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Some New Identities on the (h,q)-Genocchi Numbers and Polynomials with Weight α

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Abstract: In this paper, we deal with (h,q)-Genocchi numbers and polynomials with weight α. We also derive some new properties. Also, we introduce not only new but also interesting properties of (h,q)-Genocchi numbers with weight α by using the fermionic p-adic q-integral on $\mathbb{Z}_p$ and the weighted q-Bernstein polynomials.

Keywords: (h,q)-Genochhi numbers and polynomials with weight α, weighted Bernstein polynomials, fermionic p-adic q-integral on $\mathbb{Z}_p$.
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1 Introduction and Notations

Let p be a fixed odd prime number. Throughout this paper we use the following notations. By $\mathbb{Z}_p$ we denote the ring of p-adic rational integers, $\mathbb{Q}$ denotes the field of rational numbers, $\mathbb{Q}_p$ denotes the field of p-adic rational numbers, and $\mathbb{C}_p$ denotes the completion of algebraic closure of $\mathbb{Q}_p$. Let $\mathbb{N}$ be the set of natural numbers and $\mathbb{N}^* = \mathbb{N}\cup\{0\}$. The normalized p-adic absolute value is defined by

$$|p|_p = \frac{1}{p}.$$

In this paper, we will assume that $|q-1|_p < 1$ as an indeterminate. Let $UD(\mathbb{Z}_p)$ be the space of uniformly differentiable functions on $\mathbb{Z}_p$. For $f \in UD(\mathbb{Z}_p)$, the fermionic p-adic q-integral on $\mathbb{Z}_p$ is defined by T. Kim:

$$I_{-q}(f) = \int_{\mathbb{Z}_p} f(\xi)\, d\mu_{-q}(\xi) = \lim_{n\to\infty} \frac{1}{[p^n]_{-q}} \sum_{\xi=0}^{p^n-1} q^{\xi} f(\xi)(-1)^{\xi} \tag{1}$$

(for more information, see [28], [29] and [30]).

From (1), we easily see that

$$qI_{-q}(f_1) + I_{-q}(f) = [2]_q f(0) \tag{2}$$

where $f_1(x) := f(x+1)$ (for details, see[2-40]).

Let $C([0,1])$ be the space of continuous functions on $[0,1]$. For $C([0,1])$, the weighted q-Bernstein operator for f is defined by

$$\mathscr{B}_{n,q}^{(\alpha)}(f,x) = \sum_{k=0}^{n} f\left(\frac{k}{n}\right) B_{k,n}^{(\alpha)}(x \mid q) = \sum_{k=0}^{n} f\left(\frac{k}{n}\right) \binom{n}{k} [x]_{q^\alpha}^{k} [1-x]_{q^{-\alpha}}^{n-k}$$

where $n,\ k \in \mathbb{N}^*$. Here $B_{k,n}^{(\alpha)}(x \mid q)$ are called the weighted q-Bernstein polynomials and defined by

$$B_{k,n}^{(\alpha)}(x \mid q) = \binom{n}{k} [x]_{q^\alpha}^{k} [1-x]_{q^{-\alpha}}^{n-k},\ x \in [0,1] \tag{3}$$

(for more information, see [3], [32], [38] and [39]).

As it is well known, the familiar Genocchi polynomials are defined by means of the following generating function:

$$\sum_{n=0}^{\infty} G_n(x) \frac{t^n}{n!} = e^{G(x)t} = \frac{2t}{e^t+1} e^{xt}. \tag{4}$$

where $G^n(x) := G_n(x)$, symbolically. For $x = 0$ in (4), we have to $G_n(0) := G_n$, which are called Genocchi numbers and given by

$$e^{Gt} = \sum_{n=0}^{\infty} G_n \frac{t^n}{n!} = \frac{2t}{e^t+1}. \tag{5}$$

In [4], the q-Genocchi numbers are given by

$$G_{0,q} = 0 \text{ and } q(qG_q+1)^n + G_{n,q} = \begin{cases} [2]_q & \text{if } n = 1 \\ 0 & \text{if } n \neq 1 \end{cases}$$

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with the usual convention about replacing $(G_q)^n$ by $G_{n,q}$.

For any $n \in \mathbb{N}^*$, the (h,q)-Genocchi numbers are introduced by

$$G_{0,q}^{(h)} = 0 \text{ and } q^{h-1}\left(qG_q^{(h)}+1\right)^n + G_{n,q}^{(h)} = \begin{cases} [2]_q & \text{if } n=1 \\ 0 & \text{if } n \neq 1 \end{cases}$$

with the usual convention about replacing $\left(G_q^{(h)}\right)^n$ by $G_{n,q}^{(h)}$ (for details, see [5]).

Recently, Araci *et al*. have defined the (h,q)-Genocchi numbers with weight α as

$$\frac{\widetilde{G}_{n+1,q}^{(\alpha,h)}(x)}{n+1} = \int_{\mathbb{Z}_p} q^{(h-1)\xi}\left[x+\xi\right]_{q^\alpha}^n d\mu_{-q}(\xi). \quad (6)$$

By (6), we have the following identity:

$$\widetilde{G}_{n,q}^{(\alpha,h)}(x) = \sum_{k=0}^n \binom{n}{k} q^{\alpha k x}\widetilde{G}_{n,q}^{(\alpha,h)}\left[x\right]_{q^\alpha}^{n-k} = q^{-\alpha x}\left(q^{\alpha x}\widetilde{G}_q^{(\alpha,h)} + [x]_{q^\alpha}\right)^n \quad (7)$$

with the usual convention about replacing $\left(\widetilde{G}_q^{(\alpha,h)}\right)^n$ by $\widetilde{G}_{n,q}^{(\alpha,h)}$ is used (for details, [5]).

In this paper, we derive some new properties (h,q)-Genocchi numbers and polynomials with weight α arising from the fermionic p-adic q-integral on $\mathbb{Z}_p$ and weighted q-Bernstein polynomials.

2 On the (h,q)-Genocchi numbers and polynomials with weight α

In this section, we consider the (h,q)-Genocchi numbers and polynomials with weight α by using fermionic p-adic q-integral on $\mathbb{Z}_p$ and the weighted q-Bernstein polynomials. We now start with the following expression.

In [5], we have the (h,q)-Genocchi numbers with weight α as follows: for $\alpha \in \mathbb{N}^*$ and $n,h \in \mathbb{N}$,

$$\widetilde{G}_{0,q}^{(\alpha,h)} = 0 \text{ and } q^h\widetilde{G}_{n,q}^{(\alpha,h)}(1) + \widetilde{G}_{n,q}^{(\alpha,h)} = \begin{cases} [2]_q & \text{if } n=1, \\ 0 & \text{if } n \neq 1. \end{cases} \quad (8)$$

By (7) and (8), we obtain the following corollary.

Corollary 1. *For $\alpha \in \mathbb{N}^*$ and $n,h \in \mathbb{N}$, then we have*

$$\widetilde{G}_{0,q}^{(\alpha,h)} = 0 \text{ and } q^{h-\alpha}\left(q^\alpha\widetilde{G}_q^{(\alpha,h)}+1\right)^n + \widetilde{G}_{n,q}^{(\alpha,h)} = \begin{cases} [2]_q & \text{if } n=1, \\ 0 & \text{if } n \neq 1. \end{cases} \quad (9)$$

By (6), we get symmetric property by the following basic applications:

$$\begin{aligned}\frac{\widetilde{G}_{n+1,q^{-1}}^{(\alpha,h)}(1-x)}{n+1} &= \int_{\mathbb{Z}_p} q^{(1-h)\xi}\left[1-x+\xi\right]_{q^{-\alpha}}^n d\mu_{-q^{-1}}(\xi) \\ &= (-1)^n q^{h+\alpha n-1}\int_{\mathbb{Z}_p} q^{(h-1)\xi}\left[x+\xi\right]_{q^\alpha}^n d\mu_{-q}(\xi)\end{aligned}$$

Thus, we obtain the following theorem.

Theorem 1. *The following identity*

$$\widetilde{G}_{n+1,q^{-1}}^{(\alpha,h)}(1-x) = (-1)^n q^{h+\alpha n-1}\widetilde{G}_{n+1,q}^{(\alpha,h)}(x) \quad (10)$$

is true.

By using (7), (8) and (9), we compute

$$\begin{aligned} q^{2\alpha}\widetilde{G}_{n,q}^{(\alpha,h)}(2) &= \left(q^{2\alpha}\widetilde{G}_q^{(\alpha,h)} + [2]_{q^\alpha}\right)^n \quad (11)\\ &= \sum_{l=0}^n \binom{n}{l} q^{\alpha l}\left(q^\alpha\widetilde{G}_q^{(\alpha,h)}+1\right)^l \\ &= nq^{2\alpha-h}\left([2]_q - \widetilde{G}_{1,q}^{(\alpha,h)}\right) - q^{\alpha-h}\sum_{l=2}^n \binom{n}{l} q^{\alpha l}\widetilde{G}_{l,q}^{(\alpha,h)} \\ &= nq^{2\alpha-h}[2]_q + q^{2\alpha-2h}\widetilde{G}_{n,q}^{(\alpha,h)} \text{ if } n>1. \end{aligned}$$

After the above applications, we procure the following theorem.

Theorem 2. *For $n>1$, then we have*

$$\widetilde{G}_{n,q}^{(\alpha,h)}(2) = nq^{-h}[2]_q + q^{-2h}\widetilde{G}_{n,q}^{(\alpha,h)}.$$

We need the following equality for sequel of this paper:

$$[1-x]_{q^{-\alpha}}^n = \left(\frac{1-q^{-\alpha(1-x)}}{1-q^{-\alpha}}\right)^n = (-1)^n q^{n\alpha}[x-1]_{q^\alpha}^n. \quad (12)$$

Now also, by using (12), we consider the following

$$\begin{aligned} & q^{h-1}\int_{\mathbb{Z}_p} q^{(h-1)\xi}\left[1-\xi\right]_{q^{-\alpha}}^n d\mu_{-q}(\xi) \\ &= (-1)^n q^{h+n\alpha-1}\int_{\mathbb{Z}_p} q^{(h-1)\xi}\left[\xi-1\right]_{q^\alpha}^n d\mu_{-q}(\xi) \\ &= (-1)^n q^{h+n\alpha-1}\frac{\widetilde{G}_{n+1,q}^{(\alpha,h)}(-1)}{n+1}. \end{aligned}$$

By considering last identity and (10), we get the following theorem.

Theorem 3. *The following identity holds true:*

$$\int_{\mathbb{Z}_p} q^{(h-1)(\xi+1)}\left[1-\xi\right]_{q^{-\alpha}}^n d\mu_{-q}(\xi) = \frac{\widetilde{G}_{n+1,q^{-1}}^{(\alpha,h)}(2)}{n+1}. \quad (13)$$

From (13), we have the following

$$\int_{\mathbb{Z}_p} q^{(h-1)\xi}\left[1-\xi\right]_{q^{-\alpha}}^n d\mu_{-q}(\xi) = [2]_q + q^{h+1}\frac{\widetilde{G}_{n+1,q^{-1}}^{(\alpha,h)}}{n+1}.$$

Thus, we obtain the following theorem.

Theorem 4. *The following identity*

$$\int_{\mathbb{Z}_p} q^{(h-1)\xi}\left[1-\xi\right]_{q^{-\alpha}}^n d\mu_{-q}(\xi) = [2]_q + q^{h+1}\frac{\widetilde{G}_{n+1,q^{-1}}^{(\alpha,h)}}{n+1} \quad (14)$$

is true.

3 Some new identities on the (h,q) -Genocchi numbers with weight α

In this section, we introduce the new identities of the (h,q)-Genocchi numbers with weight α, that is, we derive some interesting relations.

For $x \in [0,1]$, we recall the definition of weighted q-Bernstein polynomials as follows:

$$B_{k,n}^{(\alpha)}(x \mid q) = \binom{n}{k} [x]_{q^\alpha}^k [1-x]_{q^{-\alpha}}^{n-k}, \text{ where } n,k \in \mathbb{Z}_+. \tag{15}$$

By expression (15), we have the symmetry property of weighted q-Bernstein polynomials, as follows:

$$B_{k,n}^{(\alpha)}(x \mid q) = B_{n-k,n}^{(\alpha)}\left(1-x \mid \frac{1}{q}\right), \text{ (for details, see [32]).} \tag{16}$$

Thus, (14), (15) and (16), we see that

$$\begin{aligned}
I_1 &= \int_{\mathbb{Z}_p} q^{(h-1)x} B_{k,n}^{(\alpha)}(x \mid q)\, d\mu_{-q}(x) \\
&= \binom{n}{k} \int_{\mathbb{Z}_p} q^{(h-1)x} [x]_{q^\alpha}^k [1-x]_{q^{-\alpha}}^{n-k} d\mu_{-q}(x) \\
&= \binom{n}{k} \sum_{l=0}^{k} \binom{k}{l} (-1)^{k+l} \int_{\mathbb{Z}_p} q^{(h-1)x} [1-x]_{q^{-\alpha}}^{n-l} d\mu_{-q}(x) \\
&= \binom{n}{k} \sum_{l=0}^{k} \binom{k}{l} (-1)^{k+l} \left\{ [2]_q + q^{h+1} \frac{\widetilde{G}_{n-l+1,q^{-1}}^{(\alpha,h)}}{n-l+1} \right\} \\
&= \begin{cases} [2]_q + q^{h+1} \frac{\widetilde{G}_{n+1,q^{-1}}^{(\alpha,h)}}{n+1} & \text{if } k=0, \\ \binom{n}{k} \sum_{l=0}^{k} \binom{k}{l} (-1)^{k+l} \left\{ [2]_q + q^{h+1} \frac{\widetilde{G}_{n-l+1,q^{-1}}^{(\alpha,h)}}{n-l+1} \right\} & \text{if } k \neq 0. \end{cases}
\end{aligned}$$

On the other hand, for n, $k \in \mathbb{Z}_+$ with $n > k$, we compute

$$\begin{aligned}
I_2 &= \int_{\mathbb{Z}_p} q^{(h-1)x} B_{k,n}^{(\alpha)}(x \mid q)\, d\mu_{-q}(x) \\
&= \binom{n}{k} \int_{\mathbb{Z}_p} q^{(h-1)x} [x]_{q^\alpha}^k [1-x]_{q^{-\alpha}}^{n-k} d\mu_{-q}(x) \\
&= \binom{n}{k} \sum_{l=0}^{n-k} \binom{n-k}{l} (-1)^l \int_{\mathbb{Z}_p} q^{(h-1)x} [x]_{q^\alpha}^{l+k} d\mu_{-q}(x) \\
&= \binom{n}{k} \sum_{l=0}^{n-k} \binom{n-k}{l} (-1)^l \frac{\widetilde{G}_{l+k+1,q}^{(\alpha,h)}}{l+k+1}.
\end{aligned}$$

Equating I_1 and I_2, we have the following theorem.

Theorem 5. *The following identity holds true:*

$$\sum_{l=0}^{n-k} \binom{n-k}{l} (-1)^l \frac{\widetilde{G}_{l+k+1,q}^{(\alpha,h)}}{l+k+1} = \begin{cases} [2]_q + q^{h+1} \frac{\widetilde{G}_{n+1,q^{-1}}^{(\alpha,h)}}{n+1} & \text{if } k=0, \\ \sum_{l=0}^{k} \binom{k}{l} (-1)^{k+l} \left\{ [2]_q + q^{h+1} \frac{\widetilde{G}_{n-l+1,q^{-1}}^{(\alpha,h)}}{n-l+1} \right\} & \text{if } k \neq 0. \end{cases}$$

Let $n_1, n_2, k \in \mathbb{Z}_+$ with $n_1 + n_2 > 2k$. Then, we derive the followings

$$\begin{aligned}
I_3 &= \int_{\mathbb{Z}_p} q^{(h-1)x} B_{k,n_1}^{(\alpha)}(x \mid q) B_{k,n_2}^{(\alpha)}(x \mid q)\, d\mu_{-q}(x) \\
&= \binom{n_1}{k}\binom{n_2}{k} \sum_{l=0}^{2k} \binom{2k}{l} (-1)^{2k+l} \int_{\mathbb{Z}_p} q^{(h-1)x} [1-x]_{q^{-\alpha}}^{n_1+n_2-l} d\mu_{-q}(x) \\
&= \left(\binom{n_1}{k}\binom{n_2}{k} \sum_{l=0}^{2k} \binom{2k}{l} (-1)^{2k+l} \left([2]_q + q^{h+1} \frac{\widetilde{G}_{n_1+n_2-l+1,q^{-1}}^{(\alpha,h)}}{n_1+n_2-l+1} \right) \right) \\
&= \begin{cases} [2]_q + q^{h+1} \frac{\widetilde{G}_{n_1+n_2+1,q^{-1}}^{(\alpha,h)}}{n+1} & \text{if } k=0, \\ \binom{n}{k} \sum_{l=0}^{2k} \binom{2k}{l} (-1)^{2k+l} \left\{ [2]_q + q^{h+1} \frac{\widetilde{G}_{n_1+n_2-l+1,q^{-1}}^{(\alpha,h)}}{n_1+n_2-l+1} \right\} & \text{if } k \neq 0. \end{cases}
\end{aligned}$$

In other words, by using the binomial theorem, we can derive the following equation.

$$\begin{aligned}
I_4 &= \int_{\mathbb{Z}_p} q^{(h-1)x} B_{k,n_1}^{(\alpha)}(x \mid q) B_{k,n_2}^{(\alpha)}(x \mid q)\, d\mu_{-q}(x) \\
&= \prod_{i=1}^{2} \binom{n_i}{k} \sum_{l=0}^{n_1+n_2-2k} \binom{n_1+n_2-2k}{l} (-1)^l \int_{\mathbb{Z}_p} q^{(h-1)x} [x]_{q^\alpha}^{2k+l} d\mu_{-q}(x) \\
&= \prod_{i=1}^{2} \binom{n_i}{k} \sum_{l=0}^{n_1+n_2-2k} \binom{n_1+n_2-2k}{l} (-1)^l \frac{\widetilde{G}_{l+2k+1,q}^{(\alpha,h)}}{l+2k+1}.
\end{aligned}$$

Combining I_3 and I_4, we state the following theorem.

Theorem 6. *For $n_1, n_2, k \in \mathbb{Z}_+$ with $n_1 + n_2 > 2k$, we have*

$$\sum_{l=0}^{n_1+n_2-2k} \binom{n_1+n_2-2k}{l} (-1)^l \frac{\widetilde{G}_{l+2k+1,q}^{(\alpha,h)}}{l+2k+1} = \begin{cases} [2]_q + q^{h+1} \frac{\widetilde{G}_{n_1+n_2+1,q^{-1}}^{(\alpha,h)}}{n_1+n_2+1} & \text{if } k=0, \\ \sum_{l=0}^{2k} \binom{2k}{l} (-1)^{2k+l} \left\{ [2]_q + q^{h+1} \frac{\widetilde{G}_{n_1+n_2-l+1,q^{-1}}^{(\alpha,h)}}{n_1+n_2-l+1} \right\} & \text{if } k \neq 0. \end{cases}$$

For $x \in \mathbb{Z}_p$ and $s \in \mathbb{N}$ with $s \geq 2$, let $n_1, n_2, \ldots, n_s, k \in \mathbb{Z}_+$ with $\sum_{l=1}^{s} n_l > sk$. Then we take the fermionic p-adic q-integral on $\mathbb{Z}_p$ for the weighted q-Bernstein polynomials of degree n as follows:

$$\begin{aligned}
I_5 &= \int_{\mathbb{Z}_p} q^{(h-1)x} \left\{ \prod_{i=1}^{s} B_{k,n_i}^{(\alpha)}(x \mid q) \right\} d\mu_{-q}(x) \\
&= \prod_{i=1}^{s} \binom{n_i}{k} \int_{\mathbb{Z}_p} [x]_{q^\alpha}^{sk} [1-x]_{q^{-\alpha}}^{n_1+n_2+\ldots+n_s-sk} q^{(h-1)x} d\mu_{-q}(x) \\
&= \prod_{i=1}^{s} \binom{n_i}{k} \sum_{l=0}^{sk} \binom{sk}{l} (-1)^{l+sk} \int_{\mathbb{Z}_p} [1-x]_{q^{-\alpha}}^{n_1+n_2+\ldots+n_s-l} q^{(h-1)x} d\mu_{-q}(x) \\
&= \begin{cases} [2]_q + q^{h+1} \frac{\widetilde{G}_{n_1+n_2+\ldots+n_s+1,q^{-1}}^{(\alpha,h)}}{n_1+n_2+\ldots+n_s+1} & \text{if } k=0, \\ \prod_{i=1}^{s} \binom{n_i}{k} \sum_{l=0}^{sk} \binom{sk}{l} (-1)^{sk+l} \left\{ [2]_q + q^{h+1} \frac{\widetilde{G}_{n_1+n_2+\ldots+n_s-l+1,q^{-1}}^{(\alpha,h)}}{n_1+n_2+\ldots+n_s-l+1} \right\} & \text{if } k \neq 0. \end{cases}
\end{aligned}$$

On the other hand, from the definition of weighted q-Bernstein polynomials and the binomial theorem, we easily get

$$\begin{aligned}
I_6 &= \int_{\mathbb{Z}_p} q^{(h-1)x} \left\{ \prod_{i=1}^{s} B_{k,n_i}^{(\alpha)}(x \mid q) \right\} d\mu_{-q}(x) \\
&= \prod_{i=1}^{s} \binom{n_i}{k} \sum_{l=0}^{n_1+\ldots+n_s-sk} \binom{\sum_{d=1}^{s}(n_d-k)}{l} (-1)^l \int_{\mathbb{Z}_p} [x]_{q^\alpha}^{sk+l} q^{(h-1)x} d\mu_{-q}(x) \\
&= \prod_{i=1}^{s} \binom{n_i}{k} \sum_{l=0}^{n_1+\ldots+n_s-sk} \binom{\sum_{d=1}^{s}(n_d-k)}{l} (-1)^l \frac{\widetilde{G}_{l+sk+1,q}^{(\alpha,h)}}{l+sk+1}.
\end{aligned}$$

Equating I_5 and I_6, we discover the following theorem.

Theorem 7. *For $s \in \mathbb{N}$ with $s \geq 2$, let $n_1, n_2, \ldots, n_s, k \in \mathbb{Z}_+$ with $\sum_{l=1}^{s} n_l > sk$. Then, we have*

$$\sum_{l=0}^{n_1+\ldots+n_s-sk} \binom{\sum_{d=1}^{s}(n_d-k)}{l} (-1)^l \frac{\widetilde{G}_{l+sk+1,q}^{(\alpha,h)}}{l+sk+1}$$

$$= \begin{cases} [2]_q + q^{h+1} \frac{\widetilde{G}_{n_1+n_2+\ldots+n_s+1,q^{-1}}^{(\alpha,h)}}{n_1+n_2+\ldots+n_s+1} & \text{if } k=0, \\ \sum_{l=0}^{sk} \binom{sk}{l} (-1)^{sk+l} \left\{ [2]_q + q^{h+1} \frac{\widetilde{G}_{n_1+n_2+\ldots+n_s-l+1,q^{-1}}^{(\alpha,h)}}{n_1+n_2+\ldots+n_s-l+1} \right\} & \text{if } k \neq 0. \end{cases}$$

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