Applied Math. Inf. Sciences Letters
An International Journal

Oscillation results of second order damped non-linear dynamic equation on time scales

M. Tamer Şenel

Department of Mathematics, Faculty of Science, Erciyes University, 38039, Kayseri , TURKEY

Abstract: This paper concerns the oscillation of solutions to second order non-linear dynamic equation with damping

$$(r(t)\Psi(x^{\Delta}(t))^{\Delta}+p(t)\Psi(x^{\Delta}(t))+q(t)x^{\sigma}(t)=0$$

on a time scale $\mathbb{T}$ which is unbounded above. $r(t)$, $p(t)$ and $q(t)$ are positive rd-continuous functions. $\Psi:\mathbb{T}\to\mathbb{R}$ is rd-continuous functions. Our results are new and different many known results for second order dynamic equations.

Keywords: Oscillation, Dynamic equations, Time scales

1. Introduction

The theory of time scales, which has recently received a lot of attention, was introduced by Stefan Hilger in his PhD thesis in 1988 in order to unify continuous and discrete analysis (see [1]). Since Stefan Hilger formed the definition of derivatives and integrals on time scales, several authors have expounded on various aspects of the new theory, see the paper by Agarwal, et al. ([2]) and the references cited . A book on the subject of time scales by Bohner and Peterson [3] summarizes and organizes much of time scale calculus.

A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of the real numbers $\mathbb{R}$. Since we are interested in the oscillatory of solutions near infinity, we assume that $sup\mathbb{T}=\infty$, and define the time scale interval $[t_0,\infty)_{\mathbb{T}}$ by $[t_0,\infty)_{\mathbb{T}}:=[t_0,\infty)\cap\mathbb{T}$. We assume that $\mathbb{T}$ has the topology that it inherits from the standard topology on the real numbers $\mathbb{T}$.

In this paper we shall study the oscillations of the following non-linear second order dynamic equations with damping

$$(r(t)\Psi(x^{\Delta}(t))^{\Delta}+p(t)\Psi(x^{\Delta}(t))+q(t)x^{\sigma}(t)=0, \qquad (1)$$

where $p(t),q(t)$ and $r(t)$ are positive rd-continuous functions.

In the last few years, much interest has focused on obtaining sufficient conditions for the oscillation/nonoscillation of solutions of different classes of dynamic equations on time scales, and we refer the reader to the papers [4-21].

Agarwal et al. ([4]), have considered the second order perturbed dynamic equation

$$(r(t)(x^{\Delta}(t))^{\gamma})^{\Delta}+F(t,x(t))=G(t,x(t),x^{\Delta}(t)), \qquad (2)$$

where $\gamma\in\mathbb{N}$ is odd and they have interested in asymptotic behavior of solutions of equation (2). In [5], Saker and et al. considered the non-linear dynamic equation

$$(a(t)x^{\Delta}(t))^{\Delta}+p(t)x^{\Delta^{\sigma}}(t)+q(t)f(x^{\sigma}(t))=0$$

when $a(t),p(t),r(t)$ are positive rd-continuous functions. They gave some sufficient conditions for oscillation. The authors supposed that $uf(u)>0, f(u)/u\geq K>0$ and $f'(u)\geq k$ for $u\neq 0$.

In this paper, by employing the Riccati transformation technique we will establish some sufficient conditions for the oscillation of (1). The paper is organized as follows: In Section 2, we develop the Riccati transformation technique to give some sufficient conditions for the oscillation of all solutions of (1). In Section 3, we establish some sufficient conditions for oscillation of Eq. (1) with $p(t)=0$.

We will use some of following assumptions:
(H_1) $r(t),p(t)$, and $q(t)$ are positive real-valued rd-functions,
$(H_2)\Psi:\mathbb{T}\to\mathbb{R}$, $\frac{\Psi(u)}{|u|}\geq\kappa$ for $\kappa>0$, $u\neq 0$,
(H_3) $\int_{t_0}^{\infty}(\frac{1}{r(t)}e_{-\frac{p}{r}}(t,t_0))\Delta t=\infty$.

Our attention is restricted to those solutions of (1) which

* Corresponding author: e-mail: senel@erciyes.edu.tr

exist on some half-line $[t_x,\infty)$ and satisfy $sup\{|x(t)| : t > \mathbb{T}\} > 0$ for any $T \ge t_x$. We assume the standing hypothesis that (1) does possess such solutions. A solution *x(t)* of (1) is said to be oscillatory if it is neither eventually positive nor eventually negative, otherwise it is nonoscillatory. The equation itself is called oscillatory if all its solutions are oscillatory.

2. Main results

Theorem 2.1. Assume that $(H_1)-(H_3)$ holds. Furthermore, assume that there exist a positive real rd-functions differentiable functions $z(t)$ such that

$$\limsup_{t\to\infty}\int_{t_0}^{t}\left[z(s)q(s)-\frac{\kappa r(s)A^2(s)}{4z(s)}\right]\Delta s=\infty, \tag{3}$$

where

$$A(t)=\left[z^{\Delta}(t)-\frac{z(t)p(t)}{r(t)}\right],$$

then every solution of (1) is oscillatory.

Proof. Suppose to the contrary that $x(t)$ is a nonoscillatory solution of (1). Without loss of generality, we may assume that $x(t) > 0$ for $t \ge t_1 > t_0$. We shall consider only this case, since in view of (H_2), the proof of the case when $x(t)$ is eventually negative is similar. Now, we claim that $x^{\Delta}(t)$ has a fixed sign on the interval $[t_2,\infty)$ for some $t_2 \ge t_1$. From (1), since $q(t) > 0$, we have

$$(r(t)\Psi(x^{\Delta}(t))^{\Delta}+p(t)\Psi(x^{\Delta}(t))=-q(t)x^{\sigma}(t)<0,$$

i.e.,

$$(r(t)\Psi(x^{\Delta}(t))^{\Delta}+p(t)\Psi(x^{\Delta}(t))<0.$$

By setting

$$y(t)=r(t)\Psi(x^{\Delta}(t)),$$

we immediately see that,

$$y^{\Delta}(t)+\frac{p(t)y(t)}{r(t)}<0,$$

which implies that

$$\left(y(t)e_{-\frac{p}{r}}\right)^{\Delta}<0.$$

Then $y(t)e_{-\frac{p}{r}}$ is decreasing and thus $y(t)$ is eventually of one sing. Then $x^{\Delta}(t)$ has a fixed sing for all sufficiently large t and we have one of the following:
First, we consider $x^{\Delta}(t) \ge 0$ on $[t_2,\infty)$ for some $t_2 \ge t_1$. Then in view of (1) we have

$$x(t)>0, x^{\Delta}(t)\ge 0, (r(t)\Psi(x^{\Delta}(t))^{\Delta}\le 0, t\ge t_2. \tag{4}$$

Define the function $w(t)$ by Riccati substitution

$$w(t):=z(t)\frac{r(t)\Psi(x^{\Delta}(t))}{x(t)}, t\ge t_2 \tag{5}$$

Then $w(t) > 0$, and satisfies

$$w^{\Delta}(t)=\left[r(t)\Psi(x^{\Delta}(t))\right]^{\sigma}\left[\frac{z(t)}{x(t)}\right]^{\Delta}+\frac{z(t)}{x(t)}\left[r(t)\Psi(x^{\Delta}(t))\right]^{\Delta}$$

In view of (1) and (5), we see that for $t \ge t_3$

$$w^{\Delta}(t)=\frac{z^{\Delta}(t)-z(t)x^{\Delta}(t)}{x(t)x^{\sigma}(t)}\left[r(t)\Psi(x^{\Delta}(t))\right]^{\sigma}+\frac{z(t)}{x(t)}\left[-p(t)\Psi(x^{\Delta}(t))-q(t)x^{\sigma}(t)\right] \tag{6}$$

However from (4),

$$r(t)\Psi(x^{\Delta}(t))\ge(r(t)\Psi(x^{\Delta}(t)))^{\sigma},\ x^{\sigma}(t)\ge x(t). \tag{7}$$

Using (7) and (H_2) in (6), we have

$$w^{\Delta}(t)\le z^{\Delta}(t)\frac{w^{\sigma}(t)}{z^{\sigma}(t)}-\frac{z(t)}{x^{\sigma}(t)}p(t)\Psi(x^{\Delta}(t))-z(t)\frac{q(t)x^{\sigma}(t)}{x^{\sigma}(t)}-z(t)\frac{x^{\Delta}(t)}{(x^{\sigma}(t))^2}[r(t)\Psi(x^{\Delta}(t))]^{\sigma}$$

$$w^{\Delta}(t)\le z^{\Delta}(t)\frac{w^{\sigma}(t)}{z^{\sigma}(t)}-\frac{z(t)p(t)}{r(t)}\frac{w^{\sigma}(t)}{z^{\sigma}(t)}-z(t)q(t)-z(t)\frac{(w^{\sigma}(t))^2}{\kappa(z^{\sigma}(t))^2r(t)}$$

$$w^{\Delta}(t)\le -z(t)q(t)+\left[z^{\Delta}(t)-\frac{z(t)p(t)}{r(t)}\right]\frac{w^{\sigma}(t)}{z^{\sigma}(t)}-z(t)\frac{(w^{\sigma}(t))^2}{\kappa(z^{\sigma}(t))^2r(t)}, \tag{8}$$

$$w^{\Delta}(t)\le -z(t)q(t)+A(t)\frac{w^{\sigma}(t)}{z^{\sigma}(t)}-z(t)\frac{(w^{\sigma}(t))^2}{\kappa(z^{\sigma}(t))^2r(t)}, \tag{9}$$

where

$$A(t)=\left[z^{\Delta}(t)-\frac{z(t)p(t)}{r(t)}\right].$$

Then

$$w^{\Delta}(t)\le -z(t)q(t)+\frac{\kappa r(t)A^2(t)}{4z(t)}-\left[\sqrt{\frac{z(t)}{\kappa r(t)}}\frac{w^{\sigma}(t)}{z^{\sigma}(t)}-\frac{1}{2}\sqrt{\frac{\kappa r(t)}{z(t)}}A(t)\right]^2,$$

$$w^{\Delta}(t)\le z(t)q(t)-\frac{\kappa r(t)A^2(t)}{4z(t)}.$$

Integration from t_3 to t, we obtain

$$w(t)-w(t_3)\le-\int_{t_3}^{t}\left[z(s)q(s)-\frac{\kappa r(s)A^2(s)}{4z(s)}\right]\Delta s$$

which yields

$$\int_{t_3}^{t}\left[z(s)q(s)-\frac{\kappa r(s)A^2(s)}{4z(s)}\right]\Delta s\le w(t_3)-w(t)<w(t_3), t\ge t_3$$

for all large t. This is contrary to (3).
Next, we consider $x^{\Delta}(t) < 0$ for $t \geq t_2 \geq t_1$.
Define the function $u(t) = -r(t)\Psi(x^{\Delta}(t))$. The from (1) and (H_3), we have

$$u^{\Delta}(t) + \frac{p(t)}{r(t)}u(t) \geq 0 \Rightarrow u(t) \geq u(t_2)e_{-\frac{p}{r}}(t,t_2),$$

Thus

$$-r(t)\Psi(x^{\Delta}(t)) \geq u(t_2)e_{-\frac{p}{r}}(t,t_2).$$

$$\Psi(x^{\Delta}(t)) \leq -u(t_2)\left(\frac{1}{r(t)}e_{-\frac{p}{r}}(t,t_2)\right).$$

from (H_3) there is a $\kappa > 0$, so that

$$\kappa x^{\Delta}(t) \leq -u(t_2)\left(\frac{1}{r(t)}e_{-\frac{p}{r}}(t,t_2)\right). \tag{10}$$

Integrating (10) from t_2 to t, we have

$$x(t) - x(t_2) \leq \frac{r(t_2)\Psi(x(t_2))}{\kappa}\int_{t_2}^{t}\left(\frac{1}{r(t)}e_{-\frac{p}{r}}(t,t_2)\right)\Delta s.$$

$$x(t) \leq x(t_2) + \frac{r(t_2)\Psi(x(t_2))}{\kappa}\int_{t_2}^{t}\left(\frac{1}{r(t)}e_{-\frac{p}{r}}(t,t_2)\right)\Delta s.$$

so condition (H_3) implies that x(t) is eventually negative, which is a contradiction. The proof is complete.

Corollary 2.2. Assume that $(H_1)-(H_3)$ hold. If

$$\limsup_{t\to\infty}\int_{t_0}^{t}\left[q(s) - \frac{\kappa p^2(s)}{4r(s)}\right]\Delta s = \infty \tag{11}$$

then every solution (1) is oscillatory.
Example 2.3. Consider the dynamic equation

$$\left(t\Psi(x^{\Delta}(t))\right)^{\Delta} + \left(\Psi(x^{\Delta}(t))\right) + \frac{1}{t}x^{\sigma}(t) = 0, \ \ t > 0$$

where $r(t) = t, \ p(t) = 1, \ \ q(t) = \frac{1}{t}$,
$\Psi(x^{\Delta}(t)) = (x^{\Delta}(t))^{2k+1}$, $k \in \mathbb{N}$. All conditions of Corollary 2.2 and $(H_1)-(H_3)$ are satisfied. Hence it is oscillatory.

Corollary 2.4. Assume that $(H_1)-(H_3)$ hold. If

$$\limsup_{t\to\infty}\int_{t_0}^{t}\left[s^{\gamma}q(s) - \frac{\kappa(r(s)(s^{\gamma})^{\Delta} - s^{\gamma}p(s))^2}{4r(s)}\kappa s^{-\gamma}\right]\Delta s = \infty \tag{12}$$

then every solution (1) is oscillatory.

Corollary 2.5. Assume that $(H_1)-(H_3)$ hold. If

$$\limsup_{t\to\infty}\int_{t_0}^{t}\left[Z(s,t_0)q(s) - \frac{\kappa r(s)}{4Z(s,t_0)}\left((Z(s,t_0))^{\Delta} - \frac{Z(s,t_0)p(s)}{r(s)}\right)^2\right]\Delta s = \infty,$$

where $Z(t,t_0) = \int_{t_0}^{t}\frac{1}{r(s)}\Delta s$, then every solution (1) is oscillatory.
Now, let us introduce the class of functions $\mathbb{R}$ which will be extensively used in the sequel. Let $\mathbb{D}_0 \equiv \{(t,s) \in \mathbb{T}^2 : t > s \geq t_0\}$ and $\mathbb{D} \equiv \{(t,s) \in \mathbb{T}^2 : t \geq s \geq t_0\}$. The function $H \in C_{rd}(\mathbb{D},\mathbb{R})$ is said belongs to the class $\mathfrak{R}$ if
(i) $H(t,t) = 0, \ t \geq t_0, \ H(t,s) > 0$, on $\mathbb{D}_0$,
(ii) H has a continuous Δ-partial derivative $H_s^{\Delta}(t,s)$ on $\mathbb{D}_0$ with respect to the second variable.(H is rd-continuous function if H is rd-continuous function in t and s.)

Theorem 2.6. Assume that $(H_1)-(H_3)$ hold.Let z(t) be positive real rd-functions differentiable function and let $H : \mathbb{D} \to \mathbb{R}$ be rd-continuous function such that H belongs to the class $\mathfrak{R}$ and where

$$\limsup_{t\to\infty}\frac{1}{H(t,t_0)}\int_{t_0}^{t}\left[H(t,s)z(s)q(s) - \frac{\kappa r(s)(\varphi(t,s))^2}{4z(s)H(t,s)}\right]\Delta s = \infty, \tag{13}$$

$$\varphi(t,s) = z^{\sigma}(s)H_s^{\Delta}(t,s) + H(t,s)A(s).$$

Then every solution of (1) is oscillatory.

Proof. Suppose to the contrary that x(t) is a nonoscillatory solution of (1) and let $t_1 \geq t_0$ be such that $x(t) \neq 0$ for all $t \geq t_1$, so without loss of generality, we may assume that x(t) is an eventually positive solution of (1) with $x(t) > 0$ for all $t \geq t_1$ sufficiently large. In view of Theorem 2.1 we see that $x^{\Delta}(t)$ is eventually negative or eventually positive. If $x^{\Delta}(t)$ is eventually negative, we are then back to second case of Theorem 2.1 and we obtain a contradiction. If $x^{\Delta}(t)$ is eventually positive, we assume that there exists $t_2 \geq t_1$ such that $x^{\Delta}(t) \geq 0$ for $t_2 \geq t_1$ and proceed as in the proof of first of Theorem 2 . From (9), it follows that

$$w^{\Delta}(t) \leq -z(t)q(t) + A(t)\frac{w^{\sigma}(t)}{z^{\sigma}(t)} - z(t)\frac{(w^{\sigma}(t))^2}{\kappa(z^{\sigma}(t))^2r(t)}, \tag{14}$$

we multiply to (14) to $H(t,s)$ then

$$H(t,s)w^{\Delta}(t) \leq -H(t,s)z(t)q(t) + H(t,s)A(t)\frac{w^{\sigma}(t)}{z^{\sigma}(t)} - H(t,s)z(t)\frac{(w^{\sigma}(t))^2}{\kappa(z^{\sigma}(t))^2r(t)},$$

$$H(t,s)z(t)q(t) \leq -H(t,s)w^{\Delta}(t) + H(t,s)A(t)\frac{w^{\sigma}(t)}{z^{\sigma}(t)} - H(t,s)z(t)\frac{(w^{\sigma}(t))^2}{\kappa(z^{\sigma}(t))^2r(t)},$$

Using the integration by parts formula, we have

$$\int_{t_2}^{t} H(t,s)z(s)q(s)\Delta s \le -H(t,t)w(t)+H(t,t_2)w(t_2) + \int_{t_2}^{t} H_s^{\Delta}(t,s)w^{\sigma}(s)\Delta s + \int_{t_2}^{t} H(t,s)A(s)\frac{w^{\sigma}(s)}{z^{\sigma}(s)}\Delta s - \int_{t_2}^{t} H(t,s)z(s)\frac{((w^{\sigma}(s))^2}{\kappa(z^{\sigma}(s))^2 r(s)}\Delta s,$$

where $H(t,t)=0$, we obtain

$$\int_{t_2}^{t} H(t,s)z(s)q(s)\Delta s \le H(t,t_2)w(t_2)+\int_{t_2}^{t}\left[z^{\sigma}(s)H_s^{\Delta}(t,s) + H(t,s)A(s)\right]\frac{w^{\sigma}(s)}{z^{\sigma}(s)}\Delta s - \int_{t_2}^{t} H(t,s)z(s)\frac{((w^{\sigma}(s))^2}{\kappa(z^{\sigma}(s))^2 r(s)}\Delta s,$$

$$\int_{t_2}^{t} H(t,s)z(s)q(s)\Delta s \le H(t,t_2)w(t_2)+\int_{t_2}^{t}\varphi(t,s)\frac{w^{\sigma}(s)}{z^{\sigma}(s)}\Delta s - \int_{t_2}^{t} H(t,s)z(s)\frac{((w^{\sigma}(s))^2}{\kappa(z^{\sigma}(s))^2 r(s)}\Delta s.$$

Therefore, by completing the square as in Theorem 2.1, we obtain

$$\int_{t_2}^{t} H(t,s)z(s)q(s)\Delta s \le H(t,t_2)w(t_2) + \int_{t_2}^{t}\frac{\kappa r(s)}{4z(s)H(t,s)}\varphi^2(t,s)\Delta s - \int_{t_2}^{t}\left[\sqrt{\frac{H(t,s)z(s)}{\kappa r(s)}}\frac{w^{\sigma}(s)}{z^{\sigma}(s)} - \frac{1}{2}\sqrt{\frac{\kappa r(s)}{z(s)H(t,s)}}\,\varphi(t,s)\right]^2\Delta s.$$

Hence, we obtain

$$\int_{t_2}^{t} H(t,s)z(s)q(s)\Delta s \le H(t,t_2)w(t_2) + \int_{t_2}^{t}\frac{\kappa r(s)}{4z(s)H(t,s)}\varphi^2(t,s)\Delta s.$$

Then for all $t\ge t_2$, we have

$$\int_{t_2}^{t}\left[H(t,s)z(s)q(s)-\frac{\kappa r(s)}{4z(s)H(t,s)}\varphi^2(t,s)\right]\Delta \le H(t,t_2)w(t_2)$$

and this implies that

$$\limsup_{t\to\infty}\frac{1}{H(t,t_2)}\int_{t_2}^{t}\left[H(t,s)z(s)q(s)-\frac{\kappa r(s)}{4z(s)H(t,s)}\varphi^2(t,s)\right]\Delta s \le w(t_2),$$

which contradicts (13). The proof is complete.
The consequences of Theorem 2.6, we get the following.

Corollary 2.7. Suppose that the assumptions of Theorem 2.6 hold. If

$$\limsup_{t\to\infty}\frac{1}{H(t,t_2)}\int_{t_2}^{t} H(t,s)\left[q(s)-\frac{\kappa r(s)}{4z(s)}\left(\frac{H_s^{\Delta}(t,s)}{H(t,s)} - \frac{p(s)}{r(s)}\right)^2\right]\Delta s=\infty,$$

then every solution of (1) is oscillatory.

Corollary 2.8. Let the assumption (13) in Theorem 2.6 be replaced by

$$\limsup_{t\to\infty}\frac{1}{H(t,t_0)}\int_{t_0}^{t} H(t,s)z(s)q(s)=\infty,$$

$$\limsup_{t\to\infty}\frac{1}{H(t,t_0)}\int_{t_0}^{t}\left[\frac{\kappa r(s)}{4z(s)H(t,s)}\left(H(t,s)A(s) + z^{\sigma}(s)H_s^{\Delta}(t,s)\right)^2\right]\Delta s<\infty,$$

then every solution of (1) is oscillatory.

Remarks 2.9. [3, Remarks 2.3] Let $H(t,s)=(t-s)^n$, $(t,s)\in\mathbb{D}$ with $n>1$, we see that H belongs to the class $\mathfrak{R}$. Hence
$((t-s)^n)^{\Delta}\le -n(t-\sigma(s))^{n-1}$.

Corollary 2.10. Assume that $(H_1)-(H_3)$ hold.Let $z(t)$ be positive real rd-functions differentiable function . If

$$\limsup_{t\to\infty}\frac{1}{t^n}\int_{t_0}^{t}\left[(t-s)^n z(s)q(s)-\frac{\kappa r(s)\phi^2(t,s)}{4z(s)(t-s)^n}\right]\Delta s=\infty,$$

where
$\phi(t,s)=(t-s)^nA(s)+nz^{\sigma}(t)(t-\sigma(s))^{n-1}, t\ge s\ge t_0,\ n>1,$
then equation (1) is oscillatory on $[t_0,\infty)$.

3. Equation (1) with $p(t)=0$.

We establish some sufficient conditions for oscillation of Eq. (1) with $p(t)=0$.

Theorem 3.1 Assume that $(H_1)-(H_3)$ hold. Furthermore, assume that there exists a positive real rd-continuous function $z(t)$ such that

$$\limsup_{t\to\infty}\int_{t_0}^{t}\left[z(s)q(s)-\frac{\kappa r(s)}{4z(s)}A^2(s)\right]\Delta s=\infty \qquad (15)$$

then every solution of Eq. (1) is oscillatory.

Proof. Suppose to the contrary that $x(t)$ is a nonoscillatory solution of (1) and let $t_1 \geq t_0$ be such that $x(t) \neq 0$ for all $t \geq t_1$, so without loss of generality, we may assume that $x(t)$ is an eventually positive solution of (1) with $x(t) > 0$ for all $t \geq t_1$ sufficiently large. In view of Theorem 2.1 we see that $x^{\Delta}(t)$ is eventually negative or eventually positive. If $x^{\Delta}(t)$ is eventually negative, we are then back to second case of Theorem 2.1 and we obtain a contradiction. If $x^{\Delta}(t)$ is eventually positive, we assume that there exists $t_2 \geq t_1$ such that $x^{\Delta}(t) \geq 0$ for $t_2 \geq t_1$ and proceed as in the proof of first case of Theorem 2.1. From (9), we have

$$w^{\Delta}(t) \leq -z(t)q(t) + A(t)\frac{w^{\sigma}(t)}{z^{\sigma}(t)} - z(t)\frac{1}{\kappa(z^{\sigma}(t))^2 r(t)}(w^{\sigma}(t))^2, \tag{16}$$

where

$$A(t) = z^{\Delta}(t) - \frac{z(t)}{r(t)}.$$

The proof is similar to that of Theorem 2.1 and hence is omitted.

Corollary 3.2. Assume that $(H_1)-(H_3)$ hold. If

$$\limsup_{t\to\infty} \int_{t_0}^{t} \left[q(s) - \frac{\kappa}{4r(s)}\right] \Delta s = \infty \tag{17}$$

then equation (1) is oscillatory.

Theorem 3.3. Assume that $(H_1)-(H_3)$ hold.Let $z(t)$ be positive real rd-functions differentiable function and let $H : \mathbb{D} \to \mathbb{R}$ be rd-continuous function such that H belongs to the class $\mathfrak{R}$. If

$$\limsup_{t\to\infty} \frac{1}{H(t,t_0)} \int_{t_0}^{t} \left[H(t,s)z(s)q(s) - \frac{\kappa r(s)C^2(t,s)}{4z(s)H(t,s)}\left(z^{\Delta}(s) - \frac{z(s)}{r(s)}\right)^2\right] \Delta s = \infty,$$

where

$$C(t,s) = z^{\Delta}(s)H_s^{\Delta}(t,s) + H(t,s),$$

then equation (1) is oscillatory.

Corollary 2.4. Assume that $(H_1)-(H_3)$ hold. Let $z(t) = 1$. If

$$\limsup_{t\to\infty} \frac{1}{H(t,t_0)} \int_{t_0}^{t} H(t,s)\left(q(s) - \frac{\kappa}{4r(s)}\right) \Delta s = \infty,$$

then every solution of (1) is oscillatory.

Acknowledgement

This work was supported by Research Fund of the Erciyes University. Project Number: FBA-11-3391.

References

[1] S. Hilger, Analysis on measure chains a unified approach to continuous and discrete calculus, Results Math. 18 , 18-56, (1990).
[2] R.P. Agarwal, M. Bohner, D. O'Regan, A. Peterson, Dynamic equations on time scales: A survey, in: R.P. Agarwal, M. Bohner, D. O'Regan (Eds.), Special Issue on Dynamic Equations on Time Scales , J. Comput. Appl. Math. 141 (12) , 1-26, (2002).
[3] M. Bohner, A. Peterson, Dynamic Equations on Time Scales: An Introduction with Applications, Birkhäuser, Boston, 2001.
[4] R.P. Agarwal, M. Bohner, S.H. Saker, Oscillation criteria for nonlinear perturbed dynamic equations of second-order on time scales , J. Appl. Math. and Computing, 20, No.1-2, 133-147, (2006).
[5] Samir H. Saker, Ravi P. Agarwal, Donal O'Regan, Oscillation of second-order damped dynamic equations on time scales, J. Math. Anal. and App., 330 , 1317-1337, (2007).
[6] M. Bohner, L. Erbe, A. Peterson, Oscillation for second order dynamic equations on time scale, J. Math., Anal., App., 301 , 491-507, (2005) .
[7] L. Erbe, A. Peterson, Riccati equations on a measure chain, in: G.S. Ladde, N.G. Medhin, M. Sambandham (Eds.), Proceedings of Dynamic Systems and Applications, vol. 3, Dynamic publishers, Atlanta, pp. 193-199, (2001).
[8] S.R. Grace, R.P. Agarwal, B. Kaymakçalan, W. Sae-jie, Oscillation theorems for second order nonlinear dynamic equations, J. Appl. Math. Comput. 32 , 205-218, (2010).
[9] Taher S. Hassan, Lynn Erbe, Allan Peterson, Oscillation Theorems of Second Order Superlinear Dynamic Equations with Damping on Time Scales, Com. Math. Appl., 59 , 550-558, (2010).
[10] M. Bohner, S.H. Saker, Oscillation of second order nonlinear dynamic equations on time scales, Rocky Mountain J. Math. 34, 1239-1254, (2004).
[11] M. Bohner, S.H. Saker, Oscillation criteria for perturbed nonlinear dynamic equations, Math. Comput. Modelling, 40 , 249-260, (2004).
[12] L. Erbe, Oscillation criteria for second order linear equations on a time scale, Canad. Appl. Math. Quart., 9 , 1-31, (2001).
[13] S.R. Grace, Oscillation Theorems for nonlinear differential equations of second order, J. of Math., Anal., App., 171 , 220-241, (1992).
[14] L. Erbe, A. Peterson, Boundedness and oscillation for nonlinear dynamic equations on a time scale, Proc. Amer. Math. Soc., 132 , 735-744, (2004).
[15] L. Erbe, A. Peterson, S.H. Saker, Oscillation criteria for second-order nonlinear dynamic equations on time scales, J. London Math. Soc. 67, 701-714, (2003).
[16] L. Erbe, A. Peterson, S.H. Saker, Asymptotic behavior of solutions of a third-order nonlinear dynamic equation on time scales, J. Comput. Appl. Math. 181 , 92-102, (2005).
[17] S.H. Saker, Oscillation of nonlinear dynamic equations on time scales, Appl. Math. Comput. 148 , 81-91, (2004).
[18] S.H. Saker, Oscillation criteria of second-order half-linear dynamic equations on time scales, J. Comput. Appl. Math. 177 , 375-387, (2005).
[19] C.C. Yeh, Oscillation theorems for nonlinear second order differential equations with damped term, Proc. Amer. Math. Soc. 84 , 397-402, (1982).

[20] Şenel, M. T., ”Oscillation theorems for second-order damped dynamic equation on time scales”, AIP Conference Proceedings (ISI) , 251-254, DOI: 10.1063/1.4747688, (2012).
[21] Şenel, M. T., Oscillation theorems for dynamic equation on time scales, Bull. Math. Anal. Appl., 3, no.4, 101-105, (2011).