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Optimal Payment time for a Retailer with exponential demand under permitted credit period by the Wholesaler

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Abstract: In paper Jamal et al.[32] developed a retailer's model for optimal cycle time and payment times for a retailer' in a deteriorating - item inventory situation where a wholesaler allows a specified credit period to the retailer for payment without any charge. This paper is the extension of Jamal et al. [32] paper. In competitive business environment the most important objectives for companies is to reduce replenishment cost. This paper considers the inventory policy for non deteriorating items with exponential demand rate. The demand rate is assumed to be increasing function of time .In this study a retailer's model for optimal cycle time and optimal payment times for a retailer have been developed, where a supplier allows a particular credit period to the buyer for payment without any extra charge. The supplier and buyer system is modeled as a optimal profit problem to determine the optimal payment time under different parameters under the same condition. The numerical technique method is used to solve the problem for finding optimal payment time and optimal inventory cycle time. The results are discussed with the help of sensitivity analysis of the optimal solution with respect to the different parameters of the system is given.

Keywords: Inventory, exponential demand, supplier, buyer, credit period, payment

1 Introduction

Demand plays an important factor in inventory management. Four types of demand are basically assumed in inventory model, i.e. constant demand, time dependent demand, probabilistic demand and stock-dependent demand. Initially the demand rate of the item was assumed to be constant. However in real market situation, demand is not constant with respect to time. The EOQ model is generally used to find the optimal order quantity in order to minimize the total inventory cost. The EOQ model assumes that the entire order for an item is received into inventory at one given time. In recent years, large numbers of research papers/articles have been presented by researchers in different areas in real life problems, for controlling inventory. The most important concern of the management is to decide when and how much to manufacture so that the total cost associated with the inventory system should be minimum. In classical EOQ model the demand rate is assumed to be a constant or time-dependent. Inventories are often replenished periodically at a certain production rate which is seldom infinite. Operational processes such as inventory management and mass customization must be effective in improving the firm's inventory performance. The main aim of companies is to meet demand on time providing high quality service. In today's business world buyer promotional activity has become more common. For example free goods, displays, advertising and so on. The promotion policy is very important for the buyer residual costs may be incurred by too many promotions while too few may result in lower sales revenue.

Goyal [1] developed an EOQ model under permissible delay in payments. He ignored the difference between the purchase cost and selling price, and concluded that the economic replenishment interval and order quantity generally increases marginally under permissible delay period. Goyal's [1] model was corrected by Dave [2] by assuming the fact that the selling price is higher than its purchase cost. Teng and Chung [3] determined economic production quantity in an inventory model for deteriorating items. Sridharan [4] developed an inventory model for non-deteriorating items. Jamal et al. [5] developed optimal payment time for a retailer under permitted delay of payment by the wholesaler. Hwang and Shinn [6] presented retailer's pricing and lot sizing policy for exponentially deteriorating products under the

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condition of permissible delay in payment by considering demand rate is a function of the selling price and optimal retailer price and lot size simultaneously. Huang [7] presented retailers inventory system as a cost minimization problem to determine the retailer's optimal inventory as a cost minimization problem to determine the retailer's optimal inventory cycle time and optimal order quantity is obtained. Chung and Huang [8] developed an economic production quantity model (EPQ) for a retailer where the suppler offers a permissible delay in payments.

All the above researchers established their EOQ and EPQ inventory models under constant demand rate. But in real life demand rate is not always constant, it vary with time. Teng et al. [9] developed economic order quantity model with trade credit financing for non- decreasing demand. Tripathy [10] developed ordering policy for linear deteriorating items for declining demand with permissible delay in payments. Tripathi [11] developed an inventory model with shortage, time - dependent demand rate and quantity dependent permissible delay in payment .Sarkar [12] developed an EOQ model for finite replenishment rate where demand rate and deterioration rate are both time dependent. Sana [13] presented an EOQ model over an infinite time horizon for perishable item where demand is price dependent and partial backorder is permitted. Teng [14] developed an EPQ model with investments on imperfect production process under limited capacity.

All the above articles/ papers are based on the assumption that the cost due to inventory system remains constant over the period. This assumption may not be true in the real life, as many countries contain annual inflation rate. As inflation increases the value of money goes down which erodes the future worth of saving and forces one for more current spending. Therefore the effect of inflation cannot be ignored.Silver and Meal [15] were first developed the EOQ model for the case of varying demand. Aggarwal et al. [16] established an inventory model for exponential demand rate and it is increasing function of time. Gupla and Vrat [17] presented an inventory model for stock- dependent demand rate. Buzacott [18] developed EOQ model with inflation subject to different types of pricing policies. Bose et al. [19] developed a model on deteriorating items with linear time dependent demand rate and shortages under inflation and time discounting. Jaggi et al. [20] presented a paper optimal order policy for deteriorating items with inflation induced demand using a discounted cash flow (DCF) approach over a finite planning horizon. Tripathi et al. [21] developed a cash flow oriented EOQ model of deteriorating items with time - dependent demand rate under permissible delay in payments. Jaggi et al. [22] developed EOQ model credit financing in economic ordering policies of deteriorating items using discounted cash flow (DCF) approach. Teng [23] developed discounted cash flow analysis on inventory control under various suppliers' trade credits. Chang et al. [24] presented an EOQ model for deteriorating items under trade credits. An EOQ model for deteriorating items was developed by Ouyang et al. [25]. Hou and Lin [26] developed a cash flow oriented EOQ model with deteriorating items under permissible delay in payments. Jaggi et al. [27] developed EOQ model retailer's optimal ordering policy under two stage trade credit financing by considering] demand rate to be a function of credit period offered by the retailer to the customers using discounted cash flow (DCF) approach.

It has been observed that the demand is usually influenced by the amount of stock displayed in the shelves; i.e. the demand rate may go up and down if the on- hand inventory level increases or decreases. Ray and Chaudhuri [28] developed an EOQ model with stock- dependent demand, shortage, inflation and time discounting. Hou [29] developed an EOQ model for deteriorating items with stock- dependent consumption rate and shortages under inflation and time discounting and obtained the total cost function is convex. Alfares [30] developed inventory model with stock - level dependent demand rate and variable holding cost. EOQ models with general demand and holding cost function was developed by Goh [31].

In this paper the demand rate is considered as exponential time dependent. The main objective of this paper is to obtain minimum total profit Z(P,T).This paper is the extension of Jamal et al. [32] in which deterioration and demand rate both are constant. The remainder of the paper is organized as follows. Relevant notation and assumptions are given in the next section 2. This is followed by mathematical formulation in section 3. The determination of optimal solution is presented in Section 4. Computational results are given in section 5. Finally, suggestions and concluding remarks are given in last section 6

2 Notation and assumption

The following notation is used through this paper:

A : the ordering cost of inventory (dollars/ order)
c : the unit purchase cost per item (dollars/ order)
λ_0 : the initial demand rate (i.e. at $t = 0$)
i : the inventory carrying cost rate
I_e : the interest earned per dollar per unit time
I_p : the interest paid per dollar per unit time (dollars/ dollar-year)
I : interest payable per cycle
E : total interest earned per cycle
M : permissible delay fixed by the wholesaler in settling the account
P : payment time of the retailer
Q : order quantity (unit per order)
s : the selling price (dollar/unit)
T : the length of the inventory cycle (time units)
$I(t)$: the inventory level at time $'t'$
$Z(P,T)$: the total variable profit per cycle per unit time

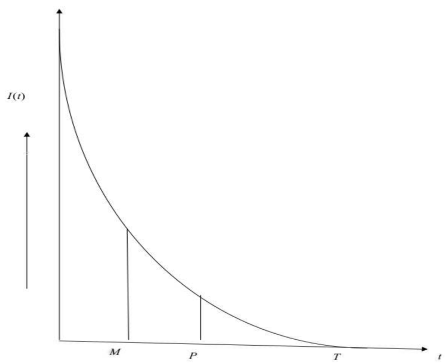

Fig. 1: The vendor and buyer inventory system

HC	: holding cost function
p	: selling price
SR	: sales revenue
$z(P,T)$	: the variable profit per cycle
$D \equiv D(t) \equiv \lambda_0 e^{\alpha t}$	: demand rate

In addition the following assumptions are being made to develop aforesaid model:

(1)The demand rate is exponentially increasing with time and is represented by $D \equiv D(t) \equiv \lambda_0 e^{\alpha t}$, where α is constant and $0 \leq \alpha \leq 1$.
(2)Shortages are not allowed.
(3)A single item is considered over the fixed period T unit of time.
(4)Lead time is zero.
(5)The replenishment occurs instantaneously at an infinite rate.

3 Mathematical Formulation

The inventory depletes due to the demand only. The wholesaler and retailer's model works on a number of system parameters of which inflation is an important factor. The inventory level at time 't' $I(t)$ during the time period $(0 \leq t \leq T)$is given by:

$$\frac{dI(t)}{dt} = -D(t) = -\lambda_0 e^{\alpha t}, (0 \leq t \leq T) \tag{1}$$

With the boundary condition $I(T) = 0$.

The total profit may be evaluated under different situations. The variable profit is a function of sales revenue, ordering cost, carrying cost, interest earned and interest payable.

(a)The sales revenue SR is given by

$$SR = p\int_0^T \lambda_0 e^{\alpha t} dt = \frac{\lambda_0 p}{\alpha}\left(e^{\alpha T} - 1\right) \tag{2}$$

(b)In most inventory systems the ordering cost of raw material is fixed at A dollars/order.
(c)The holding cost or carrying cost is a function of average inventory and it is given by

$$HC = ic\int_0^T I(t)dt = \frac{ic\lambda_0}{\alpha^2}\left\{(\alpha T - 1)e^{\alpha T} + 1\right\} \tag{3}$$

(d)The net cost of the unpaid inventory at time 't' is the cost of current inventory at time 't', minus the profit on the amount sold during time M, minus the interest earned from the sales revenue during time M. The extra amount that can be paid off in the profit on the amount sold after the permissible delay time M. Therefore, the interest payable per cycle for the inventory is given by

$$\begin{aligned} I = {} & I_p \int_M^P \left\{ cI(t) - (s-c)\int_0^M \lambda_0 e^{\alpha t} dt - sI_e \int_0^M \lambda_0 e^{\alpha t}.tdt \right\} dt \\ & - (s-c)I_p \int_0^{P-M} \lambda_0 e^{\alpha t}.tdt \\ = {} & \frac{\lambda_0 I_p}{\alpha}\left[c\left\{ e^{\alpha t}(P-M) - \frac{e^{\alpha P} - e^{\alpha M}}{\alpha} \right\} \right. \\ & \left. - \left\{ (s-c)\left(e^{\alpha M} - 1\right) - \frac{sI_e}{\alpha}\left(\alpha M e^{\alpha M} - e^{\alpha M} + 1\right) \right\}(P-M) \right] \\ & - \frac{(s-c)I_p\lambda_0}{\alpha^2}\left\{ (\alpha P - \alpha M - 1)e^{\alpha(P-M)} + 1 \right\} \end{aligned} \tag{4}$$

Interest earned per cycle is the interest earned during the positive inventory, and it is given by

$$\begin{aligned} E = {} & sI_e \left(\int_0^M \lambda_0 e^{\alpha t}.tdt + \int_0^{T-P} \lambda_0 e^{\alpha t}.tdt \right) \\ = {} & \frac{\lambda_0 sI_e}{\alpha}\left\{ Me^{\alpha M} + (T-P)e^{\alpha(T-P)} + \frac{2 - e^{\alpha M} - e^{\alpha(T-P)}}{\alpha} \right\} \end{aligned} \tag{5}$$

The total variable cost per cycle $z(P,T)$ is given by

$$z(P,T) = SR - A - HC - I + E \tag{6}$$

The variable profit per unit time $Z(P,T)$ is

$$Z(P,T) = \frac{z(P,T)}{T} \tag{7}$$

Theoretically, the optimum value of 'P' and 'T' can be obtained from (7), by differentiating partially with respect to 'P' and 'T' and putting the partial derivatives to zero, but it is difficult to evaluate these equations for finding exact optimal value of $'P'$ and 'T'. In practice $\alpha T < 1$. Thus using the series expansion of exponential terms and ignoring third and higher power of αT and αM etc. Equation (7) becomes

$$Z(P,T) \approx p\lambda_0\left(1+\frac{\alpha T}{2}\right)-\frac{A}{T}-\frac{ic\lambda_0(1+\alpha T)T}{2} -\frac{\lambda_0 I_p}{T}\left[cT\left(1+\frac{\alpha T}{2}\right)-\frac{s(P+M)}{2} -\frac{\alpha(\mathrm{s-c})}{2}\left\{M^2+(P-M)^2\right\}+\frac{sI_eM^2(1+\alpha M)}{2}\right](P-M) +\frac{\lambda_0 sI_e}{2T}\left\{M^2(1+\alpha M)+(T-P)^2+\alpha(T-P)^3\right\} \tag{8}$$

4 Determination of Optimal solution

Taking the first and second derivatives of Equation (8) with respect to 'P' and 'T' respectively, we obtain

$$\frac{\partial Z(P,T)}{\partial P}=\frac{\lambda_0}{2T}\left[I_p\left\{2sP+\alpha(s-c)\left(3P^2-6PM+4M^2\right) -cT(2+\alpha T)-sI_eM^2(1+\alpha M)\right\} -\mathrm{sI_e}(T-P)(2+3\alpha T-3\alpha P)\right] \tag{9}$$

$$\frac{\partial Z(P,T)}{\partial T}=\frac{1}{2T^2}\left[2A-\lambda_0 I_p\left\{s(P+M)+\alpha(s-c)\left(2M^2+P^2-2PM\right)-sI_eM^2(1+\alpha M)\right\}(P-M) -\lambda_0 sI_e\left\{M^2(1+\alpha M)+P^2(1-\alpha P)\right\}\right]+\alpha\lambda_0 T(sI_e-ic) +\frac{\lambda_0}{2}\left\{p\alpha+sI_e-ic-\alpha cI_p(P-M)-3sI_e\alpha P\right\} \tag{10}$$

$$\frac{\partial^2 Z(P,T)}{\partial P^2}=\frac{\lambda_0}{T}\left[\mathrm{s}\left(\mathrm{I_p}+\mathrm{I_e}\right) +3\alpha\left\{(s-c)I_p(P-M)+sI_e(T-P)\right\}\right]>0 \tag{11}$$

$$\frac{\partial^2 Z(P,T)}{\partial T\partial P}=-\frac{\lambda_0}{2T^2}\left[I_p\left\{2sP+\alpha(s-c)\left(3P^2-6PM+4M^2\right)+\alpha cT^2+sI_eM^2(1+\alpha M)\right\} +sI_e(P+T)(2+3\alpha T-3\alpha P)\right]<0 \tag{12}$$

$$\frac{\partial^2 Z(P,T)}{\partial T^2}=\frac{1}{T^3}\left[\lambda_0 I_p\left\{s(P+M)+\alpha(s-c)\left(2M^2+P^2-2PM\right)-sI_eM^2(1+\alpha M)\right\}(P-M) +\lambda_0 sI_e\left\{M^2(1+\alpha M)+P^2(1-\alpha P)\right\}-2A\right]+\alpha\lambda_0(sI_e-ic) \tag{13}$$

Since $\left(\frac{\partial^2 Z(P,T)}{\partial T^2}\right)\left(\frac{\partial^2 Z(P,T)}{\partial P^2}\right)-\left(\frac{\partial^2 Z(P,T)}{\partial T\partial P}\right)^2>0$, $\left(\frac{\partial^2 \mathrm{Z(P,T)}}{\partial \mathrm{T}^2}\right)>0$, $\left(\frac{\partial^2 \mathrm{Z(P,T)}}{\partial \mathrm{P}^2}\right)>0$, and hence, optimal solution gives the minimum value of total profit.

The appropriate values of the decision variables that minimize the total profit function lead to the solution of the problem. From Equations (9) and (10) it is clear that the response surface of the total profit function $Z(P,T)$ in (8) is convex in $'P'$ and $'T'$ which minimize variable profit $Z(P,T)$ can be obtained by solving equations $\frac{\partial Z(P,T)}{\partial P}=0$, and $\frac{\partial \mathrm{Z(P,T)}}{\partial \mathrm{T}}=0$, simultaneously within a stated range.

Putting $\frac{\partial Z(P,T)}{\partial P}=0$, and $\frac{\partial Z(P,T)}{\partial T}=0$ from (9) and (10) we get

$$I_p\left\{2sP+\alpha(s-c)(3P^2-6PM+4M^2)-cT(2+\alpha T) -sI_eM^2(1+\alpha M)\right\}-sI_e(T-P)(2+3\alpha T-3\alpha P)=0 \tag{14}$$

$$2\alpha\lambda_0(sI_e-ic)T^3+\lambda_0\left\{p\alpha-ic-c\alpha I_p(P-M) +sI_e(1-3\alpha P)\right\}T^2+2A-\lambda_0 I_p\left\{s(P+M)+\alpha(s-c) \left(2M^2+P^2-2PM\right)-sI_eM^2(1+\alpha M)\right\}(P-M) -\lambda_0 sI_e\left\{M^2(1+\alpha M)+P^2(1-\alpha P)\right\}=0 \tag{15}$$

From Equations (14) and (15) we obtain optimal 'P' and 'T' simultaneously. Equations (14) and (15) are non linear simultaneous equations. So, it is not easy to evaluate the closed form solution directly. Mathematica 7 is used for finding optimal numerical values of $P=P^*$ and $T=T^*$ simultaneously.

5 Computational Results

The mathematica Software is used for finding the optimal payment period 'P' and optimal cycle time 'T'. Let $\lambda_0=$ 1000 units/year, $A=200$ dollars/order, $p=100$/unit/year, $i=0.12$ / year, $I_e=0.13$ year, and $s=1.2c$. In this model we assumed $M\le P^*\le T^*$.

All the observations in following Tables done by the assumption $M\le P^*\le T^*$. Table 1 and 2 are constructed to study the effects of payment interest rate I_p, earned interest I_e unit, ordering cost c ,permissible delay time M on payment delay period P, inventory cycle time T,α, and total profit $Z(P,T)$. Different parametric values are used in constructing these values are given in vector forms as $I_e=(0.08,0.1,0.12,0.13,0.15), c=(20.60,100,140,200)$ dollars/unit, $M=(0,15,30,45)$ days, and $\alpha=(0.01,0.02,0.03,0.04,0.05)$. It should be noted here that the time units used for P,M, and T in the model are in 'years' while for ease of convenience, the units exhibited in the example are in 'day'.

The blank spaces mentioned in tables, below shows that results are valid according to assumption $M\le P^*\le T^*$.

Table 1.(a): Optimal delay with fixed $I_p = 0.20$ and $I_e = 0.13$

$\alpha = 0.01$					$\alpha = 0.02$				
c	M	T*	P*	Z(P,T)	c	M	T*	P*	Z(P,T)
20	0	106	96	98630.3	20	0	119	107	98783.2
	15	107	96	98789.5		15	120	108	98942.8
	30	108	97	98938.5		30	121	109	99093.5
	45	110	99	99078.3		45	123	111	99235.8
60	0	58	52	97465.0	60	0	59	53	97544.6
	15	58	52	97931.0		15	60	54	98010.8
	30	54	63	98344.0		30	62	56	98426.4
	45	63	57	98711.1		45	65	58	98797.7
100	0	44	40	96688.1	100	0	45	40	96747.9
	15	45	40	97450.4		15	46	41	97511.3
	30	47	42	98102.5		30	48	43	98166.7
	45	50	45	98662.8		45	51	46	98732.3
140	0	37	33	96061.1	140	0	37	34	96111.1
	15	38	34	97113.5		15	38	35	97164.8
	30	40	36	97987.0		30	41	37	98042.7
	45	----	----	--------		45	-----	----	---------
200	0	31	28	95274.2	200	0	31	28	95315.7
	15	32	29	96751.3		15	32	29	96794.3
	30	35	31	97933.5		30	35	32	97981.4
	45	----	----	---------		45	----	-----	----------

Table 2: Optimal delay with fixed $I_p = 0.20$ and $\alpha = 0.05$

$I_e = 0.08$					$I_e = 0.10$				
c	M	T*	P*	Z(P,T)	c	M	T*	P*	Z(P,T)
20	0	227	201	99431.2	20	0	228	204	99433.3
	15	229	203	99591.7		15	229	205	99594.3
	30	233	207	99744.7		30	233	208	99749.0
	45	239	212	99890.7		45	238	213	99897.6
60	0	66	58	97793.6	60	0	66	58	97795.4
	15	67	59	98246.3		15	67	59	98254.5
	30	70	62	98619.8		30	70	62	98650.0
	45	75	67	98935.5		45	74	66	98990.4
100	0	47	42	96929.1	100	0	47	42	96931.2
	15	49	43	97658.3		15	49	43	97675.4
	30	53	47	98219.0		30	52	46	98277.5
	45	59	52	98647.3		45	57	51	98765.1
140	0	39	34	96259.4	140	0	39	35	96261.9
	15	40	36	97252.4		15	40	36	97281.2
	30	45	40	97970.5		30	44	39	98066.1
	45	51	46	98485.4		45	50	45	98673.8
200	0	32	34	95435.7	200	0	32	28	95438.5
	15	33	36	96809.2		15	33	30	96855.8
	30	39	40	97724.5		30	34	38	97882.5
	45	----	----	------		45	-----	----	---------
$I_e = 0.12$					**$I_e = 0.15$**				
c	M	T*	P	Z(P,T)	c	M	T*	P*	Z(P,T)
20	0	228	206	99435.2	20	0	229	208	99437.6
	15	239	207	99596.7		15	230	209	99599.9
	30	232	213	99753.0		30	232	211	99758.6
	45	236	213	99904.3		45	234	213	99013.9
60	0	66	58	97796.9	60	0	66	60	97708.9
	15	67	60	98262.5		15	66	60	98274.3
	30	69	62	98676.8		30	68	61	97717.1
	45	72	65	99046.1		45	70	64	99131.7
100	0	47	42	4535.24	100	0	47	43	96935.4
	15	48	43	3628.82		15	48	44	97717.3
	30	51	46	98336.6		30	50	45	98426.6
	45	56	50	98886.2		45	52	47	99075.5
140	0	39	35	96264.0	140	0	39	35	96266.7
	15	40	36	97308.9		15	40	36	97350.2
	30	43	39	97308.9		30	44	40	98312.9
	45	----	----	------		45	----	-----	-------
200	0	32	29	95441.0	200	0	32	29	95444.2
	15	33	30	96902.4		15	33	30	96972.4
	30	37	33	98044.6		30	35	32	98296.6
	45	-----	-----	---------		45	------	-----	--------

Table 1.(b): Optimal delay with fixed $I_p = 0.20$ and $I_e = 0.13$

$\alpha = 0.03$					$\alpha = 0.04$				
c	M	T*	P*	Z(P,T)	c	M	T*	P*	Z(P,T)
20	0	138	124	98957.4	20	0	168	152	99165.2
	15	138	125	99117.7		15	169	152	99326.1
	30	140	125	99270.1		30	171	154	99480.6
	45	140	128	99415.3		45	174	157	99629.2
60	0	61	55	97626.1	60	0	63	57	97710.3
	15	62	56	98093.1		15	64	58	98178.3
	30	64	57	98511.3		30	66	59	98599.2
	45	67	60	98886.8		45	69	62	98978.9
100	0	46	41	96808.7	100	0	46	42	96870.9
	15	46	42	97573.2		15	47	43	97636.4
	30	49	44	98232.0		30	50	45	98298.6
	45	52	47	98803.0		45	53	48	98875.0
140	0	38	34	96161.8	140	0	39	35	96213.0
	15	39	37	97216.8		15	39	35	97269.4
	30	41	30	98098.7		30	42	37	98155.3
	45	----	----	-------		45	-----	----	---------
200	0	31	28	95357.5	200	0	32	28	95399.6
	15	32	29	96837.7		15	33	29	96881.5
	30	35	32	98029.6		30	36	32	98078.3
	45	----	----	---------		45	----	-----	----------
$\alpha = 0.05$					**$\alpha = 0.05$**				
20	0	228	206	99436.0	140	0	39	35	96264.9
	15	230	208	99597.8		15	40	36	97322.7
	30	232	210	99754.9		30	43	38	98212.6
	45	236	213	99907.5		45	-----	-----	----------
60	0	66	59	97797.6	200	0	32	29	95442.1
	15	66	60	98266.5		15	33	30	96925.7
	30	68	62	98690.2		30	36	33	98127.3
	45	71	65	99074.4		45	----	----	---------
100	0	47	43	96933.9					
	15	48	43	97700.7					
	30	51	46	98366.4					
	45	54	49	98948.2					

All the above observations sum up as follows:
From Table 1 it is observed that the payment period 'P', the inventory cycle time 'T' and total profit $Z(P,T)$ increases with the increase of permissible delay time 'M' . The payment period 'P' and the cycle time 'T' become longer with increase of 'α'. It is also clear from Table 1 that the payment delay period has a direct relationship and the total profit has a direct relationship with 'α'.
Table 2, shows that both cycles time 'T' and payment period 'P' tend to increase as the earned interest rate I_e increase. It also indicates that there is increase of total inventory profit with increase of earned interest rate I_e. It is observed from all the above tables that the increase of 'α' results increase in $P = P^*, T = T^*$ and $Z(P,T) = Z(P^*,T^*)$.
It is also observed from all the above tables that the increase of unit purchase cost 'c' results decrease in $P = P^*, T = T^*$ and $Z(P,T) = Z(P^*,T^*)$.
Note: If $\alpha = 0$, then this model becomes Jamal et al. [32] model for zero deterioration rate.

6 Conclusion and future Research

This paper addresses a retailer's model for optimal strategy for payment time. A model for optimal cycle and payment time for a retailer in an inventory situation with

exponential time dependent product where a vendor allows a specified credit period to the buyer for payment without any interest. In this paper we adopted an exponential demand with respect to time i.e. $R(t) = \lambda_0 e^{\alpha t}, \lambda_0 > 0, 0 \le \alpha \le 1$, implying exponential increase in the demand. An exponential demand rate being very high, it is unclear whether the real market demand of any product can be rise exponentially. The exponential time dependent rate takes place in the case of the seasonal products, disaster, earthquake, natural calamities etc.
Test results shows that the total profit increases and the optimal payment time and cycle time becomes shorter as the unit selling price increases relative to unit purchase cost, which indicates that the buyer should settle his account relatively soon. We can also see that the payment time reduces in general as the difference between payment time and earned interest rates increases. However the total cost increases. Mathematica software is used for finding optimal solutions. Truncated Taylor's series is used in exponential terms for finding closed form optimal solutions.
Numerical results show that the total inventory profit increases with increase permissible delay period 'M'. Also total inventory profit increases with increase of unit cost per item. In addition the increase of interest earned per dollar I_p results, increase of cycle time period and payment period 'P' and total inventory profit. Higher value of credit period implies higher value of total inventory profit.
The proposed model can be extended in several ways. For instance, we may extend the non-deterioration rate to time dependent deterioration rate. In addition, we could consider the demand as a function of quantity, stock-dependent, selling price, allow for shortages, cash discount etc.

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Appendix

The solution of (1) is given by $I(t) = \frac{\lambda_0}{\alpha}\left(e^{\alpha T} - e^{\alpha t}\right), (0 \le t \le T)$ with the condition $I(T) = 0$ and order quantity is $Q = I(0) = \frac{\lambda_0}{\alpha}\left(e^{\alpha T} - 1\right)$

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