	0.0233	0.0235	1.0693	2.4957	1.4264	1.2711	2.3757	1.1046	1.3000	2.4204	1.1205
	r3	0.0274	0.0209	0.0248	0.0244	0.0246	1.0707	2.4990	1.4283	1.4716	2.1478	0.6762	1.4007	2.1234	0.7227
	r4	0.0325	0.0243	0.0294	0.0290	0.0293	1.0340	2.4133	1.3794	1.3646	2.0460	0.6814	1.5357	2.2400	0.7044
27	r1	0.0374	0.0284	0.0290	0.0262	0.0280	1.2691	2.8067	1.5376	1.5867	2.4085	0.8218	1.3468	2.5402	1.1934
	r2	0.0287	0.0219	0.0237	0.0223	0.0232	1.1996	2.6528	1.4533	1.5218	2.2687	0.7469	1.2817	2.4609	1.1792
	r3	0.0240	0.0188	0.0203	0.0193	0.0200	1.1976	2.6485	1.4509	1.5208	2.2651	0.7443	1.2841	2.2609	0.9769
	r4	0.0269	0.0209	0.0233	0.0223	0.0229	1.1676	2.5822	1.4146	1.4909	2.2055	0.7146	1.2546	2.1550	0.9004
30	r1	0.0384	0.0294	0.0302	0.0273	0.0292	1.3234	2.7987	1.4752	1.6386	2.3092	0.6706	1.3914	2.2738	0.8824

Table 4: Risk of the parameter α under GELF for specified value of loss parameter δ using Prior 1 and Prior 2 respectively.

m	RI	δ=0.5					δ=-0.5				
		$\hat{\alpha}_M$	$\hat{\alpha}_{BS1}$	$\hat{\alpha}_{BS2}$	$\hat{\alpha}_{BG1}$	$\hat{\alpha}_{BG2}$	$\hat{\alpha}_M$	$\hat{\alpha}_{BS1}$	$\hat{\alpha}_{BS2}$	$\hat{\alpha}_{BG1}$	$\hat{\alpha}_{BG2}$
12	r1	0.0179	0.0115	0.0162	0.0146	0.0173	0.0213	0.0132	0.0190	0.0143	0.0194
	r2	0.0323	0.0212	0.0288	0.0257	0.0301	0.0400	0.0253	0.0352	0.0270	0.0357
	r3	0.0338	0.0223	0.0301	0.0269	0.0314	0.0418	0.0265	0.0368	0.0283	0.0373
	r4	0.0426	0.0284	0.0379	0.0336	0.0392	0.0538	0.0344	0.0472	0.0365	0.0478
15	r1	0.0100	0.0068	0.0094	0.0086	0.0100	0.0114	0.0076	0.0107	0.0083	0.0109
	r2	0.0211	0.0147	0.0197	0.0177	0.0205	0.0251	0.0170	0.0233	0.0182	0.0236
	r3	0.0201	0.0140	0.0188	0.0169	0.0196	0.0239	0.0162	0.0222	0.0173	0.0225
	r4	0.0297	0.0210	0.0275	0.0247	0.0284	0.0363	0.0248	0.0333	0.0263	0.0337
18	r1	0.0069	0.0049	0.0066	0.0060	0.0069	0.0077	0.0054	0.0074	0.0058	0.0075
	r2	0.0140	0.0101	0.0134	0.0122	0.0140	0.0162	0.0115	0.0155	0.0123	0.0157
	r3	0.0144	0.0105	0.0138	0.0126	0.0144	0.0168	0.0120	0.0161	0.0128	0.0163
	r4	0.0198	0.0146	0.0189	0.0172	0.0196	0.0236	0.0169	0.0224	0.0179	0.0226
24	r1	0.0046	0.0034	0.0041	0.0037	0.0041	0.0048	0.0035	0.0043	0.0036	0.0043
	r2	0.0061	0.0047	0.0059	0.0055	0.0061	0.0067	0.0051	0.0066	0.0054	0.0066
	r3	0.0064	0.0049	0.0062	0.0057	0.0064	0.0071	0.0054	0.0069	0.0057	0.0069
	r4	0.0080	0.0061	0.0077	0.0071	0.0079	0.0089	0.0068	0.0087	0.0071	0.0087
27	r1	0.0046	0.0035	0.0040	0.0035	0.0039	0.0046	0.0035	0.0040	0.0035	0.0040
	r2	0.0046	0.0036	0.0043	0.0039	0.0043	0.0048	0.0037	0.0045	0.0038	0.0045
	r3	0.0042	0.0032	0.0039	0.0035	0.0039	0.0043	0.0033	0.0041	0.0035	0.0041
	r4	0.0050	0.0039	0.0048	0.0044	0.0048	0.0054	0.0041	0.0051	0.0043	0.0052
30	r1	0.0042	0.0032	0.0036	0.0032	0.0036	0.0041	0.0032	0.0036	0.0032	0.0036

Table 5: Risk of the reliability function under GELF for specified value of loss parameter δ using Prior 1 and Prior 2 respectively.

m	RI	δ=0.5					δ=-0.5				
		$\hat{R}_M$	$\hat{R}_{BS1}$	$\hat{R}_{BS2}$	$\hat{R}_{BG1}$	$\hat{R}_{BG2}$	$\hat{R}_M$	$\hat{R}_{BS1}$	$\hat{R}_{BS2}$	$\hat{R}_{BG1}$	$\hat{R}_{BG2}$
12	r1	0.0106	0.0071	0.0097	0.0071	0.0097	0.0121	0.0079	0.0110	0.0079	0.0110
	r2	0.0198	0.0133	0.0177	0.0133	0.0177	0.0234	0.0153	0.0206	0.0153	0.0206
	r3	0.0207	0.0140	0.0185	0.0140	0.0185	0.0245	0.0160	0.0216	0.0160	0.0216
	r4	0.0265	0.0179	0.0235	0.0179	0.0235	0.0319	0.0208	0.0280	0.0208	0.0280
15	r1	0.0057	0.0041	0.0055	0.0041	0.0055	0.0063	0.0045	0.0060	0.0045	0.0060
	r2	0.0125	0.0090	0.0117	0.0090	0.0117	0.0143	0.0101	0.0134	0.0101	0.0134
	r3	0.0119	0.0086	0.0112	0.0086	0.0112	0.0137	0.0096	0.0128	0.0096	0.0128
	r4	0.0180	0.0130	0.0167	0.0130	0.0167	0.0211	0.0148	0.0194	0.0148	0.0194
18	r1	0.0038	0.0029	0.0037	0.0029	0.0037	0.0042	0.0031	0.0041	0.0031	0.0041
	r2	0.0081	0.0061	0.0079	0.0061	0.0079	0.0091	0.0068	0.0088	0.0068	0.0088
	r3	0.0084	0.0064	0.0081	0.0064	0.0081	0.0095	0.0071	0.0091	0.0071	0.0091
	r4	0.0117	0.0089	0.0113	0.0089	0.0113	0.0134	0.0100	0.0128	0.0100	0.0128
24	r1	0.0023	0.0018	0.0021	0.0018	0.0021	0.0024	0.0018	0.0022	0.0018	0.0022
	r2	0.0034	0.0027	0.0033	0.0027	0.0033	0.0036	0.0029	0.0036	0.0029	0.0036
	r3	0.0035	0.0028	0.0035	0.0028	0.0035	0.0038	0.0030	0.0038	0.0030	0.0038
	r4	0.0044	0.0036	0.0044	0.0036	0.0044	0.0048	0.0039	0.0048	0.0039	0.0048
27	r1	0.0022	0.0017	0.0020	0.0017	0.0020	0.0023	0.0017	0.0020	0.0017	0.0020
	r2	0.0024	0.0019	0.0022	0.0019	0.0022	0.0025	0.0020	0.0024	0.0020	0.0024
	r3	0.0021	0.0017	0.0020	0.0017	0.0020	0.0022	0.0018	0.0021	0.0018	0.0021
	r4	0.0027	0.0021	0.0026	0.0021	0.0026	0.0028	0.0022	0.0027	0.0022	0.0027
30	r1	0.0020	0.0015	0.0017	0.0015	0.0017	0.0020	0.0015	0.0018	0.0015	0.0018

Table 6: Risk of the hazard function under GELF for specified value of loss parameter δ using Prior 1 and Prior 2 respectively.

m	RI	δ=0.5					δ=-0.5				
		$\hat{H}_M$	$\hat{H}_{BS1}$	$\hat{H}_{BS2}$	$\hat{H}_{BG1}$	$\hat{H}_{BG2}$	$\hat{H}_M$	$\hat{H}_{BS1}$	$\hat{H}_{BS2}$	$\hat{H}_{BG1}$	$\hat{H}_{BG2}$
12	r1	0.0107	0.0067	0.0097	0.0078	0.0101	0.0121	0.0074	0.0108	0.0078	0.0110
	r2	0.0175	0.0117	0.0158	0.0131	0.0162	0.0202	0.0133	0.0182	0.0138	0.0183
	r3	0.0182	0.0123	0.0165	0.0137	0.0169	0.0211	0.0139	0.0190	0.0145	0.0191
	r4	0.0218	0.0152	0.0199	0.0165	0.0202	0.0255	0.0174	0.0231	0.0179	0.0232
15	r1	0.0067	0.0044	0.0061	0.0050	0.0063	0.0073	0.0047	0.0067	0.0049	0.0068
	r2	0.0124	0.0086	0.0116	0.0096	0.0119	0.0141	0.0096	0.0131	0.0100	0.0132
	r3	0.0119	0.0082	0.0111	0.0092	0.0114	0.0135	0.0091	0.0125	0.0095	0.0127
	r4	0.0165	0.0118	0.0154	0.0129	0.0157	0.0189	0.0133	0.0176	0.0138	0.0177
18	r1	0.0050	0.0035	0.0046	0.0038	0.0046	0.0053	0.0036	0.0049	0.0038	0.0050
	r2	0.0088	0.0062	0.0084	0.0070	0.0086	0.0098	0.0069	0.0094	0.0071	0.0094
	r3	0.0090	0.0064	0.0086	0.0072	0.0088	0.0100	0.0071	0.0096	0.0073	0.0096
	r4	0.0117	0.0086	0.0112	0.0095	0.0114	0.0133	0.0096	0.0127	0.0099	0.0128
24	r1	0.0043	0.0033	0.0036	0.0030	0.0034	0.0042	0.0032	0.0036	0.0031	0.0036
	r2	0.0045	0.0034	0.0043	0.0036	0.0043	0.0048	0.0036	0.0046	0.0036	0.0046
	r3	0.0048	0.0036	0.0045	0.0038	0.0045	0.0051	0.0038	0.0048	0.0038	0.0048
	r4	0.0057	0.0043	0.0054	0.0046	0.0054	0.0061	0.0045	0.0058	0.0046	0.0058
27	r1	0.0048	0.0037	0.0040	0.0033	0.0037	0.0045	0.0035	0.0038	0.0034	0.0037
	r2	0.0041	0.0032	0.0036	0.0031	0.0035	0.0041	0.0032	0.0037	0.0031	0.0036
	r3	0.0036	0.0028	0.0032	0.0027	0.0032	0.0036	0.0028	0.0033	0.0028	0.0033
	r4	0.0042	0.0032	0.0038	0.0032	0.0037	0.0043	0.0033	0.0039	0.0033	0.0039
30	r1	0.0046	0.0037	0.0039	0.0032	0.0036	0.0042	0.0034	0.0036	0.0033	0.0036

Table 7: Risk of the parameter α, reliability function, hazard function and corresponding interval estimates under Prior 0

m	RI	MLE			Risk Under SELF						Asymptotic interval		
		$\hat{\alpha}_M$	$\hat{R}_M$	$\hat{H}_M$	$\hat{\alpha}_{BS0}$	$\hat{\alpha}_{BG0}$	$\hat{R}_{BS0}$	$\hat{R}_{BG0}$	$\hat{H}_{BS0}$	$\hat{H}_{BG0}$	$\hat{\alpha}_L$	$\hat{\alpha}_U$	Length
18	r1	0.1764	0.0060	0.0279	0.1953	0.2089	0.0069	0.0070	0.0330	0.0312	1.0034	2.5863	1.5829
	r2	0.3171	0.0117	0.0435	0.3193	0.3516	0.0125	0.0127	0.0406	0.0453	0.8642	2.2258	1.3617
	r3	0.3231	0.0120	0.0438	0.3161	0.3499	0.0124	0.0125	0.0405	0.0454	0.8587	2.2169	1.3582
	r4	0.4245	0.0162	0.0553	0.4274	0.4632	0.0170	0.0172	0.0529	0.0583	0.7976	2.05902	1.2614
24	r1	0.1460	0.0042	0.0302	0.1481	0.1419	0.0042	0.0041	0.0376	0.0300	1.2171	2.7431	1.5260
	r2	0.1602	0.0054	0.0259	0.1563	0.1640	0.0054	0.0054	0.0267	0.0259	1.1148	2.5133	1.3985
	r3	0.1682	0.0056	0.0274	0.1710	0.1793	0.0060	0.0060	0.0281	0.0275	1.1044	2.4854	1.3810
	r4	0.2009	0.0069	0.0325	0.1884	0.2007	0.0068	0.0068	0.0287	0.0295	1.0710	2.4112	1.3402
30	r1	0.1532	0.0039	0.0384	0.1631	0.1502	0.0040	0.0040	0.0457	0.0362	1.3632	2.8056	1.4424

Table 8: Table represents the risk of the estimators under GELF using Prior 0.

m	RI	δ=0.5			δ=-0.5		
		$\hat{\alpha}_M$	$\hat{\alpha}_{BS0}$	$\hat{\alpha}_{BG0}$	$\hat{\alpha}_M$	$\hat{\alpha}_{BS0}$	$\hat{\alpha}_{BG0}$
18	r1	0.0069	0.0077	0.0093	0.0086	0.0086	0.0092
	r2	0.0140	0.0143	0.0173	0.0167	0.0167	0.0179
	r3	0.0144	0.0140	0.0170	0.0162	0.0162	0.0174
	r4	0.0198	0.0199	0.0236	0.0236	0.0236	0.0251
24	r1	0.0046	0.0046	0.0050	0.0047	0.0047	0.0049
	r2	0.0061	0.0059	0.0069	0.0064	0.0064	0.0068
	r3	0.0064	0.0066	0.0078	0.0073	0.0073	0.0078
	r4	0.0080	0.0075	0.0089	0.0084	0.0084	0.0089
30	r1	0.0042	0.0045	0.0045	0.0044	0.0044	0.0044
Reliability Function							
m	RI	$\hat{R}_M$	$\hat{R}_{BS0}$	$\hat{R}_{BG0}$	$\hat{R}_M$	$\hat{R}_{BS0}$	$\hat{R}_{BG0}$
18	r1	0.0038	0.0045	0.0045	0.0047	0.0050	0.0050
	r2	0.0081	0.0088	0.0088	0.0094	0.0100	0.0100
	r3	0.0084	0.0086	0.0086	0.0091	0.0097	0.0097
	r4	0.0117	0.0124	0.0124	0.0134	0.0142	0.0142
24	r1	0.0023	0.0024	0.0024	0.0024	0.0025	0.0025
	r2	0.0034	0.0034	0.0034	0.0034	0.0036	0.0036
	r3	0.0035	0.0039	0.0039	0.0040	0.0042	0.0042
	r4	0.0044	0.0044	0.0044	0.0046	0.0048	0.0048
30	r	0.0020	0.0021	0.0021	0.0021	0.0021	0.0021
Hazard Function							
m	RI	$\hat{H}_M$	$\hat{H}_{BS0}$	$\hat{H}_{BG0}$	$\hat{H}_M$	$\hat{H}_{BS0}$	$\hat{H}_{BG0}$
18	r1	0.0050	0.0055	0.0056	0.0059	0.0057	0.0058
	r2	0.0088	0.0083	0.0092	0.0099	0.0092	0.0096
	r3	0.0090	0.0082	0.0092	0.0098	0.0091	0.0095
	r4	0.0117	0.0111	0.0122	0.0134	0.0126	0.0130
24	r1	0.0043	0.0047	0.0042	0.0042	0.0045	0.0043
	r2	0.0045	0.0044	0.0045	0.0047	0.0045	0.0046
	r3	0.0048	0.0047	0.0049	0.0051	0.0050	0.0051
	r4	0.0057	0.0051	0.0055	0.0057	0.0054	0.0056
30	r	0.0046	0.0054	0.0046	0.0045	0.0049	0.0047

Table 9: Table represents the estimate of the estimators under SELF and corresponding interval estimates for Data Set I when actual reliability and hazard function at T=8 are 0.3127107 and 0.2181016 respectively.

m	Schemes	MLE			Estimates Under SELF			Interval Estimates of the parameter					
								Asymptotic interval			HPD Interval		
					Jeffrey Prior						Jeffrey Prior		
		$\hat{\alpha}_M$	$\hat{R}_M$	$\hat{H}_M$	$\hat{\alpha}_{BS0}$	$\hat{R}_{BS0}$	$\hat{H}_{BS0}$	$\hat{\alpha}_L$L	$\hat{\alpha}_U$	Length	$\hat{\alpha}_L$	$\hat{\alpha}_U$	Length
40	5*2,0*38	4.2751	0.4140	0.2753	4.3035	0.4141	0.2787	2.9501	5.6001	2.6500	3.0318	5.5598	2.5281
	0*19,5*2,0*19	2.6382	0.2809	0.2041	2.6450	0.2806	0.2049	1.8205	3.4559	1.6353	1.8657	3.4609	1.5951
	0*38,5*2	2.3808	0.2574	0.1946	2.3753	0.2561	0.1948	1.6429	3.1187	1.4758	1.6738	3.0889	1.4151
25	5*5,0*20	3.7625	0.3752	0.2508	3.7473	0.3713	0.2524	2.2875	5.2376	2.9501	2.5317	5.3820	2.8504
	0*10,5*5,0*10	2.1295	0.2337	0.1858	2.1468	0.2342	0.1870	1.2946	3.9643	2.6697	1.2764	3.9305	2.6542
	0*20,5*5	1.7693	0.1984	0.1738	1.7761	0.1983	0.1744	1.0757	3.4629	2.3873	1.1145	3.4870	2.3724
15	7*5,0*10	3.6483	0.3662	0.2456	3.6605	0.3631	0.2495	1.8018	5.4947	3.6929	2.0927	5.6611	3.5684
	0*5,7*5,0*5	1.7042	0.1919	0.1717	1.7047	0.1906	0.1723	0.8417	3.5667	2.7250	0.8785	3.5366	2.6580
	0*10,7*5	1.3458	0.1548	0.1607	1.3428	0.1537	0.1609	0.6646	3.0569	2.3922	0.7191	3.0949	2.3758

Table 10: Estimates of the parameter, reliability function and hazard function under GELF for different variation of loss parameter δ, censoring scheme and m for fixed values of n=50 when data I is considered where actual reliability and hazard function at T=8 are 0.3127107 and 0.2181016 respectively.

m	δ	Removals	Estimates Under GELF					
			Under estimation(-ve δ)			Over estimation(+ve δ)		
			$\hat{\alpha}_{BG0}$	$\hat{R}_{BG0}$	$\hat{H}_{BG0}$	$\hat{\alpha}_{BG0}$	$\hat{R}_{BG0}$	$\hat{H}_{BG0}$
40	(-1.5,1.5)	5*2,0*38	4.3289	0.4141	0.2797	4.1678	0.4137	0.2734
		0*19,5*2,0*19	2.6613	0.2806	0.2052	2.5458	0.2792	0.2028
		0*38,5*2	2.3893	0.2561	0.1951	2.3002	0.2558	0.1936
	(-1.0,1.0)	5*2,0*38	4.2974	0.4136	0.2784	4.1755	0.4126	0.2737
		0*19,5*2,0*19	2.6286	0.2791	0.2043	2.5695	0.2792	0.2032
		0*38,5*2	2.3822	0.2568	0.1951	2.3287	0.2574	0.1944
	(-0.5,0.5)	5*2,0*38	4.2406	0.4113	0.2758	4.1830	0.4113	0.2738
		0*19,5*2,0*19	2.6368	0.2813	0.2049	2.5831	0.2794	0.2035
		0*38,5*2	2.3417	0.2543	0.1939	2.3183	0.2551	0.1938
25	(-1.5,1.5)	5*5,0*20	3.7848	0.3713	0.2537	3.5429	0.3703	0.2460
		0*10,5*5,0*10	2.1695	0.2342	0.1873	2.0279	0.2336	0.1852
		0*20,5*5	1.7941	0.1983	0.1746	1.6732	0.1966	0.1729
	(-1.0,1.0)	5*5,0*20	3.7567	0.3720	0.2528	3.6029	0.3715	0.2479
		0*10,5*5,0*10	2.1266	0.2324	0.1862	2.0399	0.2327	0.1852
		0*20,5*5	1.7627	0.1970	0.1740	1.6891	0.1969	0.1732
	(-0.5,0.5)	5*5,0*20	3.7115	0.3714	0.2514	3.6613	0.3734	0.2501
		0*10,5*5,0*10	2.1072	0.2326	0.1861	2.0618	0.2319	0.1852
		0*20,5*5	1.7471	0.1971	0.1738	1.7141	0.1974	0.1735
15	(-1.5,1.5)	7*5,0*10	3.7163	0.3631	0.2513	3.4101	0.3668	0.2430
		0*5,7*5,0*5	1.7340	0.1906	0.1727	1.5693	0.1922	0.1713
		0*10,7*5	1.3649	0.1537	0.1611	1.2258	0.1538	0.1601
	(-1.0,1.0)	7*5,0*10	3.6909	0.3652	0.2511	3.4015	0.3621	0.2420
		0*5,7*5,0*5	1.7188	0.1921	0.1728	1.6068	0.1917	0.1715
		0*10,7*5	1.3486	0.1543	0.1611	1.2718	0.1556	0.1608
	(-0.5,0.5)	7*5,0*10	3.5853	0.3617	0.2471	3.4528	0.3612	0.2433
		0*5,7*5,0*5	1.6743	0.1906	0.1720	1.6267	0.1911	0.1716
		0*10,7*5	1.3198	0.1537	0.1607	1.2823	0.1543	0.1606

Table 11: Table represents the estimates of the estimators under SELF and corresponding interval estimates for Data Set II.

m	Scheme	MLE			Estimates Under SELF			Interval Estimates of the parameter					
					Jeffrey Prior			Asymptotic interval			HPD Interval		
											Jeffrey Prior		
		$\hat{\alpha}_M$	$\hat{R}_M$	$\hat{H}_M$	$\hat{\alpha}_{BS0}$	$\hat{R}_{BS0}$	$\hat{H}_{BS0}$	$\hat{\alpha}_L$	$\hat{\alpha}_U$	Length	$\hat{\alpha}_L$	$\hat{\alpha}_U$	Length
72	0*72	60.097	0.587	0.054	60.103	0.585	0.054	46.214	73.980	27.766	46.897	74.118	27.221
50	2*11,0*39	75.710	0.672	0.074	76.116	0.670	0.077	54.722	96.698	41.976	57.228	98.353	41.126
	1*22,0*28	69.267	0.639	0.065	69.543	0.637	0.067	50.065	88.469	38.404	52.055	89.305	37.250
	1*8,0*14,1*6,0*14,1*8	59.369	0.582	0.053	59.081	0.577	0.053	42.911	75.827	32.916	42.142	74.496	32.354
40	2*16,0*24	75.318	0.670	0.074	75.399	0.665	0.076	51.974	98.662	46.687	54.691	100.033	45.342
	1*32,0*8	62.827	0.603	0.057	62.819	0.599	0.058	43.355	82.299	38.944	44.816	82.032	37.216
	2*4,0*8,2*4,0*8,2*4,0*8,2*4	61.705	0.596	0.055	62.021	0.594	0.057	42.581	80.830	38.249	43.660	82.728	39.068
20	4*13,0*7	67.242	0.628	0.062	67.678	0.621	0.066	37.769	96.715	58.946	39.736	97.339	57.603
	13,0*8,13*2,0*8,13	59.188	0.581	0.053	59.040	0.572	0.055	33.245	85.130	51.886	34.373	85.402	51.029
	0*7,13*2,0*10,26	37.544	0.424	0.033	37.736	0.422	0.034	21.088	54.000	32.912	23.149	56.114	32.965

References

[1] Cohen A. C. (1963) Progressively censored samples in life testing. Technometrics 5:327-339.

[2] Balakrishnan N, Sandhu R. A.(1995) A simple simulation algorithm for generating progressive Type II censored samples. Am Stat 49(2):229-230.

[3] Aggarwala R. (2001) Progressive censoring. In: Balakrishnan N., Rao CR (eds) Handbook of statistics 20: advances in reliability.

[4] Varian H. (1975) A Bayesian approach to real estate assessment. North Holland, Amsterdam 195-208.

Table 12: Estimates of the parameter under GELF for different variation of loss parameter δ, censoring scheme and m for fixed values of n=50 when data II is considered.

m	δ	Scheme	Estimates Under GELF					
			Jeffrey Prior (Under estimation)			Jeffrey Prior (Over estimation)		
			$\hat{\alpha}_{BG0}$	$\hat{R}_{BG0}$	$\hat{H}_{BG0}$	$\hat{\alpha}_{BG0}$	$\hat{R}_{BG0}$	$\hat{H}_{BG0}$
	(-1.5,1.5)		60.3123	0.5847	0.0545	59.1114	0.5847	0.0528
72	(-1.0,1.0)	0*72	59.9411	0.5836	0.0540	59.0898	0.5836	0.0528
	(-0.5,0.5)		59.8566	0.5844	0.0538	59.4443	0.5844	0.0532
		2*11,0*39	76.4704	0.6697	0.0775	73.8506	0.6676	0.0716
	(-1.5,1.5)	1*22,0*28	69.8797	0.6368	0.0673	67.4712	0.6349	0.0629
		1*8,0*14,1*6,0*14,1*8	59.3738	0.5774	0.0537	58.2658	0.5813	0.0520
		2*11,0*39	75.6291	0.6673	0.0758	73.6483	0.6647	0.0716
50	(-1.0,1.0)	1*22,0*28	69.1669	0.6345	0.0662	67.5154	0.6331	0.0631
		1*8,0*14,1*6,0*14,1*8	59.0141	0.5773	0.0531	58.3834	0.5806	0.0522
		2*11,0*39	75.2980	0.6671	0.0752	74.6044	0.6677	0.0734
	(-0.5,0.5)	1*22,0*28	68.9430	0.6352	0.0656	68.0966	0.6345	0.0640
		1*8,0*14,1*6,0*14,1*8	58.8805	0.5780	0.0530	58.2955	0.5781	0.0522
		2*16,0*24	75.8813	0.6649	0.0774	72.9847	0.6649	0.0704
!	(-1.5,1.5)	1*32,0*8	63.2000	0.5989	0.0586	60.9512	0.5989	0.0550
		2*4,0*8,2*4,0*8,2*4,0*8,2*4	62.4222	0.5940	0.0577	60.0398	0.5940	0.0540
		2*16,0*24	75.2884	0.6647	0.0758	73.4835	0.6647	0.0714
40	(-1.0,1.0)	1*32,0*8	63.0244	0.6001	0.0582	61.4799	0.6001	0.0558
		2*4,0*8,2*4,0*8,2*4,0*8,2*4	61.6291	0.5918	0.0565	60.0618	0.5918	0.0542
		2*16,0*24	75.0576	0.6658	0.0749	74.1497	0.6658	0.0728
	(-0.5,0.5)	1*32,0*8	62.7552	0.6008	0.0576	62.0322	0.6008	0.0565
		2*4,0*8,2*4,0*8,2*4,0*8,2*4	61.8813	0.5956	0.0567	61.0963	0.5956	0.0555
		4*13,0*7	68.5131	0.6214	0.0678	63.3679	0.6214	0.0584
	(-1.5,1.5)	13,0*8,13*2,0*8,13	59.8016	0.5722	0.0557	55.1454	0.5722	0.0494
		0*7,13*2,0*10,26	37.9761	0.4199	0.0340	35.2787	0.4199	0.0325
		4*13,0*7	67.0350	0.6178	0.0652	63.6191	0.6178	0.0590
20	(-1.0,1.0)	13,0*8,13*2,0*8,13	59.4543	0.5750	0.0550	56.4984	0.5750	0.0509
		0*7,13*2,0*10,26	37.4116	0.4189	0.0337	35.5664	0.4189	0.0326
		4*13,0*7	65.5627	0.6146	0.0625	63.9605	0.6146	0.0596
	(-0.5,0.5)	13,0*8,13*2,0*8,13	58.6522	0.5745	0.0539	57.0785	0.5745	0.0517
		0*7,13*2,0*10,26	37.2682	0.4215	0.0336	36.3379	0.4215	0.0331

[5] Viveros R., Balakrishnan N. (1994) Interval estimation of life characteristics from progressively Censored data. Technometric 36:84-91.

[6] Dey S. 2007. Inverted exponential distribution as a life distribution model from a Bayesian viewpoint, Data Sci. J. 6: 107-113.

[7] Lin C., Duran B. S. and Lewis T. O. 1989. Inverted gamma as a life distribution. Micro electron. Reliab. 29 (4):619-626.

[8] Prakash G. 2009. Some estimation procedures for the inverted exponential distribution. The South Pacific Journal of Natural Science, 27:71-78.

[9] Singh S. K., Singh U. and Kumar D. 2013. Bayes estimators of the reliability function and parameter of inverted exponential distribution using informative and non-informative priors. Journal of Statistical Computation and Simulation, 83(12): 2258-2269.

[10] T. Bjerkedal, Acquisition of resistance in guinea pigs infected with different doses of virulent tubercle bacilli, Am. J. Hyg. 72 (1960), pp. 130-148.

[11] A. F. M. Smith and G. O. Roberts. Bayesian computation via the Gibbs sampler and related Markov Chain Monte Carlo methods. Journal of the Royal Statistical Society: Series B, 55:3-23, 1993.

[12] S. K. Upadhyay, N. Vasishta, and A. F. M. Smith. Bayes inference in life testing and reliability via Markov Chain Monte Carlo simulation. Sankhya A, 63:15-40, 2001.

[13] Singh S. K., Singh Umesh, and Sharma V. K., Bayesian prediction of future observations from inverse Weibull distribution based on type-ii hybrid censored sample, International Journal of Advanced Statistics and Probability, vol. 1, pp. 32-43, 2013.

[14] Chen, M.-H., Shao, Q.-M., 1999. Monte Carlo estimation of Bayesian credible and HPD intervals. Journal of Computational and Graphical Statistics 8, 69-92.

[15] Kundu, D., Pradhan, B., 2009. Estimating the parameters of the generalized exponential distribution in presence of hybrid censoring. Communications in Statistics-Theory and Methods 38, 203-2041.

[16] Pundir P. S., Singh B. P. and Maheshwari S. On hybrid censored inverted exponential distribution, International Journal of Current Research, Vol. 6, Issue, 01, pp. 4539-4544, January, 2014.
[17] H. Krishna and Kapil Kumar, Reliability estimation in Lindley distribution with progressively type II right censored sample, Mathematics and Computers in Simulation 82 (2011) 281-294.
[18] S. K. Singh, Umesh Singh and V. K. Sharma, Bayesian Estimation and Prediction for Flexible Weibull Model under Type-II Censoring Scheme,Journal of Probability and Statistics Volume 2013 (2013), Article ID 146140.