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Comparisons of Fuzzy Time Series and Hybrid Grey Model for Non-stationary Data Forecasting

Hsien-Lun Wong [1] and Jau-Min Shiu [2]
[1] Institute of Management, Minghsin University of Science and Technology, HsinChu, 30401, Taiwan
[2] Institute of Business Administration, Minghsin University of Science and Technology, HsinChu, 30401, Taiwan
[1] *Corresponding author: Email: alan@mail.must.edu.tw*

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Abstract: Fuzzy time series (FTS) and Grey model (GM) have been widely applied to forecasting problem in many fields. Rather than traditional econometric model, the two models have no prerequisites for time series normality or error calibration. In this paper, the prediction performance of FTS- heuristic model, two-factor model, Markov model and GM- GM(1,1), GM-Markov, GM-Fourier is investigated. The comparison of the models is based on forecasting error of time series. The high noise data, Taiwan export amount and foreign exchange rate obtained from AEROM, Taiwan are used for models' test. The results illustrate that the two forecasting models are appropriate for non-stationary time series. Among these models proposed, GM-Fourier residual modified model has best predictive performance. The study provides a beneficial reference of hybrid grey-based model in time series prediction.

Keywords: Fuzzy time series; Heuristic model, Two-factor model, GM(1,1), GM-Markov model, GM-Fourier model, Taiwan exports.

1 Introduction

Forecasting tasks are an important characteristic of decision-making process for management in complex and uncertain environment. As the innovation of the computer science, the methodology of forecasting tasks becomes indispensable for researchers. Artificial intelligence and statistical based models are two main techniques for time series forecasting in the past literature. Neural network based models are widely used as the artificial intelligence approaches.[1-7] These models use a great deal of training data for robust generalization; however, the applicable data are not easy to define and select. Another method is the application of genetic algorithm. [8-10] The model has the practical limitations because of the noise and dimensionality of data which may interfere with the learning of patterns.

Traditional econometric models (ARIMA or regression model) are regarded as statistical based models. [11-13] These methods need statistical tools for parameter calibration and assume time series normality. Where time series is confirmed to be non-stationary, it is difficult for traditional models to generate a stationary series, then resulting in explaining too much of the variation in the data with the model.

In recent years, fuzzy set theory has been successfully applied to forecasting problems. [14-29] Most of these models have shown to outperform their conventional counterparts. Since Zadeh (1965) defined "a fuzzy algorithm is an ordered set of fuzzy instructions which upon execution yield an approximate solution to a specified problem", [14] the concept of fuzzy set has risen frequently in many applications, such as artificial intelligent, and management decision and so on. Fuzzy time series (FTS) model can be classified a time series clustering method or pattern recognition method for decreasing dimension of data. The reason of the popularity is that FTS model needs no prerequisites and possibility of modeling by missing data.

Grey system theory is an interdisciplinary scientific area that was first proposed by Deng (1982).[31] Since then, the model was become quite popular to deal with the systems which have limited information and knowledge. As superiority to statistical models, grey model (GM) requires only limited amount of data to evaluate the behavior of unknown systems.[32] Today, the GM has shown good prediction accuracy and results in various areas.[33-37] As grey forecasting model is constructed of exponential function, which may still have worse curve-fittings effects, especially in random data. To increase the prediction precision, some techniques are integrated into the GM.[35,38,39]

In real world, many existing economic data are nonlinear or high noise due to complex environment, so these non-stationary time series are particularly important. With non-stationary data, there is a major problem of spurious regression.[40] Clearly, FTS and GM models are suitable to deal with time series with the characteristic of limited, linguistic and missing parts; however, little research have paid attention to discuss the availability of the models. For the purpose, the prediction performance of listed FTS: heuristic model, two-factor model, Markov model and hybrid GM: GM(1,1) model, GM-Markov model, GM-Fourier model is investigated. The comparison of the models is based on forecasting error of time series. The high noise data, Taiwan export amount and foreign exchange rate obtained from AEROM, Taiwan are used for models' test.

The next section presents the formulation of FTS and GM, and model algorithm. Section three compares the results of empirical tests with models. Finally, section four contains conclusions and recommendations.

2 Research Methodology

Suppose that there is a relationship between the series $F(t-1)$ and $F(t)$, defined as $F(t) = F(t-1) \times R(t-1,t)$, where "×" is an operator. Further, the fuzzy logical relationship between the series $F(t)$ and $F(t-1)$ is expressed as $F(t-1) \rightarrow F(t)$. Let $F(t) = A_i$ and $F(t-1) = A_j$, so the fuzzy logical relationship A_i and A_j exists, where A_i is the left-hand side, A_j the right-hand side. The fuzzy logical relationship can be further integrated into fuzzy groups because of the same left-hand sides. For example,

$$A \quad (1)$$

Therefore, these fuzzy logical relationships can be expressed as follows:

$$A \quad (2)$$

In a heuristic model, the parameters of the heuristic function are the fuzzy logical relationship groups and the model variables. From these fuzzy logical relationship groups, the heuristic function uses the variables to select proper fuzzy sets to establish heuristic fuzzy logical relationship groups.

All the fuzzy sets $A_1, A_2, A_3, \ldots, A_k$ are well ordered. This greatly facilitates the selection of proper fuzzy sets by the heuristic function. Suppose $F(t-1) = A_i$ and the fuzzy logical relationship group for A_i is $A_i \rightarrow A_{j1}, A_{j2}, A_{j3} \ldots$. Proper fuzzy sets, $A_{p1}, A_{p2}, A_{p3}, \ldots, A_{pk}$, can be selected by the heuristic function $h(x)$, $h(x, A_{j1} A_{j2.} A_{j3} \ldots) = A_{p1}, A_{p2}, A_{p3}, \ldots, A_{pk}$, where x is the heuristic variable; $A_{p1}, A_{p2}, A_{p3}, \ldots, A_{pk}$ are selected from $A_{j1}, A_{j2}, A_{j3} \ldots$, by the heuristic function. A heuristic fuzzy logical relationship group is obtained as below:

The algorithm for a heuristic fuzzy time series can be presented as follows:

Step1. Decide the universe of discourse and the length of interval.

Step2. Define the fuzzy sets and establish the fuzzy logical relationships.

Step3. Define the second and third variables.

Step4. Redefine the fuzzy logical groups and the threshold auto regression model.

Step5. Forecast and defuzzify.

2.1 Markov fuzzy time series model

Let U be the universe of discourse with $G = (\mu_1, \ldots, \mu_r)$ and $H = (v_1, \ldots, v_r)$, and $\{P_i, i = 1, 2, \ldots, r$ defined as ordered partition sets of U. μi and v_i are membership functions on the universe of the fuzzy set U. The fuzzy group relation between G and H is defined as

$$R \quad (3)$$

where '∘' is the max-min operator, T the transpose, R_{ij} the membership function between G and H.

2.1.1 Markov fuzzy relation matrix

It is assumed that $\{F(X(t))\}$ is a fuzzy auto-regressive process of order one, meaning that for any t, $F(X_t)$ is determined completely by $F(X_{t-1})$. Let the membership function of $F(X_t)$ be $\mu_i(X_t)$, $i = 1,2,\dots,r$, then the Markov fuzzy relation matrix is represented by

$$R^* = [R^*_{ij}]_{r\times r} = \max_{2\le t\le n}[\min((\mu_i(X_{t-1}),\mu_j(X_t))]_{r\times r} \quad (4)$$

2.1.2 P-th order fuzzy auto-regressive model

If the fuzzy time series model is expressed as below:

$$F(t) = F(t-1)\circ F(t-2)\circ \dots \circ F(t-p) \times R^* \quad (5)$$

then we call it a p-th order fuzzy autoregressive model, where R^* is the fuzzy relation matrix between $F(t)$ and $F(t-1), F(t-2),\dots,F(t-p)$. Assume that the $(FX_{1,t}, FX_{2,t},\dots, FX_{k,t})$ is a first order fuzzy auto regression

$$(FX_{1,t}, FX_{2,t},\dots,FX_{k,t}) = (FX_{1,t-1}, FX_{2,t-1},\dots,FX_{k,t-1})\begin{bmatrix} R_{11} & \cdots & R_{1k}\\ \vdots & \ddots & \vdots\\ R_{k1} & \cdots & R_{kk}\end{bmatrix} \quad (6)$$

where R_{ij} is the Markov fuzzy relation matrix. As the series $(FX_{1,t}, FX_{2,t},\dots,FX_{k,t})$is determined by the series $(FX_{1,t-1}, FX_{2,t-1},\dots,FX_{k,t-1})$, then equation (6) defines the Markov multi-variant fuzzy process.

The algorithm for a Markov fuzzy time series model is as follows:
Step 1. Define the universe of discourse.
Step 2. Maximize fuzzy membership function.
Step3. Decide the order of the fuzzy autoregressive model.
Step 4. Compute Markov fuzzy relation matrix.
Step 5. Forecast and defuzzify.

2.2 Two-factor fuzzy time series model

For this model, it is assumed that two fuzzy time series, $F(t)$ and $G(t)$, are the main factors relevant to forecasting. Two-factor criterion vectors are defined as follows:

$$C(t) = f(t-1) = [C_1, C_2,\dots,C_m],\ 0 \le C_j \le 1 \quad (7)$$

$$S(t) = g(t-1) = [S_1, S_2,\dots,S_m],\ 0 \le S_j \le 1 \quad (8)$$

The operation matrix is defined as follows:

$$O^w(t) = \begin{bmatrix} f(t-2)\\ f(t-3)\\ \vdots\\ f(t-w)\end{bmatrix} = \begin{bmatrix} O_{11} & O_{12} & \cdots & O_{1m}\\ O_{21} & O_{22} & \cdots & O_{2m}\\ \vdots & \vdots & \vdots & \vdots\\ O_{(w-1)1} & \cdots & \cdots & O_{(w-1)m}\end{bmatrix}$$

$$0 \le O_{ij} \le 1, 1 \le i \le w-1, 1 \le j \le m \quad (9)$$

where $f(t-1)$ is the fuzzified change in the first factor $F(t)$ from time $(t-1)$ to $(t-2)$; g$(t-1$the fuzzified change at time $t-1$ of the second-factor $G(t)$; m the number of intervals in the universe of discourse; w the window basis.

2.2.1 Two-factor fuzzy relationship matrix

$$R(t) = O^w(t)\otimes S(t)\otimes C(t) = \begin{bmatrix} O_{11}\times S_1\times C_1 & O_{12}\times S_2\times C_2 & \cdots & O_{1m}\times S_m\times C_m\\ O_{21}\times S_1\times C_1 & O_{22}\times S_2\times C_2 & \cdots & O_{2m}\times S_m\times C_m\\ \vdots & \vdots & \vdots & \vdots\\ O_{(w-1)1}\times S_1\times C_1 & O_{(w-1)2}\times S_2\times C_2 & \cdots & O_{(w-1)m}\times S_m\times C_m\end{bmatrix}$$

$$0 \le O_{ij} \le 1, 1 \le i \le w-1, 1 \le j \le m,\quad 0 \le S_j \le 1, 0 \le C_j \le 1 \quad (10)$$

where $R_{ij} = O^w(t)\times S(t)\times C(t)$, " $\times$ " is the multiplication operator. From the matrix model (10), the fuzzified variation $F(t)$ can be described as follows:

$$f(t) = \begin{bmatrix} \max(R_{11}, R_{21},\dots,R_{(w-1)1}), \max(R_{12}, R_{22},\dots,R_{(w-1)2}),\dots,\\ \max(R_{1m}, R_{2m},\dots,R_{(w-1)m})\end{bmatrix} \quad (11)$$

The algorithm for the two-factor fuzzy time series model can be represented as follows:
Step 1. Decide the first factor of time series and calculate its variation.
Step 2. Decide universe of discourse and length of intervals.
Step 3. Determine the second factor $G(t)$ as defined in steps (1) and (2).
Step 4. Decide the window basis of the two-factor fuzzy relationship matrix.
Step 5. Compute the fuzzified two-factor variation and defuzzify.

2.3 GM (1,1) model

The grey forecasting model was proposed to deal with the systems which have only limited information and knowledge available[30-31]. In recent years, the grey forecasting model has been successfully applied to various fields and has satisfactory results. The GM (1,1) model constructed of single variable for forecasting accumulates the original time series for more regular one, then uses differential equation for the regular data prediction. Finally, the inverse accumulated operation is applied to get the prediction value of original time series data. Suppose $x^{(0)} = \{x^{(0)}(1),\ x^{(0)}(2), \ldots, x^{(0)}(n)\}$ be an original time series, then accumulated generating operation is formulated as $x^{(1)} = \{x^{(1)}(1), x^{(1)}(2),\ \ldots, x^{(1)}(n)\}$. The basic GM(1,1) forecasting model is defined as follows:

$$\hat{x}^{(0)}(k) = \left[\hat{x}^{(0)}(1) - \frac{b}{a}\right] \cdot\ (1 - e^{a}) \cdot\ e^{-a(k-1)}$$
k=1,2,…,n (12)

where $\hat{x}^{(0)}(k)$ is the predictive value of GM(1,1) model; $x^{(1)}(1) = x^{(0)}(1)$; a and b are the developing coefficients and grey inputs, and they can be obtained as follows:

$$p = \begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y_n \tag{13}$$

$$B = \begin{bmatrix} -\frac{1}{2}\left[x^{(1)}(1) + x^{(1)}(2)\right] \cdots 1 \\ -\frac{1}{2}\left[x^{(1)}(2) + x^{(1)}(3)\right] \cdots 1 \\ \vdots \\ -\frac{1}{2}\left[x^{(1)}(n-1) + x^{(1)}(n)\right] \cdots 1 \end{bmatrix} \tag{14}$$

$$Y_n = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(k) \end{bmatrix} \tag{15}$$

After the parameters a and b obtained (13), the predictive value of sequence is $\hat{x}^{(0)}(k) = (\hat{x}^{(0)}(1), \hat{x}^{(0)}(2),\ \hat{x}^{(0)}(3), \ldots, \hat{x}^{(0)}(n))$ by substituting k = 1, 2,…, n into Eq. (12).

2.3.1 Grey-Markov model

A Markov chain is a particular type of stochastic phenomenon and it has been used to explain a variety of disciplines. [41] There are two main characteristics for Markov process: state space and state transition matrices, which can provide the expectations of the possible correction for prediction of the value for the next step. A Markov chain must satisfy the Markov property, meaning the next state of the probability depends on the current state, not past state. The transition probability of state to state is expressed as follows:

$$P_{ij}^{(m)} = \frac{M_{ij}^{(m)}}{M_i^{(m)}} \tag{16}$$

where $P_{ij}^{(m)}$ is the state transition probability from state i to state j after m steps; $M_{ij}^{(m)}$ is the number of state transition from i to j; $M_i^{(m)}$ is the total number of state i. The transition matrix of a stochastic system with k-state and m-step is defined as follows:

$$R^{(m)} = \begin{bmatrix} P_{11}^{(m)} & P_{12}^{(m)} & \cdots & P_{1k}^{(m)} \\ P_{21}^{(m)} & P_{22}^{(m)} & \cdots & P_{2k}^{(m)} \\ \vdots & \vdots & \vdots & \vdots \\ P_{k1}^{(m)} & P_{k2}^{(m)} & \cdots & P_{kk}^{(m)} \end{bmatrix} \tag{17}$$

where $R^{(m)}$ is the transition matrix reflects the transition rule in a system. If the states all come from a state space k, a finite set, then the Markov chain called a finite state Markov chain. Using the transition matrix, the predictive value of time series by GM-Markov model is obtained as follows:

$$\hat{X}_{k+1} = \frac{1}{2}\left(L_{k+1,j}, U_{k+1,j}\right) \tag{18}$$

2.3.2 Grey- Fourier model

Grey-Fourier model is used to improve the modeling accuracy of grey models by modifying the error residuals in GM(1,1), several research have been discussed in the literature. The concept of Fourier series is to transform the residual of GM(1,1) into frequency spectra and select the low-frequency terms, filtering out high-frequency and remaining the characteristics of GM(1,1). [42] Suppose E is error terms defined as $E(j) = \hat{x}^{(0)}(j) - x^{(0)}(j), j = 2,3, \ldots, n,$ the GM-Fourier model can be expressed as follows:

$$\widehat{X_a}^{(0)}(j) = \hat{X}^{(0)}(j) - E_a(j) \quad j = 2,3, \ldots, n \tag{19}$$

$$E_a(j) \cong \frac{1}{2} a_0 + \sum_{i=1}^{k_a} a_i \cos\left(\frac{2\pi \cdot i}{T_a} j\right) + b_i \sin\left(\frac{2\pi \cdot i}{T_a} j\right) \tag{20}$$

where $T_a = n - 1$, meaning the length of residual series, and $k_a = \frac{n-1}{2} - 1$, the minimum deployment frequency of Fourier series, will be an integer number. By using the least-squares method, the parameters a_0, a_i and b_i for i=1,2,3…, k_a in the Eq.20 can be estimated as follows:

$$C_a = (P_a^T P_a)^{-1} P_a^T E_a \tag{21}$$

where the C and P matrix are shown as follows:

$$C_a = [a_0, a_1, b_1, a_2, b_2, \dots, a_{k_a}, b_{k_a}]^T \tag{22}$$

$$P_a = \begin{bmatrix} \frac{1}{2}, & cos\left(\frac{1\cdot 2\pi \cdot 2}{T_a}\right), & sin\left(\frac{1\cdot 2\pi \cdot 2}{T_a}\right), & \cdots, & cos\left(\frac{k_a\cdot 2\pi \cdot 2}{T_a}\right), & sin\left(\frac{k_a\cdot 2\pi \cdot 2}{T_a}\right) \\ \frac{1}{2}, & cos\left(\frac{1\cdot 2\pi \cdot 3}{T_a}\right), & sin\left(\frac{1\cdot 2\pi \cdot 3}{T_a}\right), & \cdots, & cos\left(\frac{k_a\cdot 2\pi \cdot 3}{T_a}\right), & sin\left(\frac{k_a\cdot 2\pi \cdot 3}{T_a}\right) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{1}{2}, & cos\left(\frac{1\cdot 2\pi \cdot n}{T_a}\right), & sin\left(\frac{1\cdot 2\pi \cdot n}{T_a}\right), & \cdots, & cos\left(\frac{k_a\cdot 2\pi \cdot n}{T_a}\right), & sin\left(\frac{k_a\cdot 2\pi \cdot n}{T_a}\right) \end{bmatrix} \tag{23}$$

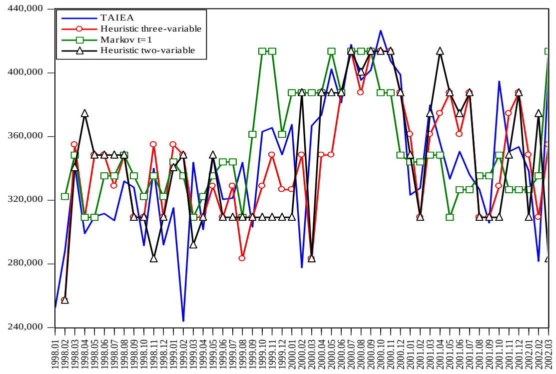

Figure.1. Pattern of actual value and predicted value with three FTS models

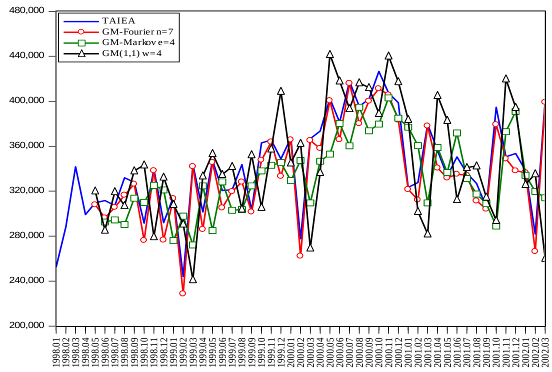

Figure.2. Pattern of actual value and predicted value with three GM models

Table 1. Forecasting error of the FTS and GM models for Taiwan exports

Sample	Period	Error	Fuzzy two-factor w=2	Fuzzy heuristic three-variable	Fuzzy Markov t=1	GM(1,1) w=4	GM-Markov e=4	GM-Fourier n=7
n=16	1998.01-	MSE	2.60×10^9	1.31×10^9	1.04×10^9	1.95×10^9	1.27×10^9	1.20×10^8
	1999.04	MAPE	13.59%	9.62%	8.46%	11.46%	10.06%	2.89%
n=32	1998.01-	MSE	2.34×10^9	1.39×10^9	1.29×10^9	2.05×10^9	1.38×10^9	1.20×10^8
	2000.08	MAPE	12.42%	9.02%	8.67%	11.31%	9.41%	2.65%
n=48	1998.01-	MSE	2.17×10^9	1.24×10^9	1.11×10^9	2.56×10^9	1.57×10^9	1.20×10^8
	2001.12	MAPE	11.48%	8.33%	7.99%	11.33%	8.80%	2.54%

3 Empirical Results and Discussion

In this study, the time series data were monthly export totals for Taiwan and foreign exchange rate. Taking the Taiwan exports as the first-factor, we define foreign exchange rate the second-factor for fuzzy multivariable model test. As the correlation coefficient of two factors is 0.98811 and p= 0.0001 (<0.01), meaning that their positive correlation is statistically significant. That makes sure the foreign exchange rate is suitable for second-factor use. After computation, the value of ADF is -3.9979, indicating that the type of Taiwan exports test is recognized as pure random walk. Similarly, the ADF of foreign exchange rate is -3.966412, which is shaped as pure random walk. The data is from January, 1998 to December, 2002 with totally 48 dataset for each, which were divided into three sections of unequal period for comparison. The first section covers January, 1998 to April, 1999 (16 months); the second section is January, 1998 to August, 2000 (32 months); the third is January, 1998 to December, 2001 (48 months). The values of MSE and MAPE are used to evaluate the forecasting performance of model.

The final results of FTS model for Taiwan exports are shown in Figure 1 and Table1. Figure 1 shows the pattern of actual value and forecasting value with three FTS models. From the Table 1, it is noted that the smallest MSE/MAPE between forecasting value and actual value is fuzzy Markov model with 1.04×10^9/8.46%, followed by fuzzy heuristic model with 1.24×10^9/8.33%, and then by fuzzy two-factor 2.17×10^9/10.15%.

The results of hybrid GM model for Taiwan exports are shown in Figure 2 and Table1. Figure 2 shows the pattern of actual value and forecasting value with hybrid GM models. Table 1 indicates that the smallest MSE/MAPE for grey models is GM-Fourier with 1.20×10^8/2.54%, followed by GM-Markov with 1.27×10^9/8.80% and then by GM(1,1) with 1.95×10^9/11.3%.

The comparisons of forecasting error of both the FTS and GM models are shown in Table 1. It should be noted that the GM-Fourier model has the lowest MSE/MAPE with 1.20×10^8/2.54% of all the forecasting models, followed by fuzzy Markov model with 1.04×10^9/8.46%, fuzzy heuristic model with 1.24×10^9/8.33%, GM-Markov model 1.27×10^9/8.80%. As the value of MAPE is lower than 10%, these four models are regarded as good data-fitting.

For the FTS models, it is interesting to note that the longer the length of data, the lower MSE of the model, except fuzzy Markov model. On the contrast, the result for GM model is that the longer the length of data, the bigger MSE, except GM-Fourier model. The findings are worthy to note that both fuzzy Markov model and GM-Fourier model are more stable in data fitting than other models proposed, meaning that the two models are suitable for applicability in stochastic time series.

4 Conclusions

In this paper, the prediction performance of three FTS and hybrid GM models is investigated. The Taiwan exports and foreign exchange rates are used for model comparisons. Based on the MSE/MAPE of model, the empirical results illustrate that both the FTS and GM models are appropriate for non-stationary time series. However, the GM-Fourier residual model has a better forecasting accuracy than other methods proposed, regardless of time series. As a result, it is an effective strength of GM-Fourier model used for forecasting purpose in uncertain environment. Finally, although the GM-Fourier model is more accurate for the Taiwan exports example, further testing would be required to determine the generality of the result, such as the type of time series.

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Dr. Hsien-Lun Wong is an associate professor of Institute of Management, Minghsin University of Science and Technology, Taiwan. He obtained his PhD degree from National Chiai Tung University in July, 2006 and is current a member of the Chinese Institute of Transportation. His research fields are transportation and logistics planning, marketing research and forecasting technology.

Mr. Jay-Min Shih obtained his MBA degree from Minchin University of Science and Technology, Taiwan, in July, 2011. His research interests are forecasting system, data mining and consumer behavior.