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Bayes And Empirical Bayes Estimators based on Generalized Half Logistic Records Data

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Abstract: This study considers the estimation problem for the parameter and reliability function of Generalized Half logistic distribution under record Data. We use the maximum likelihood and Bayesian procedures to obtain the estimators of parameter and reliability function of Generalized Half logistic distribution. We also obtain the Empirical Bayesian Estimators for the parameter and reliability function of Generalized Half logistic distribution and considered the problem of predicting future record in a Bayesian and Empirical Bayesian approaches. Comparisons are made between the different estimators based on a simulation study.

Keywords: Generalized half logistic distribution, records data, Bayesian estimation, empirical Bayesian estimation

1 Introduction

Let $X = |Z|$, where Z is the standard logistic random variable, X is called the folded or half logistic random variable. The density function of half logistic distribution is a monotonic decreasing function of x is $[0,\infty)$ and has an increasing hazard rate. The generalized versions of half logistic distribution namely Type-I and TypeII were considered along with point estimation of scale parameters and estimation of stress strength reliability based on complete sample by Ramakrishna [7]. Arora et al. [6] considered maximum likelihood estimators of the generalized half logistic distribution under type I progressive censoring with changing failure rates. Azimi et al.[5] obtained Bayes estimators of the parameter and reliability function of generalized half logistic distribution by taking progressive type II censored sample using different loss functions such as LINEX, precautionary and entropy loss functions. The cumulative distribution function (cdf), and probability density function (pdf), of the generalized half logistic distribution with parameter $\beta > 0$ are

$$F(x|\beta) = 1 - \left[\frac{2e^{-x}}{1+e^{-x}}\right]^{\beta}, \; x > 0 \tag{1}$$

$$f(x|\beta) = \frac{\beta\,(2e^{-x})^{\beta}}{(1+e^{-x})^{\beta+1}} \tag{2}$$

The reliability function $R(t)$, at mission time t is given by

$$R(t) = \left[\frac{2e^{-t}}{1+e^{-t}}\right]^{\beta}. \tag{3}$$

Record values and the associated statistics are of interest and important in many real life applications. In industry and reliability studies, many products fail under stress. Chandler [10] introduced the study of record values and documented many of the basic properties of records. Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed (iid) random variables with cumulative distribution function(cdf)$F(x)$ and probability density function(pdf) $f(x)$, for $n \geq 1$, define

$$G(1) = 1, \quad G(n+1) = \min\left\{ j : j > G(n), X_j > X_{G(n)} \right\}$$

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The sequence $\{X_{G(n)}\}$ is known as upper record statistics (record times). For more details and applications in the record values, see Ahsanullah [4] and Arnold et al.[3].

Let $\mathbf{X} = \{X_{G(1)} = X_1, X_{G(2)} = X_2, ..., X_{G(n)} = X_n\}$ be the first n upper record values arising from a sequences of i.i.d. Generalized Half logistic variables with pdf (2), and distribution function cdf (1). The likelihood function, (see Ahsanullah [4]) is given by

$$L(\mathbf{x}|\beta) = f\left(X_{G(n)}|\beta\right) \prod_{i=1}^{n-1} \frac{f\left(X_{G(i)}|\beta\right)}{1-F\left(X_{G(i)}|\beta\right)} \tag{4}$$

It follows from (1), (2) and (4) that

$$L(\mathbf{x}|\beta) = \beta^n (u(x_n))^\beta \tag{5}$$

Where

$$u(x_n) = \frac{2e^{-x_n}}{1+e^{-x_n}}$$

The maximum likelihood estimator (MLE) of β is

$$\hat{\beta} = -\frac{n}{\ln u(x_n)}$$

It is clear that the MLE of reliability function, $R(t)$, can be obtained by $\hat{R} = \left[\frac{2e^{-t}}{1+e^{-t}}\right]^{\hat{\beta}}$.

2 Bayesian Estimation

2.1 Prior and Posterior Distributions

Here, we consider the natural conjugate family of prior densities for β as the following form

$$\pi(\beta) = \frac{b^a}{\Gamma(a)} \beta^{a-1} e^{-b\beta}. \tag{6}$$

where a and b are specified positive constants. Combining the likelihood function (5) with the prior pdf (6), via Bayes theorem, results in the posterior density function of β

$$\pi(\beta|\mathbf{x}) = \frac{(b-\ln u(x_n))^{n+a}}{\Gamma(n+a)} \beta^{n+a-1} e^{-\beta(b-\ln u(x_n))}. \tag{7}$$

Substituting $\beta = \frac{-\ln R}{\ln \frac{1+e^{-t}}{2e^{-t}}}$ into (7) , we can obtain the posterior density function of $R = R(t)$ as

$$\pi(R|\mathbf{x}) = \frac{1}{\Gamma(n+a)} \left(\frac{b-\ln u(x_n)}{\ln T}\right)^{n+a} R^{\frac{b-\ln u(x_n)}{\ln T}-1} (-\ln R)^{n+a-1}$$

where $0 < R < 1$ and $T = \frac{1+e^{-t}}{2e^{-t}}$.

2.2 Bayesian Estimation Under Minimum Expected Loss Function

In this and next two subsections we present the Bayesian estimation for the parameter β and reliability function based on upper record values. In Bayesian analysis, widely used loss function is a quadratic loss function given by

$$L(\beta,d) = w(d-\beta)^2$$

If $w = 1$, it reduces to squared error loss function and for $w = \beta^{-2}$,it becomes

$$L(\beta,d) = \beta^{-2}(d-\beta)^2$$

known as Minimum Expected Loss Function introduced by Rao Tummala and Sathe [9]. The Bayesian estimators of β under Minimum Expected Loss Function are given by

$$\hat{\beta}_{ME} = \frac{E(\beta^{-1}|\mathbf{x})}{E(\beta^{-2}|\mathbf{x})}$$

$$= \frac{n+a-2}{b-\ln u(x_n)} \tag{8}$$

And the Bayes estimator of $R(t)$ at time t under Minimum Expected Loss function is

$$\hat{R}_{ME} = \left(1 + \frac{\ln T}{b-\ln u(x_n)+\ln T}\right)^{n+a} \tag{9}$$

2.3 Bayes Estimator Under General Entropy Loss Function

In many practical situations, it appears to be more realistic to express the loss in terms of the ratio $\frac{d}{\beta}$. In this case, Calabria and Pulcini [2] point out that a useful asymmetric loss function is the general entropy loss function,

$$L(\delta) = [\delta^q - q\ln(\delta) - 1], \delta = \frac{d}{\beta}. \tag{10}$$

The magnitude of q reflects the degree of asymmetry. The Bayes estimator of β under the GELF is given by

$$\hat{\beta}_{GE} = \left[E\left(\beta^{-q}|\mathbf{x}\right)\right]^{-\frac{1}{q}} \tag{11}$$

Therefore we have

$$\hat{\beta}_{GE} = \left(\frac{\Gamma(n+a-q)}{\Gamma(n+a)}\right)^{-\frac{1}{q}} \times \frac{1}{b-\ln u(x_n)}. \tag{12}$$

and

$$\hat{R}_{GE} = \left(\frac{b-\ln u(x_n)}{b-\ln u(x_n)+q\ln T}\right)^{-\frac{n+a}{q}} \tag{13}$$

2.4 Bayesian Estimation Under Linex Loss Function

Another asymmetric loss function given by Varian [11], known as Linex loss function, is defined as

$$L(\Delta) = e^{k\Delta} - k\Delta - 1, \quad \Delta = d - \beta,\ k \neq 0 \tag{14}$$

The sign and magnitude of the k reflect the deviation and degree of asymmetry, respectively. The Bayesian estimator of β under this loss function is given by

$$\hat{\beta}_L = \frac{-1}{k}\ln E\left[\exp(-k\beta)\right]$$

Therefore we obtain Bayesian estimator of the parameter β and reliability function $R(t)$ under Linex loss function as the following form

$$\hat{\beta}_L = -\frac{n+a}{k}\ln\left(\frac{b-\ln u(x_n)}{b-\ln u(x_n)+k}\right) \tag{15}$$

$$\hat{R}_L = -\frac{1}{k}\ln\left(\sum_{s=0}^{\infty}\frac{(-k)^s}{s!}\left(\frac{b-\ln u(x_n)}{b-\ln u(x_n)+s\ln T}\right)^{n+a}\right) \tag{16}$$

3 Empirical Bayes Estimation

In the parametric empirical Bayes method an estimate of the hyperparameter is usually obtained as a maximum likelihood estimate or a method of moments estimate (Carlin and Louis [8], p.62). Here we estimate the unknown hyperparameter β based on the method of maximum likelihood estimate. Assume that the conjugate family of prior distributions for β is the family of gamma distributions, with known parameter a and unknown parameter b. When the prior parameter b is unknown,we may use the empirical Bayes approach to get its estimate. the margin density function is

$$f(\mathbf{x}) = \int L(\mathbf{x}|\beta)\pi(\beta)d\beta = \int \beta^n (u(x_n))^\beta \frac{b^a}{\Gamma(a)}\beta^{a-1}e^{-b\beta}d\beta = \frac{\Gamma(n+a)}{\Gamma(a)} \frac{b^a}{(b-\ln u(x_n))^{n+a}}$$

Based on $f(\mathbf{x})$ the MLE of b is

$$\hat{b} = -\frac{a}{n}\ln u(x_n) \tag{17}$$

Substituting $\hat{b}$ into (8) and (9) the empirical Bayes estimations of parameter β and reliability function $R(t)$ under minimum expected loss function are obtained

$$\hat{\beta}_{EME} = \frac{n(n+a-2)}{-(a+n)\ln u(x_n)} \tag{18}$$

and

$$\hat{R}_{EME} = \left(1+\frac{n\ln T}{-(a+n)\ln u(x_n)+n\ln T}\right) \tag{19}$$

Also empirical Bayes estimations of parameter β and reliability function $R(t)$ Under general entropy loss function are obtained

$$\hat{\beta}_{EGE} = \left(\frac{\Gamma(n+a-q)}{\Gamma(n+a)}\right)^{-\frac{1}{q}} \times \frac{-n}{(a+n)\ln u(x_n)} \tag{20}$$

$$\hat{R}_{EGE} = \left(\frac{(a+n)\ln u(x_n)}{(a+n)\ln u(x_n)-nq\ln T}\right)^{-\frac{n+a}{q}} \tag{21}$$

And for Linex loss function we have

$$\hat{\beta}_{EL} = -\frac{n+a}{k}\ln\left(\frac{(a+n)\ln u(x_n)}{(a+n)\ln u(x_n)-nk}\right) \tag{22}$$

$$\hat{R}_{EL} = -\frac{1}{k}\ln\left(\sum_{s=0}^{\infty}\frac{(-k)^s}{s!}\left(\frac{(a+n)\ln u(x_n)}{(a+n)\ln u(x_n)-ns\ln T}\right)^{n+a}\right) \tag{23}$$

4 Prediction Of The Future Records

In the context of prediction of the future records, the prediction intervals provide bounds to contain the results of a future record, based upon the results of the previous record observed from the same distribution. This section is devoted to deriving Bayes predictive density function, which is necessary to obtain bounds for predictive interval of future records. Let $\mathbf{X} = \left\{X_{G(1)} = X_1, X_{G(2)} = X_2, ..., X_{G(n)} = X_n\right\}$ with $X_{G(1)} \leq X_{G(2)} \leq ... \leq X_{G(n)}$ be the first n observed upper record values from the Generalized Half logistic distribution. Based on such a record sample, Bayesian prediction is needed for the sth record $X_{G(s)}$, $1<n<s$. For the convenience of notation, let $X_{G(n)} = x_n$ and $X_{G(s)} = y$ The conditional pdf of y for given X_n is given by Ahsanullah [4] in the form

$$f(y|x_n,\beta) = \frac{[w(y)-w(x_n)]^{s-n-1}}{\Gamma(s-n)}\frac{f(y)}{1-F(x_n)} \tag{24}$$

Where $w(\cdot) = -\ln[1-F(\cdot)]$. For Generalized Half logestic distribution with (pdf) and (cdf) given by (2) and (1) respectively, the function $f(y|x_n,\beta)$ becomes

$$f(y|x_n,\beta) = \frac{\beta^{s-n}}{\Gamma(s-n)}(\ln g(y,x_n))^{s-n-1}(g(y,x_n))^{-\beta} \times \frac{1}{1+e^{-y}} \tag{25}$$

Where

$$g(y,x_n) = \frac{e^{-x_n}(1+e^{-y})}{e^{-y}(1+e^{-x_n})}$$

The Bayes predictive density function of y given x_n is given by

$$f(y|x_n) = \int f(y|x_n,\beta)\pi(\beta|\mathbf{x})d\beta \tag{26}$$

Substituting (7) and (25) in (26), we get

$$f(y|x_n) = \frac{(b-\ln u(x_n))^{n+a}}{B(n+a,s-n)} \times \frac{(\ln g(y,x_n))^{s-n-1}}{(b+\ln\frac{1+e^{-y}}{2e^{-y}})^{s+a}} \times \frac{1}{1+e^{-y}} \tag{27}$$

where $B(\cdot,\cdot)$ is the beta function. It follows that the lower and upper $100\alpha\%$ prediction bounds for $Y = X_{G(s)}$, given the previous data $\mathbf{x}$, can be derived using the predictive survival function defined by

$$\mathbf{Pr}(Y \geq \lambda|\mathbf{x}) = \int_{\lambda}^{\infty} f(y|x_n)dy \tag{28}$$

The predictive bounds of a two-sided interval with cover α , for the future record Y may be thus obtained by solving the following two equations, for lower (LL) and upper (UL) limits:

$$\mathbf{Pr}(Y > LL|\mathbf{x}) = \frac{1+\alpha}{2}, \qquad \mathbf{Pr}(Y > UL|\mathbf{x}) = \frac{1-\alpha}{2}$$

In most case, we will predict the first unobserved record value $X_{G(n+1)}$,the predictive survival function for $Y = X_{G(n+1)}$ is given from (27) and (28) by settings $s = n+1$, simplifies to

$$\mathbf{Pr}(Y \geq \lambda|\mathbf{x}) = \left(\frac{b-\ln u(x_n)}{b+\ln\frac{1+e^{-\lambda}}{2e^{-\lambda}}}\right)^{n+a} \tag{29}$$

thus, a $100\alpha\%$, Bayesian prediction interval for $Y = X_{G(n+1)}$, satisfying

$$LL = \ln\left(2\exp\left\{\frac{b-\ln u(x_n)}{\left(\frac{1+\alpha}{2}\right)^{\frac{1}{n+a}}} - b\right\} - 1\right) \tag{30}$$

and

$$UL = \ln\left(2\exp\left\{\frac{b-\ln u(x_n)}{\left(\frac{1-\alpha}{2}\right)^{\frac{1}{n+a}}} - b\right\} - 1\right) \tag{31}$$

4.1 Empirical Bayes Prediction Interval

substituting $\hat{b}$ into (30) and (31), we obtain the empirical Bayes one-step prediction interval as the following form

$$LL_E = \ln\left(2\exp\left\{\frac{a}{n}\ln u(x_n)\left(1-\frac{a+n}{a\left(\frac{1+\alpha}{2}\right)^{\frac{1}{n+a}}}\right)\right\} - 1\right) \tag{32}$$

and

$$UL_E = \ln\left(2\exp\left\{\frac{a}{n}\ln u(x_n)\left(1-\frac{a+n}{a\left(\frac{1-\alpha}{2}\right)^{\frac{1}{n+a}}}\right)\right\} - 1\right) \tag{33}$$

5 Simulation study

In this section, the maximum likelihood, Bayes (Minimum Expected Loss, LINEX, General Entropy) and Empirical Bayes Estimators are compared based on a Monte Carlo simulation study and also compute the Bayes and Empirical Bayes prediction interval for the future upper record value. we used the following steps to generate a Recored values from the Generalized Half Logistic distribution and compute different estimates.

1.For a given values of prior parameters a and b (Table 1), we generate β from the prior density (6).
2.For given β obtained in step(1), we generate $n = (4, 7, 10)$ upper record values from the Generalized Half logistic distribution with pdf (2) using $x_i = \ln\left(2(1-U_i)^{-\frac{1}{\beta}} - 1\right)$, where U_i are independent $uniform(0,1)$ random variables.
3.We obtained the estimates $N = 2000$ times and calculated the Estimated Risk (ER) given by

$$ER = \frac{1}{2000}\sum_{i=1}^{2000}(\hat{\gamma}_i - \gamma)^2$$

where $\hat{\gamma}$ is an estimate of γ.

Table 1: True values for simulation .

a	b	β	k	q	t	$R(t)$
2	1	0.998182	1	1	0.5	1.324361

Table 2: Averaged values of Estimated Risk for estimates of the parameter β.

n	$ER(\hat{\beta})$	$ER(\hat{\beta}_{ME})$	$ER(\hat{\beta}_L)$	$ER(\hat{\beta}_{GE})$	$ER(\hat{\beta}_{EME})$	$ER(\hat{\beta}_{EL})$	$ER(\hat{\beta}_{EGE})$
4	0.60702	0.16958	0.09114	0.11290	0.33807	0.32493	0.42270
7	0.15894	0.14083	0.08439	0.10342	0.18408	0.13721	0.16002
10	0.09972	0.12485	0.07800	0.09432	0.14710	0.10101	0.11830

Table 3: Averaged values of Estimated Risk for estimates of the $R(t)$.

n	$ER(\hat{R})$	$ER(\hat{R}_{ME})$	$ER(\hat{R}_L)$	$ER(\hat{R}_{GE})$	$ER(\hat{R}_{EME})$	$ER(\hat{R}_{EL})$	$ER(\hat{R}_{EGE})$
4	0.34548	0.01300	0.28649	0.17297	0.07741	0.21409	0.23131
7	0.29785	0.01133	0.26478	0.02497	0.08867	0.17295	0.02100
10	0.28123	0.01002	0.25510	0.01341	0.09343	0.16091	0.01363

6 Real data

We apply the proposed procedures by considering a real data given by Hinkley [12], which represents the thirty successive values of March precipitation (in inches) in Minneapolis/StPau over a period of 30 years. The Torabi and Bagheri [13] showed that the data fits well the extended generalized half logistic distribution. 0.77, 1.74, 0.81, 1.20, 1.95, 1.20, 0.47, 1.43, 3.37, 2.20, 3.00, 3.09, 1.51, 2.10, 0.52, 1.62, 1.31, 0.32, 0.59, 0.81, 2.81, 1.87, 1.18, 1.35, 4.75, 2.48, 0.96, 1.89, 0.90, 2.05

If only the five upper record values have been observed, these are(seo et al. [11])

$$0.77, 1.74, 1.95, 3.37, 4.75$$

Table 4: Bayes and Empirical Bayes prediction interval for the future upper record value x_{n+1}.

			Bayes			Empirical Bayes		
α	n	x_n	LL	UL	*Width*	LL_E	UL_E	*Width*
90%	4	2.2258	2.2603	4.6730	*2.4127*	2.2491	3.8966	*1.6475*
	7	3.3788	3.4067	5.2711	*1.8644*	3.3995	4.7848	*1.3853*
	10	5.8200	5.8506	7.8444	*1.9938*	5.8465	7.5681	*1.7216*
95%	4	2.2258	2.2428	5.4112	*3.1684*	2.2372	4.3993	*2.1621*
	7	3.3788	3.3925	5.8001	*2.4076*	3.3889	5.1779	*1.7890*
	10	5.8200	5.8351	8.3887	*2.5536*	5.8330	8.0380	*2.2050*
99%	4	2.2258	2.2291	7.4834	*5.2543*	2.2280	5.8040	*3.5760*
	7	3.3788	3.3815	7.1946	*3.8131*	3.3808	6.2129	*2.8321*
	10	5.8200	5.8229	9.7805	*3.9576*	5.8225	9.2398	*3.4173*

Using the formulae presented in Section 1, the maximum likelihood estimates of β and reliability function $R(0.5)$ are $\hat{\beta} = 1.229871$ and $\hat{R} = 0.7078611$, respectively. Using results of Sections 1 and 2 and prior parameters a and b (Table 1), different Bayes and Empirical Bayes estimates for the parameter and reliability function are computed and presented in Tables 5 and 6.

Using the result of Section 4, the 90%, 95% and 99%, Bayesian prediction intervals for the sixth record are (4.794986, 7.998011), (4.772166, 8.966793)and (4.754382, 11.62267), respectively. Similarly, the 90%, 95% and 99%, empirical Bayes one-step prediction intervals for the sixth record are(4.792214,7.798273), (4.7708,8.707398) and (4.754112, 11.19964), respectively.

Table 5: Bayes and Empirical Bayes estimates of the parameter β, (Hinkley data).

$\hat{\beta}_{ME}$	$\hat{\beta}_{L}$	$\hat{\beta}_{GE}$	$\hat{\beta}_{EME}$	$\hat{\beta}_{EL}$	$\hat{\beta}_{EGE}$
0.8243388	0.9892066	1.068253	0.8784792	1.054175	1.133021

Table 6: Bayes and Empirical Bayes estimates of the reliability function $R(t)$, (Hinkley data).

$\hat{R}_{ME}$	$\hat{R}_{L}$	$\hat{R}_{GE}$	$\hat{R}_{EME}$	$\hat{R}_{EL}$	$\hat{R}_{EGE}$
1.354185	1.401091	1.303358	1.047037	1.401091	1.250818

7 Conclusion

Based on upper record values, the present paper proposes Bayesian and Empirical Bayesian approaches to estimate the parameter and reliability function for the Generalized Half Logistic distribution .We also considered the problem of predicting future record in a Bayesian and Empirical Bayesian approaches. The Bayes estimators are obtained using both symmetric and asymmetric loss functions. There are some conclusions which have been noticed as follows

1. Table 2 shows that the Bayes estimates of parameter β under the LINEX loss function has the smallest Estimated Risk (ER) as compared with other estimates (Bayes estimates under the General Entropy and Minimum Expected loss functions, maximum likelihood estimator (MLEs) and different Empirical Bayes estimates). Also, the Estimated Risk decrease as n increases.
2. Table 3 shows that the Bayes estimates of reliability function under the Minimum Expected loss has the smallest Estimated Risk (ER) as compared with other estimates (Bayes estimates under the General Entropy and LINEX loss functions, maximum likelihood estimator (MLEs) and different Empirical Bayes estimates). Also, the Estimated Risk decrease as n increases.

3.Table 4 shows that for different size n, The length of the Empirical Bayes prediction interval for the future upper record value always shorter than the Bayes prediction interval.

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