Information Sciences Letters
An International Journal

http://dx.doi.org/10.12785/isl/030304

Fixed Point Theorems for Nonself Contraction Mappings in N-Normed Spaces

Mehmet Kir and Hukmi Kiziltunc*

Department of Mathematics, Faculty of Science, Ataturk University, 25240, Erzurum, Turkey

Received: 30 Oct. 2013, Revised: 11 Jul. 2014, Accepted: 12 Jul. 2014
Published online: 1 Sep. 2014

Abstract: The fixed point theorems of contraction mappings in $n-$normed spaces was studied. [M. Kir and H. Kiziltunc, On Fixed Point Theorems For Contraction Mappings in n-Normed Spaces, (AMISL)]. By the same motivation, we will derive some fixed point theorems for nonself contraction mappings in n-Banach spaces which we stated in the present paper.

Keywords: $n-$normed spaces, $n-$Banach spaces, fixed point, contraction mappings, nonself mappings, retractions.

1 Introduction and Preliminaries

In 1963, S.Gahler introduced the concept of 2-normed space. Since 1963, S. Gähler, Y. J. Cho, R. W. Frees, C. R. Diminnie, R. E. Ehret, K. Iséki, A. White and many others have studied on both 2-normed spaces and 2-metric spaces. Also, H. Gunawan and M. Mashadi [3] defined n-normed space.

It is well known that Banach's contraction mapping theorem is one of the most important cornerstones of functional analysis. Due to the importance, many celabrated mathematicians, as Rhoades and Berinde, have investigated generalizations of Banach fixed point theorem.

A mapping $T : X \to X$ where (X,d) is a metric space, is said to be a contraction if there exists $k \in [0,1)$ such that for all $x,y \in X$,

$$d(Tx,Ty) \leq kd(x,y). \tag{1}$$

If the metric space (X,d) is complete then the mapping satisfying (1) has a unique fixed point. Also, Inequality (1) implies continuity of T.

Berinde [9] unify many fixed point theorems of contraction mappings in one theorem by using a function φ defined as following:

Definition 1. *[9] Let $\varphi : \mathbb{R}_+ \to \mathbb{R}_+$ be a function. In connection with the function φ we consider the following properties:*

(i_φ) φ is monotone increasing, i.e., $t_1 \geqslant t_2$ implies $\varphi(t_1) \geqslant \varphi(t_2)$;
(ii_φ) $\varphi(t) < t$ for all $t > 0$;
(iii_φ) $\varphi(0) = 0$;
(iv_φ) φ is continuous;
(v_φ) $\{\varphi^n(t)\}$ converges to 0 for all $t \geq 0$;
(vi_φ) $\sum_{n=0}^{\infty} \varphi^n(t)$ converges for all $t > 0$;
(vii_φ) $t - \varphi(t) \to 0$ as $t \to \infty$;
$(viii_\varphi)$ φ is subaddive.

Berinde gave some important relationships between above conditions.
1) (i_φ) and (ii_φ) imply (iii_φ);
2) (ii_φ) and (iv_φ) imply (iii_φ);
3) (i_φ) and (v_φ) imply (ii_φ).

Definition 2. *[9] A function φ satisfying (i_φ) and (v_φ) is said to be a comparison function.*

Also, Berinde gave following results:

Lemma 1. *[9]*
1) *Any comparison function satisfies (iii_φ);*
2) *Any comparison function satisfying $(viii_\varphi)$ satisfies (iv_φ), too;*
3) *If φ is a comparison function, then, for any $k \in \mathbb{N}^*$, φ^k is a comparison function, too;*
4) *If φ is a comparison function, then the function $s : \mathbb{R}_+ \to \mathbb{R}_+$*

$$s(t) = \sum_{k=0}^{\infty} \varphi^k(t)$$

* Corresponding author e-mail: mehmetkir04@gmail.com

satisfies (i_φ) and (iii_φ).

We can give some examples of function φ as follows;

1. $\varphi:\mathbb{R}_+\to\mathbb{R}_+$, $\varphi(t)=kt$, $k\in[0,1)$, satisfies all the conditions (i_φ) - $(viii_\varphi)$.

2. $\varphi:\mathbb{R}_+\to\mathbb{R}_+$, $\varphi(t)=\frac{t}{t+1}$, satisfies (i_φ), (v_φ) and (vii_φ).

In fixed point theory there are many interation schema. In this paper we will use Picard iteration schema defined as following:

Definition 3.*Let E be any set and $T:E\to E$ a selfmap. For any given $x\in E$, we define $T^n(x)$ inductively by $T^0(x)=x$ and $T^{n+1}(x)=T(T^n(x))$; we recall $T^n(x)$ the n^{th} iterative of x under T. For any $x_0\in X$, the sequence $\{x_n\}_{n\geq 0}\subset X$ given by*

$$x_n=Tx_{n-1}=T^nx_0,\ \ n=1,2,...$$

is called the sequence of successive approximations with the initial value x_0. It is also known as the Picard iteration starting at x.

Now, we will give some definitions and results in n-normed spaces.

Definition 4. *[3] Let $n\in\mathbb{N}$ and E be a real vector space of dimension $d\geq n$. A real valued function $\|\cdot,\cdots,\cdot\|$ on E^n satisfying the following*

$n_1)$ $\|x_1,\cdots,x_n\|=0$ if and only if $x_1,...,x_n$ are linearly dependent;

$n_2)$ $\|x_1,\cdots,x_n\|$ is invariant under permutation;

$n_3)$ $\|x_1,\cdots,x_{n-1},cx_n\|=|c|\,\|x_1,\cdots,x_{n-1},x_n\|$ for all $c\in\mathbb{R}$,

$n_4)$ $\|x_1,\cdots,x_{n-1},y+z\|\leq\|x_1,\cdots,x_{n-1},y\|+\|x_1,\cdots,x_{n-1},z\|$,

is called an $n-norm$ on E and the pair $(E,\|\cdot,\cdots,\cdot\|)$ is called $n-normed$ space.

Definition 5. *[3] A sequence $\{x_n\}$ in a n-normed space $(E,\|\cdot,\cdots,\cdot\|)$ is said to be a Cauchy sequence if $\lim\limits_{m,n\to\infty}\|x_n-x_m,x_2,\cdots,x_n\|=0$ for all $x_2,...,x_n\in E$.*

Definition 6. *[3] A sequence $\{x_n\}$ in a n-normed space $(E,\|\cdot,\cdots,\cdot\|)$ is said to be convegent if there is a point x in E such that $\lim\limits_{n\to\infty}\|x_n-x,x_2,\cdots,x_n\|=0$ for all $x_2,...,x_n$ in E. if $\{x_n\}$ converges to x then, we write $x_n\to x$ as $n\to\infty$.*

Definition 7. *[3] A linear n-normed space is said to be complete if every Cauchy sequence is convergent to an element of E. A complete n-normed space E is called n-Banach space.*

First time, the authors [7] introduced the some concept of fixed point theory for $n-$normed spaces as following:

Definition 8. *[7] Let E be a linear n-normed space then the mapping $T:E\to E$ is said to be a contraction if there exist some $k\in[0,1)$ such that*

$$\|Tx-Ty,x_2,\cdots,x_n\|\leq k\,\|x-y,x_2,\cdots,x_n\|\quad,\ \textit{for all}\ x,y,x_2,...,x_n\in E. \tag{2}$$

Definition 9. *[7] Let E be a linear n-normed space then the mapping $T:E\to E$ is called contractive if*

$$\|Tx-Ty,x_2,\cdots,x_n\|<\|x-y,x_2,\cdots,x_n\|\quad,\ \textit{for all}\ x,y,z\in E. \tag{3}$$

Theorem 1. *Let $(E,\|.,\cdots,.\|)$ be a linear n-Banach space and K be a nonempty closed and bounded subset of E. A selfmap $T:K\to K$ be contraction then T has a unique fixed point in K.*

Definition 10. *[7] Let $(E,\|\cdot,\cdots,\cdot\|)$ be a linear n-normed space. A mapping $T:E\to E$ is said to be a $\varphi-contraction$ if there exists a comparison function $\varphi:\mathbb{R}_+\to\mathbb{R}_+$ such that*

$$\|Tx-Ty,x_2,\cdots,x_n\|\leq\varphi(\|x-y,x_2,\cdots,x_n\|)\quad,\ \textit{for all}\ x,y,x_2,...,x_n\in E. \tag{4}$$

*Remark.*In Definition 10, if we take $\varphi(t)=kt$ with $k\in[0,1)$, we obtain definition of contraction mappings to n-normed spaces. It is clear that Definition 10 is an extension of Definition 8.

Theorem 2. *Let $(E,\|.,\cdots,.\|)$ be a linear n-Banach space and K be a nonempty closed and bounded subset of E. A selfmap $T:K\to K$ be $\varphi-$contraction then T has a unique fixed point in K.*

Definition 11. *[7] Let $(E,\|\cdot,\cdots,\cdot\|)$ be a linear n-normed space, C be a subset of E then the closure of C is $\overline{C}=\{x\in E$; there is a sequence x_n of C such that $x_n\to x\}$. We say, C is sequentially closed if $C=\overline{C}$.*

Definition 12. *[7] Let $(E,\|\cdot,\cdots,\cdot\|)$ be a linear n-normed space, B be a nonempty subset of E and $e\in B$ then B is said to be e-bounded if there exist some $M>0$ such that $\|e,x_2,\cdots,x_n\|\leq M$ for all $x_2,\cdots,x_n\in B$. If for all $e\in B$, B is e-bounded then B is called a bounded set.*

From Theorem 1 and Theorem 2, it is clear that the authors [7] showed a contraction mapping and a φ-contraction mapping have a unique fixed point in n-Banach spaces.

In this study, we'll discuss similar theorems for nonself contraction mapping and nonself φ-contraction mapping. As a result, we will obtain the fixed point theorems for nonself mappings in n-Banach spaces.

2 The Concept of Retraction For Nonself-Mappings in n-Normed Space

Iterative techniques for nonexpansive and asymptotically nonexpansive mappings in Banach space including Mann type and Ishikawa type iteration processes have been studied extensively by various authors;[11]-[19]. However, if the domain of T, $D(T)$, is a proper subset of E (and this is the case in several applications) and T maps

$D(T)$ into E, then the iteration processes of Mann type and Ishikawa type have been studied by the authors mentioned above, their modifications introduced may fail to be well defined (for more detail [8]-[19]).

A subset K of E is said to be a retract of E if there exists a continuous map $P:E\to K$ such that $Px=x$, for all $x\in K$. Every closed convex subset of a uniformly convex Banach space is a retract. A map $P:E\to K$ is said to be a retraction if $P^2=P$. It follows that if a map P is a retraction, then $Py=y$ for all $y\in R(P)$, the range of P.

The concept of retraction for nonself-mappings was firstly introduced by Chidume [19] as the generalization of nonexpansive self-mappings.

In this section, we established fixed point theorems for non-self mappings applying the definition of retraction to n-Banach spaces.

Lemma 2. *Let E be a real n-normed space and K be a nonempty subset of E. Let $P:E\to K$ be a nonexpansive retraction of E onto K and $T:E\to K$ be a nonself contraction mapping with $k\in[0,1)$. Then $PT:K\to K$ is a contraction mapping with $k\in[0,1)$.*

Proof.For all $x,y\in K$, and for all fixed $x_2,\cdots,x_n\in K$, we have

$$\begin{aligned}\|(PT)x-(PT)y,x_2,\cdots,x_n\| &\le \|Tx-Ty,x_2,\cdots,x_n\|\\ &\le k\|x-y,x_2,\cdots,x_n\|.\end{aligned}$$

Thus, the mapping PT is a contraction with $k\in[0,1)$, too.

Theorem 3. *Let K be a closed and bounded subset of n-Banach space E, $P:E\to K$ be a nonexpansive retraction of E onto K . Let a nonself-map $T:E\to K$ be a nonself contraction mapping. If a sequence $\{a_n\}_{n=1}^{\infty}\subset K$ be defined as follow;*

$$a_n=(PT)^n a_0 \ , a_0\in K. \tag{5}$$

Then, the sequence $\{a_n\}_{n=1}^{\infty}$ has a unique fixed point in K.

Proof.Let $a_0\in K$ and $\{a_n\}$ be a sequence defined by (5) in K. Note that T is a contraction then there exists $k\in[0,1)$ such that $\|Tx-Ty,x_2,\cdots,x_n\|\le k\|x-y,x_2,\cdots,x_n\|$ for all $x,y,x_2,\cdots,x_n\in K$.

From Lemma 2, we have

$$\begin{aligned}\|(PT)^n a_0-(PT)^n a_1,x_2,\cdots,x_n\| &\le k\|(PT)^{n-1}a_0-(PT)^{n-1}\\ &\quad a_1,x_2,\cdots,x_n\|\\ &\le k^2\|(PT)^{n-2}a_0-(PT)^{n-2}\\ &\quad a_1,x_2,\cdots,x_n\|\\ &\ \ \vdots\\ &\le k^n\|x_0-x_1,x_2,\cdots,x_n\| \end{aligned}\tag{6}$$

Now, we show that $\{a_n\}$ is a Cauchy sequence in K. For all $n,m>0$ with $m>n$, taking $m=n+p$ for all $x_2,\cdots,x_n\in K$, we have

$$\begin{aligned}\|a_n-a_m,x_2,\cdots,x_n\| &= \|a_n-a_{n+p},x_2,\cdots,x_n\|\\ &= \|(a_n-a_{n+1})+(a_{n+1}-a_{n+2})+\\ &\quad ...+(a_{n+p-1}-a_{n+p}),x_2,\cdots,x_n\|\\ &\le \|a_n-a_{n+1},x_2,\cdots,x_n\|\\ &\quad +\|a_{n+1}-a_{n+2},x_2,\cdots,x_n\|\\ &\quad +...+\|a_{n+p-1}-a_{n+p},x_2,\cdots,x_n\|\\ &= \|(PT)^n a_0-(PT)^n a_1,x_2,\cdots,x_n\|\\ &\quad +\|(PT)^{n+1}a_0-(PT)^{n+1}a_1,x_2,\cdots,x_n\|\\ &\quad +...+\|(PT)^{n+p-1}a_0-(PT)^{n+p-1}\\ &\quad a_n,x_2,\cdots,x_n\|\\ &\le k^n\|a_0-a_1,x_2,\cdots,x_n\|\\ &\quad +k^{n+1}\|a_0-a_1,x_2,\cdots,x_n\|\\ &\quad +...+k^{n+p-1}\|a_0-a_1,x_2,\cdots,x_n\|\\ &\le \frac{k^n}{1-k}\|a_0-a_1,x_2,\cdots,x_n\|.\end{aligned}\tag{7}$$

Note that E is bounded then there exists $M>0$ such that

$$\|a_0-a_1,x_2,\cdots,x_n\|\le M \quad \text{for all } x_2,\cdots,x_n\in E. \tag{8}$$

Substituting (8) into (7) we obtain that

$$\|a_n-a_m,x_2,\cdots,x_n\|\le\frac{k^n}{1-k}M. \tag{9}$$

In equation (9) if we take $m,n\to\infty$, we have

$$\lim_{n,m\to\infty}\|a_n-a_m,x_2,\cdots,x_n\|=0\,. \tag{10}$$

The equation (10) implies that $\{a_n\}_{n=1}^{\infty}$ is a Cauchy sequence in K . Note that K closed and bounded so $\{a_n\}_{n=1}^{\infty}$ converges to any $a\in K$. Also, PT is continuous, we have

$$(PT)a=\lim_{n\to\infty}(PT)a_n=\lim_{n\to\infty}a_{n+1}=a. \tag{11}$$

Thus, $a\in K$ is a fixed point of PT.

Now, we have to prove that the fixed point is unique. Let $a'\in K$ with $a'\ne a$ assume that $(PT)a'=a'$ then, we obtain

$$\begin{aligned}\left\|a-a',x_2,\cdots,x_n\right\| &= \left\|(PT)a-(PT)a',x_2,\cdots,x_n\right\|\\ &\le k\left\|a-a',x_2,\cdots,x_n\right\|.\end{aligned}\tag{12}$$

The (12) implies $k\ge1$ but this case is a contradition for $k\in[0,1)$.

This implies that $a=a'$. Hence, the fixed point is unique. This completes the proof.

Our next theorem is an extension of Theorem 3.

Lemma 3. *Let E be a real n-normed space and K be a nonempty subset of E. Let $P:E\to K$ be a nonexpansive retraction of E onto K and $T:E\to K$ be a nonself φ-contraction mapping. Then, $PT:K\to K$ is a φ-contraction mapping.*

Proof.For all $x,y\in K$, *and for all fixed* $x_2,\cdots,x_n\in K$ *,using definition of* φ *function, we have*

$$\begin{aligned}\|(PT)x-(PT)y,x_2,\cdots,x_n\| &\le \|Tx-Ty,x_2,\cdots,x_n\|\\ &\le \varphi(\|x-y,x_2,\cdots,x_n\|) \quad (13)\end{aligned}$$

This implies that the mapping PT *is* φ*-contraction mapping.*

Theorem 4. *Let* K *be a closed and bounded subset of n-Banach space* E, $P:E\to K$ *be a nonexpansive retraction of* E *onto* K. *Let a nonself-map* $T:E\to K$ *be a* φ*-contraction mapping. Let a sequence* $\{a_n\}_{n=1}^{\infty}\subset K$ *be defined as follows;*

$$a_n=(PT)^n a_0 \ \ , a_0\in K. \tag{14}$$

Then, $\{a_n\}_{n=1}^{\infty}$ *has a unique fixed point in* K.

Proof.Let $a_0\in K$ *and* $\{a_n\}$ *be a sequence defined by (14) in* K. *Note that* T *is a* φ*-contraction then there exists a comparison function* φ *such that*

$$\|Tx-Ty,x_2,\cdots,x_n\|\le\varphi(\|x-y,x_2,\cdots,x_n\|) \quad \text{for all } x,y,x_2,\cdots,x_n\in K. \tag{15}$$

Also, we have

$$\begin{aligned}\|(PT)^n a_0-(PT)^n a_1,x_2,\cdots,x_n\| &\le \varphi(\|(PT)^{n-1}a_0-(PT)^{n-1}a_1,x_2,\cdots,x_n\|)\\ &\le \varphi^2(\|(PT)^{n-2}a_0-(PT)^{n-2}a_1,x_2,\cdots,x_n\|)\\ &\ \ \vdots\\ &\le \varphi^n(\|a_0-a_1,x_2,\cdots,x_n\|) \quad (16)\end{aligned}$$

Now, we show that $\{a_n\}$ *is a Cauchy sequence in* K. *For all* $n,m>0$ *with* $m>n$, *taking* $m=n+p$ *for all* $x_2,\cdots,x_n\in K$ *then, we have*

$$\begin{aligned}\|a_n-a_m,x_2,\cdots,x_n\| &= \|a_n-a_{n+p},x_2,\cdots,x_n\|\\ &= \|(a_n-a_{n+1})+(a_{n+1}-a_{n+2})+\dots\\ &\quad +(a_{n+p-1}-a_{n+p}),x_2,\cdots,x_n\|\\ &\le \|a_n-a_{n+1},x_2,\cdots,x_n\|\\ &\quad +\|a_{n+1}-a_{n+2},x_2,\cdots,x_n\|\\ &\quad +\dots+\|x_{an+p-1}-a_{n+p},x_2,\cdots,x_n\|\\ &= \|(PT)^n a_0-(PT)^n a_1,x_2,\cdots,x_n\|\\ &\quad +\|(PT)^{n+1}a_0-(PT)^{n+1}a_1,x_2,\cdots,x_n\|\\ &\quad +\dots+\|(PT)^{n+p-1}a_0-(PT)^{n+p-1}a_n,x_2,\cdots,x_n\|\\ &\le \varphi^n(\|a_0-a_1,x_2,\cdots,x_n\|)+\varphi^{n+1}(\|a_0-a_1,x_2,\cdots,x_n\|)\\ &\quad +\dots+\varphi^{n+p-1}(\|a_0-a_1,x_2,\cdots,x_n\|) \quad (17)\end{aligned}$$

Note that E *is bounded then there exists* $M>0$ *such that*

$$\|a_0-a_1,x_2,\cdots,x_n\|\le M \quad \text{for all } x_2,\cdots,x_n\in E. \tag{18}$$

Substituting (18) into (17) we obtain that

$$\|a_n-a_m,x_2,\cdots,x_n\|\le\varphi^n(M)+\varphi^{n+1}(M)+\cdots \tag{19}$$

In (19) if we take $m,n\to\infty$*,we have*

$$\lim_{n,m\to\infty}\|a_n-a_m,x_2,\cdots,x_n\|=0. \tag{20}$$

The inequality (20) implies that $\{a_n\}_{n=1}^{\infty}$ *is a Cauchy sequence in* K. *Note that* K *closed and bounded so* $\{a_n\}_{n=1}^{\infty}$ *converges to any* $a\in K$. *Also,* PT *is continuous, we have*

$$(PT)a=\lim_{n\to\infty}(PT)a_n=\lim_{n\to\infty}a_{n+1}=a. \tag{21}$$

Now, we have to prove that the fixed point is unique. Let $a'\in K$ *with* $a'\ne a$ *assume that* $(PT)a'=a'$ *then, we obtain*

$$\begin{aligned}\left\|a-a',x_2,\cdots,x_n\right\| &= \left\|(PT)a-(PT)a',x_2,\cdots,x_n\right\|\\ &\le k\left\|a-a',x_2,\cdots,x_n\right\|. \quad (22)\end{aligned}$$

The (22) implies $k\ge1$ *but this case is a contradition for* $k\in[0,1)$. *This implies that* $a=a'$. *Hence, the fixed point is unique. This completes the proof.*

Corollary 1.*It is clear that if we take* $\varphi(t)=kt$ *where* $k\in[0,1)$ *then we obtain Theorem 3.*

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