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A Perishable Inventory System with Repeated Customers and Server Interruptions

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Abstract: In this article, we present a continuous review perishable inventory system with a service facility. The service facility consists of a single server and a finite waiting room. If the arriving primary customer finds the waiting hall is empty and the server is idle, he immediately joins the service. The demanded items are issued by a server after some random time due to service on it. We assume that the service may interrupted due to some physical phenomena and the service resumes after repair. An arriving primary customer finds the waiting room is full is permitted to enter into orbit otherwise the customer waits for his service in the waiting hall. The customers in the orbit are called repeated customers and they retry for their service after some random time. The inventory is replenished according to an (s,S) ordering policy. The joint probability distribution of the number of customers in the waiting area, the number of customers in the orbit and the inventory level is obtained for the steady state case. Some important system performance measures in the steady state are derived. Several numerical examples are presented to illustrate the effect of the system parameters.

Keywords: Continuous review inventory system, Perishable item, Service facility, (s,S) policy, Repeated customer, Service interruption, Repair.

1 Introduction

Inventory models considered in the literature, the demanded items are directly issued from the stock, if available. The demands that occurred during stock out period are either not satisfied (lost sales case) or satisfied only after the replenishment of the ordered items (backlog case). Latter it is assumed either all (full backlog case) or only a fixed number of demands (partial backlogging) that occurred during stock out period are satisfied. The case of inventories maintained at service facilities, the demanded items are issued to the customers only after some service is performed on it. In this situation the items are issued not immediately to the demand but after a random time of service. This forces the formation of queues in this type of models. This necessitates the study of both the inventory level and the queue length joint distributions. Study of such models is beneficial to organizations which

– Provide service to customers by using items from a stock.
– Maintain stock of items each of which needs service such as assembly or initialization or installation, etc.

Examples for the first type include firms that are engaged in servicing consumer products such as Television sets, Computers, etc., and for the second type include firms that supply bicycles which need assembly of its parts, that supply food items which need heating or garnishing and that computers which need installation of basic services.

The concept of retrial demands in inventory was introduced by Artalejo et al. [1]. They have assumed Poisson demand, exponential lead time and exponential retrial time. In that work, the authors proceeded with an algorithmic analysis of the system. Ushakumari [16] considered a retrial inventory system with classical retrial policy. As a variant, we consider a continuous review retrial inventory system with server interruptions in this paper. For a brief review of retrial queues, see Artalejo ([2], [3]), Artalejo et al. [4], Artalejo et al. [5], Falin [8] and Falin and Templeton [9].

Krishnamoorthy and Anbazhagan [11] analyzed a perishable queueing-inventory system with N policy, Poisson arrivals, exponential distributed lead times and service times. The joint probability distributions of the number of customers in the system and the inventory

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level were obtained in the steady state case. Two other papers where an inventory model with service time is considered, are by Krishnamoorthy et.al ([12], [13]). The article [12] is considered for an inventory model with instantaneous replenishment and the service process is subject to interruptions. The discussion in [13] is an inventory model with positive lead time, server interruptions and an orbit of infinite capacity, where no waiting space is provided for customers, other than for the one whose service gets interrupted.

In this paper, we consider a continuous review (s,S) retrial inventory system with server interruptions, in which the server provides K types of heterogeneous service and each arriving customer has the option of choosing either type of service. The joint probability distribution of the number of customers in the waiting hall, number of customers in the orbit and the inventory level is obtained in the steady state case. Various system performance measures are derived and the total expected cost rate is calculated.

The rest of the paper is organized as follows. In the next section, we describe the mathematical model and the notations used in this paper are defined. Analysis of the model and the steady state solution of the model are dealt with in section 3. Some key system performance measures are derived in section 4. In section 5, we calculate the total expected cost rate. In section 6, we provide some interesting numerical examples. The last section is meant for conclusion.

2 Mathematical model

We consider a continuous review inventory system with a stock of maximum S units. The system consists of a single server and a finite waiting hall of size N. The primary customers arrive according to a Poisson process with parameter $\lambda(>0)$. The items are issued by a server to the customer after some service time due to the service performed on the items. The service time follows a negative exponential distribution with parameter μ. An arriving primary customer finds the waiting hall is empty and the server is idle, he immediately joins the service. We assume K types of services are available at service facility. The customer chooses type i service with probability $p_i,\ i=1,2,\ldots,K$ and $\sum_{i=1}^{K} p_i = 1$. Any arriving customer, who finds the waiting room is full, are permitted to enter into orbit of finite size M. The customers in the orbit are called repeated customers. They retry for their service after some random time. We assume the time between two successive retrials is an exponential random variable with parameter θ. In this article, we assumes the classical retrial policy. That is, the rate of retrial, when the repeated attempt in an interval $(t,t+dt)$ given that there are i customers in the orbit at time t is $i\theta + o(dt)$.

While the server serves a customer, the service may get interrupted with the interruption process governed by a Poisson process with parameter ν_1. It is assumed that the server is under interruption, no further interruption can befall the server. On completion of an interruption the service resumes, with the duration of an interruption exponentially distributed with parameter ν_2. The demanding customers receive their service one by one and they demand a single item. The operating policy is (s,S) ordering policy. According to that, an order for $Q(=S-s>s+1)$ items are placed whenever the inventory level drops to s and the items are received only after a random time which has exponential distribution with parameter $\beta(>0)$. The life time of each item has negative exponential distribution with parameter $\gamma(>0)$. We have assumed that an item cannot be perished while in service. The customer finds both the waiting hall and the orbit full, is considered to be lost. Various stochastic processes involved in the system are independent.

2.1 Notations

$\mathbf{0}$: Zero matrix

$[A]_{ij}$: entry at $(i,j)^{th}$ position of a matrix A

$$H(x) : \begin{cases} 1, \text{ if } x \geq 0, \\ 0, \text{ otherwise.} \end{cases}$$

$$\delta_{ij} : \begin{cases} 1, \text{ if } j = i \\ 0, \text{ otherwise} \end{cases}$$

$\bar{\delta}_{ij} : 1 - \delta_{ij}$

$k \in V_i^j : k = i, i+1, \ldots j$

$$Y(t) : \begin{cases} 0, \text{ if the server is idle at time t} \\ 1, \text{ if the server is busy at time t} \\ 2, \text{ if the server is on interruption at time t} \end{cases}$$

$$\underset{i=j}{\overset{k}{\Omega}} c_i : \begin{cases} c_j c_{j-1} \cdots c_k & \text{if } j \geq k \\ 1 & \text{if } j < k \end{cases}$$

$\mathbf{e}^T : (1,1,\ldots,1)$

3 Analysis

Let $L(t)$, $Y(t)$, $X(t)$ and $Z(t)$ respectively, denote the inventory level, the server status, the number of customers (waiting and being served) in the waiting room and the number of customers in the orbit at time t. From the assumptions made on the input and output processes, it can be shown that the stochastic process $\{(L(t),Y(t),X(t),Z(t)), t \geq 0\}$ is a continuous time Markov chain with state space given by E.

where $E = E_1 \cup E_2 \cup E_3$ with

$$E_1 : \{(0,0,i_3,i_4) \mid i_3 = 0,1,2,\ldots,N,\ i_4 = 0,1,2,\ldots,M,\}$$
$$E_2 : \{(i_1,0,0,i_4) \mid i_1 = 1,2,\ldots,S,\ i_4 = 0,1,2,\ldots,M,\}$$
$$E_3 : \{(i_1,i_2,i_3,i_4) \mid i_1 = 1,2,\ldots,S,\ i_2 = 1,2,\ i_3 = 1,2,\ldots,N, i_4 = 0,1,2,\ldots,M\}$$

The infinitesimal generator of this process is

$$\Theta = (\ a((i_1,i_2,i_3,i_4);(j_1,j_2,j_3,j_4))\ ),\quad (i_1,i_2,i_3,i_4),(j_1,j_2,j_3,j_4) \in E$$

The infinitesimal generator Θ can be written in terms of sub matrices $\Theta_{i_1 j_1}$, namely,

$$\Theta = ((\Theta_{i_1 j_1})),$$

where

$$\Theta_{i_1 j_1} = \begin{cases} A_{i_1} & j_1 = i_1,\ i_1 = 0,1,2,\ldots,S \\ B_{i_1} & j_1 = i_1 - 1,\ i_1 = 1,2,\ldots,S-1,S \\ C & j_1 = i_1 + Q,\ i_1 = 1,2,\ldots,s, \\ C_1 & j_1 = i_1 + Q,\ i_1 = 0, \\ \mathbf{0} & \text{Otherwise.} \end{cases}$$

More explicitly,

$$\Theta = \begin{array}{c} S \\ S-1 \\ \vdots \\ s+1 \\ s \\ s-1 \\ \vdots \\ 1 \\ 0 \end{array} \begin{pmatrix} A_S & B_S & & & & & & & \\ & A_{S-1} & B_{S-1} & & & & & & \\ & & \cdots & & & & & & \\ & & & \cdots & A_{s+1} & B_{s+1} & & & \\ C & & & & & A_s & B_s & & \\ & C & & & & & A_{s-1} & & \\ & & \cdots & & & & \cdots & & \\ & & C & & & & & \cdots & A_1 \ B_1 \\ & & & C_1 & & & & & A_0 \end{pmatrix}$$

where

$$[C]_{i_2 j_2} = \begin{cases} C_{00}^{(0)} & j_2 = i_2, \quad i_2 = 0 \\ C_{11}^{(0)} & j_2 = i_2, \quad i_2 = 1 \\ C_{22}^{(0)} & j_2 = 2, \quad i_2 = 2 \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[C_{00}^{(0)}]_{i_3 j_3} = \begin{cases} W_{00}^{(0)} & j_3 = 0, \quad i_3 = 0 \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[W_{00}^{(0)}]_{i_4 j_4} = \begin{cases} \beta, & j_4 = i_4, \quad i_4 \in V_0^M \\ 0, & \text{otherwise.} \end{cases}$$

$$[C_{11}^{(0)}]_{i_3 j_3} = \begin{cases} W_{00}^{(0)} & j_3 = i_3, \quad i_3 \in V_1^N \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[C_{22}^{(0)}]_{i_3 j_3} = \begin{cases} W_{00}^{(0)} & j_3 = i_3, \quad i_3 \in V_1^N \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[C_1]_{i_2 j_2} = \begin{cases} C_{00}^{(1)} & j_2 = 0, \quad i_2 = 1 \\ C_{01}^{(1)} & j_2 = 1, \quad i_2 = 0 \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[C_{00}^{(1)}]_{i_3 j_3} = \begin{cases} W_{00}^{(0)} & j_3 = 0, \quad i_3 = 0 \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[C_{01}^{(1)}]_{i_3 j_3} = \begin{cases} W_{00}^{(0)} & j_3 = i_3, \quad i_3 \in V_1^N \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[B_1]_{i_2 j_2} = \begin{cases} B_{00}^{(1)} & j_2 = 0, \quad i_2 = 0 \\ B_{10} & j_2 = 0, \quad i_2 = 1 \\ B_{11} & j_2 = 1, \quad i_2 = 1 \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[B_{00}^{(1)}]_{i_3 j_3} = \begin{cases} W_1 & j_3 = 0, \quad i_3 = 0 \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[W_1]_{i_4 j_4} = \begin{cases} \gamma, & j_4 = i_4, \quad i_4 \in V_0^M \\ 0, & \text{otherwise.} \end{cases}$$

$$[B_{10}]_{i_3 j_3} = \begin{cases} C_2 & j_3 = 0, \quad i_3 = 1 \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[C_2]_{i_4 j_4} = \begin{cases} \mu & j_4 = i_4, \quad i_4 \in V_0^M \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[B_{11}]_{i_3 j_3} = \begin{cases} C_2 & j_3 = i_3 - 1,\ i_3 \in V_2^M \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

For $i_1 = 2,3,\ldots,S,$

$$[B_{i_1}]_{i_2 j_2} = \begin{cases} B_{00}^{(i_1)} & j_2 = 0, \quad i_2 = 0 \\ B_{10}^{(i_1)} & j_2 = 0, \quad i_2 = 1 \\ B_{11}^{(i_1)} & j_2 = 1, \quad i_2 = 1 \\ B_{22}^{(i_1)} & j_2 = 2, \quad i_2 = 2 \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[B_{00}^{(i_1)}]_{i_3 j_3} = \begin{cases} W_{i_1} & j_3 = i_3, \quad i_3 = 0 \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[W_{i_1}]_{i_2 j_2} = \begin{cases} i_1\gamma, & j_4 = i_4, \quad i_4 \in V_0^M \\ 0, & \text{otherwise.} \end{cases}$$

$$[B_{10}^{(i_1)}]_{i_3 j_3} = \begin{cases} C_2 & j_3 = i_3, \quad i_3 = 1 \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[B_{11}^{(i_1)}]_{i_3 j_3} = \begin{cases} T_{(i1-1)} & j_3 = i_3, \quad i_3 \in V_1^N \\ C_2 & j_3 = i_3 - 1,\ i_3 \in V_2^N \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[T_{(i_1-1)}]_{i_4j_4} = \begin{cases} (i_1-1)\gamma, \ j_4 = i_4, & i_4 \in V_0^M \\ 0, & \text{otherwise.} \end{cases}$$

$$[A_0]_{i_2j_2} = \begin{cases} A_{00}^{(0)} \ j_2 = 0, & i_2 = 0 \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[A_{00}^{(0)}]_{i_3j_3} = \begin{cases} C_3 \ j_3 = i_3+1, & i_3 \in V_0^{N-1} \\ C_4 \ j_3 = i_3, & i_3 \in V_0^{N-1} \\ C_5 \ j_3 = i_3, & i_3 = N \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[C_3]_{i_4j_4} = \begin{cases} \lambda, \ j_4 = i_4, & i_4 \in V_0^M \\ 0, & \text{otherwise.} \end{cases}$$

$$[C_4]_{i_4j_4} = \begin{cases} -(\lambda+\beta), \ j_4 = i_4, & i_4 \in V_0^M \\ 0, & \text{otherwise.} \end{cases}$$

$$[C_5]_{i_4j_4} = \begin{cases} \lambda, & j_4 = i_4+1, i_4 \in V_0^{M-1} \\ -(\bar{\delta}_{i_4M}\lambda \\ +\beta), & j_4 = i_4, i_4 \in V_0^M \\ 0, & \text{otherwise.} \end{cases}$$

For $i_1 = 1,2,3,\ldots,S$,

$$[A_{i_1}]_{i_2j_2} = \begin{cases} A_{00}^{(i_1)} \ j_2 = 0, & i_2 = 0 \\ A_{01} \ j_2 = 1, & i_2 = 0 \\ A_{11}^{(i_1)} \ j_2 = 1, & i_2 = 1 \\ A_{12} \ j_2 = 2, & i_2 = 1 \\ A_{21} \ j_2 = 1, & i_2 = 2 \\ A_{22}^{(i_1)} \ j_2 = 2, & i_2 = 2 \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[A_{00}^{(i_1)}]_{i_3j_3} = \begin{cases} D^{(i_1)} \ j_3 = i_3, & i_3 = 0 \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[D^{(i_1)}]_{i_4j_4} = \begin{cases} -(\lambda+ \\ i_1\gamma+i_3\theta+\beta), & j_4 = i_4, i_4 \in V_0^M \\ 0, & \text{otherwise.} \end{cases}$$

$$[A_{10}]_{i_3j_3} = \begin{cases} C_6 \ j_3 = 1, & i_3 = 0 \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[C_6]_{i_4j_4} = \begin{cases} \lambda \ \ j_4 = i_4, & i_4 \in V_0^M \\ i_4\theta \ j_4 = i_4-1, & i_4 \in V_1^M \\ 0, & \text{otherwise.} \end{cases}$$

$$[A_{11}^{(i_1)}]_{i_3j_3} = \begin{cases} C_3 \ \ j_3 = i_3+1, & i_3 \in V_1^{N-1} \\ E^{(i_1)} \ j_3 = i_3, & i_3 \in V_1^{N-1} \\ G^{(i_1)} \ j_3 = i_3, & i_3 = N \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[E^{(i_1)}]_{i_4j_4} = \begin{cases} -(\lambda+(i_1-1)\gamma+ \\ H(s-i_1)\beta+ \\ \nu_1+\mu), & j_4 = i_4, i_4 \in V_0^M \\ 0, & \text{otherwise.} \end{cases}$$

$$[G^{(i_1)}]_{i_4j_4} = \begin{cases} \lambda, & j_4 = i_4+1, i_4 \in V_0^{M-1} \\ -(\bar{\delta}_{i_4M}\lambda+(i_1-1)\gamma+ \\ H(s-i_1)\beta+\nu_1+\mu), & j_4 = i_4, i_4 \in V_0^M \\ 0, & \text{otherwise.} \end{cases}$$

$$[A_{12}]_{i_3j_3} = \begin{cases} C_7 \ j_3 = i_3, & i_3 \in V_1^N \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[C_7]_{i_4j_4} = \begin{cases} \nu_1 \ j_4 = i_4, & i_4 \in V_0^M \\ 0, & \text{otherwise.} \end{cases}$$

$$[A_{21}]_{i_3j_3} = \begin{cases} C_8 \ j_3 = i_3, & i_3 \in V_1^N \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[C_8]_{i_4j_4} = \begin{cases} \nu_2 \ j_4 = i_4, & i_4 \in V_0^M \\ 0, & \text{otherwise.} \end{cases}$$

$$[A_{22}^{(i_1)}]_{i_3j_3} = \begin{cases} C_3 \ \ j_3 = i_3+1, & i_3 \in V_1^{N-1} \\ H^{(i_1)} \ j_3 = i_3, & i_3 \in V_1^{N-1} \\ J^{(i_1)} \ j_3 = i_3, & i_3 = N \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[H^{(i_1)}]_{i_4j_4} = \begin{cases} -(\lambda+(i_1-1)\gamma+ \\ H(s-i_1)\beta+\nu_2), & j_4 = i_4, i_4 \in V_0^M \\ 0, & \text{otherwise.} \end{cases}$$

$$[J^{(i_1)}]_{i_4j_4} = \begin{cases} \lambda, & j_4 = i_4+1, i_4 \in V_0^{M-1} \\ -(\bar{\delta}_{i_4M}\lambda+(i_1-1)\gamma+ \\ H(s-i_1)\beta+\nu_2), & j_4 = i_4, i_4 \in V_0^M \\ 0, & \text{otherwise.} \end{cases}$$

It may be noted that the matrices A_{i_1}, $i_1 = 1,2,\ldots,S$, C and B_{i_1}, $i_1 = 2,3,\ldots,S$, B_1, B_2 and C are square matrices of size $(2N+1)(M+1)$. $C_{00}^{(0)}$, $W_{00}^{(0)}$, $C_{00}^{(1)}$, C_2, C_3, C_4, C_5, C_6, C_7, C_8, $B_{00}^{(i_1)}$, W_{i_1}, $D^{(i_1)}$, $A_{00}^{(i_1)}$, $E^{(i_1)}$, $G^{(i_1)}$, $H^{(i_1)}$, $J^{(i_1)}$, $i_1 = 1,2,\ldots,S$ and $T_{(i_1-1)}$, $i_1 = 2,3,\ldots,S$ are square matrices of size $(M+1)$. C_{01}, B_{11}, A_{12}, A_{21}, $C_{11}^{(0)}$, $C_{22}^{(0)}$, $A_{11}^{(i_1)}$, $A_{22}^{(i_1)}$, $B_{11}^{(i_1)}$, and $B_{22}^{(i_1)}$, $i_1 = 1,2,\ldots,S$ are square matrices of size $N(M+1)$. A_0 and $A_{00}^{(0)}$ are square matrices of size $(N+1)(M+1)$. B_1, C_1, $C_{01}^{(1)}$, B_{10} and A_{01} are matrices of size $(2N+1)(M+1)\times(N+1)(M+1)$, $(N+1)(M+1)\times(2N+1)(M+1)$, $(N+1)(M+1)\times N(M+1)$, $N(M+1)\times M+1$ and $M+1\times N(M+1)$ respectively.

3.1 Steady state analysis

It can be seen from the structure of Θ that the homogeneous Markov process $\{(L(t),Y(t),X(t),Z(t)),t\geq 0\}$ on the finite space E is irreducible. Hence the limiting distribution

$$\phi^{(i_1,i_2,i_3,i_4)} = \lim_{t\to\infty} Pr[L(t)=i_1, Y(t)=i_2, X(t)=i_3, Z(t)=i_4 | L(0), Y(0), X(0), Z(0)],$$

exists. Let

$$\Phi = (\phi^{(\mathbf{0})}, \phi^{(\mathbf{1})}, \ldots, \phi^{(\mathbf{S})})$$

partitioning the vector, $\phi^{(\mathbf{i_1})}$ as follows:

$$\begin{aligned}
\phi^{(\mathbf{0})} &= (\phi^{(\mathbf{0,0})}),\\
\phi^{(\mathbf{0,0})} &= (\phi^{(\mathbf{0,0,0})}, \phi^{(\mathbf{0,0,1})}, \ldots, \phi^{(\mathbf{0,0,N})}),\\
\phi^{(\mathbf{i_1})} &= (\phi^{(\mathbf{i_1,0})}, \phi^{(\mathbf{i_1,1})}, \phi^{(\mathbf{i_1,2})}), \quad i_1 = 1,2,\ldots,S;\\
\phi^{(\mathbf{i_1,0})} &= (\phi^{(\mathbf{i_1,0,0})}), \quad i_1 = 1,2,\ldots,S;\\
\phi^{(\mathbf{i_1,1})} &= (\phi^{(i_1,1,1)}, \phi^{(i_1,1,2)}, \ldots, \phi^{(i_1,1,N)}), i_1 = 1,2,\ldots,S;\\
\phi^{(\mathbf{i_1,2})} &= (\phi^{(i_1,2,1)}, \phi^{(i_1,2,2)}, \ldots, \phi^{(i_1,2,N)}), i_1 = 1,2,\ldots,S;
\end{aligned}$$

which is partitioned as follows,

$$\begin{aligned}
\phi^{(\mathbf{0,0,i_3})} &= (\phi^{(0,0,i_3,0)}, \phi^{(0,0,i_3,1)}, \ldots, \phi^{(0,0,i_3,M)}),\\
&\quad i_3 = 0,1,2,\ldots,N;\\
\phi^{(\mathbf{i_1,0,0})} &= (\phi^{(i_1,0,0,0)}, \phi^{(i_1,0,0,1)}, \phi^{(i_1,0,0,2)}, \ldots, \phi^{(i_1,0,0,M)}),\\
&\quad i_1 = 1,2,\ldots,S;\\
\phi^{(\mathbf{i_1,1,i_3})} &= (\phi^{(i_1,1,i_3,0)}, \phi^{(i_1,1,i_3,1)}, \ldots, \phi^{(i_1,1,i_3,M)}),\\
&\quad i_1 = 1,2,\ldots,S;\ i_1 = 1,2,\ldots,N;\\
\phi^{(\mathbf{i_1,2,i_3})} &= (\phi^{(i_1,2,i_3,0)}, \phi^{(i_1,2,i_3,1)}, \ldots, \phi^{(i_1,2,i_3,M)}),\\
&\quad i_1 = 1,2,\ldots,S;\ i_1 = 1,2,\ldots,N;
\end{aligned}$$

where $\phi^{(i_1,i_2,i_3,i_4)}$ denotes the steady state probability for the state (i_1,i_2,i_3,i_4) of the process, exists and is given by

$$\Phi\Theta = 0 \quad \text{and} \quad \sum\sum\sum_{(i_1,i_2,i_3,i_4)} \phi^{(i_1,i_2,i_3,i_4)} = 1. \tag{1}$$

The first equation of the above yields the following set of equations:

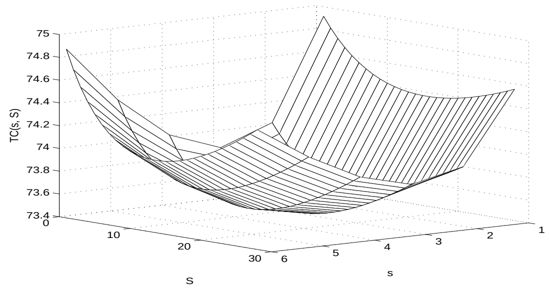
$$\begin{aligned}
\phi^{(i_1)}B_{i_1} + \phi^{(i_1-1)}A_{i_1-1} &= 0,\\
&\quad i_1 = 1,2,\ldots,Q,\\
\phi^{(i_1)}B_{i_1} + \phi^{(i_1-1)}A_{i_1-1} + \phi^{(i_1-1-Q)}C_1 &= 0,\\
&\quad i_1 = Q+1, (*)\\
\phi^{(i_1)}B_{i_1} + \phi^{(i_1-1)}A_{i_1-1} + \phi^{(i_1-1-Q)}C &= 0,\\
&\quad i_1 = Q+2, Q+3, \ldots, S,\\
\phi^{(S)}A_S + \phi^{(s)}C &= 0.
\end{aligned}$$

After lengthy simplifications, the above equations, except $(*)$, yields

$$\begin{aligned}
\phi^{(i_1)} &= (-1)^{Q-i_1}\phi^{(Q)} \underset{j=Q}{\overset{i_1+1}{\Omega}} B_j A_{j-1}^{-1},\\
&\quad i_1 = Q-1, Q-2, \ldots, 0\\
&= (-1)^{2Q-i_1+1}\phi^{(Q)} \times \sum_{j=0}^{S-i_1}\left[\left(\underset{k=Q}{\overset{s+1-j}{\Omega}} B_k A_{k-1}^{-1}\right) C A_{S-j}^{-1} \left(\underset{l=S-j}{\overset{i_1+1}{\Omega}} B_l A_{l-1}^{-1}\right)\right],\\
&\quad i_1 = S, S-1, \ldots, Q+1
\end{aligned}$$

where $\phi^{(Q)}$ can be obtained by solving,

$$\phi^{(Q+1)}B_{Q+1} + \phi^{(Q)}A_Q + \phi^{(0)}C_1 = 0 \text{ and } \sum_{i_1=0}^{S}\phi^{(i_1)}\mathbf{e} = 1,$$



$\lambda = 1.2,\ \beta = 2,\ \gamma = 0.3,\ \theta = 3,\ \mu = 2,\ \nu_1 = 1,\ \nu_2 = 0.5,$
$p_1 = 0.2,\ p_2 = 0.5,\ p_3 = 0.3,\ K = 3,\ N = 6,\ M = 4,$
$c_h = 0.01,\ c_s = 50,\ c_p = 0.5,\ c_l = 0.3,\ c_0 = 2,\ c_i = 6,$
$c_r = 2,\ c_w = 0.4$

Fig. 1: A three dimensional plot of the cost function $TC(S,s,N,M)$

thatis

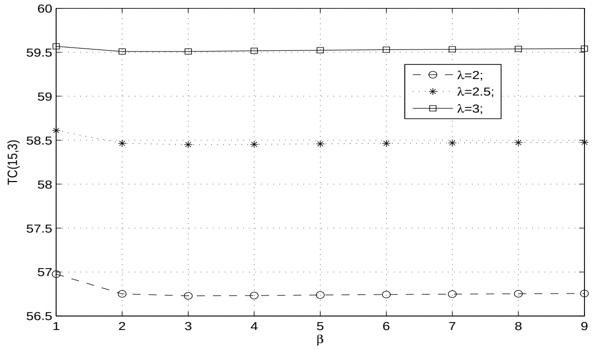
$$\phi^{(Q)}\left[(-1)^Q\sum_{j=0}^{s-1}\left[\left(\underset{k=Q}{\overset{s+1-j}{\Omega}} B_k A_{k-1}^{-1}\right) C A_{S-j}^{-1}\left(\underset{l=S-j}{\overset{Q+2}{\Omega}} B_l A_{l-1}^{-1}\right)\right] \times B_{Q+1} + A_Q + \left\{(-1)^Q \underset{j=Q}{\overset{1}{\Omega}} B_j A_{j-1}^{-1}\right\} C\right] = 0,$$

and

$$\phi^{(Q)}\left[\sum_{i_1=0}^{Q-1}\left((-1)^{Q-i_1}\underset{j=Q}{\overset{i_1+1}{\Omega}} B_j A_{j-1}^{-1}\right) + I + \sum_{i_1=Q+1}^{S}\sum_{j=0}^{S-i_1}(-1)^{2Q-i_1+1}\left(\underset{k=Q}{\overset{s+1-j}{\Omega}} B_k A_{k-1}^{-1}\right) C \times A_{S-j}^{-1}\left(\underset{l=S-j}{\overset{i_1+1}{\Omega}} B_l A_{l-1}^{-1}\right)\right]\mathbf{e} = 1.$$

4 System performance measures

In this section some performance measures of the system under consideration in the steady state are derived.



$\gamma=0.3,\ \theta=3,\ \mu=2,\ \nu_1=1,\ \nu_2=0.5,\ p_1=0.2,\ p_2=0.5,$
$p_3=0.3,\ K=3,\ N=6,\ M=4,\ c_h=0.01,\ c_s=50,$
$c_p=0.5,\ c_l=0.3,\ c_0=2,\ c_i=6,\ c_r=2,\ c_w=0.4$

Fig. 2: Variation of β vs λ on the cost function

4.1 Expected inventory level

Let η_{IL} denote the average inventory level in the steady state. Then

$$\eta_{IL}=\sum_{i_1=1}^{S}\sum_{i_4=0}^{M} i_1\phi^{(i_1,0,0,i_4)}+\sum_{i_1=1}^{S}\sum_{i_2=1}^{2}\sum_{i_3=1}^{N}\sum_{i_4=0}^{M} i_1\phi^{(i_1,i_2,i_3,i_4)}$$

4.2 Expected reorder rate

Let η_{RR} denote the expected reorder rate in the steady state. Then

$$\eta_{RR}=\sum_{i_4=0}^{M}(s+1)\gamma\phi^{(s+1,0,0,i_4)}+\sum_{i_2=1}^{2}\sum_{i_3=1}^{N}\sum_{i_4=0}^{M}s\gamma\phi^{(s+1,i_2,i_3,i_4)}+\sum_{i_3=1}^{N}\sum_{i_4=0}^{M}\{\sum_{r=1}^{K}p_r\mu\}\phi^{(s+1,1,i_3,i_4)}$$

4.3 Expected perishable rate

Let η_{PR} denote the expected perishable rate in the steady-state. Then

$$\eta_{PR}=\sum_{i_1=1}^{S}\sum_{i_4=0}^{M}i_1\gamma\phi^{(i_1,0,0,i_4)}+\sum_{i_1=1}^{S}\sum_{i_2=1}^{2}\sum_{i_3=1}^{N}\sum_{i_4=0}^{M}(i_1-1)\gamma\phi^{(i_1,i_2,i_3,i_4)}$$

4.4 Expected interruption rate

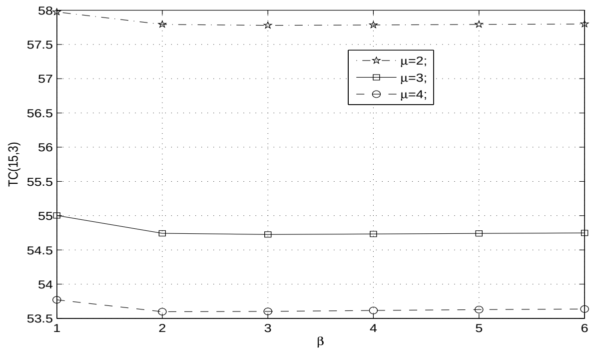
Let η_{INTR} denote the effective interruption rate in the steady state. Then

$$\eta_{INTR}=\sum_{i_1=1}^{S}\sum_{i_3=1}^{N}\sum_{i_4=0}^{M}\nu_1\phi^{(i_1,1,i_3,i_4)}$$

4.5 Expected repair rate

Let η_{REP} denote the effective repair rate is given by

$$\eta_{REP}=\sum_{i_1=1}^{S}\sum_{i_3=1}^{N}\sum_{i_4=0}^{M}\nu_2\phi^{(i_1,2,i_3,i_4)}$$



$\lambda=1.2,\ \gamma=0.3,\ \theta=3,\ \nu_1=1,\ \nu_2=0.5,\ p_1=0.2,\ p_2=0.5,$
$p_3=0.3,\ K=3,\ N=6,\ M=4,\ c_h=0.01,\ c_s=50,\ c_p=0.5,$
$c_l=0.3,\ c_0=2,\ c_i=6,\ c_r=2,\ c_w=0.4$

Fig. 3: Variation of β vs μon the cost function

4.6 Expected number of customers lost

Let η_{LL} denote the expected number of customers lost in the steady-state. Then

$$\eta_{LL}=\lambda\phi^{(0,0,N,M)}+\sum_{i_1=1}^{S}\sum_{i_2=1}^{2}\lambda\phi^{(i_1,i_2,N,M)}$$

4.7 Expected number of customers in the orbit

Let η_{OO} denote the expected number of customers in the orbit. Then

$$\eta_{OO}=\sum_{i_3=0}^{N}\sum_{i_4=1}^{M}i_4\phi^{(0,0,i_3,i_4)}+\sum_{i_1=1}^{S}\sum_{i_4=1}^{M}i_4\phi^{(i_1,0,0,i_4)}+\sum_{i_1=1}^{S}\sum_{i_2=1}^{2}\sum_{i_3=1}^{N}\sum_{i_4=1}^{M}i_4\phi^{(i_1,i_2,i_3,i_4)}$$

4.8 Expected waiting time in the waiting room

Let η_{WW} denote the expected waiting time of the primary customers in the waiting room. Then

$$\eta_{WW} = \frac{\Gamma}{\eta_{AR}},$$

where

$$\Gamma = \sum_{i_3=1}^{N}\sum_{i_4=0}^{M} i_3\phi^{(0,0,i_3,i_4)} + \sum_{i_1=1}^{S}\sum_{i_2=1}^{2}\sum_{i_3=1}^{N}\sum_{i_4=0}^{M} i_3\phi^{(i_1,i_2,i_3,i_4)}$$

and the effective arrival rate (Ross [15]), η_{AR} is given by

$$\eta_{AR} = \sum_{i_3=0}^{N-1}\sum_{i_4=0}^{M} \lambda\phi^{(0,0,i_3,i_4)} + \sum_{i_1=1}^{S}\sum_{i_4=0}^{M} \lambda\phi^{(i_1,0,0,i_4)} + \sum_{i_1=1}^{S}\sum_{i_2=1}^{2}\sum_{i_3=1}^{N-1}\sum_{i_4=0}^{M} \lambda\phi^{(i_1,i_2,i_3,i_4)}$$

4.9 Probability that server is busy

Let P_B denote probability that server is busy is given by

$$P_B = \sum_{i_1=1}^{S}\sum_{i_3=1}^{N}\sum_{i_4=0}^{M} \phi^{(i_1,1,i_3,i_4)}$$

4.10 Probability that server is idle

Let P_{ID} denote probability that server is idle is given by

$$P_{ID} = \sum_{i_1=1}^{S}\sum_{i_4=0}^{M} \phi^{(i_1,0,0,i_4)} + \sum_{i_3=0}^{N}\sum_{i_4=0}^{M} \pi^{(0,0,i_3,i_4)}$$

4.11 Probability that server is on interruption

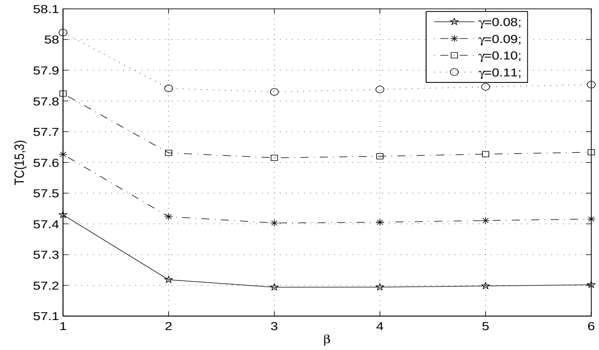
Let P_{IN} denote probability that server is on interruption is given by

$$P_{IN} = \sum_{i_1=1}^{S}\sum_{i_3=1}^{N}\sum_{i_4=0}^{M} \phi^{(i_1,2,i_3,i_4)}$$

4.12 The overall rate of retrials

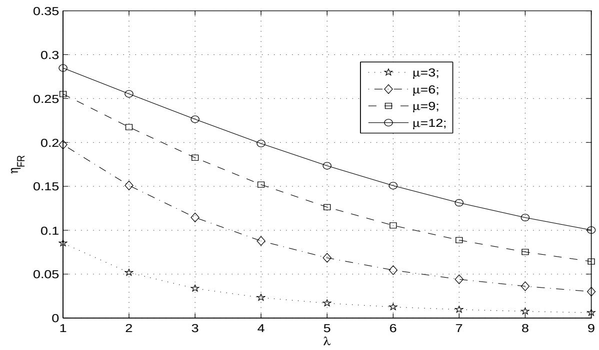
The overall rate of retrials at which the orbiting customers request his demand is given by

$$\eta_{RT} = \sum_{i_3=0}^{N}\sum_{i_4=1}^{M} (i_4\theta)\phi^{(0,,0,i_3,i_4)} + \sum_{i_1=1}^{S}\sum_{i_4=1}^{M} (i_4\theta)\phi^{(i_1,0,0,i_4)} + \sum_{i_1=1}^{S}\sum_{i_2=1}^{2}\sum_{i_3=1}^{N}\sum_{i_4=1}^{M} (i_4\theta)\phi^{(i_1,i_2,i_3,i_4)}$$



$\lambda = 1.2, \theta = 3, \mu = 2, \nu_1 = 1, \nu_2 = 0.5, p_1 = 0.2, p_2 = 0.5, p_3 = 0.3, K = 3, N = 6, M = 4, c_h = 0.01, c_s = 50, c_p = 0.5, c_l = 0.3, c_0 = 2, c_i = 6, c_r = 2, c_w = 0.4$

Fig. 4: Variation of β vs γ on the cost function



$\beta = 2, \gamma = 0.3, \theta = 3, \nu_1 = 1, \nu_2 = 0.5, p_1 = 0.2, p_2 = 0.5, p_3 = 0.3, K = 3, N = 6, M = 4, c_h = 0.01, c_s = 50, c_p = 0.5, c_l = 0.3, c_0 = 2, c_i = 6, c_r = 2, c_w = 0.4$

Fig. 5: Variation of λ vs μ on η_{FR}

4.13 The successful retrial rate

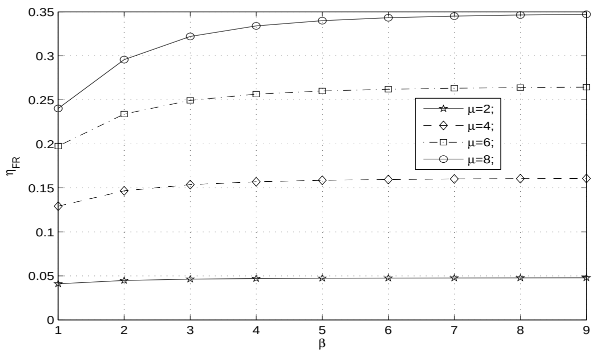
The rate at which the orbiting customers successfully receive his demands is given by

$$\eta_{SR} = \sum_{i_1=1}^{S}\sum_{i_4=1}^{M} (i_4\theta)\pi^{(i_1,0,0,i_4)}$$

4.14 Fraction of successful rate of retrials

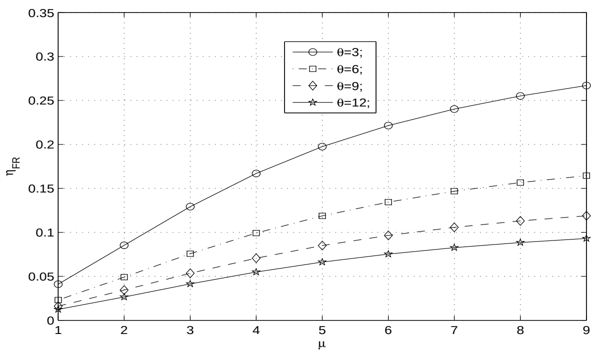
The fraction of successful rate of retrials is given by

$$\eta_{FR} = \frac{\eta_{SR}}{\eta_{OR}}$$



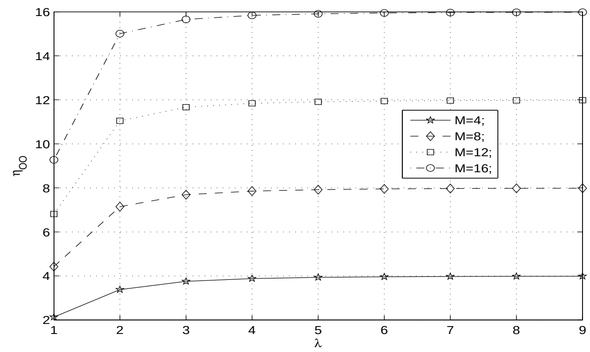
$\lambda=1.2,\ \gamma=0.3,\ \theta=3,\ \nu_1=1,\ \nu_2=0.5,\ p_1=0.2,\ p_2=0.5,\ p_3=0.3,\ K=3,\ N=6,\ M=4,\ c_h=0.01,\ c_s=50,\ c_p=0.5,\ c_l=0.3,\ c_0=2,\ c_i=6,\ c_r=2,\ c_w=0.4$

Fig. 6: Variation of β vs μ on η_{FR}



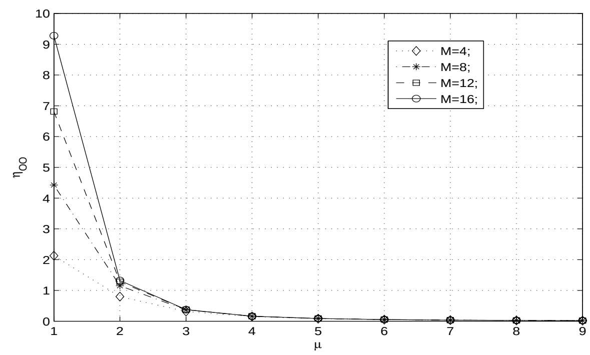
$\lambda=1.2,\ \beta=2,\ \gamma=0.3,\ \nu_1=1,\ \nu_2=0.5,\ p_1=0.2,\ p_2=0.5,\ p_3=0.3,\ K=3,\ N=6,\ M=4,\ c_h=0.01,\ c_s=50,\ c_p=0.5,\ c_l=0.3,\ c_0=2,\ c_i=6,\ c_r=2,\ c_w=0.4$

Fig. 7: Variation of μ vs θ on η_{FR}



$\beta=2,\ \gamma=0.3,\ \theta=3,\ \mu=2,\ \nu_1=1,\ \nu_2=0.5,\ p_1=0.2,\ p_2=0.5,\ p_3=0.3,\ K=3,\ N=6,\ c_h=0.01,\ c_s=50,\ c_p=0.5, c_l=0.3,\ c_0=2,\ c_i=6,\ c_r=2,\ c_w=0.4$

Fig. 8: Variation of λ vs M on η_{OO}



$\lambda=1.2,\ \beta=2,\ \gamma=0.3,\ \theta=3, \nu_1=1,\ \nu_2=0.5,\ p_1=0.2,\ p_2=0.5,\ p_3=0.3,\ K=3,\ N=6,\ c_h=0.01,\ c_s=50,\ c_p=0.5,\ c_l=0.3,\ c_0=2,\ c_i=6,\ c_r=2,\ c_w=0.4$

Fig. 9: Variation of μ vs M on η_{OO}

5 Cost analysis

To compute the total expected cost per unit time (total expected cost rate), the following costs, are considered,

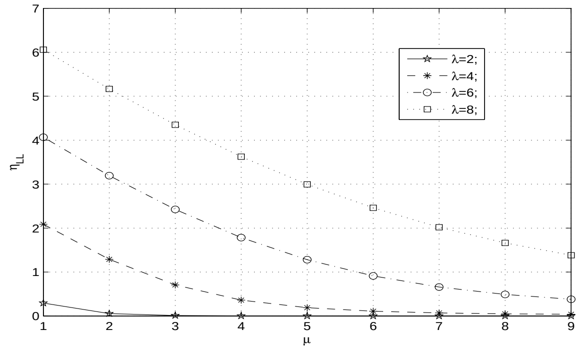
c_h : The inventory carrying cost per unit item per unit time
c_s : Set up cost (ordering cost) per order
c_p : Failure cost per unit item per unit time
c_w : Waiting time cost of a primary customer per unit time
c_o : Waiting time cost of a orbiting customer per unit time
c_l : cost due to loss of customers per unit per unit time,
c_i : cost per interruption per unit time,
c_r : cost per repair per unit time,

The long run total expected cost rate is given by

$$TC(s,S) = c_h\eta_{IL} + c_s\eta_{RR} + c_p\eta_{PR} + c_w\eta_{WW} + c_o\eta_{OO} + c_l\eta_{LL} + c_i\eta_{INTR} + c_r\eta_{REP}$$

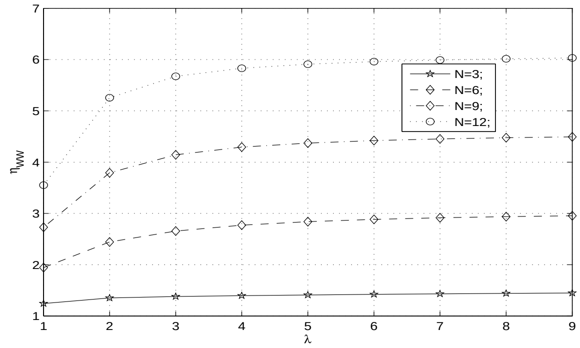
Substituting the values of η's we get TC(S, s, N, M)=

$$c_s\sum_{i_4=0}^{M}(s+1)\gamma\phi^{(s+1,0,0,i_4)} + c_s\sum_{i_3=1}^{N}\sum_{i_2=1}^{2} s\gamma\phi^{(s+1,i_2,i_3,i_4)} +$$
$$c_s\sum_{i_3=1}^{N}\{\sum_{r=1}^{K}p_r\mu\}\phi^{(s+1,1,i_3,i_4)} + c_o\sum_{i_4=1}^{M}\sum_{i_3=0}^{N} i_4\phi^{(0,0,i_3,i_4)} +$$
$$c_o\sum_{i_1=1}^{S}\sum_{i_4=1}^{M}\sum_{i_2=1}^{2}\sum_{i_3=1}^{N} i_4\phi^{(i_1,i_2,i_3,i_4)} + c_o\sum_{i_1=1}^{S}\sum_{i_4=1}^{M} i_4\phi^{(i_1,0,0,i_4)} +$$
$$c_h\sum_{i_1=1}^{S}\sum_{i_4=0}^{M} i_1\phi^{(i_1,0,0,i_4)} + c_h\sum_{i_1=1}^{S}\sum_{i_2=1}^{2}\sum_{i_3=1}^{N}\sum_{i_4=0}^{M} i_1\phi^{(i_1,i_2,i_3,i_4)} +$$



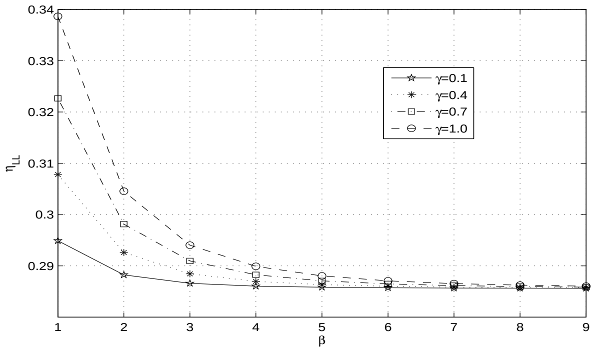
$\beta=2,\ \gamma=0.3,\ \theta=3,\ \nu_1=1,\ \nu_2=0.5,\ p_1=0.2,\ p_2=0.5,\ p_3=0.3,\ K=3,\ N=6,\ M=4,\ c_h=0.01,\ c_s=50,\ c_p=0.5,\ c_l=0.3,\ c_0=2,\ c_i=6,\ c_r=2,\ c_w=0.4$

Fig. 10: Variation of μ vs λ on η_{LL}



$\lambda=1.2,\ \beta=2,\ \gamma=0.3,\ \theta=3,\ \mu=2,\ \nu_1=1,\ \nu_2=0.5,\ p_1=0.2,\ p_2=0.5,\ p_3=0.3,\ K=3,\ N=6,\ M=4,\ c_h=0.01,\ c_s=50,\ c_p=0.5,\ c_l=0.3,\ c_0=2,\ c_i=6,\ c_r=2,\ c_w=0.4$

Fig. 12: Variation of λ vs N on η_{WW}



$\lambda=1.2,\ \theta=3,\ \mu=2,\ \nu_1=1,\ \nu_2=0.5,\ p_1=0.2,\ p_2=0.5,\ p_3=0.3,\ K=3,\ N=6,\ M=4,\ c_h=0.01,\ c_s=50,\ c_p=0.5,\ c_l=0.3,\ c_0=2,\ c_i=6,\ c_r=2,\ c_w=0.4$

Fig. 11: Variation of β vs γ on η_{LL}

$$c_p\sum_{i_1=1}^{S}\sum_{i_4=0}^{M} i_1\gamma\phi^{(i_1,0,0,i_4)} + c_p\sum_{i_2=1}^{2}\sum_{i_3=1}^{N}(i_1-1)\gamma\phi^{(i_1,i_2,i_3,i_4)} +$$

$$c_l\lambda\phi^{(0,0,N,M)} + c_l\sum_{i_1=1}^{S}\sum_{i_2=1}^{2}\lambda\phi^{(i_1,i_2,N,M)} +$$

$$c_r\sum_{i_1=1}^{S}\sum_{i_3=1}^{N}\sum_{i_4=0}^{M}\nu_2\phi^{(i_1,2,i_3,i_4)} + c_i\sum_{i_1=1}^{S}\sum_{i_3=1}^{N}\sum_{i_4=0}^{M}\nu_1\phi^{(i_1,1,i_3,i_4)} +$$

$$c_w\frac{\Gamma}{\eta_{AR}}c_i$$

Due to the complex form of the limiting distribution, it is difficult to discuss the properties of the cost function analytically. Hence, a detailed computational study of the cost function is carried out numerically.

6 Numerical Illustrations

In this section, we discuss some numerical examples that reveal the possible convexity of the total expected cost rate. A typical three dimensional plot of the total expected cost function is given in Figure 1. We have studied the effect of varying the cost and other system parameters on the optimal values and the system performance measures and also the results agreed with what one would expect.

*Example 1.*First, we explore the behaviour of the cost function by considering as the function of two variables by fixing the others at a constant level. Tables $1-5$, give the total expected cost rate as a function of $TC(S,s,6,4)$, $TC(S,3,4,M)$, $TC(34,s,N,9)$ $TC(S,3,N,4)$, and $TC(34,s,4,M)$. Towards this end, we first fix the parameter and cost values as $\lambda=1.2,\ \beta=2,\ \gamma=0.3,\ \theta=3,\ \mu=2,\ \nu_1=1,\ \nu_2=0.5,\ p_1=0.2,\ p_2=0.5,\ p_3=0.3,\ c_h=0.01,\ c_s=50,\ c_p=0.5,\ c_l=0.3,\ c_0=2,\ c_i=6,\ c_r=2,\ c_w=0.4$. In each table, underlined value denotes the row minimum and in **bold** faced value denotes the column minimum. Hence the both underlined and **bold** faced value refers the optimal value of the function.

*Example 2.*Here, we study the effect of the primary demand rate λ, the service time μ, the lead time rate β, the retrial demand rate θ and the perishable rate γ on the total expected cost rate. From figure 2 to figure 4, we observe the following:

- The optimal expected cost rate increases when λ increases.
- As is to be expected, β increases the total expected cost rate decreases.
- Again the optimal expected cost rate increases when μ and γ increase.

Table 1: Variation of S and s on total expected cost rate

S \ s	1	2	3	4	5
31	74.447710	73.623077	73.434792	73.526976	73.771360
32	74.422260	73.607249	73.420889	73.510470	73.748832
33	74.402000	73.596821	73.412924	73.500686	73.734050
34	74.386477	73.591299	**73.410327**	**73.496951**	73.726207
35	74.375293	**73.590245**	73.412597	73.498678	**73.724601**
36	**74.368091**	73.593274	73.419293	73.505350	73.728620
37	74.368555	73.600042	73.430025	73.516514	73.737730

Table 2: Variation of S and M on total expected cost rate

S \ M	3	4	5	6	7
38	90.760815	90.760717	90.760707	90.760709	90.760710
39	90.735353	90.735259	90.735250	90.735252	90.735254
40	90.719468	90.719378	90.719370	90.719372	90.719373
41	**90.712454**	**90.712369**	**90.712361**	**90.712363**	**90.712365**
42	90.713677	90.713596	90.713589	90.713591	90.713593
43	90.722559	90.722483	90.722476	90.722478	90.722479
44	90.738576	90.738504	90.738497	90.738499	90.738501

*Example 3.*In this example, we look at the impact of the primary demand rate λ, the service time μ, the lead time rate β, the retrial demand rate θ and the perishable rate γ on the fraction, η_{FR} of successful retrial from the orbit. From figure 5 to figure 7, we observe the following:

–The fraction of successful rate of retrial, η_{FR} decreases when the primary arrival rate increases.
–As is to be expected as the mean retrial time decreases, η_{FR} decreases.
–When β and μ increase η_{FR} increases.

*Example 4.*In this example, we illustrate of the effect of the primary demand rate λ, the service time μ, the lead time rate β, the retrial demand rate θ and the perishable rate γ on η_{OO}. From figure 8 and figure 9, we observe the following:

–As is to be expected as the primary arrival rate increases, η_{OO} increases.
–When μ increase η_{OO} decreases.

*Example 5.*In this example, we monitored the effect of the primary demand rate λ, the service time μ, the lead time rate β, the retrial demand rate θ and the perishable rate γ on η_{LL}. From figure 10 and figure 11, we observe the following:

–As is to be expected as the primary arrival rate increases, we lost more customers.
–And also we restrict our customer loss by increasing the service rate
–The customer loss direct proportional to the life time of the item and inversely to the β.

*Example 6.*In this example, we calculate the effect of the primary demand rate λ on η_{ww}. From figure 12, we observe the following:

–The expected waiting time increases when as the primary arrival rate increases.
–As is to be expected the waiting time increases when N increases.

Table 3: Variation of s and N on total expected cost rate

s \ N	2	3	4	5	6
2	75.561406	75.537180	75.581821	75.645130	75.714295
3	74.866968	74.813387	74.835157	74.876939	74.925165
4	74.584315	74.508598	74.511703	74.538037	74.571493
5	**74.541601**	**74.449009**	**74.437206**	**74.451010**	**74.473402**
6	74.654382	74.548528	74.524776	74.528235	74.541612
7	74.877131	74.760501	74.726987	74.721804	74.727578
8	75.184026	75.058348	75.016652	75.004119	75.005342

Table 4: Variation of S and N on total expected cost rate

S \ N	4	5	6	7	8
27	87.493601	85.841454	85.817954	86.099374	86.524422
28	87.463552	85.813757	85.792280	86.074663	86.500125
29	87.444781	85.796910	85.777199	86.060403	86.486202
30	**87.436156**	**85.789827**	**85.771646**	**86.055546**	**86.481609**
31	87.436697	85.791563	85.774699	86.059179	86.485440
32	87.445544	85.801291	85.785551	86.070506	86.496905
33	87.461946	85.818288	85.803495	86.088827	86.515310

Table 5: Variation of s and M on total expected cost rate

s \ M	23	24	25	26	27
7	57.704030	57.703918	57.703901	57.703974	57.704129
8	57.682001	57.681882	57.681859	57.681925	57.682075
9	57.675840	57.675715	57.675685	57.675746	57.675891
10	**57.685810**	**57.685679**	**57.685646**	**57.685703**	**57.685844**
11	57.712258	57.712126	57.712091	57.712146	57.712285
12	57.755635	57.755504	57.755469	57.755525	57.755666
13	57.816500	57.816374	57.816344	57.816406	57.816552

7 Conclusion

The stochastic model discussed here is useful in studying a perishable inventory system at a service facility with server interruptions, repeated customers and (s,S) policy. The joint probability distribution of the number of customers in the waiting hall, number of customers in the orbit and the inventory level is derived in the steady state. Various system performance measures are derived and the long-run total expected cost rate is calculated. By assuming a suitable cost structure on the inventory system, we have presented extensive numerical illustrations to show the effect of change of values for constants on the total expected cost rate. The authors are working in the direction of MAP (Markovian arrival process) arrival for the customers and service times follow PH-distributions.

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