Quantum Physics Letters
*An International Journal*

http://dx.doi.org/10.12785/qpl/030101

# Topological and Non-Topological Soliton Solutions of the Coupled Klein-Gordon-Schrodinger and the Coupled Quadratic Nonlinear Equations

*Mozhgan Akbari*[*]

Department of Mathematics, Faculty of Mathematical Sciences, University of Guilan, Rasht P.O. Box 1914, Rasht, Iran

Received: 3 Sep. 2013, Revised: 6 Jan. 2014, Accepted: 14 Jan. 2014
Published online: 1 Apr. 2014

**Abstract:** In this paper, two solitary wave solutions are obtained for the Coupled Klein-Gordon-Schrodinger and Coupled quadratic nonlinear equations by the ansatz method. Both topological and non-topological solitary wave solutions are obtained.

**Keywords:** Coupled Klein-Gordon-Schrodinger equation, Coupled quadratic equation, Topological and non-topological solitary wave solutions

## 1 Introduction

Nonlinear partial differential equations (NPDEs) are widely used to describe complex phenomena in various fields of science, especially in physics. Therefore solving nonlinear problems play an important role in nonlinear sciences. Many effective methods of obtaining explicit solutions of NPDEs have been presented such as the tanh-function method and its various extension [1,2], the Jacobi elliptic function expansion method [3], the homogeneous balance method [4], the F-expansion method and its extension [5], $(\frac{G'}{G})$-expansion method [6], the modified simple equation method [7,8], the semi-inverse variational principle [9] the solitary wave ansatz method [10,11,12,13,14,15,16,17,18,19,20,21] and so on. It is very interesting to note that the solitary wave ansatz method has been successfully applied to many kinds of NLPDEs with constant and varying coefficients, such as, for example, the $K(m,n)$ equation [14,21], the BBM equation [20], the $B(m,n)$ equation [17], the nonlinear Schrodinger's equation [14,18] and many others. This new method has been proved by many to be reliable, effective and powerful. The aim of this paper is to extend the solitary wave ansatz method of finding new soliton solutions for the Coupled Klein-Gordon-Schrodinger equation

$$u_{tt} - c^2 u_{xx} + u + |v|^2 = 0, \quad (1)$$

$$iv_t + v_{xx} + uv = 0. \quad (2)$$

and the Coupled quadratic nonlinear equation

$$iu_t + u_{xx} - u + u^* v = 0, \quad (3)$$

$$2iv_t + v_{xx} - \alpha v + \frac{1}{2}u^2 = 0. \quad (4)$$

## 2 Non-Topological solitary wave

In this section, we will calculate the non-topological solitary wave solution of the Coupled Klein-Gordon-Schrodinger equation and the Coupled quadratic nonlinear equation, using the solitary wave ansatz.

### 2.1 Non-Topological soliton solution of the Coupled Klein-Gordon-Schrodinger equation

For solving the Coupled Klein-Gordon-Schrodinger equations (1) and (2) we use a solitary wave ansatz of the form

$$u(x,t) = A_1 sech^{p_1} \tau, \quad (5)$$

$$v(x,t) = A_2 sech^{p_2} \tau e^{i\eta}, \quad (6)$$

* Corresponding author e-mail: m-akbari@guilan.ac.ir

where

$$\tau = B(x - qt). \tag{7}$$

and

$p_1 > 0, p_2 > 0$

for solitary waves to exist. Here, in (5) and (6) $A_1, A_2$ and $B$ are free parameters, while $q$ is the velocity of the soliton. The unknown exponents $p_1$ and $p_2$ will be determined, The phase component of (6) is given by

$$\eta = -\kappa x + \omega t + \theta, \tag{8}$$

Where $\kappa$ represents the soliton frequency, $\omega$ is the soliton wave number, and $\theta$ is the phase constant. Thus from (5) and (6) we have:

$$\begin{aligned} u_{tt} &= p_1^2 q^2 A_1 B^2 sech^{p_1}\tau \\ &\quad - p_1(p_1+1) q^2 A_1 B^2 sech^{(p_1+2)}\tau, \end{aligned} \tag{9}$$

$$\begin{aligned} u_{xx} &= p_1^2 A_1 B^2 sech^{p_1}\tau \\ &\quad - p_1(p_1+1) A_1 B^2 sech^{(p_1+2)}\tau \end{aligned} \tag{10}$$

$$|v|^2 = A_2^2 sech^{2p_2}\tau, \tag{11}$$

$$v_t = \{p_2 q A_2 B sech^{p_2}\tau tanh\tau + i\omega A_2 sech^{p_2}\tau\} e^{i\eta}, \tag{12}$$

$$\begin{aligned} v_{xx} &= \{p_2^2 A_2 B^2 sech^{p_2}\tau \\ &\quad - p_2(p_2+1) A_2 B^2 sech^{(p_2+2)} + 2i\kappa p_2 A_2 B\tau \\ &\quad sech^{p_2}\tau tanh\tau - A_2\kappa^2 sech^{p_2}\tau\} e^{i\eta}, \end{aligned} \tag{13}$$

$$uv = A_1 A_2 sech^{(p_1+p_2)}\tau, \tag{14}$$

Now substituting (9)-(14) into (1) and (2) gives

$$\begin{aligned} &(p_1^2 A_1 B^2 (q^2 - c^2) + A_1) sech^{p_1}\tau \\ &- p_1(p_1+1) A_1 B^2 (q^2 - c^2) sech^{(p_1+2)}\tau \\ &+ A_2^2 sech^{2p_2}\tau = 0, \end{aligned} \tag{15}$$

$$\begin{aligned} &i p_2 A_2 B (q + 2\kappa) sech^{p_2}\tau tanh\tau \\ &+ (-\omega A_2 + p_2^2 A_2 B^2 - A_2\kappa^2) sech^{p_2}\tau \\ &- p_2(p_2+1) A_2 B^2 sech^{(p_2+2)}\tau \\ &+ A_1 A_2 sech^{(p_1+p_2)}\tau = 0. \end{aligned} \tag{16}$$

By setting the imaginary part to zero in (16) we get

$$\kappa = -\frac{q}{2} \tag{17}$$

By balancing the power of $sech^{(p_2+2)}\tau$ and $sech^{(p_1+p_2)}\tau$ in (16) we have:

$$p_1 = 2.$$

By balancing the power of $sech^{(p_1+2)}\tau$ and $sech^{2p_2}\tau$ in (15) we get:

$$p_2 = 2.$$

Now, from (15) and (16), setting the coefficients of the linearly independent functions $sech^{(p_i+j)}\tau$ to zero, where $i = 1,2$ and $j = 0,2$, gives

$$\begin{aligned} &p_1^2 A_1 B^2 (q^2 - c^2) + A_1 = 0, \\ &- p_1(p_1+1) A_1 B^2 (q^2 - c^2) + A_2^2 = 0, \\ &-\omega A_2 + p_2^2 A_2 B^2 - A_2\kappa^2 = 0, \\ &- p_2(p_2+1) A_2 B^2 + A_1 A_2 = 0. \end{aligned}$$

Solving the above equations yields

$$B = \sqrt{\frac{-1}{4(q^2 - c^2)}}, A_1 = -\frac{3}{2}\frac{1}{q^2 - c^2},$$

$$A_2 = \frac{3}{2}\sqrt{\frac{-1}{q^2 - c^2}}, \omega = \frac{-1}{q^2 - c^2} - \frac{q^2}{4},$$

which shows that the solitary waves will exist for

$$\frac{1}{q^2 - c^2} < 0.$$

Thus, the solitary wave solution of the Coupled Klein-Gordon-Schrodinger equation is given by

$$\begin{aligned} u(x,t) &= -\frac{3}{2}\frac{1}{q^2 - c^2} sech^2(x - qt), \\ v(x,t) &= \frac{3}{2}\sqrt{\frac{-1}{q^2 - c^2}} sech^2(x - qt) \\ &\quad \times e^{i(\frac{q}{2}x + (\frac{-1}{q^2 - c^2} - \frac{q^2}{4})t + \theta)}. \end{aligned}$$

### 2.2 Non-Topological soliton solution of the Coupled quadratic nonlinear equation

For solving the Coupled quadratic nonlinear equations (3) and (4) we use a solitary wave ansatz of the form

$$u(x,t) = A_1 sech^{p_1}\tau e^{i\eta}, \tag{18}$$

$$v(x,t) = A_2 sech^{p_2}\tau e^{2i\eta}, \tag{19}$$

where

$$\tau = B(x - qt). \tag{20}$$

and

$p_1 > 0, \quad p_2 > 0$

for solitary waves to exist. Here, in (18) and (19) $A_1, A_2$ and $B$ are free parameters, while $q$ is the velocity of the soliton. The unknown exponents $p_1$ and $p_2$ will be determined, The phase component of (18) and (19) is given by

$$\eta = -\kappa x + \omega t + \theta, \tag{21}$$

Where $\kappa$ represents the soliton frequency, $\omega$ is the soliton wave number, and $\theta$ is the phase constant. Thus from (18) and (19) we have:

$$u_t = \{p_1 q A_1 B sech^{p_1}\tau tanh\tau + i\omega A_1 sech^{p_1}\tau\}e^{i\eta}, \quad (22)$$

$$u_{xx} = \{p_1^2 A_1 B^2 sech^{p_1}\tau - p_1(p_1+1)A_1B^2 sech^{(p_1+2)}\tau + 2i\kappa p_1 A_1 B sech^{p_1}\tau tanh\tau - A_1\kappa^2 sech^{p_1}\tau\}e^{i\eta}, \quad (23)$$

$$u^* v = A_1 A_2 sech^{p_1+p_2}\tau e^{i\eta}, \quad (24)$$

$$v_t = \{p_2 q A_2 B sech^{p_2}\tau tanh\tau + 2i\omega A_2 sech^{p_2}\tau\}e^{2i\eta}, \quad (25)$$

$$v_{xx} = \{p_2^2 A_2 B^2 sech^{p_2}\tau - p_2(p_2+1)A_2B^2 sech^{(p_2+2)}\tau + 4i\kappa p_2 A_2 B sech^{p_2}\tau tanh\tau - 4A_2\kappa^2 sech^{p_2}\tau\}e^{2i\eta}, \quad (26)$$

$$u^2 = A_1^2 sech^{2p_1}\tau e^{2i\eta}, \quad (27)$$

Now substituting (22)-(27) into (3) and (4) gives

$$ip_1A_1B(q+2\kappa) + sech^{p_1}\tau tanh\tau + (-\omega A_1 + p_1^2 a_1 B^2 - A_1)sech^{p_1}\tau - p_1(p_1+1)A_1B^2 sech^{(p_1+2)}\tau + A_1A_2 sech^{(p_1+p_2)}\tau = 0, \quad (28)$$

$$2ip_2A_2B(q+2\kappa)sech^{p_2}\tau tanh\tau + (-4\omega A_2 + p_2^2A_2B^2 - 4A_2\kappa^2 - A_2\alpha)sech^{p_2}\tau - p_2(p_2+1) A_2B^2 sech^{(p_2+2)}\tau + \frac{1}{2}A_1^2 sech^{2p_1}\tau = 0. \quad (29)$$

By setting the imaginary part to zero in (28) and (29) we get

$$\kappa = -\frac{q}{2} \quad (30)$$

By balancing the power of $sech^{(p_1+2)}\tau$ and $sech^{(p_1+p_2)}\tau$ in (28) we have:

$$p_2 = 2.$$

By balancing the power of $sech^{(p_2+2)}\tau$ and $sech^{2p_1}\tau$ in (29) we get:

$$p_1 = 2.$$

Now, from (28) and (29), setting the coefficients of the linearly independent functions $sech^{(p_i+j)}\tau$ to zero, where $i = 1,2$ and $j = 0,2$, gives

$$-\omega A_1 + p_1^2 A_1 B^2 - A_1\kappa^2 - A_1 = 0,$$
$$-p_1(p_1+1)A_1B^2 + A_1A_2 = 0,$$
$$-4\omega A_2 + p_2^2 A_2 B^2 - 4A_2\kappa^2 - A_2\alpha = 0,$$
$$-p_2(p_2+1)A_2B^2 + \frac{1}{2}A_1^2 = 0.$$

Solving the above equations yields

$$B = \sqrt{\frac{\omega+\kappa^2+1}{4}}, \; A_1 = \frac{3}{\sqrt{2}}(\omega+\kappa^2+1),$$
$$A_2 = \frac{3}{2}(\omega+\kappa^2+1), \; \omega = -\frac{1}{3}(\frac{3}{4}q^2+\alpha-1)$$

which shows that the solitary waves will exist for

$$\omega + \kappa^2 + 1 > 0.$$

Thus, the solitary wave solution of the Coupled quadratic nonlinear equation is given by

$$u(x,t) = \frac{3}{\sqrt{2}}(\omega+\kappa^2+1)sech^2\tau e^{i(\frac{q}{2}x - \frac{1}{3}(\frac{3}{4}q^2+\alpha-1)t+\theta)},$$
$$v(x,t) = \frac{3}{2}(\omega+\kappa^2+1)sech^2\tau e^{2i(\frac{q}{2}x - \frac{1}{3}(\frac{3}{4}q^2+\alpha-1)t+\theta)}.$$

## 3 Topological solitary wave

In this section, we will calculate the topological solitary wave solution of the Coupled Klein-Gordon- Schrodinger equation and the Coupled quadratic nonlinear equation, using the solitary wave ansatz.

### *3.1 Topological soliton solution of the Coupled Klein-Gordon-Schrodinger equation*

To start off, the hypothesis is taken to be

$$u(x,t) = A_1 tanh^{p_1}\tau, \quad (31)$$
$$v(x,t) = A_2 tanh^{p_2}\tau e^{i\eta}, \quad (32)$$

where

$$\tau = B(x-qt). \quad (33)$$

and

$$p_1 > 0, \;\; p_2 > 0$$

for solitary waves to exist. Here, in (31) and (32) $A_1, A_2$ and $B$ are free parameters, while $q$ is the velocity of the soliton. The unknown exponents $p_1$ and $p_2$ will be determined, The phase component of (32) is given

$$\eta = -\kappa x + \omega t + \theta, \quad (34)$$

Where $\kappa$ represents the soliton frequency, $\omega$ is the soliton wave number, and $\theta$ is the phase constant. Thus from (31) and (32) we have:

$$u_{tt} = A_1p_1B^2(p_1-1)q^2 tanh^{p_1-2}\tau - 2p_1^2q^2A_1B^2 tanh^{p_1}\tau + A_1p_1B^2(p_1+1)q^2tanh^{p_1+2}\tau,$$
$$u_{xx} = -A_1p_1B^2(p_1-1)tanh^{p_1-2}\tau + 2p_1^2A_1B^2tanh^{p_1}\tau - A_1p_1(p_1+1)B^2tanh^(p_1+2)\tau,$$
$$|v|^2 = A_2^2 tanh^{2p_2}\tau,$$
$$v_t = \{-p_2qA_2Btanh^{p_2-1}\tau + p_2qA_2Btanh^{p_2+1}\tau + i\omega A_2 tanh^{p_2}\tau\}e^{i\eta},$$
$$v_{xx} = \{p_2A_2B^2(p_2-1)tanh^{p_2-2}\tau + (-2A_2p_2^2B^2 - A_2\kappa^2)tanh^{p_2}\tau + p_2(p_2+1)A_2B^2tanh^{(p_2+2)}\tau - 2i\kappa p_2A_2Btanh^{p_2-1}\tau + 2ip_2A_2Btanh^{p_2+1}\tau\}e^{i\eta},$$
$$uv = A_1A_2 tanh^{(p_1+p_2)}\tau,$$

Now substituting the above expressions into (1) and (2) gives

$$A_1p_1B^2(p_1-1)(q^2+c^2)tanh^{p_1-2}\tau+(-2p_1^2A_1B^2(q^2+c^2)+A_1)tanh^{p_1}\tau+A_1p_1B^2(p_1+1)(q^2+c^2)tanh^{p_1+2}\tau+A_2^2tanh^{2p_2}\tau=0, \quad (35)$$

$$-ip_2A_2B(q+2\kappa)tanh^{(p_2-1)}\tau+ip_2A_2B(q+2\kappa)tanh^{(p_2+1)}\tau+(-\omega A_2-2A_2p_2^2B^2-A_2\kappa^2)tanh^{p_2}\tau+A_2p_2B^2(p_2-1)tanh^{(p_2-2)}\tau+p_2(p_2+1)A_2B^2tanh^{(p_2+2)}\tau+A_1A_2tanh^{(p_1+p_2)}\tau=0. \quad (36)$$

By setting the imaginary part to zero (36) we get

$$\kappa=-\frac{q}{2} \quad (37)$$

By balancing the power of $tanh^{(p_1+2)}\tau$ and $tanh^{(p_1+p_2)}\tau$ in (36) we have:

$$p_1=2.$$

By balancing the power of $tanh^{(p_1+2)}\tau$ and $tanh^{2p_2}\tau$ in (35) we have:

$$p_2=2.$$

Now, from (35) and (36), setting the coefficients of the linearly independent functions $tanh^{(p_i+j)}\tau$ to zero, where $i=1,2$ and $j=0,\pm1$, gives

$$-2p_1^2A_1B^2(q^2+c^2)+A_1=0,$$
$$A_1p_1B^2(p_1+1)(q^2+c^2)+A_2^2=0,$$
$$-\omega A_2-2A_2p_2^2B^2-A_2\kappa^2=0,$$
$$p_2(p_2+1)A_2B^2+A_1A_2=0.$$

Solving the above equations yields

$$A_1=\frac{-3}{4(q^2+c^2)},\quad A_2=\frac{3}{4}\sqrt{\frac{1}{(q^2+c^2)}}$$

$$B=\sqrt{\frac{1}{8(q^2+c^2)}},\quad \omega=-\frac{1}{(q^2+c^2)}+\frac{q^2}{4}.$$

Hence, finally the topological solitary wave solution to the Coupled Klein-Gordon-Schrodinger equation is given by

$$u(x,t)=\frac{-3}{4(q^2+c^2)}tanh^2[\sqrt{\frac{1}{8(q^2+c^2)}}(x-qt)],$$

$$v(x,t)=\frac{3}{4}\sqrt{\frac{1}{(q^2+c^2)}}tanh^2[\sqrt{\frac{1}{8(q^2+c^2)}}(x-qt)]\times e^{i(-\frac{q}{2}x+(-\frac{1}{(q^2+c^2)}+\frac{q^2}{4})t+\theta)}.$$

### 3.2 Topological soliton solution of the Coupled quadratic nonlinear equation

To start off, the hypothesis is taken to be

$$u(x,t)=A_1tanh^{p_1}\tau e^{i\eta}, \quad (38)$$

$$v(x,t)=A_2tanh^{p_2}\tau e^{2i\eta}, \quad (39)$$

where

$$\tau=B(x-qt). \quad (40)$$

and

$p_1>0,\ \ p_2>0$

for solitary waves to exist. Here, in (38) and (39) $A_1,A_2$ and $B$ are free parameters, while $q$ is the velocity of the soliton. The unknown exponents $p_1$ and $p_2$ will be determined, The phase component of (38) and (39) is given by

$$\eta=-\kappa x+\omega t+\theta, \quad (41)$$

Where $\kappa$ represents the soliton frequency, $\omega$ is the soliton wave number, and $\theta$ is the phase constant. Thus from (38) and (39) we have:

$$u_t=\{-A_1p_1Bqtanh^{p_1-1}\tau+p_1qA_1Btanh^{p_1+1}\tau+i\omega A_1tanh^{p_1}\tau\}e^{i\eta},$$
$$u_{xx}=\{A_1p_1B^2(p_1-1)tanh^{p_1-2}\tau+(-2p_1^2A_1B^2-A_1\kappa^2)tanh^{p_1}\tau+2iA_1\kappa p_1tanh^{p_1+1}\tau\}e^{i\eta},$$
$$u^*v=A_1A_2tanh^{(p_1+p_2)}\tau e^{i\eta},$$
$$v_t=\{-p_2qA_2Btanh^{p_2-1}\tau+p_2qA_2Btanh^{p_2+1}\tau+2i\omega A_2tanh^{p_2}\tau\}e^{2i\eta},$$
$$v_{xx}=\{p_2A_2B^2(p_2-1)tanh^{p_2-2}\tau+(-2A_2p_2^2B^2-4A_2\kappa^2)tanh^{p_2}\tau+p_2(p_2+1)A_2B^2tanh^{(p_2+2)}\tau-4i\kappa p_2A_2Btanh^{p_2-1}\tau+4i\kappa p_2A_2B\,tanh^{p_2+1}\tau\}e^{2i\eta},$$
$$u^2=A_1^2tanh^{2p_1}\tau e^{2i\eta},$$

Now substituting the above expressions into (3) and (4) gives

$$-iA_1p_1B(q+2\kappa)tanh^{p_1-1}\tau+ip_1A_1B(q+2\kappa)tanh^{p_1+1}\tau+(-\omega A_1-2p_1^2A_1B^2-A_1\kappa^2-A_1)tanh^{p_1}\tau+A_1p_1B^2(p_1-1)tanh^{p_1-2}\tau+A_1p_1B^2(p_1+1)tanh^{p_1+2}+A_1A_2tanh^{p_1+p_2}\tau=0, \quad (42)$$

$$-2ip_2A_2B(q+2\kappa)tanh^{(p_2-1)}\tau+2ip_2A_2B(q+2\kappa)tanh^{(p_2+1)}\tau+(-2\omega A_2-2A_2p_2^2B^2-4A_2\kappa^2-\alpha A_2)tanh^{p_2}\tau+A_2p_2B^2(p_2-1)tanh^{p_2-2}\tau+p_2(p_2+1)A_2B^2tanh^{(p_2+2)}\tau+\frac{1}{2}A_1^2tanh^{2p_1}\tau=0. \quad (43)$$

By setting the imaginary part to zero in (42) and (43) we get

$$\kappa=-\frac{q}{2} \quad (44)$$

By balancing the power of $tanh^{(p_1+2)}\tau$ and $tanh^{(p_1+p_2)}\tau$ in (42) we have:

$$p_1=2.$$

By balancing the power of $tanh^{(p_1+2)}\tau$ and $tanh^{2p_2}\tau$ in (43) we have:

$$p_2=2.$$

Now, from (42) and (43), setting the coefficients of the linearly independent functions $tanh^{(p_i+j)}\tau$ to zero, where $i = 1,2$ and $j = 0,\pm 1$, gives

$$-\omega A_1 - 2p_1^2 A_1 B^2 - A_1\kappa^2 - A_1 = 0,$$
$$A_1 p_1 B^2 (p_1+1) + A_1 A_2 = 0,$$
$$-2\omega A_2 - 2A_2 p_2^2 B^2 - 4A_2\kappa^2 - \alpha A_2 = 0,$$
$$p_2(p_2+1)A_2 B^2 + \frac{1}{2}A_1^2 = 0.$$

Solving the above equations yields

$$A_1 = -\frac{3}{4}(\kappa^2+\omega+1),\ A_2 = \frac{3}{4}\sqrt{-2(\kappa^2+\omega+1)},$$
$$B = \sqrt{\frac{-1}{8(\kappa^2+\omega+1)}},\ \omega = (3\kappa^2+\alpha-1)$$

which shows that the solitary waves will exist for

$$\kappa^2+\omega+1 < 0$$

Thus, the solitary wave solution of the Coupled quadratic nonlinear equation is given by

$$u(x,t) = -\frac{3}{4}(\kappa^2+\omega+1)tanh^2[\sqrt{\frac{-1}{8(\kappa^2+\omega+1)}}(x-qt)]e^{i(\frac{q}{2}x+(\frac{3}{4}q^2+\alpha-1)t+\theta)},$$
$$v(x,t) = \frac{3}{4}\sqrt{-2(\kappa^2+\omega+1)}tanh^2[\sqrt{\frac{-1}{8(\kappa^2+\omega+1)}}(x-qt)]e^{2i(\frac{q}{2}x+(\frac{3}{4}q^2+\alpha-1)t+\theta)},$$

## 4 Conclusion

In this paper, we have used the solitary wave ansatz method to obtain the topological and non-topological soliton solution of the Coupled Klein-Gordon-Schrodinger and Coupled quadratic nonlinear equations. It should be noted that solitary wave ansatz method is a powerful efficient method to obtain exact topological and non-topological soliton solutions for nonlinear partial differential equations.

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