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Some Ratio-cum-product Type Estimators for Population Mean Under Double Sampling in the Presence of Non-response

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Abstract: In this paper, we have proposed some ratio-cum-product type estimators for population mean of the study variable y in the presence of non-response using auxiliary information under double sampling. The expressions of mean squared error (MSE) of the proposed estimators are derived under double (two-stage) sampling. In addition, an empirical study is carried out to show the properties of the proposed estimators..

Keywords: Auxiliary variable, double sampling, mean Square error, efficiency.

1 Introduction

In surveys covering socio-economic studies, several variables are considered simultaneously. While conducting a households survey for the study of several variables information is in most of the cases, not obtained from all the units in the survey even after some call backs. An estimate obtained from such incomplete data is misleading, especially when the respondents differ from non-respondents because the estimate can be biased (see [7]). [3] suggested a technique for sub-sampling the non-respondents in order to adjust for non-response in mail surveys. In estimating population parameters use of auxiliary information improves precision of an estimate when auxiliary variable x is highly correlated with the study variable y (see [15,14]). Several authors including [2], [9,10], [4,5] and [16,17] discussed the problem of estimating the population mean $\bar{Y}$ of study variable y when the population mean $\bar{X}$ of auxiliary variable x is known in the presence of non response. Also some other authors including [11], [13] have discussed the problem of estimating the population mean under double sampling with non- response.

Let y be the study variable and x_1, x_2 be the auxiliary variables. Let $\bar{Y}$, $\bar{X_1}$ and $\bar{X_2}$ are the population means of study and auxiliary variables, respectively. Here, we assume that $\bar{X_2}$ is known and $\bar{X_1}$ is unknown. By using simple random sampling without replacement (SRSWOR), we draw a preliminary sample of size n' from the population of size N and on the basis of n' units we estimate the unknown population mean $\bar{X_1}$ as $\bar{x_1}'$.

Now, we draw a subsample of size n from the preliminary sample of size n' using SRSWOR and we observe that n_1 units respond and n_2 units do not respond in the sample of size n for the study variable y. Using [3] technique of sub sampling, a sub sample of r units is selected from the n_2 non respondent units and enumerate completely by direct interview, such that $r = (n_2/L), L > 1$, where L is the inverse sampling rate. Here we assume that response is obtained for all the r units. Thus we have $(n_1 + r)$ observation on study variable y. Using [3] technique, the estimator for population mean using $(n_1 + r)$ observations on study variable y is given by

$$\bar{y}^* = \frac{n_1\bar{y}_1 + n_2\bar{y}_{r2}}{n} \tag{1}$$

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where $\bar{y}_1$ and $\bar{y}_{r2}$ denote the sample means of y based on n_1 and r units respectively.
The estimator $\bar{y}^*$ is unbiased and has variance

$$V(\bar{y}^*) = \frac{1-f}{n}S_y^2 + \frac{L-1}{n}KS_{y_2}^2 \quad (2)$$

where, $f = \frac{n}{N}$, $K = \frac{N_2}{N}$, S_y^2 and S_{y2}^2 are the population mean squares of y for the entire population and for the non-responding part of the population, respectively.
[6] proposed a ratio type estimator of population mean using available information on two auxiliary variables in the presence of non response, given by

$$\bar{y}_R = \bar{y}^* \left(\frac{\bar{x}'_1}{\bar{x}_1}\right)\left(\frac{\bar{X}_2}{\bar{x}'_2}\right) \quad (3)$$

The mean square error (MSE) expression of (3) is given by

$$MSE\,(\bar{y}_R) = \bar{Y}^2\left[f_1C_y^2 + (f_1 - f_2)C_{x_1}^2 + f_2C_{x_2}^2 - 2(f_1 - f_2)\rho_{yx_1}C_yC_{x_1} - \rho_{yx_2}C_yC_{x_2}\right] + \frac{L-1}{n}KS_{y2}^2 \quad (4)$$

[12] ratio-cum-product estimator of population mean in double sampling under non-response using two auxiliary variables, is given by

$$\bar{y}_{rpd} = \bar{y}^* \left(\frac{\bar{x}'_1}{\bar{x}_1}\right)^{\alpha_1}\left(\frac{\bar{x}'_2}{\bar{X}_2}\right)^{\alpha_2} \quad (5)$$

where α_1 and α_2 are constants.
The MSE expression of (5) is given as

$$MSE\,(\bar{y}_{rpd}) = \bar{Y}^2\left[f_1C_y^2(1-\rho_{yx_1}^2) + f_2C_y^2(\rho_{yx_1}^2 - \rho_{yx_2}^2)\right] + \frac{L-1}{n}KS_{y_2}^2 \quad (6)$$

In this paper, motivated by [8] ratio-cum-product type estimator is presented in the presence of non-response under double sampling scheme. The expressions for the bias and mean square error of the proposed estimator are obtained and compared with relevant estimators. The expression of minimum variance has been obtained for the optimum value of n, n' and L for fixed cost $C \le C_0$ and for the specified variance $V = V'_0$.

2 Proposed Estimator

Motivated by [8], we have proposed some ratio-cum-product type estimators under double sampling, defined as

$$T_1 = \bar{y}^* \left\{\frac{\bar{x}'_2 + \alpha_{21}(\bar{X}_2 - \bar{x}'_2)}{\bar{x}_1 + \alpha_{11}(\bar{x}'_1 - \bar{x}_1)}\right\}\left\{\frac{\bar{x}'_1}{X_2}\right\} \quad (7)$$

$$T_2 = \bar{y}^* \left\{\frac{\bar{x}'_1}{\bar{x}_1 + \alpha_{12}(\bar{x}'_1 - \bar{x}_1)}\right\}\left\{\frac{\bar{X}_2}{\bar{x}'_2 + \alpha_{22}(\bar{X}_2 - \bar{x}'_2)}\right\} \quad (8)$$

$$T_3 = \bar{y}^* \left\{\frac{\bar{x}_1 + \alpha_{13}(\bar{x}'_1 - \bar{x}_1)}{\bar{x}'_1}\right\}\left\{\frac{\bar{x}'_2 + \alpha_{23}(\bar{X}_2 - \bar{x}'_2)}{X_2}\right\} \quad (9)$$

$$T_4 = \bar{y}^* \left\{\frac{\bar{x}_1 + \alpha_{14}(\bar{x}'_1 - \bar{x}_1)}{\bar{x}'_2 + \alpha_{24}(X_2 - \bar{x}'_2)}\right\}\left\{\frac{\bar{X}_2}{\bar{x}'_1}\right\} \quad (10)$$

To obtain the bias and MSE expressions of the estimators $T_i (i = 1,2,3,4)$ to the first degree of approximation, we define
$e_0 = \frac{\bar{y}^* - \bar{Y}}{\bar{Y}}$, $e_1 = \frac{\bar{x}_1 - \bar{X}_1}{\bar{X}_1}$, $e'_1 = \frac{\bar{x}'_1 - \bar{X}_1}{\bar{X}_1}$, $e'_2 = \frac{\bar{x}'_2 - \bar{X}_2}{\bar{X}_2}$
such that, $E(e_0) = E(e_1) = E(e'_1) = E(e'_2) = 0$
Also, $E(e_0^2) = f_1C_y^2 + \frac{L-1}{n}KC_{y_2}^2$, $E(e_1^2) = f_1C_{x_1}^2$, $E(e_1'^2) = f_2C_{x_1}^2$, $E(e_2'^2) = f_2C_{x_2}^2$,
$E(e_0e_1) = f_1\rho_{yx_1}C_yC_{x_1}$, $E(e_0e'_1) = f_2\rho_{yx_1}C_yC_{x_1}$, $E(e_0e'_2) = f_2\rho_{yx_2}C_yC_{x_2}$, $E(e_1e'_1) = f_2C_{x_1}^2$,
$E(e_1e'_2) = f_2\rho_{x_1x_2}C_{x_1}C_{x_2}$, $E(e'_1e'_2) = f_2\rho_{x_1x_2}C_{x_1}C_{x_2}$
$C_y = \frac{S_y}{\bar{Y}}$, $C_{x_1} = \frac{S_{x_1}}{\bar{X}_1}$, $C_{x_2} = \frac{S_{x_2}}{\bar{X}_2}$, $C_{y_2} = \frac{S_{y_2}}{\bar{Y}}$, $f_1 = \frac{1}{n} - \frac{1}{N}$, $f_2 = \frac{1}{n'} - \frac{1}{N}$

$S_y^2=\frac{1}{N-1}\sum_{i=1}^{N}(Y_i-\bar{Y})^2$, $S_{x_1}^2=\frac{1}{N-1}\sum_{i=1}^{N}(X_{1i}-\bar{X}_1)^2$, $S_{x_2}^2=\frac{1}{N-1}\sum_{i=1}^{N}(X_{2i}-\bar{X}_2)^2$, $S_{y_2}^2=\frac{1}{N_2-1}\sum_{i=1}^{N_2}(Y_{2i}-\bar{Y}_2)^2$

Expressing (7) in terms of e's, we have

$$T_1=\bar{Y}(1+e_0)\left\{\frac{\bar{X}_2(1+e_2')+\alpha_{21}(\bar{X}_2-\bar{X}_2(1+e_2'))}{\bar{X}_1(1+e_1)+\alpha_{11}\left(\bar{X}_1(1+e_1')-\bar{X}_1(1+e_1)\right)}\right\}\left\{\frac{\bar{X}_1(1+e_1')}{\bar{X}_2}\right\}$$
$$T_1=\bar{Y}(1+e_0)(1+e_1')(1+(1-\alpha_{21})e_2')(1+e_1+\alpha_{11}(e_1'-e_1))^{-1} \tag{11}$$

Expanding the right hand side of (11) and retaining terms up to second degrees of e's, we have

$$T_1=\bar{Y}\left[1+(1-\alpha_{11})^2e_1^2-(1-\alpha_{11})\{e_1+e_0e_1+e_1e_1'\}+(1-\alpha_{21})\{e_2'+e_1'e_2'+e_0e-2'\}-\alpha_{11}\{e_1'+e_1'^2+e_0e_1'\}\right.$$
$$\left.+\alpha_{11}^2e_1'^2+2\alpha_{11}(1-\alpha_{11})e_1e_1'-\alpha_{11}(1-\alpha_{21})e_1'e_2'+e_1'+e_0e_1'+e_0\right] \tag{12}$$

Taking expectations of both sides of (12) and then subtracting $\bar{Y}$ from both sides, we get the bias of the estimator T_1 up to the first order of approximation as

$$Bias(T_1)=\bar{Y}\left[(1-\alpha_{11})^2f_1C_{x_1}^2-(1-\alpha_{11})\rho_{yx_1}C_yC_{x-1}(f_1+f_2)+(1-\alpha_{21})f_2C_{x_2}\{\rho_{x_1x_2}C_{x_1}+\rho_{yx_2}C_y\}\right.$$
$$-\alpha_{11}f_2C_{x_1}\{C_{x_1}+\rho_{yx_1}C_y\}+\alpha_{11}^2f_2C_{x_1}^2+2\alpha_{11}(1-\alpha_{11})f_2C_{x_1}^2-\alpha_{11}(1-\alpha_{21})f_2\rho_{x_1x_2}C_{x_1}C_{x_2}$$
$$\left.+f_2\rho_{yx_1}C_yC_{x_1}\right] \tag{13}$$

From (12), we have

$$\left(T_1-\bar{Y}\right)\cong\bar{Y}\left[e_0+(1-\alpha_{11})e_1'-(1-\alpha_{11})e_1+(1-\alpha_{21})e_2'\right] \tag{14}$$

Squaring both sides of (14) and then taking expectations, we get the MSE of T_1 up to the first order of approximation as

$$MSE(T_1)=\bar{Y}^2\left[f_1C_y^2+(1-\alpha_{11})^2C_{x_1}^2(f_1-f_2)+(1-\alpha_{21})^2f_2C_{x_2}^2-2(1-\alpha_{11})\rho_{yx_1}C_yC_{x_1}(f_1-f_2)\right.$$
$$\left.+2(1-\alpha_{21})f_2\rho_{yx_2}C_yC_{x_2}\right]+\frac{L-1}{n}KS_{y_2}^2 \tag{15}$$

Partially differentiating (15) with respect to α_{11} and α_{21} and equating to zero, we get the optimum values of α_{11} and α_{21} , as

$$\alpha_{11(opt)}=1-\rho_{yx_1}\frac{C_y}{C_{x_1}}\quad and\quad \alpha_{21(opt)}=1+\rho_{yx_2}\frac{C_y}{C_{x_2}}$$

Similarly, we get the bias and MSE expressions of the estimators T_2, T_3 and T_4 respectively, as

$$Bias(T_2)=\bar{Y}\left[(1-\alpha_{12})^2f_1C_{x_1}^2+(1-\alpha_{22})^2f_2C_{x_2}^2+(1-\alpha_{12})(1-\alpha_{22})f_2\rho_{x_1x_2}C_{x_1}C_{x_2}\right.$$
$$+\alpha_{12}(1-\alpha_{12})f_2\{C_{x_1}^2+\rho_{x_1x_2}C_{x_1}C_{x_2}+\rho_{yx_1}C_yC_{x_1}\}-(1-\alpha_{12})\{f_2C_{x_1}^2+f_1\rho_{yx_1}C_yC_{x_1}\}$$
$$\left.-(1-\alpha_{22})f_2C_{x_2}\{\rho_{x_1x_2}C_{x_1}C_{x_2}+\rho_{yx_2}C_yC_{x_2}\}-\alpha_{12}f_2C_{x_1}\{C_{x_1}\rho_{yx_1}C_y\}+f_2\rho_{yx_1}C_yC_{x_1}\right] \tag{16}$$

$$Bias(T_3)=\bar{Y}\left[f_2\{C_{x_2}^2-\rho_{yx_1}C_yC_{x_1}\}+(1-\alpha_{23})f_2\rho_{yx_2}C_yC_{x_2}+(1-\alpha_{13})C_{x_1}\{f_1\rho_{yx_1}C_y-f_2C_{x_1}\}\right.$$
$$\left.+\alpha_{13}f_2C_{x_1}\{\rho_{yx_1}C_y-C_{x_1}\}\right] \tag{17}$$

$$Bias(T_4)=\bar{Y}\left[f_2C_{x_2}^2+(1-\alpha_{24})f_2C_{x_2}^2-f_2\rho_{yx_1}C_yC_{x_1}-(1-\alpha_{14})C_{x_1}\{f_1\rho_{yx_1}C_y-f_2C_{x_1}\}\right.$$
$$\left.-(1-\alpha_{24})f_2\rho_{yx_2}C_yC_{x_2}+\alpha_{14}f_2C_{x_1}\{\rho_{yx_1}C_y-C_{x_1}\}\right] \tag{18}$$

and

$$MSE(T_2)=\bar{Y}^2\left[f_1C_y^2+(1-\alpha_{12})^2C_{x_1}^2(f_1-f_2)+(1-\alpha_{22})^2f_2C_{x_2}^2-2(1-\alpha_{12})\rho_{yx_1}C_yC_{x_1}(f_1-f_2)\right.$$
$$\left.-2(1-\alpha_{22})f_2\rho_{yx_2}C_yC_{x_2}\right]+\frac{L-1}{n}KS_{y_2}^2 \tag{19}$$

where

$$\alpha_{12(opt)}=1-\rho_{yx_1}\frac{C_y}{C_{x_1}}\quad and\quad \alpha_{22(opt)}=1-\rho_{yx_2}\frac{C_y}{C_{x_2}}$$

$$MSE(T_3) = \bar{Y}^2\left[f_1C_y^2 + (1-\alpha_{13})^2C_{x_1}^2(f_1-f_2) + (1-\alpha_{23})^2 f_2C_{x_2}^2 - 2(1-\alpha_{13})\rho_{yx_1}C_yC_{x_1}(f_1-f_2) + 2(1-\alpha_{23})f_2\rho_{yx_2}C_yC_{x_2}\right] + \frac{L-1}{n}KS_{y_2}^2 \quad (20)$$

where

$$\alpha_{13(opt)} = 1-\rho_{yx_1}\frac{C_y}{C_{x_1}} \quad and \quad \alpha_{23(opt)} = 1+\rho_{yx_2}\frac{C_y}{C_{x_2}}$$

$$MSE(T_4) = \bar{Y}^2\left[f_1C_y^2 + (1-\alpha_{14})^2C_{x_1}^2(f_1-f_2) + (1-\alpha_{24})^2 f_2C_{x_2}^2 + 2(1-\alpha_{14})\rho_{yx_1}C_yC_{x_1}(f_1-f_2) - 2(1-\alpha_{24})f_2\rho_{yx_2}C_yC_{x_2}\right] + \frac{L-1}{n}KS_{y_2}^2 \quad (21)$$

where

$$\alpha_{14(opt)} = 1+\rho_{yx_1}\frac{C_y}{C_{x_1}} \quad and \quad \alpha_{24(opt)} = 1-\rho_{yx_2}\frac{C_y}{C_{x_2}}$$

3 Determination of n', n and L (for the fixed cost $C \le C_0$)

Let C_0 be the total cost (fixed) of the survey apart from overhead cost. The expected total cost of the survey apart from overhead cost is given by

$$C' = (c_1' + c_2')n' + c_1 n + c_2 n_1 + c_3\frac{n_2}{L} \quad (22)$$

Since C' vary from sample to sample, so the expected cost can be written as:

$$C = E(C') = (c_1' + c_2')n' + n\left(c_1 + c_2w_1 + c_3\frac{w_2}{L}\right) \quad (23)$$

where
c_1' is the cost per unit of obtaining information on auxiliary variable x_1.
c_2' is the cost per unit of obtaining information on additional auxiliary variable x_2.
c_1 is the cost per unit of mailing questionnaire/visiting the unit at the subsample.
c_2 is the cost per unit of collecting, processing data obtained from n_1 responding units.
c_3 is the cost per unit of obtaining and processing data (after extra efforts) for the sub sampling units.
and $w_1 = \frac{N_1}{N}$, $w_2 = \frac{N_2}{N}$, response rate and non-response rate in the population.
The expression $Var(T_i), i = 1,2,3,4$ given by (15), (19), (20), (21), can be written as

$$Var(T_i) = \left\{\frac{1}{n}V_{0i} + \frac{1}{n'}V_{1i} + \frac{L}{n}V_{2i}\right\} + (terms\, independent\, of\; n,\, n'\, and\, L) \quad (24)$$

where V_{0i}, V_{1i} and V_{2i} are the coefficients of the terms $\frac{1}{n}$, $\frac{1}{n'}$ and $\frac{L}{n}$ in the expression for $T_i, i = 1,2,3,4$.
Let us define a function ϕ as

$$\phi = Var(T_i) + \lambda_i\left\{(c_1' + c_2')\,n' + n\left(c_1 + c_2w_1 + c_3\frac{w_2}{L}\right)\right\} \quad (25)$$

where λ_i is the Lagrange's multiplier.
Partially differentiating equation (25) with respect to n', n and L and equating to zero, we get,

$$n' = \sqrt{\frac{V_{1i}}{\lambda_i\,(c_1' + c_2')}},\; n = \sqrt{\frac{(V_{0i} + LV_{2i})}{\lambda_i\left(c_1 + c_2 + c_3\frac{w_2}{L}\right)}}\; and\; L_{opt} = \sqrt{\frac{V_{0i}w_2c_3}{V_{2i}\,(c_1 + c_2w_1)}} \quad (26)$$

Putting the value of n', n and L_{opt} from equation (26) in equation (23), we have,

$$\sqrt{\lambda_i} = \frac{1}{C}\left\{\sqrt{(V_{0i} + L_{opt}V_{2i})\left(c_1 + c_2 + c_3\frac{w_2}{L_{opt}}\right)} + \sqrt{V_{1i}\,(c_1' + c_2')}\right\} \quad (27)$$

Thus the minimum value of $Var(T_i); i = 1,2,3,4$ is given as

$$Var(T_i)_{min} = \frac{1}{C}\left\{\sqrt{(V_{0i}+L_{opt}V_{2i})\left(c_1+c_2+c_3\frac{w_2}{L_{opt}}\right)}+\sqrt{V_{1i}\left(c_1'+c_2'\right)}\right\}^2 - (terms\, independent\, of\, n,\, n'\, and\, L) \quad (28)$$

Neglecting the terms independent of n', n and L, we have,

$$Var(T_i)_{min} = \frac{1}{C}\left\{\sqrt{(V_{0i}+L_{opt}V_{2i})\left(c_1+c_2+c_3\frac{w_2}{L_{opt}}\right)}+\sqrt{V_{1i}\left(c_1'+c_2'\right)}\right\}^2 \quad (29)$$

Putting the optimum value of L from (26) to (29), we get the minimum value of $Var(T_i)$ as

$$Var(T_i)_{min} = \frac{1}{C}\left\{\sqrt{V_{0i}(c_1+c_2w_1)}+\sqrt{V_{0i}w_2c_3}+\sqrt{V_{1i}(c_1'+c_2')}\right\}^2 \quad (30)$$

4 Determination of n, $n$and L for specified variance $V = V_0'$

Let V_0' be the variance of the estimator $T_i (i = 1,2,3,4)$ fixed in advanced, so we have,

$$V_0' = \frac{V_{0i}}{n}+\frac{V_{1i}}{n'}+\frac{LV_{2i}}{n}+(terms\, independent\, of\, n,\, n'\, and\, L); i = 1,2,3,4. \quad (31)$$

To obtain the optimum values of n', n and L and minimizing the average total cast for the specified variance of the estimator T_i , we define a function ψ given as

$$\psi = \left(c_1'+c_2'\right)n' + n\left(c_1+c_2w_1+c_3\frac{w_2}{L}\right)+\mu_i\left(T_i - V_0'\right); i = 1,2,3,4. \quad (32)$$

where μ_i is the Lagrange's multiplier. Partially differentiating equation (32) with respect to n', n and L and equating to zero, we get,

$$n' = \sqrt{\frac{\mu_i V_{1i}}{\left(c_1'+c_2'\right)}},\ n = \sqrt{\frac{\mu_i\left(V_{0i}+LV_{2i}\right)}{\left(c_1+c_2+c_3\frac{w_2}{L}\right)}}\ and\ L_{opt} = \sqrt{\frac{V_{0i}w_2c_3}{V_{2i}\left(c_1+c_2w_1\right)}} \quad (33)$$

Putting the values of n', n and L_{opt} from equation (33) in equation (31), we have,

$$\sqrt{\mu_i} = \frac{\left\{\sqrt{(V_{0i}+L_{opt}V_{2i})\left(c_1+c_2+c_3\frac{w_2}{L_{opt}}\right)}+\sqrt{V_{1i}\left(c_1'+c_2'\right)}\right\}}{V_0'+(terms\, independent\, of\, n,\, n'\, and\, L)} \quad (34)$$

Thus the minimum expected total cost for the specified variance V_0' will be given by

$$C_{i(min)} = \frac{\left\{\sqrt{(V_{0i}+L_{opt}V_{2i})\left(c_1+c_2+c_3\frac{w_2}{L_{opt}}\right)}+\sqrt{V_{1i}\left(c_1'+c_2'\right)}\right\}}{V_0'+(terms\, independent\, of\, n,\, n'\, and\, L)} \quad (35)$$

Neglecting the terms independent of n', n and L, we have,

$$C_{i(min)} = \frac{\left\{\sqrt{(V_{0i}+L_{opt}V_{2i})\left(c_1+c_2+c_3\frac{w_2}{L_{opt}}\right)}+\sqrt{V_{1i}\left(c_1'+c_2'\right)}\right\}}{V_0'} \quad (36)$$

5 Empirical Study

For empirical study we consider [1] data. 25 families have been observed for the following three variabes. y: Head length of second son
x_1: Head length of first son
x_2: Head breadth of first son
$\bar{Y} = 183.84$, $\bar{X}_1 = 185.72$, $\bar{X}_2 = 151.12$, $C_y = 0.0546$, $C_{x_1} = 0.0526$, $C_{x_2} = 0.0488$
$\rho_{yx_1} = 0.7108$, $\rho_{yx_2} = 0.6932$, $\rho_{x_1x_2} = 0.7346$. Consider $n = 7$ and $n' = 10$

The table 1 given below shows the percentage relative efficiency of $\bar{y}_R, \bar{y}_{rpd}$ and $T_i (i = 1, 2, 3, 4)$ with respect to $\bar{y}^*$ for the different choice of K and L.

Table 1: Percentage relative efficiency of estimators w.r.t. $\bar{y}^*$

K	L	PRE of $\bar{Y}_R$ with respect to $\bar{Y}^*$	PRE of $\bar{Y}_{rpd}$ with respect to $\bar{Y}^*$	PRE of $T_i (i = 1, 2, 3, 4)$ with respect to $\bar{Y}^*$
0.1	2.0	166.44	180.02	180.02
	2.5	161.61	173.76	173.76
	3.0	157.43	168.41	168.41
	3.5	153.78	163.78	163.78
0.2	2.0	157.43	168.41	168.41
	2.5	150.56	159.73	159.73
	3.0	145.17	153.01	153.01
	3.5	140.81	147.65	147.65
0.3	2.0	150.56	159.73	159.73
	2.5	142.88	150.19	150.19
	3.0	137.22	143.28	143.28
	3.5	132.88	138.04	138.04
0.4	2.0	145.17	153.01	153.01
	2.5	137.22	143.28	143.28
	3.0	131.65	136.56	136.56
	3.5	127.53	131.65	131.65

6 Conclusion

In this paper, we have proposed ratio-cum-product type estimator in the presence of non-response under double sampling scheme. From the table 1 we conclude that the efficiency of proposed estimators are greater than that of the estimator proposed by [6] and it is same as the estimator proposed by [12]

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