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Explicit Travelling Wave Solutions of Two Nonlinear Evolution Equations

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Abstract: In this paper, we applied the sine-cosine method and the rational functions in exp(ksi) method for the modified Kawachara equation and the Damped Sixth-order Boussinesq Equation, respectively. New solitons solutions and periodic solutions are explicitly obtained with the aid of symbolic computation.

Keywords: Travelling wave solutions, the sine-cosine method,the rational functions in exp(ksi) method

1 Introduction

We are living in a nonlinear world. So many physical phenomenon modelled by nonlinear partial differential equations. Therefore solutions of these partial differential equations will help us to much more understanding these physical processes. In the last decades, many methods proposed for obtaining explicit traveling wave solutions of nonlinear evolution equations such as the rational functions in $exp(\xi)$ method [1], tanh method [2,3], sine-cosine method [4], the exp-function method [5], the tanh-coth method [6], the (G'/G)-expansion method [7, 9], the solitary wave ansatz method [10,17], the variational iteration method [18], the multiplier approach method [19] and so on.

In this paper, we establish solitons and periodic solutions to the modified Kawachara equation, which describes the motion of a water waves with surface tension

$$u_t + u_x + u^2 u_x + p u_{xxx} + q u_{xxxx} = 0, \tag{1}$$

p and q are constants [20] and the sixth-order Boussinesq equation with damping term

$$u_{tt} - u_{xx} - u_{xxtt} - u_{xxxxxx} - a u_{xxt} = \left(u^2\right)_{xx} \tag{2}$$

where is a real constant. It describes the bidirectional propagation of small amplitude long capillary-gravity waves on the surface of shallow water [21]. Local, global and asymptotic behavior of solution this equation studied by Polat and Pişkin [22] and blow up of the solution of this equation studied by Pişkin [23].

2 Analysis of the methods

A partial differential equation (PDE)

$$P(u, u_t, u_x, u_{xx}, \ldots) = 0 \tag{3}$$

can be converted to an ordinary differential equation (ODE)

$$Q(u, u', u'', u''', \ldots) = 0, \tag{4}$$

upon using a wave variable $u(x,t) = u(\xi)$, $\xi = x - ct$ where u' denotes $\frac{\partial u}{\partial \xi}$. Then (4) is integrated as long as all terms contain derivatives where integration constants are considered zeros.

2.1 The sine-cosine method

The sine-cosine method was developed by Wazwaz [4] and was successfully applied to nonlinear evolution equations [24,27], to nonlinear equations systems [28].

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The solutions of the reduced ODE (4) can be expressed in the form

$$u(x,t)=\lambda\cos^{\beta}(\mu\xi),\ |\xi|\leq\frac{\pi}{2\mu} \quad (5)$$

or in the form

$$u(x,t)=\lambda\sin^{\beta}(\mu\xi),\ |\xi|\leq\frac{\pi}{\mu} \quad (6)$$

where μ,λ and β are parameters that will be determined, $\xi=x-ct$, μ and c are the wave number and the wave speed, respectively.

The assumption (5) gives

$$\begin{aligned}(u^n)'(\xi)&=-n\beta\mu\lambda^n\cos^{n\beta-1}(\mu\xi)\sin(\mu\xi),\\(u^n)''(\xi)&=-n^2\beta^2\mu^2\lambda^n\cos^{n\beta}(\mu\xi)+n\mu^2\lambda^n\beta(n\beta-1)\cos^{n\beta-2}(\mu\xi),\end{aligned} \quad (7)$$

where similar equations can be obtained for the sine assumption. Substituting the sine-cosine assumptions and their derivatives into the reduced ODE gives a trigonometric equation of $\sin^R(\mu\xi)$ or $\cos^R(\mu\xi)$ terms. The parameters are then determined by first balancing the exponents of each pair of cosine to determine R. We next collect all coefficients of the same power in $\cos^k(\mu\xi)$ where these coefficients have to vanish. This gives a system of algebraic equations among the unknowns μ,λ and β that will be determined. The solutions proposed in (5) and (6) follow immediately.

2.2 The rational functions in $exp(\xi)$ method

This method firstly proposed by B. Q. Lu and et al. in 1993 [1]. Later studied by many researchers [29,30].

In this method, we shall seek a rational function type of solution for a given partial differential equation, in terms of $exp(\xi)$ of the following form

$$U=\sum_{k=0}^{m}\frac{a_k}{(1+e^{\xi})^k} \quad (8)$$

where $a_0,a_1,\ldots,a_m$ are some constants to be determined from the solution of (4).

Differentiating (8) with respect to ξ, introducing the result into (4) and setting the coefficients of the same power of equal to zero, we obtain algebraic equations. The rational function solution of the (3) can be solved by obtaining $a_0,a_1,\ldots,a_m$ from this system.

3 Application of the sine-cosine method

In this section, we will first use the sine-cosine method to develop solitary wave solutions to the modified Kawachara equation.

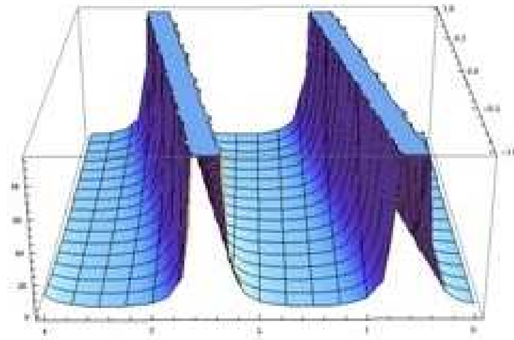

Fig. 1: The periodic solutions of (13) when c=3, p=-2.

Using the wave variable $\xi=x-ct$, (2) into an ODE

$$(1-c)u+\frac{u^2}{3}+pu''+qu^{(4)}=0 \quad (9)$$

Substituting the cosine assumption (5) into (9) gives

$$\begin{aligned}&(1-c)\lambda\cos^{\beta}(\mu\xi)+\tfrac{\lambda^3}{3}\cos^{3\beta}(\mu\xi)-p\mu^2\beta^2\lambda\cos^{\beta}(\mu\xi)+\\&p\lambda\mu^2\beta(\beta-1)\cos^{\beta-2}(\mu\xi)+q\mu^4\beta^4\lambda\cos^{\beta}(\mu\xi)\\&-2q\mu^4\lambda\beta(\beta-1)(\beta^2-2\beta+2)\cos^{\beta-2}(\mu\xi)+\\&q\mu^4\lambda\beta(\beta-1)(\beta-2)(\beta-3)\cos^{\beta-4}(\mu\xi)=0.\end{aligned} \quad (10)$$

Equating the exponents and the coefficients of like powers of cosine function leads to

$$\begin{aligned}\beta(\beta-1)(\beta-2)(\beta-3)&\neq0,\\\beta-4&=3\beta,\\(1-c)\lambda-4p\mu^2\lambda+16q\mu^4\lambda&=0,\\p\mu^2\lambda-120q\mu^4\lambda&=0,\\\tfrac{\lambda^3}{3}+120q\mu^4\lambda&=0.\end{aligned} \quad (11)$$

Solving this system (11) yields

$$\begin{aligned}&\beta=-2,\\&\mu=\mp\tfrac{1}{4}\sqrt{\tfrac{5(1-c)}{p}},p\neq0\\&\lambda=\mp\tfrac{3}{2}\sqrt{\tfrac{5(c-1)}{2}},\\&c=\tfrac{-4p^2+25q}{25q},q\neq0.\end{aligned} \quad (12)$$

This leads, for $\frac{1-c}{p}>0$, the following periodic solutions

$$u_{1,2}(x,t)=\mp\tfrac{3}{2}\sqrt{\tfrac{5(c-1)}{2}}\sec^2\left(\tfrac{1}{4}\sqrt{\tfrac{5(1-c)}{p}}(x-ct)\right),\quad \left|\tfrac{1}{4}\sqrt{\tfrac{5(1-c)}{p}}(x-ct)\right|<\tfrac{\pi}{2} \quad (13)$$

and

$$u_{3,4}(x,t)=\mp\tfrac{3}{2}\sqrt{\tfrac{5(c-1)}{2}}\csc^2\left(\tfrac{1}{4}\sqrt{\tfrac{5(1-c)}{p}}(x-ct)\right),\quad \tfrac{1}{4}\sqrt{\tfrac{5(1-c)}{p}}(x-ct)<\pi. \quad (14)$$

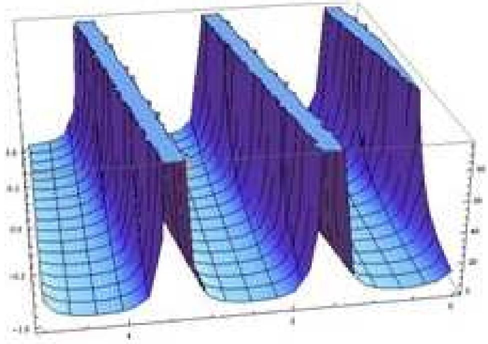

Fig. 2: The periodic solutions of (13) when c=3, p=-2.

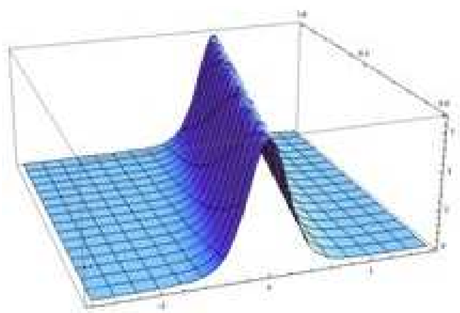

Fig. 3: The soliton solutions of (15) when c=3, p=2.

However, for $\frac{1-c}{p} < 0$, we obtained the solitons solutions

$$u_{5,6}(x,t) = \mp\frac{3}{2}\sqrt{\frac{5(c-1)}{2}}sech^2\left(\frac{1}{4}\sqrt{\frac{5(c-1)}{p}}(x-ct)\right), \tag{15}$$

and

$$u_{7,8}(x,t) = \pm\frac{3}{2}\sqrt{\frac{5(c-1)}{2}}csch^2\left(\frac{1}{4}\sqrt{\frac{5(c-1)}{p}}(x-ct)\right). \tag{16}$$

4 Application of rational function type of solution

Now, we will find a rational function type of solution to the sixth-order Boussinesq equation with damping term, in terms of $exp(\xi)$. Firstly, we make the transformation

$$u(x,t) = U(\xi), \xi = \alpha(x-\beta t) \tag{17}$$

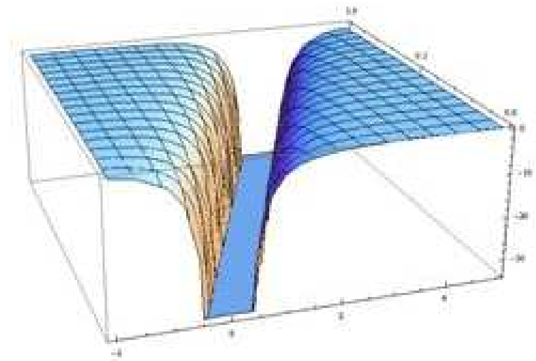

Fig. 4: The soliton solutions of (16) when c=3, p=2

and (2) becomes

$$\left(\beta^2-1\right)U'' + a\alpha\beta U''' - \alpha^2\beta^2U^{(4)} - \alpha^4U^{(6)} = \left(U^2\right)'' \tag{18}$$

Balancing $U^{(6)}$ with $\left(U^2\right)''$ in (18) gives $m=4$. So that, the rational exponential method assumes finite expansion

$$U(\xi) = a_0 + \frac{a_1}{1+e^{\xi}} + \frac{a_2}{\left(1+e^{\xi}\right)^2} + \frac{a_3}{\left(1+e^{\xi}\right)^3} + \frac{a_4}{\left(1+e^{\xi}\right)^4} \tag{19}$$

where $a_j(j=0,1,2,3,4)$ are constants to be determined later. Substituting (19) in (18) and equating the coefficients of the powers e^{ξ}, we then get the following algebraic relations:

$$-a_1 - 2a_0a_1 - a_1\alpha^4 - aa_1\alpha\beta + a_1\beta^2 - a_1\alpha^2\beta^2 = 0, \tag{20a}$$

$$\begin{aligned}&-6a_1 - 12a_0a_1 - 4a_1^2 - 4a_2 - 8a_0a_2 + 54a_1\alpha^4 - 64a_2\alpha^4 - 2a_1\alpha\beta \\ &-8aa_2\alpha\beta + 6a_1\beta^2 + 4a_2\beta^2 + 6a_1\alpha^2\beta^2 - 16a_2\alpha^2\beta^2 = 0,\end{aligned} \tag{20b}$$

$$\begin{aligned}&-14a_1 - 28a_0a_1 - 22a_1^2 - 22a_2 - 44a_0a_2 - 18a_1a_2 - 9a_3 - 18a_0a_3 \\ &-134a_1\alpha^4 + 818a_2\alpha^4 - 729a_3\alpha^4 + 8aa_1\alpha\beta - 26aa_2\alpha\beta - 27aa_3\alpha\beta + \\ &14a_1\beta^2 + 22a_2\beta^2 + 9a_3\beta^2 + 34a_1\alpha^2\beta^2 + 2a_2\alpha^2\beta^2 - 81a_3\alpha^2\beta^2 = 0,\end{aligned} \tag{20c}$$

$$\begin{aligned}&-14a_1 - 28a_0a_1 - 48a_1^2 - 48a_2 - 96a_0a_2 - 84a_1a_2 - 16a_2^2 - 42a_3 \\ &-84a_0a_3 - 32a_1a_3 - 16a_4 - 32a_0a_4 - 434a_1\alpha^4 - 588a_2\alpha^4 + \\ &4998a_3\alpha^4 - 4096a_4\alpha^4 + 34aa_1\alpha\beta - 12aa_2\alpha\beta - 78aa_3\alpha\beta \\ &-64aa_4\alpha\beta + 4a_1\beta^2 + 48a_2\beta^2 + 42a_3\beta^2 + 116a_4\beta^2 + \\ &46a_1\alpha^2\beta^2 + 132a_2\alpha^2\beta^2 - 42a_3\alpha^2\beta^2 - 256a_4\alpha^2\beta^2 = 0,\end{aligned} \tag{20d}$$

$$\begin{aligned}&-50a_1^2 - 50a_2 - 100a_0a_1 - 150a_1a_2 - 60a_2^2 - 75a_3 - 150a_0a_3 \\ &-120a_1a_3 - 50a_2a_3 - 60a_4 - 120a_0a_4 - 50a_1a_4 - 2450a_2\alpha^4 + \\ &-3675a_3\alpha^4 + 21540a_4\alpha^4 50aa_1\alpha\beta + 50aa_2\alpha\beta - 45aa_3\alpha\beta \\ &-140aa_4\alpha\beta + 50a_2\beta^2 + 75a_3\beta^2 + 60a_4\beta^2 + \\ &190a_2\alpha^2\beta^2 + 285a_3\alpha^2\beta^2 - 60a_4\alpha^2\beta^2 = 0,\end{aligned} \tag{20e}$$

$$\begin{aligned}&14a_1+28a_0a_1-20a_1^2-20a_2-40a_0a_2-120a_1a_2-80a_2^2-60a_3\\&-120a_0a_3-160a_1a_3-140a_2a_3-36a_3^2-80a_4-160a_0a_4-140a_1a_4\\&-72a_2a_4+434a_1\alpha^4+280a_2\alpha^4-5460a_3\alpha^4-25880a_4\alpha^4+34aa_1+\\&80aa_2\alpha\beta+60aa_3\alpha\beta\ \ -160a_0a_4-140a_1a_4-72a_2a+434a_1\alpha^4+\\&280a_2\alpha^4-5460a_3\alpha^4-25880a_4\alpha^4+34aa_1+80aa_2\alpha\beta+\\&60aa_3\alpha\beta-40aa_4\alpha\beta-14a_1\beta^2+20a_2\beta^2+60a_3\beta^2+80a_4\beta^2\\&-46a_1\alpha^2\beta^2+40a_2\alpha^2\beta^2+300a_3\alpha^2\beta^2+520a_4\alpha^2\beta^2=0,\end{aligned}\tag{20f}$$

$$\begin{aligned}&14a_1+28a_0a_1+6a_1^2+6a_2+12a_0a_2-30a_1a_2-40a_2^2\\&-15a_3-30a_0a_3-80a_1a_3-120a_2a_3-66a_3^2-40a_4-80a_0a_4\\&-120a_1a_4-132a_2a_4-98a_3a_4+134a_1\alpha^4+1086a_2\alpha^4+\\&3585a_3\alpha^4+8360a_4\alpha^4+8aa_1\alpha\beta+42aa_2\alpha\beta+75aa_3\alpha\beta+\\&80aa_4\alpha\beta-14a_1\alpha^2-6a_2\beta^2+15a_3\beta^2+40a_4\beta^2\\&-34a_1\alpha^2\beta^2-66a_2\alpha^2\beta^2-15a_3\alpha^2\beta^2+200a_4\alpha^2\beta^2=0,\end{aligned}\tag{20g}$$

$$\begin{aligned}&6a_1+12a_0a_1+8a_1^2+8a_2+16a_0a_2+12a_1a_2+6a_3+12a_0a_3\\&-20a_2a_3-24a_3^2-20a_1a_4-48a_2a_4-84a_3a_4-64a_4^2\\&-54a_1\alpha^4-172a_2\alpha^4-354a_3\alpha^4-600a_4\alpha^4-2aa_1\alpha\beta+\\&4aa_2\alpha\beta+18aa_3\alpha\beta+40aa_4\alpha\beta-6a_1\beta^2-8a_2\beta^2-6a_3\beta^2\\&-6a_1\alpha^2\beta^2-28a_2\alpha^2\beta^2-66a_3\alpha^2\beta^2-120a_4\alpha^2\beta^2=0,\end{aligned}\tag{20h}$$

$$\begin{aligned}&a_1+2a_0a_1+2a_1^2+2a_2+4a_0a_2+6a_1a_2+4a_2^2+3a_3+6a_0a_3\quad+\\&8a_1a_3+10a_2a_3+6a_3^2+4a_4+8a_0a_4+10a_1a_4+12a_2a_4+\ 14a_3a_4+\\&8a_4^2+a_1\alpha^4+2a_2\alpha^4+3a_3\alpha^4+4a_4\alpha^4-aa_1\alpha\beta-2aa_2\alpha\beta-3aa_3\alpha\beta\\&-4aa_4\alpha\beta++a_1\alpha^2\beta^2+2a_2\alpha^2\beta^2+3a_3\alpha^2\beta^2+4a_4\alpha^2\beta^2=0.\end{aligned}\tag{20i}$$

When the system (20) solved by aid of Mathematica, we will find the following two sets of solutions

$$\begin{aligned}&\alpha=\tfrac{-i\beta}{\sqrt{13}}\text{ or }\alpha=\tfrac{i\beta}{\sqrt{13}}\\&a_0=\tfrac{-169+169\beta^2+36\beta^4}{338},\\&a_1=0,\\&a_2=\tfrac{-840\beta^4}{169},\\&a_3=-2a_2\\&a_4=a_2\end{aligned}\tag{20}$$

Substituting (20) and (21) in (19), we obtain exact travelling wave solutions for (2) of the form

$$\begin{aligned}u_1(x,t)=&\frac{-169+169\beta^2+36\beta^4}{338}-\frac{840\beta^4}{169\left(1+e^{\frac{-i\beta}{\sqrt{13}}(x-\beta t)}\right)^2}\\&+\frac{1680\beta^4}{169\left(1+e^{\frac{-i\beta}{\sqrt{13}}(x-\beta t)}\right)^3}-\frac{840\beta^4}{169\left(1+e^{\frac{-i\beta}{\sqrt{13}}(x-\beta t)}\right)^4},\end{aligned}\tag{21}$$

and

$$\begin{aligned}u_2(x,t)=&\frac{-169+169\beta^2+36\beta^4}{338}-\frac{840\beta^4}{169\left(1+e^{\frac{i\beta}{\sqrt{13}}(x-\beta t)}\right)^2}\\&+\frac{1680\beta^4}{169\left(1+e^{\frac{i\beta}{\sqrt{13}}(x-\beta t)}\right)^3}-\frac{840\beta^4}{169\left(1+e^{\frac{i\beta}{\sqrt{13}}(x-\beta t)}\right)^4}.\end{aligned}\tag{22}$$

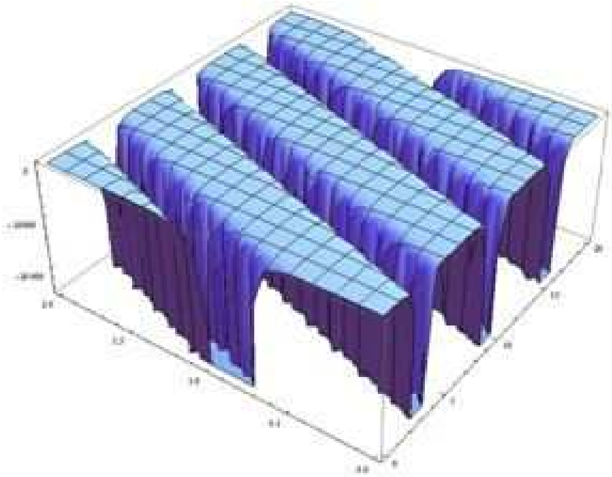

Fig. 5: The soliton solutions of (21) when $\alpha=-i,\beta=\sqrt{13}$

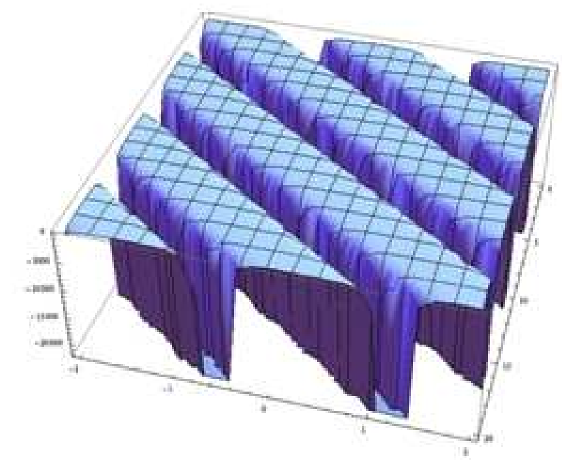

Fig. 6: The soliton solutions of (22) when $\alpha=i,\ \beta=\sqrt{13}$

5 Conclusion

The sine-cosine method and the rational functions in method were effectively used for analytic treatment of the handled equations.

In this paper, we have shown that the sixth-order Boussinesq equation with damping term possess periodic type solution and the modified Kawachara equations possess periodic and solitary type solutions. We believe that some of the obtained solutions are new.

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