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A Mixture of Modified Inverse Weibull Distribution

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Abstract: Mixture distribution of life time distribution occurs in many setting and play very important role in many practical applications. So this article is considered with the mixture of modified inverse Weibull distribution(MMIWD). Statistical properties of the model with some graphs of the density and application of real life data are discussed.

Keywords:Modified inverse Weibull distribution ; Mixture Modified inverse Weibull distribution ; Statistical properties; Reliability and failure rate functions.

1. Introduction

Jiang et al (1999) have shown that the inverse Weibull mixture model with negative weight can be represent the output of o system under certain situation. Sultem et al (2007) investigated the mixture model of two inverse Weibull distributions some properties of the model with graph and hazard rate.Karreema .A Makhrib (2012) considered with the mixture of three inverse Weibull distributions with some statistical properties, graphs and hazard rate are discussed. M. S. Khan and R. King(2012) introduced a generalized version of four parameter modified inverse Weibull distribution(MIWD) which generate the following distribution (1) modified inverse exponential distribution (2) modified inverse Rayleigh distribution (3) inverse Weibull distribution.

Here we introduced a mixture of two modified inverse Weibull distribution (MMIWD). The p.d.f of MIWD$(\alpha_1, \beta_1, \gamma_1, \xi_1, t)$ is given by

$$f_1(t) = \left[\alpha_1 + \beta_1\gamma_1\left(\frac{1}{t-\xi_1}\right)^{\beta_1-1}\right]\left(\frac{1}{t-\xi_1}\right)^2 Exp\left[-\frac{\alpha_1}{t-\xi_1} - \gamma_1\left(\frac{1}{t-\xi_1}\right)^{\beta_1}\right] \alpha_1, \beta_1, \gamma_1 > 0 \text{ and } \xi_1 < t < \infty \tag{1.1}$$

Where β_1 is the shape parameter α_1, γ_1 are the scale parameters and ξ_1 is called the location parameter also called a guarantee time/ failure free time / minimum life. With c.d.f

$$F_1(t) = Exp\left[-\frac{\alpha_1}{t-\xi_1} - \gamma_1\left(\frac{1}{t-\xi_1}\right)^{\beta_1}\right] \tag{1.2}$$

The p.d.f of MIWD$(\alpha_2, \beta_2, \gamma_2, \xi_2, t)$ is given as

$$f_2(t)=\left[\alpha_2+\beta_2\gamma_2\left(\frac{1}{t-\xi_2}\right)^{\beta_2-1}\right]\left(\frac{1}{t-\xi_2}\right)^2 Exp\left[-\frac{\alpha_2}{t-\xi_2}-\gamma_2\left(\frac{1}{t-\xi_2}\right)^{\beta_2}\right]\alpha_2,\beta_2,\gamma_2>0 \text{ and } \xi_2<t<\infty \quad (1.3)$$

with

$$\text{c.d.f } F_2(t)=Exp\left[-\frac{\alpha_2}{t-\xi_2}-\gamma_2\left(\frac{1}{t-\xi_2}\right)^{\beta_2}\right] \quad (1.4)$$

Where β_2 = Shape parameter. α_2,γ_2 = Scale parameters . ξ_2 = location parameter.

2.Mixture Modified IWD

A K-parameter Mixture distribution function is defined as

$$f(t)=\sum_{i=1}^{k}p_i f_i(t) \qquad ; \sum_{i=1}^{k}p_i=1$$

Where $f_i(t)$= ith sub population. p_i=proportion of sub model (Mixture parameter)

For k = 2

$$f(t)=p_1f_1(t)+p_2f_2(t) \qquad ; p_1+p_2=1 \quad (2.1)$$

A mixture of two inverse Weibull distribution can be obtain by using (1.1) and (1.3) in (2.1) we have

$$f(t)=p_1\left(\alpha_1+\beta_1\gamma_1\left(\frac{1}{t-\xi_1}\right)^{\beta_1-1}\right)\left(\frac{1}{t-\xi_1}\right)^2 exp\left(-\frac{\alpha_1}{t-\xi_1}-\gamma_1\left(\frac{1}{t-\xi_1}\right)^{\beta_1}\right)$$

$$+p_2\left(\alpha_2+\beta_2\gamma_2\left(\frac{1}{t-\xi_2}\right)^{\beta_2-1}\right)\left(\frac{1}{t-\xi_2}\right)^2 exp\left(-\frac{\alpha_2}{t-\xi_2}-\gamma_2\left(\frac{1}{t-\xi_2}\right)^{\beta_2}\right) \quad (2.2)$$

with c.d.f $$F(t)=p_1\ exp\left(-\frac{\alpha_1}{t-\xi_1}-\gamma_1\left(\frac{1}{t-\xi_1}\right)^{\beta_1}\right)+p_2\ exp\left(-\frac{\alpha_2}{t-\xi_2}-\gamma_2\left(\frac{1}{t-\xi_2}\right)^{\beta_2}\right) \quad (2.3)$$

Where $\alpha_1,\beta_1,\gamma_1,\xi_1,\alpha_2,\beta_2,\gamma_2,\xi_2>0\ ,\ p_1>0\ , mixture\ parameter \quad \eta<t<\infty$

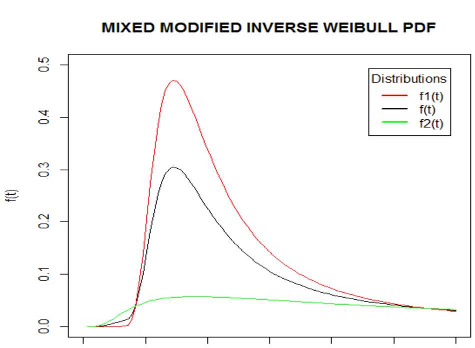


Figure1:Mixed modified inverse Weibull p.d.f

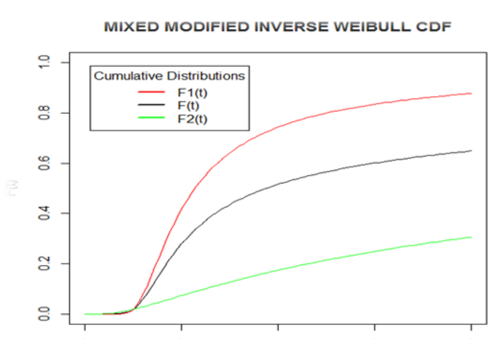


Figure2:Mixed modified inverse Weibull c.d.f

$(\boldsymbol{\alpha_1=1,\ \beta_1=3,\ \gamma_1=3\ ,\alpha_2=1\ ,\beta_2=0.5\ ,\gamma_2=3\ ,p=0.6})$

3. Reliability Analysis

The Mixture MIWD can be useful characterization of life time data analysis. The reliability function or survival function of Mixture MIWD is given by

$$R(t) = 1 - F(t)$$

$$R(t) = 1 - \left(p_1\, exp\left(-\frac{\alpha_1}{t-\xi_1} - \gamma_1\left(\frac{1}{t-\xi_1}\right)^{\beta_1}\right) + p_2\, exp\left(-\frac{\alpha_2}{t-\xi_2} - \gamma_2\left(\frac{1}{t-\xi_2}\right)^{\beta_2}\right)\right) \tag{3.1}$$

On the characterization in reliability analysis is the hazard rate defined as

$$h(t) = \frac{f(t)}{1 - F(t)}$$

The hazard function (HF) of mixture MIWD is also known as institutions failure rate, is given as under

$$h(t) = \frac{p_1\left(\alpha_1 + \beta_1\gamma_1\left(\frac{1}{t-\xi_1}\right)^{\beta_1-1}\right)\left(\frac{1}{t-\xi_1}\right)^2 exp\left(-\frac{\alpha_1}{t-\xi_1} - \gamma_1\left(\frac{1}{t-\xi_1}\right)^{\beta_1}\right) + p_2\left(\alpha_2 + \beta_2\gamma_2\left(\frac{1}{t-\xi_2}\right)^{\beta_2-1}\right)\left(\frac{1}{t-\xi_2}\right)^2 exp\left(-\frac{\alpha_2}{t-\xi_2} - \gamma_2\left(\frac{1}{t-\xi_2}\right)^{\beta_2}\right)}{1 - \left(p_1\, exp\left(-\frac{\alpha_1}{t-\xi_1} - \gamma_1\left(\frac{1}{t-\xi_1}\right)^{\beta_1}\right) + p_2\, exp\left(-\frac{\alpha_2}{t-\xi_2} - \gamma_2\left(\frac{1}{t-\xi_2}\right)^{\beta_2}\right)\right)}$$

It is important to note that the unit for h(t) is the probability of failure per unit of time, distance or cycle.

Proof:

i) If $\beta_1 = 2,\ \beta_2 = 2$ the failure rate is same as the Mixture MIRD$(\alpha_1, \beta_1, \xi_1, \alpha_2, \beta_2, \xi_2, p_1, t)$

$$h(t) = \frac{p_1\left(\alpha_1 + 2\gamma_1\left(\frac{1}{t-\xi_1}\right)\right)\left(\frac{1}{t-\xi_1}\right)^2 exp\left(-\frac{\alpha_1}{t-\xi_1} - \gamma_1\left(\frac{1}{t-\xi_1}\right)^2\right) + p_2\left(\alpha_2 + 2\gamma_2\left(\frac{1}{t-\xi_2}\right)\right)\left(\frac{1}{t-\xi_2}\right)^2 exp\left(-\frac{\alpha_2}{t-\xi_2} - \gamma_2\left(\frac{1}{t-\xi_2}\right)^2\right)}{1 - \left(p_1\, exp\left(-\frac{\alpha_1}{t-\xi_1} - \gamma_1\left(\frac{1}{t-\xi_1}\right)^2\right) + p_2\, exp\left(-\frac{\alpha_2}{t-\xi_2} - \gamma_2\left(\frac{1}{t-\xi_2}\right)^2\right)\right)}$$

ii) If $\beta_1 = 1, \beta_2 = 1$ the failure rate in same Mixture MIED$(\alpha_1, \beta_1, \xi_1, \alpha_2, \beta_2, \xi_2, p_1, t)$

$$h(t) = \frac{p_1[\alpha_1 + \gamma_1]\left(\frac{1}{t-\xi_1}\right)^2 exp\left(-\frac{\alpha_1}{t-\xi_1} - \gamma_1\left(\frac{1}{t-\xi_1}\right)\right) + p_2[\alpha_2 + 2\gamma_2]\left(\frac{1}{t-\xi_2}\right)^2 exp\left(-\frac{\alpha_2}{t-\xi_2} - \gamma_2\left(\frac{1}{t-\xi_2}\right)\right)}{1 - \left[p_1 exp\left(-\frac{\alpha_1}{t-\xi_1} - \gamma_1\left(\frac{1}{t-\xi_1}\right)\right) + p_2\, exp\left(-\frac{\alpha_2}{t-\xi_2} - \gamma_2\left(\frac{1}{t-\xi_2}\right)\right)\right]}$$

iii) If $\alpha_1 = 0, \alpha_2 = 0$ the failure in same Mixture IWD$(\beta_1, \gamma_1, \xi_1, \beta_2, \gamma_2, \xi_2, p_1, t)$

$$h(t)=\frac{p_1\left(\beta_1\gamma_1\left(\frac{1}{t-\xi_1}\right)^{\beta_1-1}\right)\left(\frac{1}{t-\xi_1}\right)^2 exp\left(-\gamma_1\left(\frac{1}{t-\xi_1}\right)^{\beta_1}\right)+p_2\left(\beta_2\gamma_2\left(\frac{1}{t-\xi_2}\right)^{\beta_2-1}\right)\left(\frac{1}{t-\xi_2}\right)^2 exp\left(-\gamma_2\left(\frac{1}{t-\xi_2}\right)^{\beta_2}\right)}{1-\left[p_1 Exp\left(-\gamma_1\left(\frac{1}{t-\xi_1}\right)^{\beta_1}\right)+p_2\, exp\left(-\gamma_2\left(\frac{1}{t-\xi_2}\right)^{\beta_2}\right)\right]}$$

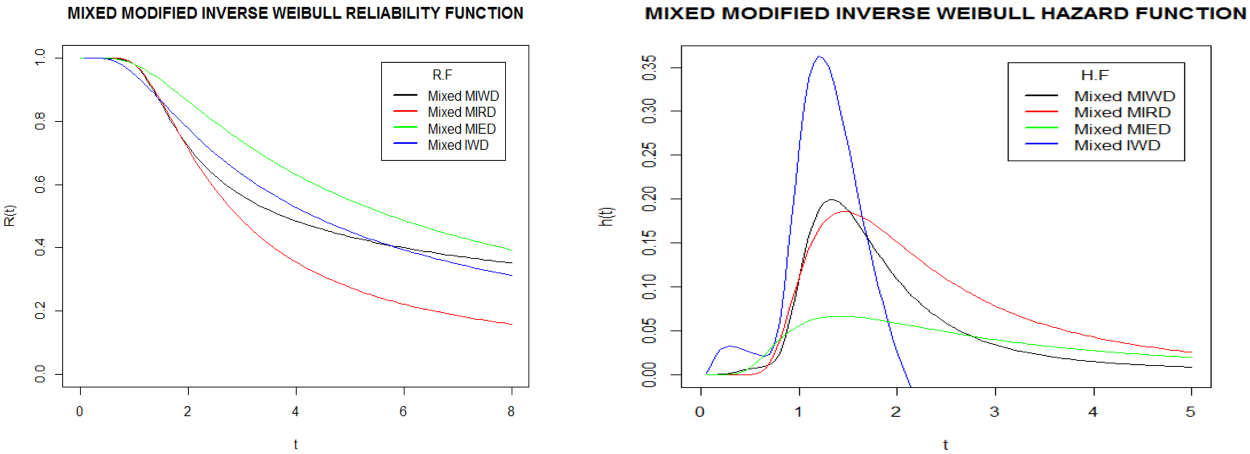


Figure3 :Mixed modified inverse Weibull Reliability & Hazard function(Mixed MIWD)

$(\boldsymbol{\alpha_1=1,\ \beta_1=3,\ \gamma_1=3\ ,\alpha_2=1\ ,\beta_2=0.5\ ,\gamma_2=3\ ,p=0.6})$

Mixed modified inverse Rayleigh Reliability& Hazard function (Mixed MIRD)

$(\boldsymbol{\alpha_1=1,\ \beta_1=2,\ \gamma_1=3\ ,\alpha_2=1\ ,\beta_2=2\ ,\gamma_2=3\ ,p=0.6})$

Mixed modified inverse Exponential Reliability & Hazard function(Mixed MIED)

$(\boldsymbol{\alpha_1=1,\ \beta_1=1,\ \gamma_1=3\ ,\alpha_2=1\ ,\beta_2=1\ ,\gamma_2=3\ ,p=0.6})$

Mixed modified inverse Weibull Reliability& Hazard function(Mixed IWD)

$(\boldsymbol{\alpha_1=0,\ \beta_1=3,\ \gamma_1=3\ ,\alpha_2=0\ ,\beta_2=0.5\ ,\gamma_2=3\ ,p=0.6})$

4. Moments

The following theorem gives the r[th] moment of Mixture MIWD$(\alpha_1,\beta_1,\xi_1,\alpha_2,\beta_2,\xi_2,p_1,t)$

Theorem:

If t has the mixture MIWD$(\alpha_1,\beta_1,\xi_1,\alpha_2,\beta_2,\xi_2,p_1,t)$ the r[th]moment of t, say $\acute{\mu}_r$is given as follows

$$\acute{\mu}_r=\begin{cases} p_1\left[\sum_{i=0}^{\infty}\frac{(-1)^i\gamma_1^i}{i!}\left(\alpha_1{}^{r-i\beta_1}\Gamma(i\beta_1-r+1)+\beta_1\gamma_1\alpha_1^{r-\beta_1(i+1)}\Gamma(\beta_1(i+1)-r)\right)\right]+ \\ p_2\left[\sum_{i=0}^{\infty}\frac{(-1)^i\gamma_2^i}{i!}\left(\alpha_2{}^{r-i\beta_2}\Gamma(i\beta_2-r+1)+\beta_2\gamma_2\alpha_2^{r-\beta_2(i+1)}\Gamma(\beta_2(i+1)-r)\right)\right] for \alpha_1,\beta_1,\gamma_1,\alpha_2,\beta_2,\gamma_2,p_1>0 \\ p_1\left[\gamma_1^{\frac{r}{\beta_1}}\Gamma\left(1-\frac{r}{\beta_1}\right)\right]+p_2\left[\gamma_2^{\frac{r}{\beta_2}}\Gamma\left(1-\frac{r}{\beta_2}\right)\right] for \alpha_1,\alpha_2=0 \\ p_1[\alpha_1^r\Gamma(1-r)]+p_2[\alpha_2^r\Gamma(1-r)] for \beta_1,\gamma_1,\beta_2,\gamma_2=0 \end{cases}$$

The proof of this theorem is provided in appendix. Based on above result, the coefficient of variation (C.V), coefficient of skewness (C.S) and coefficient of kurtosis(C.K) of mixture modified IWD can be obtain to the following relation

$$C.V = \sqrt{\frac{\mu_2}{\mu_1} - 1}$$

$$C.S = \frac{\mu_3 - 3\mu_2\mu_1 + 2\mu_1^3}{(\mu_2 - \mu_1^2)^{\frac{3}{2}}}$$

$$C.K = \frac{\mu_4 - 4\mu_3\mu_1 + 6\mu_2\mu_1^2 - 3\mu_1^4}{(\mu_2 - \mu_1^2)^2}$$

The coefficient of variation is the quantity used for measure the consistency of life time data. The coefficient of skewness is the quantity to measure the skewness of life time data. The coefficient of kurtosis is the quantity used to measure the peakness of the distribution of the life time distribution. So, the above model is helpful for assessing these characteristics.

5. Moment generating

The following gives the M.G.F of Mixture MIWD($\alpha_1, \beta_1, \xi_1, \alpha_2, \beta_2, \xi_2, p_1, t$).

Theorem:

It t has mixture MIWD($\alpha_1, \beta_1, \xi_1, \alpha_2, \beta_2, \xi_2, p_1, t$) the M.G.F of t , say $M_o(t)$ is given

$$M_o(t) = \begin{cases} p_1\left[\sum_{i=0}^{\infty}\frac{(-1)^i\gamma_1^i}{i!}\left[\frac{\alpha_1\Gamma(i\beta_1+1)}{(\alpha_1-1)^{i\beta_1+1}}\right] + \frac{\beta_1\gamma_1\Gamma\left(\beta_1(i+1)\right)}{(\alpha_1-t)^{\beta_1(i+1)}}\right] + \\ p_2\left[\sum_{i=0}^{\infty}\frac{(-1)^i\gamma_2^i}{i!}\left[\frac{\alpha_2\Gamma(i\beta_2+1)}{(\alpha_2-1)^{i\beta_2+1}}\right] + \frac{\beta_2\gamma_2\Gamma\left(\beta_2(i+1)\right)}{(\alpha_2-t)^{\beta_2(i+1)}}\right]_{for\alpha_1,\beta_1,\gamma_1,\alpha_2,\beta_2,\gamma_2,p_1>0} \\ p_1\left[\sum_{i=0}^{\infty}\frac{t^i\gamma_1^{\frac{i}{\beta_1}}}{i!}\Gamma\left(1-\frac{i}{\beta_1}\right)\right] + p_2\left[\sum_{i=0}^{\infty}\frac{t^i\gamma_2^{\frac{i}{\beta_2}}}{i!}\Gamma\left(1-\frac{i}{\beta_2}\right)\right]_{for\alpha_1,\alpha_2=0} \\ p_1\left[\frac{\alpha_1}{\alpha_1-t}\right] + p_2\left[\frac{\alpha_2}{\alpha_2-t}\right]_{for\beta_1,\gamma_1,\beta_2,\gamma_2=0} \end{cases}$$

The proof of theorem is provided in the appendix. Based on the above result the measure of central tendency, measure of dispersion and , C.V, C.K, C.S can be obtained according to the above relation

6. Application on the Lifetimes of the Electronic Components

By using the data for the lifetime of “20”electronic components that were taken from the Murthy et al (1998), we find the mixture of modified inverse Weibull distribution and other function are shown in the following tables

Table 1:Two Modified inverse Weibull Distribution With Parameters $\alpha_1 = 1, \beta_1 = 3, \gamma_1 = 3, \alpha_2 = 1, \beta_2 = 0.5, \gamma_2 = 3$

Obs. No.	x_i	$f_1(x)$	$f_2(x)$	$F_1(x)$	$F_2(x)$	$R_1(x)$	$R_2(x)$	$h_1(x)$	$h_2(x)$
1	0.03	0.0000	0.0000	0.0000	0.0000	1.0000	1.0000	0.0000	1.4E-19
2	0.12	0.0000	0.0000	0.0000	0.0000	1.0000	1.0000	0.0000	4.4E-06
3	0.22	0.0000	0.0006	0.0000	0.0000	1.0000	1.0000	0.0000	0.0006
4	0.35	0.0000	0.0056	0.0000	0.0004	1.0000	0.9996	0.0000	0.0056
5	0.73	0.0038	0.0325	0.0001	0.0076	0.9999	0.9924	0.0038	0.0327
6	0.79	0.0159	0.0361	0.0006	0.0096	0.9994	0.9904	0.0159	0.0364
7	1.25	0.4184	0.0526	0.0967	0.0307	0.9033	0.9693	0.4632	0.0543
8	1.41	0.4691	0.0550	0.1687	0.0393	0.8313	0.9607	0.5643	0.0573
9	1.52	0.4671	0.0560	0.2204	0.0454	0.7796	0.9546	0.5991	0.0587
10	1.79	0.4030	0.0570	0.3390	0.0608	0.6610	0.9392	0.6097	0.0607
11	1.8	0.4000	0.0570	0.3430	0.0613	0.6570	0.9387	0.6088	0.0607
12	1.94	0.3568	0.0569	0.3960	0.0693	0.6040	0.9307	0.5908	0.0611
13	2.38	0.2403	0.0550	0.5259	0.0940	0.4741	0.9060	0.5069	0.0607
14	2.4	0.2361	0.0549	0.5306	0.0951	0.4694	0.9049	0.5030	0.0606
15	2.87	0.1579	0.0516	0.6217	0.1201	0.3783	0.8799	0.4175	0.0587
16	2.99	0.1436	0.0508	0.6397	0.1263	0.3603	0.8737	0.3986	0.0581
17	3.14	0.1281	0.0496	0.6601	0.1338	0.3399	0.8662	0.3768	0.0573
18	3.17	0.1252	0.0494	0.6639	0.1353	0.3361	0.8647	0.3726	0.0571
19	4.72	0.0496	0.0389	0.7863	0.2034	0.2137	0.7966	0.2319	0.0488
20	5.09	0.0418	0.0368	0.8032	0.2174	0.1968	0.7826	0.2122	0.0470

Figure 4 P.d.f for two Modified inverse Weibull distribution, with $\alpha_1 = 1, \beta_1 = 3, \gamma_1 = 3, \alpha_2 = 1, \beta_2 = 1.5, \gamma_2 = 3$

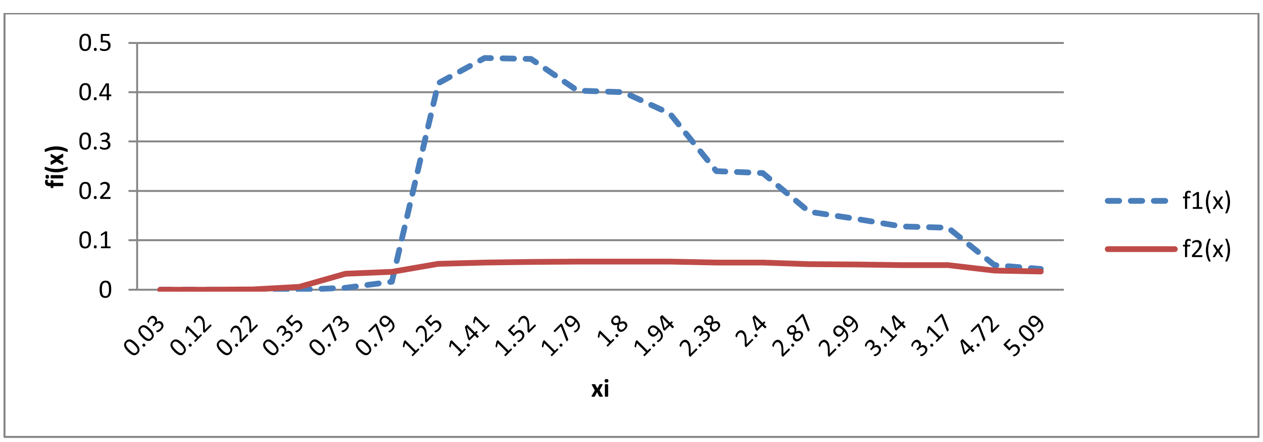


Figure 5 c.d.f for two Modified inverse Weibull distributions, with $\alpha_1 = 1, \beta_1 = 3, \gamma_1 = 3, \alpha_2 = 1, \beta_2 = 1.5, \gamma_2 = 3$

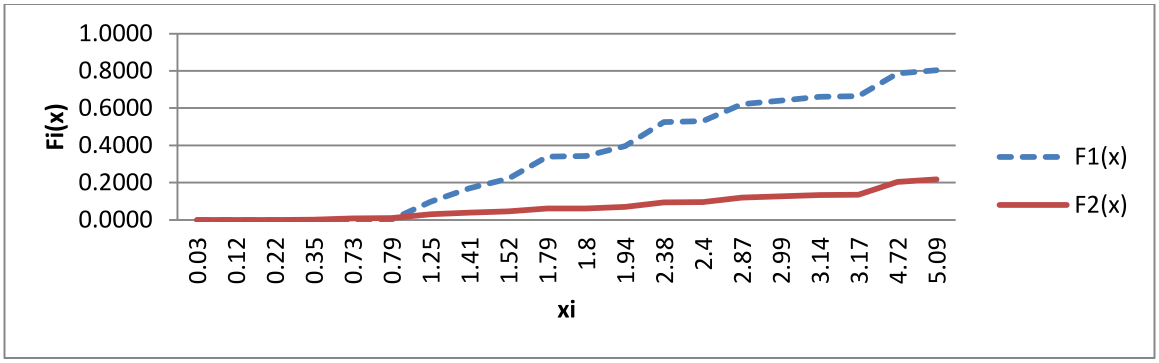


Figure 6 Reliability for two Modified inverse Weibull distribution, with $\alpha_1 = 1, \beta_1 = 3, \gamma_1 = 3, \alpha_2 = 1, \beta_2 = 1.5, \gamma_2 = 3$

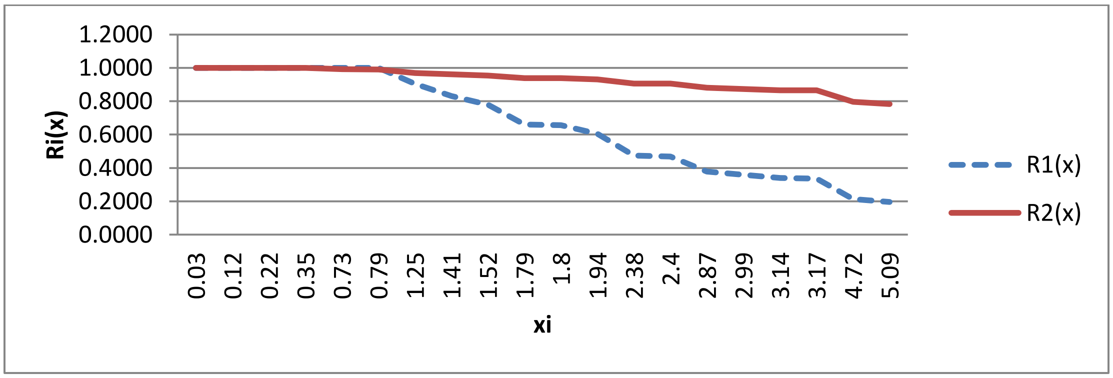


Figure 7 Failure rate for two Modified inverse Weibull distribution, with $\alpha_1 = 1, \beta_1 = 3, \gamma_1 = 3, \alpha_2 = 1, \beta_2 = 1.5, \gamma_2 = 3$

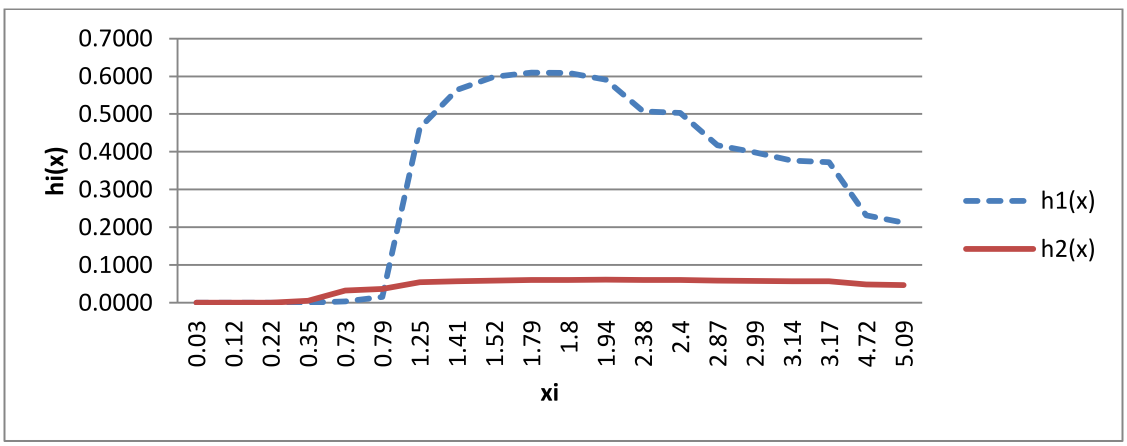


Table 2: Mixture Distribution with the parameter $\alpha_1 = 1, \beta_1 = 3, \gamma_1 = 3, \alpha_2 = 1, \beta_2 = 1.5, \gamma_2 = 3$ **and Mixing parameter;** $p1 = 0.1$

Obs. No.	xi	f(x)	F(x)	R(x)	h(x)
1	0.03	1.264E-19	9.027E-23	1	1.264E-19
2	0.12	3.957E-06	3.750E-08	1	3.957E-06
3	0.22	0.0006	1.594E-05	1.0000	0.0006
4	0.35	0.0050	0.0003	0.9997	0.0050

5	0.73	0.0296	0.0068	0.9932	0.0298
6	0.79	0.0340	0.0087	0.9913	0.0343
7	1.25	0.0892	0.0373	0.9627	0.0926
8	1.41	0.0964	0.0523	0.9477	0.1017
9	1.52	0.0971	0.0629	0.9371	0.1037
10	1.79	0.0916	0.0886	0.9114	0.1005
11	1.8	0.0913	0.0895	0.9105	0.1003
12	1.94	0.0869	0.1020	0.8980	0.0967
13	2.38	0.0735	0.1372	0.8628	0.0852
14	2.4	0.0730	0.1386	0.8614	0.0847
15	2.87	0.0623	0.1703	0.8297	0.0750
16	2.99	0.0600	0.1776	0.8224	0.0730
17	3.14	0.0575	0.1864	0.8136	0.0707
18	3.17	0.0570	0.1881	0.8119	0.0702
19	4.72	0.0399	0.2617	0.7383	0.0541
20	5.09	0.0373	0.2759	0.7241	0.0515

Table 3: Mixture Distribution with the parameter $\alpha_1 = 1, \beta_1 = 3, \gamma_1 = 3, \alpha_2 = 1, \beta_2 = 1.5, \gamma_2 = 3$and Mixing parameter;$p1 = 0.5$

Obs.No.	Xi	f(x)	F(x)	R(x)	h(x)
1	0.03	7.020E-20	5.015E-23	1	7.020E-20
2	0.12	2.198E-06	2.083E-08	1	2.198E-06
3	0.22	0.0003	8.854E-06	1.0000	0.0003
4	0.35	0.0028	0.0002	0.9998	0.0028
5	0.73	0.0182	0.0039	0.9961	0.0182
6	0.79	0.0260	0.0051	0.9949	0.0261
7	1.25	0.2355	0.0637	0.9363	0.2515
8	1.41	0.2621	0.1040	0.8960	0.2925
9	1.52	0.2616	0.1329	0.8671	0.3017
10	1.79	0.2300	0.1999	0.8001	0.2875
11	1.8	0.2285	0.2022	0.7978	0.2864
12	1.94	0.2069	0.2326	0.7674	0.2696
13	2.38	0.1477	0.3099	0.6901	0.2140
14	2.4	0.1455	0.3129	0.6871	0.2117
15	2.87	0.1048	0.3709	0.6291	0.1666
16	2.99	0.0972	0.3830	0.6170	0.1575
17	3.14	0.0888	0.3969	0.6031	0.1473
18	3.17	0.0873	0.3996	0.6004	0.1454
19	4.72	0.0442	0.4948	0.5052	0.0875
20	5.09	0.0393	0.5103	0.4897	0.0802

Table 4: Mixture Distribution with the parameter $\alpha_1 = 1, \beta_1 = 3, \gamma_1 = 3, \alpha_2 = 1, \beta_2 = 1.5, \gamma_2 = 3$and Mixing parameter;$p1 = 0.8$

Obs.No.	xi	f(x)	F(x)	R(x)	h(x)
1	0.03	2.808E-20	2.006E-23	1	2.808E-20
2	0.12	8.794E-07	8.333E-09	1	8.794E-07
3	0.22	0.0001	3.542E-06	1.0000	0.0001
4	0.35	0.0011	0.0001	0.9999	0.0011
5	0.73	0.0096	0.0016	0.9984	0.0096
6	0.79	0.0199	0.0024	0.9976	0.0200
7	1.25	0.3453	0.0835	0.9165	0.3767
8	1.41	0.3863	0.1429	0.8571	0.4507
9	1.52	0.3849	0.1854	0.8146	0.4725
10	1.79	0.3338	0.2834	0.7166	0.4658
11	1.8	0.3314	0.2867	0.7133	0.4645
12	1.94	0.2968	0.3307	0.6693	0.4435
13	2.38	0.2033	0.4395	0.5605	0.3627
14	2.4	0.1998	0.4435	0.5565	0.3591
15	2.87	0.1367	0.5214	0.4786	0.2856
16	2.99	0.1250	0.5371	0.4629	0.2701
17	3.14	0.1124	0.5548	0.4452	0.2524
18	3.17	0.1101	0.5582	0.4418	0.2491
19	4.72	0.0474	0.6697	0.3303	0.1436
20	5.09	0.0408	0.6860	0.3140	0.1298

Figure 8 p.d.f for two Mixture Modified inverse Weibull distribution, with$\alpha_1 = 1, \beta_1 = 3, \gamma_1 = 3, \alpha_2 = 1, \beta_2 = 1.5, \gamma_2 = 3$

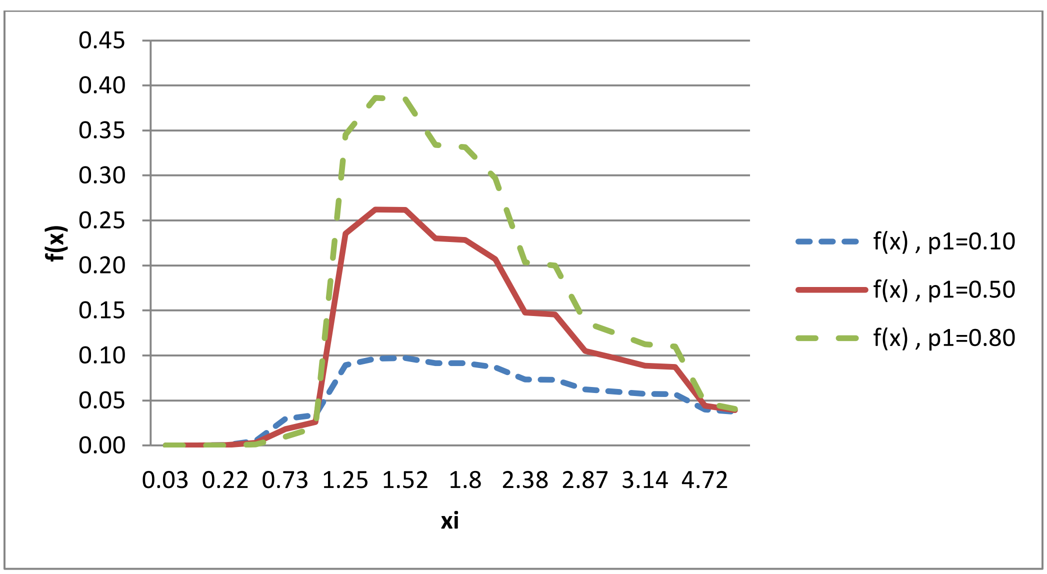


Figure 9 c.d.f for two Mixture Modified inverse Weibull distribution, with$\alpha_1 = 1, \beta_1 = 3, \gamma_1 = 3, \alpha_2 = 1, \beta_2 = 1.5, \gamma_2 = 3$

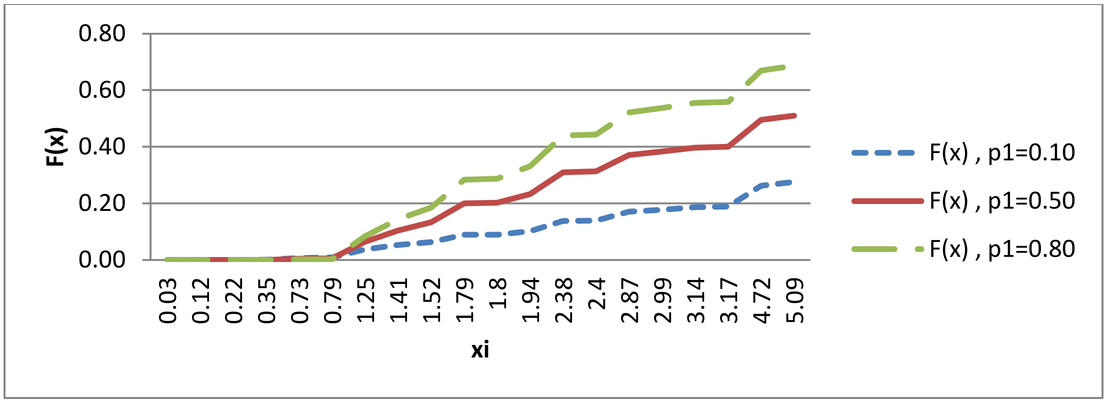


Figure 10 Reliability function for two Mixture Modified inverse Weibull distribution, with $\alpha_1 = 1, \beta_1 = 3, \gamma_1 = 3, \alpha_2 = 1, \beta_2 = 1.5, \gamma_2 = 3$

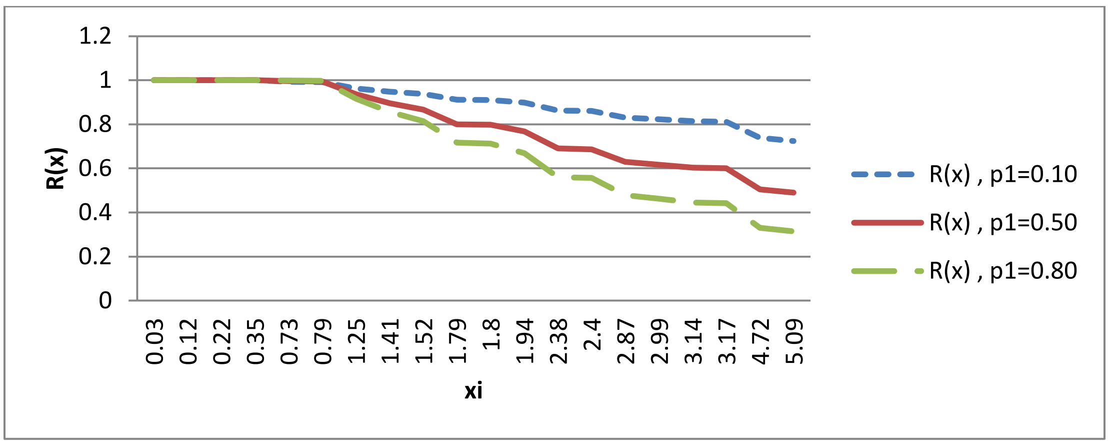
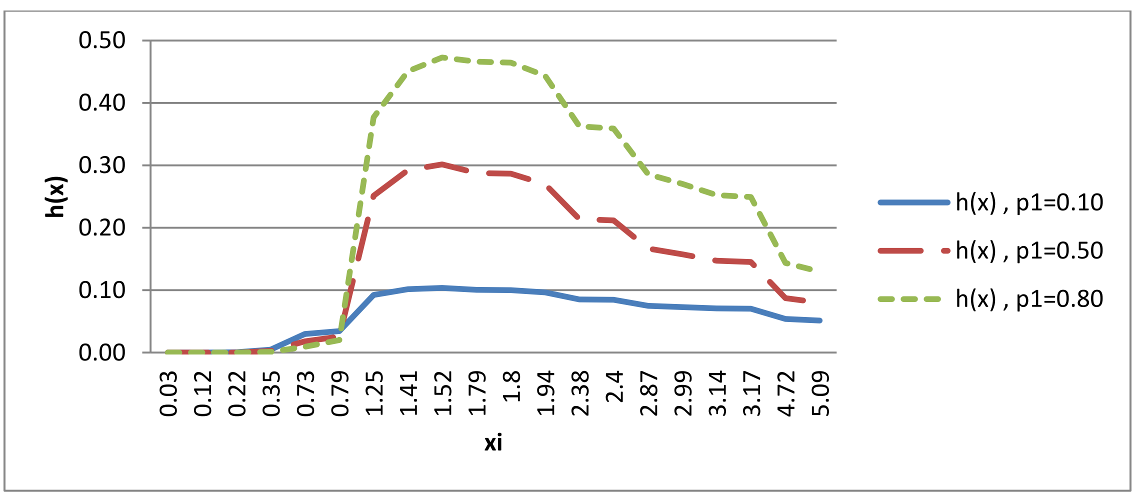


Figure 11 Failure rate for two Mixture Modified inverse Weibull distribution, with $\alpha_1 = 1, \beta_1 = 3, \gamma_1 = 3, \alpha_2 = 1, \beta_2 = 1.5, \gamma_2 = 3$



7. Conclusion

In this paper we introduced a mixture of modified inverse Weibull distribution and studies it different properties. We also drive reliability function and failure rate and graph these function to see the variation on the curve when different values are assigned to the parameters. From figure 4, we see that probability distribution in the correct shape. From figure 5 we observe that c.d.f is increasing function with maximum value is one and minimum value is zero. From figure 6 we can see that reliability function which is shows the decreasing trend . Figure 7 shows the failure rate of the data. For mixture modified inverse Weibull distribution, we can see the figure 8 which shows the p.d.f for different value of the mixing parameter. From figure 9 which is the c.d.f of mixture distribution with different value of mixing parameter and showing increasing trend while figure 10 showing the decreasing trend. Figure 11 shows the failure rate of the data.

It is observed that the proposed mixture has several desirable properties and several existing well known distribution can be obtain as special case of the new mixture. Parameter estimation through the statistical software Matlab R (2009a) and method of moment can also be applied for future. To drive and study the properties of MLEs of the parameters, Information matrix and the Bayesians analysis are required. More work in this direction should be possible.

Appendix

The proof of **theorem 3.1** starts with

$$\acute{\mu}_r = E(t^r) = \int_{\xi}^{\infty} t^r f\big(\alpha_1, \beta_1, \gamma_1, \xi_1, \alpha_2, \beta_2, \gamma_2, \xi_2, p_1, t\big)dt$$

By substituting the equation (2.2) in to above equation we have

$$\acute{\mu}_r = \int_{\xi}^{\infty} t^r \begin{bmatrix} p_1\left(\alpha_1+\beta_1\gamma_1\left(\frac{1}{t-\xi_1}\right)^{\beta_1-1}\right)\left(\frac{1}{t-\xi_1}\right)^2 exp\left(-\frac{\alpha_1}{t-\xi_1}-\gamma_1\left(\frac{1}{t-\xi_1}\right)^{\beta_1}\right)+ \\ p_2\left(\alpha_2+\beta_2\gamma_2\left(\frac{1}{t-\xi_2}\right)^{\beta_2-1}\right)\left(\frac{1}{t-\xi_2}\right)^2 exp\left(-\frac{\alpha_2}{t-\xi_2}-\gamma_2\left(\frac{1}{t-\xi_2}\right)^{\beta_2}\right)\end{bmatrix} dt$$

Case A: in this case $\xi_1, \xi_2 = 0$ and $\alpha_1, \beta_1, \gamma_1, \alpha_2, \beta_2, \gamma_2 > 0$ and $p_1 + p_2 = 1$

Using $$exp\left(-\gamma_1\left(\frac{1}{t}\right)^{\beta_1}\right) = \sum_0^{\infty}\frac{(-1)^i\gamma_1^i\left(\frac{1}{t}\right)^{i\beta_1}}{i!} \quad \text{(I)}$$ and

$$exp\left(-\gamma_2\left(\frac{1}{t}\right)^{\beta_2}\right) = \frac{\sum_0^{\infty}(-1)^i\gamma_2^i\left(\frac{1}{t}\right)^{i\beta_2}}{i!} \quad \text{(II)}$$

$$\acute{\mu}_r = \begin{bmatrix} p_1\left[\sum_0^{\infty}\frac{(-1)^i\gamma_1^i}{i!}\int_0^{\infty} t^r\left(\alpha_1+\beta_1\gamma_1\left(\frac{1}{t}\right)^{\beta_1-1}\right)\left(\frac{1}{t}\right)^2 \exp\left(-\frac{\alpha_1}{t}\right)\right]dx + \\ p_2\left[\sum_0^{\infty}\frac{(-1)^i\gamma_2^i}{i!}\int_0^{\infty} t^r\left(\alpha_2+\beta_2\gamma_2\left(\frac{1}{t}\right)^{\beta_2-1}\right)\left(\frac{1}{t}\right)^2 \exp\left(-\frac{\alpha_2}{t}\right)\right]dx\end{bmatrix}$$

$$\acute{\mu}_r = \begin{bmatrix} p_1 \sum_0^\infty \frac{(-1)^i \gamma_1^i}{i!} \left[\int_0^\infty \alpha_1 \left(\frac{1}{t}\right)^{-t+i\beta_1+2} exp\left(-\frac{\alpha_1}{t}\right) dt + \right. \\ \left. \int_0^\infty \beta_1\gamma_1 \left(\frac{1}{t}\right)^{-t+i\beta_1+\beta_1+1} exp\left(-\frac{\alpha_1}{t}\right) dt \right] + \\ p_2 \sum_0^\infty \frac{(-1)^i \gamma_2^i}{i!} \left[\int_0^\infty \alpha_2 \left(\frac{1}{t}\right)^{-t+i\beta_2+2} exp\left(-\frac{\alpha_2}{t}\right) dt + \right. \\ \left. \int_0^\infty \beta_2\gamma_2 \left(\frac{1}{t}\right)^{-t+i\beta_2+\beta_2+1} exp\left(-\frac{\alpha_2}{t}\right) dt \right] \end{bmatrix}$$

$$\acute{\mu}_r = \begin{matrix} p_1 \sum_0^\infty \frac{(-1)^i \gamma_1^i}{i!} \left[\alpha_1^{r-i\beta_1} \Gamma(i\beta_1 - r + 1) + \beta_1\gamma_1 \alpha_1^{r-\beta_1(i+1)} \Gamma(\beta_1(i+1) - r) \right] + \\ p_2 \sum_0^\infty \frac{(-1)^i \gamma_2^i}{i!} \left[\alpha_2^{r-i\beta_2} \Gamma(i\beta_2 - r + 1) + \beta_2\gamma_2 \alpha_2^{r-\beta_2(i+1)} \Gamma(\beta_2(i+1) - r) \right] \end{matrix}$$

Case B:In 2nd case $\xi_1, \xi_2 = 0$ and $\alpha_1, \alpha_2 = 0$ and $\beta_1, \gamma_1,, \beta_2, \gamma_2 > 0$

Then
$$\acute{\mu}_r = \begin{matrix} p_1 \int_0^\infty \beta_1\gamma_1 \left(\frac{1}{t}\right)^{-r+\beta_1+1} exp\left(\gamma_1 \left(\frac{1}{t}\right)^{\beta_1}\right) dt + \\ p_2 \int_0^\infty \beta_2\gamma_2 \left(\frac{1}{t}\right)^{-r+\beta_2+1} exp\left(\gamma_2 \left(\frac{1}{t}\right)^{\beta_2}\right) dt \end{matrix}$$

By substituting $w = \gamma_1 \left(\frac{1}{t}\right)^{\beta_1}$ and $v = \gamma_2 \left(\frac{1}{t}\right)^{\beta_2}$

$$\acute{\mu}_r = p_1 \left[\gamma_1^{r/\beta_1} \Gamma\left(1 - \frac{r}{\beta_1}\right)\right] + p_1 \left[\gamma_2^{r/\beta_2} \Gamma\left(1 - \frac{r}{\beta_2}\right)\right] \quad for\ r = 1,2,3,4$$

Case C: In this case we assume that $\alpha_1\alpha_2 > 0, \beta_1\beta_2 = 0, \xi_1, \xi_2 = 0$

$$\acute{\mu}_r = \begin{bmatrix} p_1 \int_0^\infty \alpha_1 t^{r-2} exp\left(-\frac{\alpha_1}{t}\right) dt + \\ p_2 \int_0^\infty \alpha_2 t^{r-2} exp\left(-\frac{\alpha_2}{t}\right) dt \end{bmatrix}$$

By substituting $w = \left(-\frac{\alpha_1}{t}\right)$ and $v = \left(-\frac{\alpha_2}{t}\right)$ we get

$$\acute{\mu}_r = p_1\left(\alpha_1^r \Gamma(1-r)\right) + p_2\left(\alpha_2^r \Gamma(1-r)\right) \qquad for\ r = 1,2,3,4$$

The proof of **theorem 3.1** starts with

$$M_x(t) = E[e^{tx}] = \int_{\xi}^{\infty} e^{tx} f\big(\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_1\gamma_2.\xi_1, \xi_2, p_1\big)\, dt$$

By substituting from equation(2.3) in to above relation and assuming the minimum life time is zero we have

$$M_x(t) = \begin{bmatrix} p_1 \int_0^{\infty} \left(\alpha_1 + \beta_1\gamma_1 \left(\frac{1}{x}\right)^{\beta_1 - 1}\right) \left(\frac{1}{x}\right)^2 exp\left(\frac{t}{x} - \frac{\alpha_1}{x} - \gamma_1 \left(\frac{1}{x}\right)^{\beta_1}\right) dx + \\ p_2 \int_0^{\infty} \left(\alpha_2 + \beta_2\gamma_2 \left(\frac{1}{x}\right)^{\beta_2 - 1}\right) \left(\frac{1}{x}\right)^2 exp\left(\frac{t}{x} - \frac{\alpha_2}{x} - \gamma_2 \left(\frac{1}{x}\right)^{\beta_2}\right) dx \end{bmatrix} \qquad \text{(M)}$$

Case A: $\xi_1, \xi_2 = 0$ and $\alpha_1, \beta_1, \gamma_1, \alpha_2, \beta_2, \gamma_2 > 0$ and $p_1 + p_2 = 1$using equation(I) and (II) in equation(M) takes the from

$$M_x(t) = \begin{bmatrix} p_1 \sum_0^{\infty} \frac{(-1)^i \gamma_1^i}{i!} \int_0^{\infty} \left(\alpha_1 + \beta_1\gamma_1 \left(\frac{1}{x}\right)^{\beta_1 - 1}\right) \left(\frac{1}{x}\right)^{i\beta_1 + 2} exp\left(\frac{t}{x} - \frac{\alpha_1}{x}\right) dx + \\ p_2 \sum_0^{\infty} \frac{(-1)^i \gamma_2^i}{i!} \int_0^{\infty} \left(\alpha_2 + \beta_2\gamma_2 \left(\frac{1}{x}\right)^{\beta_2 - 1}\right) \left(\frac{1}{x}\right)^{i\beta_2 + 2} exp\left(\frac{t}{x} - \frac{\alpha_2}{x}\right) dx \end{bmatrix}$$

$$M_x(t) = \begin{bmatrix} p_1 \sum_0^{\infty} \frac{(-1)^i \gamma_1^i}{i!} \left[\frac{\alpha_1 \Gamma(i\beta_1 + 1)}{(\alpha_1 - t)^{i\beta_1 + 1}} + \frac{\beta_1\gamma_1 \Gamma\big(\beta_1(i+1)\big)}{(\alpha_1 - t)^{\beta_1(i+1)}}\right] + \\ p_2 \sum_0^{\infty} \frac{(-1)^i \gamma_2^i}{i!} \left[\frac{\alpha_2 \Gamma(i\beta_2 + 1)}{(\alpha_2 - t)^{i\beta_2 + 1}} + \frac{\beta_2\gamma_2 \Gamma\big(\beta_2(i+1)\big)}{(\alpha_2 - t)^{\beta_2(i+1)}}\right] \end{bmatrix}$$

Case B: in this case $\xi_1, \xi_2 = 0$ and $\alpha_1, \alpha_2, = 0$, and $\beta_1, \gamma_1, \beta_2, \gamma_2 > 0$

By using the equation (I)and (II) in equation (M) takes the following form

$$M_0(t) = \begin{bmatrix} p_1 \int_0^{\infty} \beta_1\gamma_1 \left(\frac{1}{x}\right)^{\beta_1 + 1} Exp\left(tx - \gamma_1 \left(\frac{1}{x}\right)^{\beta_1 + 1}\right) dx + \\ p_2 \int_0^{\infty} \beta_2\gamma_2 \left(\frac{1}{x}\right)^{\beta_2 + 1} Exp\left(tx - \gamma_2 \left(\frac{1}{x}\right)^{\beta_2 + 1}\right) dx \end{bmatrix}$$

$$M_0(t) = p_1 \sum_{i=0}^{\infty} \frac{t^i \gamma_1^{\frac{i}{\beta_1}}}{i!} \Gamma\left(1 - \frac{i}{\beta_1}\right) + p_2 \sum_{i=0}^{\infty} \frac{t^i \gamma_2^{\frac{i}{\beta_2}}}{i!} \Gamma\left(1 - \frac{i}{\beta_2}\right)$$

Case C: in this case $\xi_1, \xi_2 = 0$ and $\alpha_1, \alpha_2, > 0$, and $\beta_1, \gamma_1, \beta_2, \gamma_2 = 0$

$$M_0(t) = \begin{bmatrix} p_1 \int_0^\infty \alpha_1 \left(\frac{1}{x}\right)^2 exp\left(\frac{t}{x} - \frac{\alpha_1}{x}\right) dx + \\ p_2 \int_0^\infty \alpha_2 \left(\frac{1}{x}\right)^2 exp\left(\frac{t}{x} - \frac{\alpha_2}{x}\right) dx \end{bmatrix}$$

$$M_0(t) = p_1 \left[\frac{\alpha_1}{\alpha_1 - t}\right] + p_2 \left[\frac{\alpha_2}{\alpha_2 - t}\right]$$

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