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Substitutable Inventory Systems with Coordinated Reorder Levels

N. Anbazhagan[1,*], *Mark Goh*[2]*and Vigneshwaran*[3]

[1] Department of Mathematics, Alagappa University, Karaikudi, India.
[2] NUS Business School, National University of Singapore, Singapore.
[3] Department of Mathematics, Thiagarajar College of Engineering, Madurai, India.

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Abstract: This paper considers a two commodity continuous review inventory system with Markovian demands. The two commodities are assumed to be both way substitutable. That is, if the inventory level of one commodity reaches zero, then a demand for this commodity will be satisfied by an item of the other commodity. A joint order is placed when the inventory level reaches to any one of the reorder levels in the set of reorder levels with some prefixed probability. The limiting probability distribution for the joint inventory levels is computed. Various operational characteristics and the expression for the long run total expected cost rate are derived.

Keywords: Two-commodity, substitutable items, Poisson demand, continuous review, coordinated reorder levels.

1 Introduction

With the advent of advanced computing systems, many organizations have increasingly use multi commodity inventory systems. Further, models were proposed with independently established reorder points. When several products compete for limited storage space, or share the same transport facility, or items are produced on (procured from) the same equipment (supplier), the above strategy overlooks the potential savings associated with the joint replenishment and cost reduction in the ordering and setup costs.

In continuous review inventory systems,Balintfy and Silver [1,2] have considered a coordinated reordering policy which is represented by the triplet of vectors $(\mathbf{S},\mathbf{c},\mathbf{s})$, where the parameters S_i, c_i and s_i(the components of **S, c** and **s** respectively) are specified for each item i with $s_i \leq c_i \leq S_i$. In this policy, if the level of commodity i at any time is below s_i, an order is placed for $S_i - s_i$ items and at the same time, for any other item $j(\neq i)$ with available inventory at or below its can-order level c_j, an order is placed so as to bring its level back to its capacity S_j. Subsequently many articles have appeared with models involving the above policy. Another article of interest is due to Federgruen [3] et al., which deals with the general case of compound Poisson demands and non-zero lead times. A review of inventory models under joint replenishment is provided by Goyal and Satir[4] .

Kalpakam and Arivarignan[5] have introduced an $(\mathbf{s},\mathbf{S})$ policy with a single reorder level **s** defined in terms of the total number of items in the stock. This policy avoids separate ordering for each commodity and hence a single processing of orders for both commodities has some advantages in situation where the procurement is made from the same suppliers, items are produced on the same machine, or items have to be supplied by the same transport facility.

Krishnamoorthy [6] et al. have considered a two commodity continuous review inventory system without lead time. In their model, each demand is for one unit of the first commodity or one unit of second commodity or one unit of each commodity 1 and 2, with prefixed probabilities.Krishnamoorthy and Varghese[7] have considered a two commodity inventory problem without lead time and with Markov shift in demand for the type of commodity namely “commodity-1”, “commodity-2” or “both commodities”, using the direct Markov renewal theoretical results. For the

* Corresponding author e-mail: n.anbazhagan.alu@gmail.com

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same problem, Sivasamy and Pandiyan[8] have derived various results by applying filtering techniques.

A natural extension of $(\mathbf{s},\mathbf{S})$ policy to two-commodity inventory system is to have two reorder levels and to place orders for each commodity independent of the other. But this policy will increase the total cost as separate processing of two orders is required.

Anbazhagan and Arivarignan [9] have considered a two commodity inventory system with independent reorder levels where a joint order for both commodities is placed only when the levels of both commodities fall below their respective reorder levels. The demand points for each commodity form independent Poisson processes and the lead time is distributed as negative exponential. They have also assumed unit demands for both commodities. Yadavalli [10,11] et al. have analyzed two commodity inventory system under various ordering policies. Sivakumar[12] et al. have considered a two commodity coordinated and individual ordering policies with renewal demands. Anbazhagan and Vigneshwaran[13] have considered a two commodity markovian inventory system with joint reorder levels.

In this article we considered a two commodity continuous review inventory system with independent reorder levels $s_i - k, k = 0,1,\cdots,r$ where a joint order for both commodities is placed only when the levels of both commodities fall below their respective reorder levels. It is assumed that the demand for commodity i is of unit size and the time points of demand occurrences form a Poisson process with parameter λ_i $(i = 1,2)$. The two commodities are assumed to be substitutable. That is, if the inventory level of one commodity reaches zero, then any demand for this commodity will be satisfied by an item of the other commodity.The lead time is assumed to be distributed as negative exponential with parameter μ_k, $k = 0,1,\cdots,r$. The joint probability distribution of the two inventory levels is obtained in the steady state case. Various measures of systems performance in the steady state are also derived.

2 Problem formulation

Consider a two commodity inventory system with capacity S_i units for commodity i, $(i = 1,2)$. It is assumed that the demands for i-th commodity are of unit size and having Poisson distribution with parameter λ_i $(i = 1,2)$. The demand process of the two commodities are further assumed to be independent. The two commodities are assumed to be substitutable. That is, if the inventory level of one commodity reaches zero, then any demand for this commodity will be satisfied by the item of the other commodity. The reordering policy is to place order for both commodities when both inventory levels are less than or equal to their respective reorder levels $s_i - k, k = 0,1,\cdots,r$, with probability p_k, $\sum p_k = 1$, $(0 \leq r \leq \min\{s_1 - 2, s_2 - 2\}$ and $S_i - s_i + k > s_i + 1, i = 1,2)$. The ordering quantity is $Q^i_{s_i-k}(= S_i - s_i + k), i = 1,2$. The lead time initiated at level $s_i - k$ is assumed to be distributed as exponential with parameter $\mu_k(> 0)$. The demands that occur during stock out periods are lost.

Let $L_i(t)$ denote the net inventory level of commodity i at time t. Then the process

$L = \{(L_1(t), L_2(t)), t \geq 0\}$ has the state space

$E = \{0,1,\cdots,S_1\}$ x $\{0,1,\cdots,S_2\}$.

The space of inventory levels of commodity 1 and 2 is shown in Figure 1.

Notations

0 : zero matrix

$1'_N$: $(1,1,\cdots,1)_{1\times N}$

$\mathbf{e}^T$: $(1,1,\ldots,1)$.

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I_N : identity matrix of order N

δ_{ij} : Kronecker delta

$$\sum_{j=0}^{i} a^j = \begin{cases} a^0 + a^1 + \cdots + a^i, & \text{if } i \text{ is nonnegative integer} \\ 0, & \text{otherwise} \end{cases}$$

$[A]_{ij}$: $(i,j)-$th element of the matrix A

$$H(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

$$\overset{k}{\underset{i=j}{\Omega}} c_i = \begin{cases} c_j c_{j-1} \cdots c_k & \text{if } j \geq k \\ 1 & \text{if } j < k \end{cases}$$

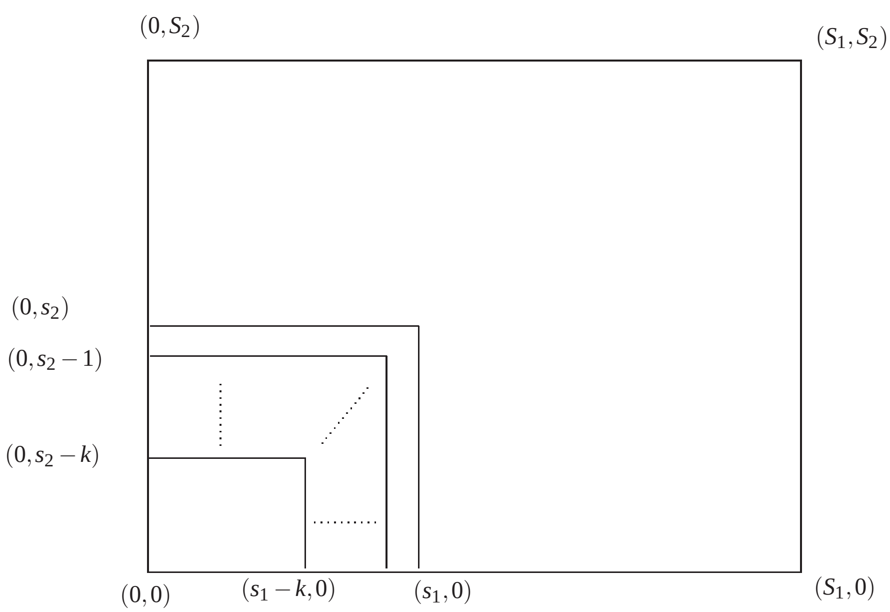

Fig. 1: Space of inventory levels of commodity 1 and 2

From the assumptions made on demand and the replenishment processes it follows that L is a Markov process. To determine the infinitesimal generator $\tilde{A} = ((\, a((i,j);(k,l))\,)), \quad (i,j),(k,l) \in E$, of this process.

Theorem 1: The infinitesimal generator of this Markov process $a((i,j),(k,l))$ is given by,

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$$
\begin{cases}
\lambda_1+\delta_{0j}\lambda_2, & k=i-1, \quad i=1,2,\cdots,S_1, & l=j, \quad j=0,1,\cdots,S_2 \\
\delta_{io}\lambda_1+\lambda_2, & k=i, \quad i=0,1,\cdots,S_1, & l=j-1, \quad j=1,2,\cdots,S_2 \\
-(\lambda_1+\lambda_2), & k=i, \quad i=s_1+1,\cdots,S_1, & l=j, \quad j=0,1,\cdots,S_2 \\
-(\lambda_1+\lambda_2), & k=i, \quad i=0,1,\cdots,s_1, & l=j, \quad j=s_2+1,\cdots,S_2 \\
-(\lambda_1+\lambda_2+ H(s_1-i-s_2+j-1)\sum_{n=0}^{s_2-j}\mu_n p_n+ H(s_2-j-s_1+i)\sum_{n=0}^{s_1-i}\mu_n p_n), & k=i, \quad i=s_1-r,\cdots,s_1 & l=j \quad j=0,1\ \cdots,s_2 \\
-((1-\delta_{0i}\delta_{0j})(\lambda_1+\lambda_2)+ H(s_1-i-s_2+j-1)\sum_{n=0}^{s_2-j}\mu_n p_n+ H(s_2-j-s_1+i)\sum_{n=0}^{r}\mu_n p_n), & k=i, \quad i=0,1,\cdots,s_1-r-1 & l=j, \quad j=0,1,\cdots,s_2 \\
\mu_{s_2-m}p_{s_2-m} & k=i+Q^1_{m-s_2+s_1}, \quad i=s_1,s_1-1,\cdots,s_1-r \text{ with } m=s_2-s_1+i,\cdots,s_2 & l=j+Q^2_m \quad j=m,m-1,\cdots,0 \\
\mu_{s_2-m}p_{s_2-m} & k=i+Q^1_{m-s_2+s_1}, \quad i=s_1-r-1,\cdots,0 \text{ with } m=s_2-r,\cdots,s_2 & l=j+Q^2_m \quad j=m,m-1,\cdots,0 \\
0, & \text{otherwise}
\end{cases}
$$

Proof:

The infinitesimal generator $a((i,j),(k,l))$ of this process can be obtained using the following arguments:

(i)Let $i>0$ and $j>0$. A demand takes the inventory level (i,j) to $(i-1,j)$ with intensity λ_1 the demand being for the first commodity or to $(i,j-1)$ with intensity λ_2 the demand being for the second commodity.
(ii)From the state (i,j), $(\leq (s_1-k,s_2-k))$ a replenishment takes the joint inventory level to $(i+Q^1_{s_1-k}, j+Q^2_{s_2-k})$ and the intensity of transition is given by μ_k, $k=0,1,\cdots,r$.
(iii)We observe that no transition other than the above is possible except $(i,j)\neq(k,l)$.
(iv)Finally the value of $a((i,j),(i,j))$ is obtained by

$$a((i,j),(i,j)) = -\sum_{\substack{k\\(k,l)\neq(i,j)}}\sum_{l} a((i,j),(k,l))$$

Hence we get the infinitesimal generator $a((i,j),(k,l))$. □

In order to write the infinitesimal generator $\tilde{A}$ in matrix form, we arrange the states in lexicographic order and group S_2+1 states as

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$$i = ((i,0),(i,1),\cdots,(i,S_2)), i = 0,1,\cdots,S_1.$$

Then the rate matrix $\tilde{A}$ has a block partitioned form with the following sub matrix $[\tilde{A}]_{ij}$ at the i-th row and j-th column position.

$$[\tilde{A}]_{ij} = \begin{cases} B & \text{if} \quad j = i-1, & i = S_1, S_1-1,\cdots,1 \\ A & \text{if} \quad j = i, & i = S_1, S_1-1,\cdots,s_1+1 \\ A_{s_1+1-i} & \text{if} \quad j = i, & i = s_1, s_1-1,\cdots,1,0 \\ M_{[j-i-Q^1_{s_1}]} & \text{if} \quad j = S_1, S_1-1,\cdots,S_1-(s_1-i), & i = s_1, s_1-1,\cdots,s_1-r \\ M_{[j-i-Q^1_{s_1}]} & \text{if} \quad j = S_1-(s_1-i)+r,\cdots,S_1-(s_1-i), & i = s_1-r-1,\cdots,1,0 \\ 0 & \text{otherwise.} \end{cases}$$

where

$$[B]_{ab} = \begin{cases} \lambda_1 & \text{if} \quad b = a,\ a = S_2, S_2-1,\cdots,1 \\ \lambda_1+\lambda_2 & \text{if} \quad b = a,\ a = 0 \\ 0 & \text{otherwise.} \end{cases}$$

$$[A]_{ab} = \begin{cases} \lambda_2 & \text{if} \quad b = a-1,\ a = S_2, S_2-1,\cdots,1 \\ -(\lambda_1+\lambda_2) & \text{if} \quad b = a, \qquad a = S_2, S_2-1,\cdots,0 \\ 0 & \text{otherwise.} \end{cases}$$

$$[M_i]_{ab} = \begin{cases} p_i\mu_i & \text{if} \quad b = Q_2+i+a \ a = s_2-i,\cdots,1,0, \\ 0 & \text{otherwise.} \end{cases}$$

with $i = 0,1,\cdots,r$

$$[A_i]_{ab} = \begin{cases} \lambda_2 & \text{if} \quad b = a-1,\ a = S_2, S_2-1,\cdots,1 \\ -(\lambda_1+\lambda_2) & \text{if} \quad b = a, \qquad a = S_2, S_2-1,\cdots,s_2+1 \\ -(\lambda_1+\lambda_2+ \\ H(i-s_2+b-2)\sum_{k=0}^{s_2-a} p_k\mu_k + \\ H(s_2-b-i+1)\sum_{k=0}^{i-1} p_k\mu_k) & \text{if} \quad b = a, \qquad a = s_2, s_2-1,\cdots,1,0 \\ 0 & \text{otherwise.} \end{cases}$$

with $i = 1,2,\cdots,r+1$

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$$[A_i]_{ab} = \begin{cases} \lambda_2 & \text{if} \quad b = a-1,\ a = S_2, S_2-1, \cdots, 1 \\ -(\lambda_1+\lambda_2) & \text{if} \quad b = a, \quad\quad a = S_2, S_2-1, \cdots, s_2+1 \\ -(\lambda_1+\lambda_2+ \\ H(i-s_2+b-2)\sum\limits_{k=0}^{s_2-a} p_k\mu_k + \\ H(s_2-b-i+1)\sum\limits_{k=0}^{r} p_k\mu_k) & \text{if} \quad b = a, \quad\quad a = s_2, s_2-1, \cdots, 1, 0 \\ 0 & \text{otherwise.} \end{cases}$$

with $i = r+2, \cdots, s_1$

$$[A_{s_1+1}]_{ab} = \begin{cases} \lambda_1+\lambda_2 & \text{if} \quad b = a-1,\ a = S_2, S_2-1, \cdots, 1 \\ -(\lambda_1+\lambda_2) & \text{if} \quad b = a, \quad\quad a = S_2, S_2-1, \cdots, s_2+1 \\ -(\lambda_1+\lambda_2+ \\ H(s_1-s_2+b-1)\sum\limits_{k=0}^{s_2-a} p_k\mu_k + \\ H(s_2-s_1-b)\sum\limits_{k=0}^{r} p_k\mu_k) & \text{if} \quad b = a, \quad\quad a = s_2, s_2-1, \cdots, 1 \\ \sum\limits_{k=0}^{r} p_k\mu_k & \text{if} \quad b = a, \quad\quad a = 0 \\ 0 & \text{otherwise.} \end{cases}$$

3 Steady state Results

It can be seen from the structure of A that the homogeneous Markov process $\{(L_1(t), L_2(t)), t \geq 0\}$ on the state space E is irreducible. Hence the limiting distribution

$$\Phi = (\phi^{(S_1)}, \phi^{(S_1-1)}, \ldots, \phi^{(0)})$$

with $\phi^{(q)} = (\phi^{(q,S_2)}, \phi^{(q,S_2-1)}, \cdots, \phi^{(q,0)})$, where $\phi^{(i,j)}$ denotes the steady state probability for the state (i,j) of the inventory level process, exists and is given by

$$\Phi\tilde{A} = 0 \quad \text{and} \sum_{(i,j)\in E}\sum \phi^{(i,j)} = 1 \tag{1}$$

Theorem 2: The steady state probability Φ is given by

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$$\phi^{(i)} = (-1)^i \phi^{(0)} \underset{m=s_1+1}{\overset{s_1+2-i}{\Omega}} A_m B^{-1}, \qquad i = 1,2,\cdots,s_1+1$$

$$\phi^{(i)} = (-1)^i \phi^{(0)} \left(\underset{m=s_1+1}{\overset{1}{\Omega}} A_m B^{-1} \right) (AB^{-1})^{i-s_1-1}, \qquad i = s_1+2,\cdots,Q^1_{s_1}$$

$$\phi^{(i)} = \phi^{(0)} \Bigg\{ (-1)^i \left(\underset{m=s_1+1}{\overset{1}{\Omega}} A_m B^{-1} \right) (AB^{-1})^{i-s_1-1} + \sum_{l=Q^1_{s_1}+1}^{i} \sum_{k=0}^{l-Q^1_{s_1}-1} (-1)^{i-l+1} [\delta_{k0} + (1-\delta_{k0})(-1)^k \underset{m=s_1+1}{\overset{s_1+2-k}{\Omega}} A_m B^{-1}] M_{[l-Q^1_{s_1}-1-k]} B^{-1} (AB^{-1})^{i-l} \Bigg\} \qquad i = Q^1_{s_1}+1,\cdots,Q^1_{s_1}+r+1$$

$$\phi^{(i)} = \phi^{(0)} \Bigg\{ (-1)^i \left(\underset{m=s_1+1}{\overset{1}{\Omega}} A_m B^{-1} \right) (AB^{-1})^{i-s_1-1} + \sum_{l=Q^1_{s_1}+1}^{Q^1_{s_1}+r+1} \sum_{k=0}^{l-Q^1_{s_1}-1} (-1)^{i-l+1} [\delta_{k0} + (1-\delta_{k0})(-1)^k \underset{m=s_1+1}{\overset{s_1+2-k}{\Omega}} A_m B^{-1}] M_{[l-Q^1_{s_1}-1-k]} B^{-1} (AB^{-1})^{i-l} + \sum_{l=Q^1_{s_1}+r+2}^{i} \sum_{k=l-Q^1_{s_1}-r-1}^{l-Q^1_{s_1}-1} (-1)^{i-l+k+1} [\underset{m=s_1+1}{\overset{s_1+2-k}{\Omega}} A_m B^{-1}] M_{[l-Q^1_{s_1}-1-k]} B^{-1} (AB^{-1})^{i-l} \Bigg\} \qquad i = Q^1_{s_1}+r+2,\cdots,S_1$$

The value of $\phi^{(0)}$ can be obtained from the relation $\sum\sum_{(i,j)\in E} \phi^{(i,j)} = 1$, as

$$\phi^{(0)} = \Bigg\{ I + \sum_{i=1}^{s_1+1} (-1)^i \left(\underset{m=s_1+1}{\overset{s_1+2-i}{\Omega}} A_m B^{-1} \right) + \sum_{i=s_1+2}^{Q^1_{s_1}} (-1)^i \left(\left(\underset{m=s_1+1}{\overset{1}{\Omega}} A_m B^{-1} \right) (AB^{-1})^{i-s_1-1} \right) +$$

$$\sum_{i=Q^1_{s_1}+1}^{Q^1_{s_1}+r+1} \Bigg((-1)^i \left(\underset{m=s_1+1}{\overset{1}{\Omega}} A_m B^{-1} \right) (AB^{-1})^{i-s_1-1} +$$

$$\sum_{l=Q^1_{s_1}+1}^{i} \sum_{k=0}^{l-Q^1_{s_1}-1} (-1)^{i-l+1} [\delta_{k0} + (1-\delta_{k0})(-1)^k \underset{m=s_1+1}{\overset{s_1+2-k}{\Omega}} A_m B^{-1}] M_{[l-Q^1_{s_1}-1-k]} B^{-1} (AB^{-1})^{i-l} \Bigg) +$$

$$\sum_{i=Q^1_{s_1}+r+2}^{S_1} \Bigg((-1)^i \left(\underset{m=s_1+1}{\overset{1}{\Omega}} A_m B^{-1} \right) (AB^{-1})^{i-s_1-1} +$$

$$\sum_{l=Q^1_{s_1}+1}^{Q^1_{s_1}+r+1} \sum_{k=0}^{l-Q^1_{s_1}-1} (-1)^{i-l+1} [\delta_{k0} + (1-\delta_{k0})(-1)^k \underset{m=s_1+1}{\overset{s_1+2-k}{\Omega}} A_m B^{-1}] M_{[l-Q^1_{s_1}-1-k]} B^{-1} (AB^{-1})^{i-l} +$$

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$$\sum_{l=Q^1_{s_1}+r+2}^{i}\sum_{k=l-Q^1_{s_1}-r-1}^{l-Q^1_{s_1}-1}(-1)^{i-l+k+1}[\underset{m=s_1+1}{\overset{s_1+2-k}{\Omega}}A_mB^{-1}]M_{[l-Q^1_{s_1}-1-k]}B^{-1}(AB^{-1})^{i-l}\Bigg)\Bigg\}^{-1}$$

Proof:

The first equation of (1) yields the following set of equations:

$$\phi^{(i)}B+\phi^{(i-1)}A_{s_1-i+2}=0, \qquad i=1,2,\cdots,s_1+1$$

$$\phi^{(i)}B+\phi^{(i-1)}A=0, \qquad i=s_1+2,s_1+3,\cdots,Q^1_{s_1}$$

$$\phi^{(i)}B+\phi^{(i-1)}A+\sum_{k=0}^{i-Q^1_{s_1}-1}\phi^{(k)}M_{[i-Q^1_{s_1}-1-k]}=0, \qquad i=Q^1_{s_1}+1,\cdots,Q^1_{s_1}+r+1$$

$$\phi^{(i)}B+\phi^{(i-1)}A+\sum_{k=i-Q^1_{s_1}-r-1}^{i-Q^1_{s_1}-1}\phi^{(k)}M_{[i-Q^1_{s_1}-1-k]}=0, \qquad i=Q^1_{s_1}+r+2,\cdots,S_1$$

$$\text{and} \qquad \phi^{(S_1)}A+\sum_{k=s_1-r}^{s_1}\phi^{(k)}M_{[s_1-k]}=0,$$

Solving the above set of equations we get the required result. □

4 System Performance Measures

In this section, some performance measures of the system are derived under consideration.

4.1 Mean Inventory Level

Let β_1 denote the average inventory level of the commodity 1 in the steady state. Then we have

$$\beta_1=\sum_{i=1}^{S_1}i\left(\sum_{j=0}^{S_2}\phi^{(i,j)}\right). \tag{2}$$

Let β_2 denote the average inventory level of the commodity 2 in the steady state. Then we have

$$\beta_2=\sum_{j=1}^{S_2}j\left(\sum_{i=0}^{S_1}\phi^{(i,j)}\right). \tag{3}$$

4.2 Mean Reorder Rate

Let β_3 denote the mean reorder rate then we have

$$\beta_3=\sum_{k=0}^{r}p_k\left(\sum_{i=0}^{s_1-k}(\delta_{i0}\lambda_1+\lambda_2)\phi^{(i,s_2-k+1)}+\sum_{i=0}^{s_2-k}(\lambda_1+\delta_{0j}\lambda_2)\phi^{(s_1-k+1,i)}\right) \tag{4}$$

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4.3 Mean Shortage level

Let β_4 denote the mean shortage level, then we have

$$\beta_4 = (\lambda_1 + \lambda_2)\phi^{(0,0)}. \tag{5}$$

5 Cost Analysis

We assume a specified cost structure for the proposed inventory system as follows:

k : ordering cost per order.
h_i : holding cost for the commodity i per unit item per unit time.
c : shortage cost per unit item.

Under the above cost structure, the expected total cost per unit time(expected total cost rate) in the steady state for this model is defined to be

$$TC(S_1,S_2,s_1,s_2,r) = h_1\beta_1 + h_2\beta_2 + k\beta_3 + c\beta_4.$$

Substituting the values for β_i's we can compute the value of $TC(S_1,S_2,s_1,s_2,r)$.

6 Numerical Illustration

As the expected total cost rate is obtained in a complex form, the convexity of the expected total cost rate cannot be studied analytically. Hence, numerical search procedures are used to find the local optimal values for (S_1,S_2) with fixed (s_1,s_2,r), s_1 with fixed (S_1,S_2,r) , s_2 with fixed (S_1,S_2,r) and r with fixed (S_1,S_2,s_1,s_2). With a large number of numerical examples it is found that the expected total cost rate in the long run is either a convex function of both S_1 and S_2 or any one of the variables r and (s_1,s_2).

Table 1 gives the expected total cost rate as a function of S_1 and S_2 by fixing constant values for the other variables and costs. After obtaining the local optima, S_1^* and S_2^*, the sensitivity analysis are carried out to see how the changes in S_1 and S_2 affect the expected total cost rate(see figure 2). For this the values of

$$\frac{TC(S_1,S_2,8,7,5)}{TC(S_1^*,S_2^*,8,7,5)}$$

by fixing the parameters and costs as $\lambda_1 = 1.5; \lambda_2 = 1.5; p_i = (0.6)(0.4)^i, i = 0,1,\cdots,r-1; p_r = 1 - \sum_{i=0}^{r-1} p_i; \mu_i = 5.2 + i(0.2), i = 0,1,\cdots,r; h_1 = 3.85; h_2 = 3.0; k = 1400; c = 13.2$, are computed.

S_1 \ S_2	21	22	23	24	25
31	1.004901	1.003075	1.002683	1.003793	1.006230
32	1.003084	1.001339	1.000775	1.001521	1.003599
33	1.002129	1.000584	**1.000000**	1.000514	1.002226
34	1.001902	1.000635	1.000150	1.000569	1.002011
35	1.002304	1.001361	1.001060	1.001499	1.002789

Table 1: Sensitivity of S_1 and S_2 on expected total cost rate

Here $S_1^* = 33$ and $S_2^* = 23$ and $TC(33,23,8,7,5) = 181.71435$. It appears that the expected total cost rate is more sensitive to the changes in S_2 than S_1.

Fixing all parameters and other cost values except s_1 and s_2, the expected total cost rates are computed as shown in tables 2 and 3 respectively. The four curves in figures 3 and 4 correspond to $(S_1,S_2) = (38,38)$, $(S_1,S_2) = (38,40)$, $(S_1,S_2) = (40,40)$ and $(S_1,S_2) = (40,38)$ represent the different convex functions of s_1 and s_2 respectively.

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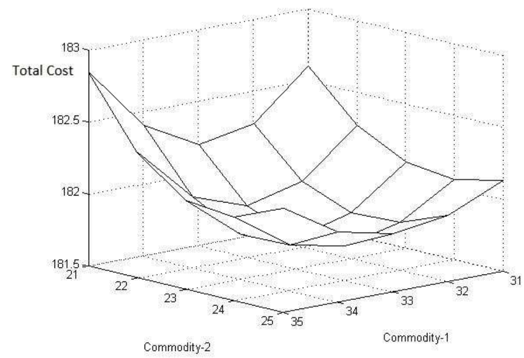

Fig. 2: Effect of S_1 and S_2 on total expected cost rate

(S_1,S_2) / s_1	(38,38)	(38,40)	(40,40)	(40,38)
10	97.994613	96.139090	96.431995	98.081569
11	97.994360	96.023943	96.289144	98.065952
12	98.000400	95.923640	96.151807	98.035658
13	98.025336	95.843428	96.028996	98.012370
14	98.075970	95.786353	95.925645	98.008380
15	98.155708	95.754412	95.844185	98.028728
16	98.267247	95.749442	95.786557	98.077060
17	98.412948	95.773273	95.754432	98.155900
18	98.595169	95.827835	95.749426	98.267220
19	98.816480	95.915211	95.773260	98.412886

Table 2: Effect of s_1 values on Expected total cost rate

(S_1,S_2) / s_2	(38,38)	(38,40)	(40,40)	(40,38)
7	102.231805	102.480491	101.029476	100.907859
8	102.085301	102.077340	100.840682	100.870128
9	102.109983	101.909026	100.778083	100.924047
10	102.253535	101.908423	100.804334	101.044312
11	102.480068	102.023224	100.894234	101.215964
12	102.768004	102.216954	101.032716	101.431622
13	103.106375	102.466501	101.212020	101.688819
14	103.492655	102.763268	101.430368	101.988307
15	103.927836	103.104920	101.688470	102.332150

Table 3: Effect of s_2 values on Expected total cost rate

In Table 4 the expected total cost rates by fixing constant values for all variables and costs except r are presented. The four curves in figure 5 correspond to $(S_1,S_2)=(45,47)$, $(S_1,S_2)=(45,45)$, $(S_1,S_2)=(47,45)$ and $(S_1,S_2)=(47,47)$ represent different convex functions of r.

Next the impact of the holding costs h_1 and h_2 on the optimal values (S_1^*,S_2^*) and the corresponding expected total cost rate are studied. For the parameters and the probability distribution $s_1=8; s_2=7 r=5; \lambda_1=1.5; \lambda_2=1.5; p_i=(0.6)(0.4)^i, i=0,1,\cdots,r-1; p_r=1-\sum_{i=0}^{r-1} p_i; \mu_i=5.2+i(0.2), i=0,1,\cdots,r; k=1400; c=13.2$, the total cost rate increases when h_1 and h_2 increase(see table 5). The impact of ordering cost per order and the shortage cost per unit item on the optimal values $(S_1^*,S2^*)$ and the corresponding expected total cost rate are studied by fixing the parameters and the probability distribution: $s_1=8; s_2=7; r=5; \lambda_1=1.5; \lambda_2=1.5; p_i=(0.6)(0.4)^i, i=0,1,\cdots,r-1; p_r=1-\sum_{i=0}^{r-1} p_i; \mu_i=5.2+i(0.2), i=0,1,\cdots,r; h_1=3.85; h_2=3$. Table 6 shows that the total cost rate increases when k and c increase.

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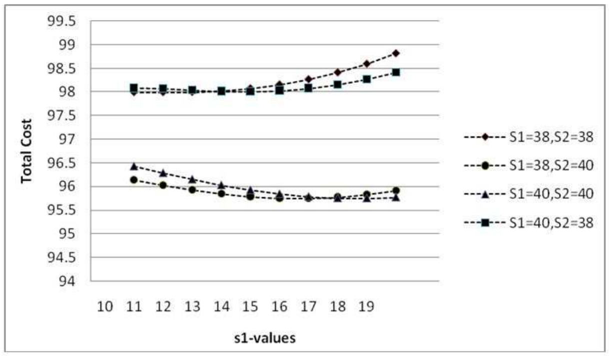
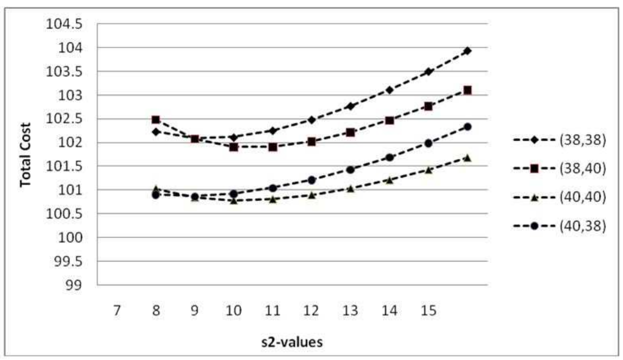


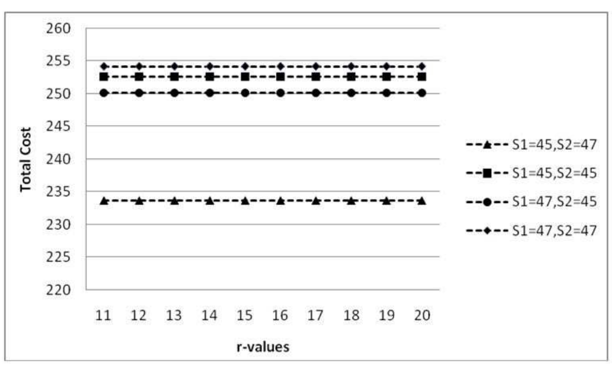
$r=5$; $s2=12$; $\lambda_1=1.9$; $\lambda_2=1$; $p_i=(0.6)(0.4)^i$, $i=0,1,\ldots r-1$; $h1=1.5$;
$h2=1$; $p_r=1-\sum_{i=0}^{r-1} p_i$; $\mu_i=5.2+i\,(0.2)$, $i=0,1,\ldots r$; $k=1800$; $c=13.2$

Fig. 3: Effect of s_1 on total expected cost rate



$r=5$; $s1=12$; $\lambda_1=1$; $\lambda_2=1.9$; $p_i=(0.6)(0.4)^i$, $i=0,1,\ldots r-1$; $h1=1$;
$h2=1.5$; $p_r=1-\sum_{i=0}^{r-1} p_i$; $\mu_i=5.2+i\,(0.2)$, $i=0,1,\ldots r$; $k=1800$; $c=13.2$

Fig. 4: Effect of s_2 on total expected cost rate



$s1=22$; $s2=22$; $\lambda_1=1.5$; $\lambda_2=1.5$; $p_i=(0.6)(0.4)^i$, $i=0,1,\ldots r-1$; $h1=3.85$;
$h2=3$; $p_r=1-\sum_{i=0}^{r-1} p_i$; $\mu_i=5.2+i\,(0.2)$, $i=0,1,\ldots r$; $k=1400$; $c=13.2$

Fig. 5: Effect of r on total expected cost rate

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(S_1, S_2) / r	(45,47)	(45,45)	(47, 45)	(47, 47)
11	233.665264632532280	252.592956907586910	250.136940405272130	254.204713535347680
12	233.665264624049230	252.592956903543780	250.136940396980090	254.204713531245800
13	233.665264622816750	252.592956902972840	250.136940395784110	254.204713530661730
14	233.665264622637580	252.592956902892440	250.136940395612160	254.204713530578770
15	233.665264622611800	252.592956902881010	250.136940395587940	254.204713530567120
16	233.665264622608160	252.592956902879620	250.136940395584360	254.204713530565530
17	233.665264622607680	252.592956902879450	250.136940395584080	254.204713530565240
18	233.665264622607540	252.592956902879370	250.136940395583990	254.204713530565160
19	233.665264622607590	252.592956902879420	250.136940395584020	254.204713530565160
20	233.665264622607590	252.592956902879420	250.136940395584020	254.204713530565270

Table 4: Effect of r on Expected total cost rate

h_2 / h_1	2.9		3.0		3.1		3.2		3.3	
3.75	178.7545		179.5042		180.2539		180.9377		181.5953	
	34	23	34	23	34	23	34	22	35	21
3.8	179.8534		180.6229		181.3726		182.0617		182.7397	
	33	23	34	23	34	23	34	22	34	22
3.85	180.9400		181.7144		182.4887		183.1857		183.8637	
	33	23	33	23	33	23	34	22	34	22
3.90	182.0266		182.8010		183.5753		184.3097		184.9877	
	33	23	33	23	33	23	34	22	34	22
3.95	183.1132		183.8876		184.6620		185.4032		186.1021	
	33	24	33	23	33	23	33	22	33	22

Table 5: Effect of holding costs h_1 and h_2 on optimal values

c / k	13.0		13.1		13.2		13.3		13.4	
1300	176.370606		176.370608		176.370610		176.370612		176.370613	
	33	23	33	23	33	23	33	23	33	23
1350	179.042477		179.042479		179.042481		179.042483		179.042485	
	33	23	33	23	33	23	33	23	33	23
1400	181.714348		181.714350		181.714352		181.714354		181.714356	
	33	23	33	23	33	23	33	23	33	23
1450	184.386219		184.386221		184.386224		184.386226		184.386228	
	33	23	33	23	33	23	33	23	33	23
1500	187.058091		187.058093		187.058095		187.058097		187.058099	
	33	23	33	23	33	23	33	23	33	23

Table 6: Effect of ordering cost and shortage cost on optimal values

7 Conclusion

In this paper, a substitutable inventory system of two commodities with reorder levels of band width r has been studied. The joint probability distribution of the inventory levels in the steady state and the stationary measures of system performances have been derived. An example has also been provided to prove the existence of local optima when the total cost function is treated as a function of two variables S_1 and S_2 or a single variable s_1 or s_2 or r. Future work will consider the demand process as renewal type.

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N. Anbazhagan is a Professor of Mathematics, Alagappa University, Karaikudi, India. He received his M.Phil and PhD in Mathematics from Madurai Kamaraj University, Madurai, India and M.Sc (Mathematics) from Cardamom Planters Association College, Bodinayakanur, India. He has received Young Scientist Award (2004) from DST, New Delhi, India; Young Scientist Fellowship (2005) from TNSCST, Chennai, India and Career Award for Young Teachers (2005) from AICTE, India. He has successfully completed one research project, funded by DST, India. His research interests include stochastic modeling, optimization techniques and inventory and queuing systems. He has published the research articles in several journals, including stochastic analysis and applications, APJOR and ORiON.

Mark Goh is a member of the National University of Singapore. A faculty of the Business School, he holds the appointments of Director (Industry Research) at TLI - Asia Pacific, a joint venture with Georgia Tech, USA; and a Principal Researcher at the Centre for Transportation Research. He was a Program Director of the Penn-State NUS Logistics Management Program. He was also Director of Supply Chain Solutions for Asia/Middle East with APL Logistics, responsible for crafting logistics engineering solutions for major MNCs in this part of the world. Other past appointments held by Dr. Goh include: Board Member of the Chartered Institute of Transport (Singapore), Chairman of the Academic Board of Examiners for the Singapore Institute of Purchasing and Materials Management, member of the Advisory Committee of the Transportation Resource Centre (NUS) and Vice President of the Operations Research Society of Singapore, Associate Senior Fellow of the Institute of Southeast Asian Studies. His other professional affiliations include membership of INFORMS, and the Academy of International Business. His biography appears in the Who's Who in Asia and the Pacific Nations, the Whos Who in the World, and in the

Outstanding People of the 20th Century. He has been involved in executive training and a consultant for organisations both in Singapore and overseas, such as PSA Corp, Siemens Nixdorf, CAAS, Fuji-Xerox AP, Hewlett-Packard Far East, DHL, and Cleanaway (China). He is currently on the editorial boards of the Journal of Supply Chain Management, Q3 Quarterly, Journal for Inventory Research, International Journal of Supply Chain Management and Advances in Management Research. His current research interests focus on supply chain strategy, performance measurement, buyer-seller relationships and reverse logistics. He has more than 130 technical papers published in internationally refereed journals and conferences.

Vigneshwaran is an Assistant Professor of Mathematics, Thiagarajar College of Engineering, Madurai, Tamil Nadu, India. He has MSc (Applied Mathematics), MPhil(Mathematics) degrees with eleven years of teaching experience. He obtained his PhD in Mathematics from the Alagappa University, Karaikudi, India. His current research interests include applied probability, inventory and queuing systems.

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