Appl. Math. Inf. Sci. 6 No. 2S pp. 445S-451S (2012)

Applied Mathematics & Information Sciences
An International Journal

A Fuzzy-Grey Model for Non-stationary Time Series Prediction

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Received June 22, 2010; Revised March 21, 2011; Accepted 11 June 2011
Published online: 1 January 2012

Abstract: In time series prediction, historical data are used as the basis of estimating future outcomes. Many methods including statistical predictive models and artificial intelligence (AI) based models have been proposed for time series prediction. When dealing with limited information, researchers tend to seek for AI-based approaches as statistical models require large samples to determine the underlying distribution. This paper introduces a novel approach using fuzzy interpolation in constructing new data points adaptively within the range of known data in the grey prediction model. Denoted as fuzzy-grey prediction models (FGPM), the proposed model can improve the prediction accuracy of conventional grey models in the application of non-stationary time series prediction. The proposed model was tested on a practical data set derived from Taiwan Stock Exchange Capitalization Weight Stock Index (TAIEX). Experimental results showed that the proposed FGPM has the ability of fitting non-stationary time series accurately and outperforms some existing methods.

Keywords: Grey Model, Fuzzy Theory, Time Series Prediction

1 Introduction

A time series is an ordered sequence of data points. Any series of measurements taken at different time intervals can be regarded as a time series. The analysis of time series is a broad field of study [1-5], ranging from science, engineering, to finance and economics. Time series prediction is to predict future values using a fitting model based on historical data. Various approaches including statistical models and artificial intelligence (AI) based models have been developed for the application of time series prediction.

The autoregressive (AR) models, the integrated (I) models, the moving average (MA) models, and their hybrid models are most widely used statistical approaches for modeling the variations of linear time series [6-8]. For financial and economic time series, the autoregressive conditional heteroskedasticity (ARCH) models and generalized ARCH (GARCH) models are two useful methods to capture the changes of variance over time [9-10]. Some variations of GARCH models have been developed in recent years [11-12].

However, methods rely on statistical models for prediction of time series may have a problem when the available data are limited as those methods do not satisfy the assumptions for which statistical models have been derived. This is because statistical models require sufficiently large sizes of sample data to determine the underlying distribution. AI-based approaches usually make fewer assumptions compared to statistical methods. Therefore, researchers tend to seek for alternatives through the use of artificial intelligence techniques.

AI-based approaches to time series analysis include artificial neural networks (ANNs) [13-14], fuzzy theory [15], hidden Markov models [16], genetic algorithms, support vector machines, and their hybrid models [17-18]. Among various approaches, ANNs are the most popular method used for prediction, and their accuracy and forecasting capabilities have been demonstrated. A major problem of ANN-based models is that they require a large number of training data and need a tedious training process. In recent years, the grey model (GM) has been demonstrated to have a good performance in time series prediction without the use of large historical data [19-24]. However, the prediction accuracy of GM is limited and unsatisfactory when the data for prediction present non-stationary characteristics.

Stationarity is a convenient assumption to describe the statistical property of a time series. Most practical time series, particularly those from economic and business domains are non-stationary due to the influence of perturbation in the considered system, which makes conventional approaches inapplicable to real-word problems. Stock market index is a highly stochastic and non-stationary financial time series. It is difficult to develop a fitting model by using conventional statistical or AI-based methods. To develop an accurate fitting method for prediction of non-stationary time series, in this study, an improved grey model incorporated with fuzzy interpolation was proposed.

2 Methodology

2.1 Grey Prediction Models

The grey model is a powerful tool in the grey system theory for estimating the behavior of partially unknown systems or systems with limited information [25]. It has been successfully applied to various fields since it was proposed by Prof. J.L. Deng in 1982 [26]. In grey system theory, GM(1,1) is the most widely used model for prediction. The first parameter in the bracket of GM(1,1) denotes the order of differential equation, and the second parameter indicates the number of variables. The basic concept of grey model is described as follows. Consider a time series with n data points,

$$x^{(0)}=(x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)) \qquad (2.1)$$

where the subscript (0) denotes the original series and $x^{(0)}(k)$ represents the k-th data point. The following procedures address the process of using GM(1,1) to make a one-step ahead prediction.

Step 1: Perform accumulated generating operation (AGO)

Before developing the fitting model, the accumulated generating operation is performed to the original time series or primitive sequence to weaken the tendency of data variation. Let $x^{(1)}(k)$ be the new time series generated by performing AGO, as listed in (2.2).

$$x^{(1)}=(x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)) \qquad (2.2)$$

where

$$x^{(1)}(k)=\sum_{i=1}^{k} x^{(0)}(i), \quad k=1, 2, 3, \cdots, n. \qquad (2.3)$$

Note that the new series $x^{(1)}$ obtained from one-time AGO (1-AGO) has the property of monotonic increasing. Obviously, this monotonic property makes $x^{(1)}$ easier to be predicted than that in the original series $x^{(0)}$.

Step 2: Modeling with a differential equation

As the solution of first-order ordinary differential equations (ODE) has the same monotonic increasing property as the series derived from AGO, the model of $x^{(1)}$ can be approximated by a differential equation as follows.

$$\frac{dx^{(1)}}{dt}+ax^{(1)}=b \qquad (2.4)$$

where a and b are model parameters need to be determined, called developing coefficient and grey input, respectively.

Step 3: Determine model parameters

The model parameters a and b in (2.4) can be estimated by introducing a new variable $z^{(1)}(k)$, called background value, in the following approximation model.

$$x^{(0)}(k)+az^{(1)}(k)=b, \; k\geq 2 \qquad (2.5)$$

where $z^{(1)}(k)$ is defined as

$$\begin{aligned} z^{(1)}(k) &= (z^{(1)}(2), z^{(1)}(3), \dots, z^{(1)}(n)), \; k = 2, 3,\dots, n \\ &= \alpha\, x^{(1)}(k)+(1-\alpha)x^{(1)}(k-1) \end{aligned} \qquad (2.6)$$

In conventional grey models, the parameter α in (2.6) is set to 0.5, and can be expressed as

$$z^{(1)}(k)=0.5\,x^{(1)}(k)+0.5x^{(1)}(k-1) \qquad (2.7)$$

Now, a and b can be estimated using the least-square technique described as follows.

At first, we rearrange (2.5) in the form of simultaneous equations.

$$\begin{cases} x^{(0)}(2)+az^{(1)}(2)=b \\ x^{(0)}(3)+az^{(1)}(3)=b \\ \quad\vdots \\ x^{(0)}(n)+az^{(1)}(n)=b \end{cases} \qquad (2.8)$$

Then, the matrix form of (2.8) is described as

$$\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \tag{2.9}$$

Let

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \cdots & \cdots \\ -z^{(1)}(n) & 1 \end{bmatrix} \tag{2.10}$$

and

$$Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \tag{2.11}$$

By applying the Moore-Penrose pseudo-inverse, parameters a and b can be determined using (2.12).

$$\begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y \tag{2.12}$$

Substitute (2.12) with (2.10)-(2.11), the model parameters can be obtained given below.

$$\begin{cases} a(\alpha) = \dfrac{-(n-1)C + BD}{(n-1)A - B^2} \\ b(\alpha) = \dfrac{AD - BC}{(n-1)A - B^2} \end{cases} \tag{2.13}$$

where

$$\begin{cases} A = \sum_{k=2}^{n} [z^{(1)}(k)]^2 \\ B = \sum_{k=2}^{n} z^{(1)}(k) \\ C = \sum_{k=2}^{n} X^{(0)}(k) \cdot z^{(1)}(k) \\ D = \sum_{k=2}^{n} X^{(0)}(k) \end{cases} \tag{2.14}$$

Step 4: Estimate data points in $x^{(1)}$ space
As the initial condition of (2.4) is $x^{(1)}(1) = x^{(0)}(1)$, once the model parameters are determined from (2.13), the solution of (2.4) can be obtained accordingly, given as follows.

$$\hat{x}^{(1)}(k) = \left(x^{(0)}(1) - \frac{b}{a} \right) e^{-a(k-1)} + \frac{b}{a} \tag{2.15}$$

where $\hat{x}^{(1)}(k)$ is defined as the estimated value of $x^{(1)}(k)$. By substituting k with n+1 into (2.15), the one-step ahead prediction in $x^{(1)}$ space can be obtained as follows.

$$\hat{x}^{(1)}(n+1) = \left(x^{(0)}(1) - \frac{b}{a} \right) e^{-a(n)} + \frac{b}{a} \tag{2.16}$$

Step 5: Transform estimated AGO series back to $x^{(0)}$ space
This step is to transform the predicted value in $x^{(1)}$ back to $x^{(0)}$ space by performing the operation of inverse AGO (IAGO) using (2.17).

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \tag{2.17}$$

The prediction process of GM(1,1) is shown in Figure 2.1.

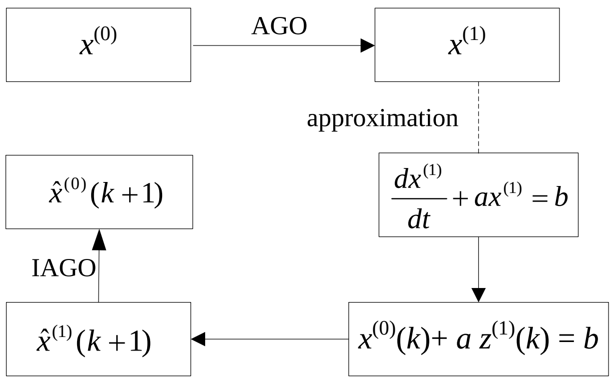

Figure 2.1: Prediction process of GM(1,1)

In conventional GM(1,1), the entire data set is used for prediction. An attempt is made to construct a rolling model for improving the prediction accuracy by using the updated data to GM(1,1) [27]. A rolling model of GM(1,1) is a technique that adopts the most recent historical data for prediction.

2.2 The Proposed Improved GM(1,1) Model

It can be seen that GM(1,1) is constructed based on the exponential function for the solution of (2.4). This prediction mechanism could cause a bad fitting when employed to a non-stationary time series. To avoid this limitation, two strategies are proposed in this paper to improve the performance of conventional GM(1,1) for fitting non-stationary time series. One is to reduce the error resulting from the setting of α =0.5 in estimating background value $z^{(1)}(k)$. The other is to provide more global extension of original time series using fuzzy interpolation technique.

2.2.1 Optimization of background value

In (2.7), conventional grey models select 0.5 as the value of α, leading to an error term in estimating $z^{(1)}(k)$ when the predicted series is non-stationary. This is because α directly controls the value of $z^{(1)}(k)$. In other words, the value of α has an impact on the accuracy of GM(1,1). In this study, instead of selecting α =0.5, the value of α is determined by minimizing the error function using (2.18).

$$\frac{dE(\alpha)}{d\alpha} = 0 \tag{2.18}$$

where $E(\alpha)$ is defined in (2.19) as the error term, used to quantify the difference between the estimated value $\hat{x}^{(0)}(k)$ and the practical value $x^{(0)}(k)$.

$$E(\alpha) = \left|\hat{x}^{(1)}(k+1) - x^{(1)}(k+1)\right| \tag{2.19}$$

where

$$\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) + \frac{b(\alpha)}{a(\alpha)}\right)e^{ak} - \frac{b(\alpha)}{a(\alpha)} \tag{2.20}$$

The prediction process of the proposed improved grey model with optimal background values is depicted in Figure 2.2.

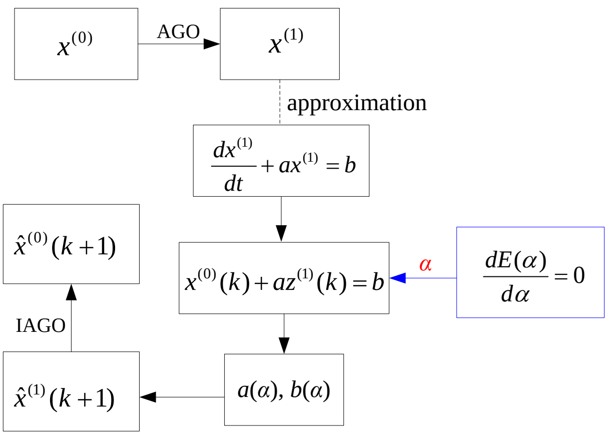

Figure 2.2: Prediction process of improved GM(1,1)

2.2.2 Fuzzy interpolation

Using the prediction model of conventional GM(1,1) for continuous time series may result in the occurrence of overshooting [28]. The overshooting phenomenon should be avoided for improving the prediction accuracy. As the second strategy, an interpolation method using fuzzy if-then rules is introduced to overcome the aforementioned problem. The aim of using interpolation is to track the original series by constructing the intermediate values from known data points.

Interpolation can be used to perform time series analysis and prediction. There are several popular interpolation models, such as cubic splines and Fourier methods. However, they are basically local extensions of given data, which can cause monotonic outcomes as they are extended [29]. To prevent the interpolation from becoming monotonic extensions, the fuzzy set theory was applied to interpolate irregularly scattered data adaptively. Fuzzy set theory offers a mathematically formalized method to handle imprecise information [30]. It is a mapping of a set of real numbers onto membership values that lie in the range [0, 1]. An element of a fuzzy set is an ordered pair containing a set element and the degree of membership in the fuzzy set. The higher membership value implies greater satisfaction.

The number of interpolative points needs to be created depends on the estimated error of GM(1,1). A rule of estimating the number of interpolative points could be stated as "When the estimated error becomes *high*, the interpolative points are subject to be *more* than those in a low estimated error interval." The purpose of using more interpolative points in bad-fit intervals is to make the fitting curve track the trend of the best fit line. As the above intuitive rule contains linguistic terms with some degree of uncertainty, such as *high* or *more*, it is preferable to deal with them using fuzzy set theory. A triangular membership function is adopted in this study for its computation speed and simplicity, as shown in Figure 2.3.

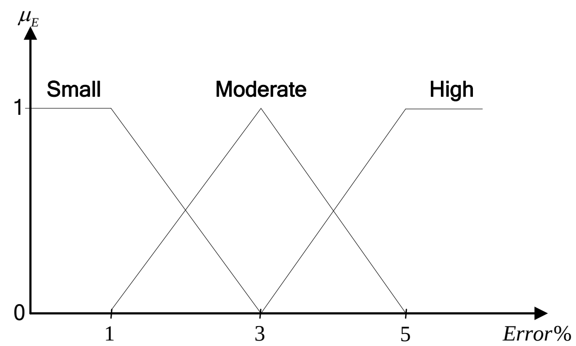

Figure 2.3: The membership function of the linguistic terms

A fuzzy rule base plays an important role in fuzzy inference. Each rule characterizes an argument-value of a respective fuzzy function in the sense that the antecedent part of the rule characterizes the argument, and the consequent part of the rule characterizes the dependent value. Fuzzy interpolation is useful for the application of sparse rule-based systems [31]. Instead of interpolating rules, in this study, we applied the concept to interpolate data points for improving the accuracy of GM(1,1). Examples of the fuzzy if-then rules are as follows.

IF estimated error is *Small*

THEN *Few* interpolative points are required.

IF estimated error is *High*

THEN *More* interpolative points are required.

IF estimated error is *Moderate*

THEN *Moderate* interpolative points are required.

In the inference procedures, the estimated errors are first fuzzified to calculate the firing strength in fuzzy rules. For simplifying the computation process, in the consequent parts of the fuzzy rules, the singleton fuzzy sets *Few*, *More*, and *Moderate* are crisply defined as 1, 3, and 5, respectively. Defuzzification is the process of converting a fuzzy set into a real value that is the best representation of the fuzzy set. The floor function of weighted average described in (2.21) is applied to derive the crisp defuzzification values.

$$f^* = \left\lfloor \frac{\sum_{j=1}^{N_R} \mu_j y_j}{\sum_{j=1}^{N_R} \mu_j} \right\rfloor \tag{2.21}$$

where μ_j and y_j are the firing strength of antecedent part and the fuzzy singleton of consequent part in the j-th fuzzy rule, respectively, and N_R represents the number of fuzzy rules. The square bracket notation in (2.21) refers to the function that maps a real number to the largest previous integer or the largest integer not greater than the real number, which can be expressed as

$$\lfloor x \rfloor = m, \text{ if and only if } m \le x < m+1 \tag{2.22}$$

The floor function applied in (2.21) is to make sure that an integer is obtained. Although the selection of membership functions and defuzzification schemes is subjective, it is not sensitive to predictive results in our experiments. Generally speaking, the incorporation of fuzzy techniques provides a flexible way for interpolating data points.

3 Experimental Results

In this section, a practical data set was employed to verify the effectiveness of the proposed approach. The test data set was obtained from the closing price of Taiwan Stock Exchange Capitalization Weight Stock Index (TAIEX) from April 27, 2009 to July 21, 2009 for consecutive 60 trading days as listed in Table 3.1. Due to page limitation, Table 3.1 only lists the first ten points of the original data and the associated predictive values by GM(1,1) and FGPM.

Table 3.1 The first ten data points and their predictive values

# of data point	Original data	Predictive value by GM(1,1)	Predictive value by FGPM
1	5705.05	5875.366	5766.173
2	5596.73	5652.993	5575.520
3	5614.06	5448.493	5572.238
4	5992.57	5547.868	5875.992
5	6330.40	6145.519	6352.264
6	6379.94	6728.679	6477.652
7	6566.70	6628.156	6522.038
8	6572.87	6666.290	6633.407
9	6583.87	6700.796	6582.265
10	6647.50	6591.670	6630.065

Figure 3.1 shows a comparison among GM(1,1), ARIMA, and the proposed FGPM model. From the results, we can observe that GM(1,1) has bad prediction accuracy around the turning points. With the proposed approach, this phenomenon can be reduced. Note that the stock index was normalized and the scale of the horizontal axis was represented by the trading day. It can be seen that the FGPM model has the ability of fitting non-stationary time series, and can provide more accurate prediction than other three models.

In order to make a further comparison in error analysis, three criteria for performance evaluation were used, namely the mean absolute percentage error (MAPE), mean square error (MSE), and root mean square error (RMSE), as listed in (2.23)-(2.25).

$$\text{MAPE}=\frac{\sum_{k=1}^{n}\left(\left|x^{(0)}(k)-\hat{x}^{(0)}(k)\right|/x^{(0)}(k)\right)}{n}\times 100\% \tag{2.23}$$

$$\text{MSE}=\frac{1}{n}\sum_{k=1}^{n}\left(x^{(0)}(k)-\hat{x}^{(0)}(k)\right)^2 \tag{2.24}$$

$$\text{RMSE}=\sqrt{\frac{\sum_{k=1}^{n}\left(x^{(0)}(k)-\hat{x}^{(0)}(k)\right)^2}{n}} \tag{2.25}$$

where n is the total number of test data.

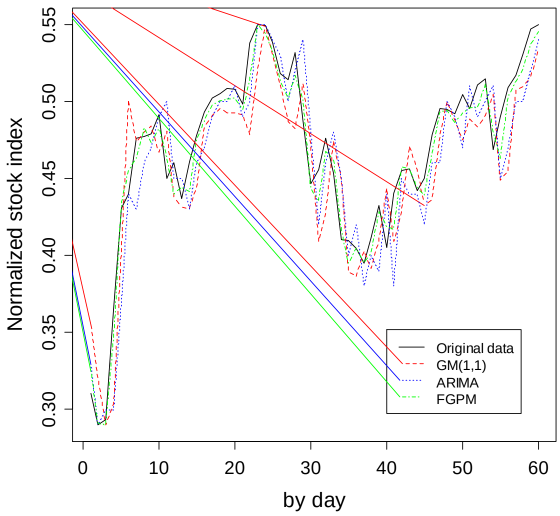

Figure 3.1: Prediction results among different predictive models

Table 3.2 lists the comparative results of predictive errors. It is observed that the proposed FGPM has the best performance among the three models.

Table 3.2 Comparative analysis of predictive errors

Evaluation criteria	GM(1,1)	FGPM	ARIMA
MAPE	5.4407	1.7194	4.3209
MSE	0.0243	0.0077	0.0193
RMSE	0.0302	0.01	0.0257

4 Conclusion

A novel model for non-stationary time series prediction has been proposed in this study. The contribution of this paper has two folds. First, the parameter α in background value is optimized to reduce the estimated error in conventional GM(1,1). Second, an interpolation method incorporated with fuzzy set theory was investigated to overcome the problem of overshooting occurrence. By combining these two strategies, the proposed model exhibits good tracking features to non-stationary time series. Experimental results showed that the proposed approach has significant improvement over some existing methods.

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