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A New Space-Filling Curve Based Method for the Traveling Salesman Problems

Yi-Chih Hsieh [1] and Peng-Sheng You [2]

[1] Department of Industrial Management, National Formosa University, Yunlin 632, Taiwan
[2] Graduate Institute of Marketing & Logistics/Transportation, National Chia-Yi University, Chia-Yi, 600, Taiwan
Corresponding author: Email: yhsieh@nfu.edu.tw

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Abstract: The Traveling Salesman Problem (TSP) is one of the most intensively studied problems in optimization and it is used as a benchmark for many optimization methods. Given a list of n cities and their pairwise distances, the TSP aims to find a shortest possible tour that visits each city exactly once. As known, there are several applications for the TSPs, including mail/product delivery, production sequencing, planning, logistics, and the manufacture of microchips etc. In addition, some distribution problem, vehicle routing problem and scheduling problem etc can be reduced into a TSP. Moreover, the TSP can be applied in the DNA sequencing. The TSP is an NP-hard problem and several heuristic approaches have been proposed for solving it approximately. As known, the space-filling curve (SFC) method is a very special heuristic approach for solving the TSP. The SFC that can transform a point of two-dimensional space in [0,1]×[0,1] into a point of one-dimensional line in [0,1] was firstly proposed by Peano in 1890. In 1988, based upon the square type SFC, Bartholdi and Platzman developed a heuristic for solving the TSPs and applied it for developing the tour of meal delivery. There are many different basic types for the recursively transformation of SFCs. In this paper, we intend to propose a new type of SFC based method for solving the TSP. Numerical results of one hundred random problems and fourteen benchmark problems show that the new SFC based method performs better and faster than the typical square SFC based method when few CPU time is allowed.

Keywords: Space-Filling Curve, Traveling Salesman Problem, Tour

1 Introduction

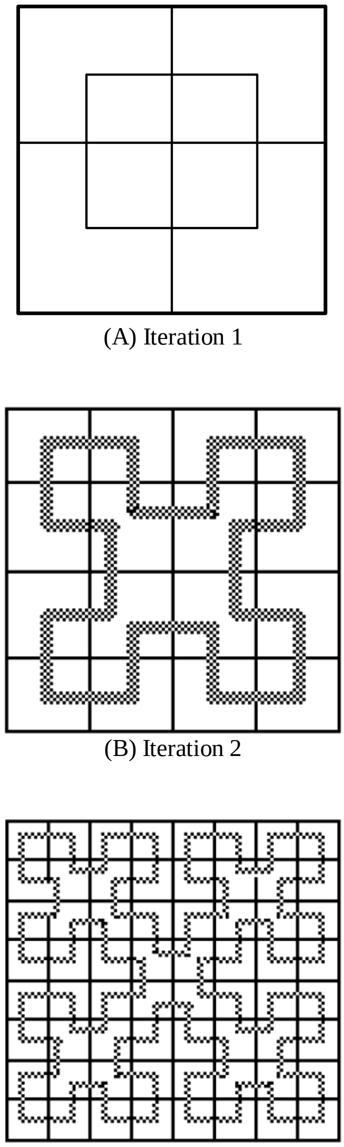
The Traveling Salesman Problem (TSP) has been first formulated as a mathematical problem by mathematician Karl Menger. It is one of the most intensively studied problems in optimization and is used as a benchmark for many optimization methods. Given a list of n cities and their pairwise distances, the TSP is to find a shortest possible tour that visits each city exactly once. In other words, TSP is to find a shortest Hamiltonian circuit in a Hamiltonian network [1]. As known, there are several applications for the TSP, including mail/product delivery, production sequencing, planning, logistics, and the manufacture of microchips etc. In addition, some distribution problem, vehicle routing problem and scheduling problem etc can be reduced into a TSP. Moreover, the TSP can be applied in the DNA sequencing. Therefore, the TSP is one of the most intensively studied problems in computational mathematics and optimizations.

The TSP is an NP-hard problem [2] and there have several heuristic approaches been proposed for solving it approximately. The variants and methods of TSPs are referred to the survey paper by Laport [2] and the excellent website of TSPLIB [3]. As known, the space-filling curve (SFC) method is a very special heuristic approach for solving the TSP. The SFC that can transform a point of two-dimensional space in [0,1]×[0,1] into a point of one-dimensional line in [0,1] was firstly proposed by Peano in 1890. In 1988, based upon the “square” type SFC, Platzman and Bartholdi [4] developed a heuristic for solving the TSPs and applied it for developing the tour of meal delivery. Additionally, there are several reference papers related to the SFC for TSPs and the other applications, including Norman and Moscato [5], Schamberger and Wierum [6], Asano et al. [7], Wang et al. [8], Platzman and Bartholdi [9]. There are many different basic "types" for the recursively transformation of space-filling curve. In this paper, we intend to propose a new type of SFC method for solving the TSP. Numerical results of several random problems and benchmark problems show that the new proposed SFC based approach performs better and faster than the typical square SFC based approach when few CPU time is allowed.

This paper is organized as follows. In Section 2, we will briefly present the SFC. In Section 3, we will develop a new SFC based method for solving the TSPs. Numerical results of the new proposed approach are reported and compared with those by the typical SFC based method in Section 4. Short conclusions are summarized in Section 5.

2 Space-Filling Curve

In 1890, Peano discovered a densely self-intersecting curve that passes through every point of the unit square to construct a continuous mapping from the unit interval onto the unit square. Peano was motivated by Georg Cantor's earlier counterintuitive result that the infinite number of points in a unit interval is the same cardinality as the infinite number of points in any finite-dimensional manifold, such as the unit square. The problem Peano solved was whether such a mapping could be continuous; i.e., a curve that fills a space. Fig. 2.1 illustrates a square type of SFC with various numbers of recursive iterations. Note that this type of SFC was applied by Platzman and Bartholdi [4] for solving TSP.



(A) Iteration 1

(B) Iteration 2

(C) Iteration 3

Figure 2.1: The "square" type of SFC used by Platzman and Bartholdi [4]

3 Method

3.1 Another Type of Space-Filling Curve

In this paper, we propose another type of SFC for solving TSP. The proposed basic type of SFC is in the form of "米", a Chinese character which literal means "rice" (see Fig. 3.1(A)). Fig. 3.1(B) and Fig. 3.1(C) illustrate the SFCs of recursive iteration 2 and iteration 3, respectively.

3.2 Mapping of Space-Filling Curve

For simplicity, we use the following three cities and the proposed SFC of iteration 1 as an ex ample (see Fig. 3.2(A)). Fig. 3.2(B) shows these three cities with the basic type of SFC. Fig. 3.2(C) illustrates the corresponding projected points (marked in red) of these three cities to their nearest axles. Thus, following the sequence of SFC in Fig. 3.3(A), we obtain the sequence of A-B-C-A as the tour for this example. More specifically, as shown in Fig. 3.2(C), there are eight axles, namely, axle 1 to axle 8. Assume that the projected points of city A on axle 1 and axle 8 are z_{A1} and z_{A8}, respectively. Let r_{A1} and r_{A8} be the distance of point A to z_{A1} and the distance of point A to z_{A8}, respectively. We suppose that:

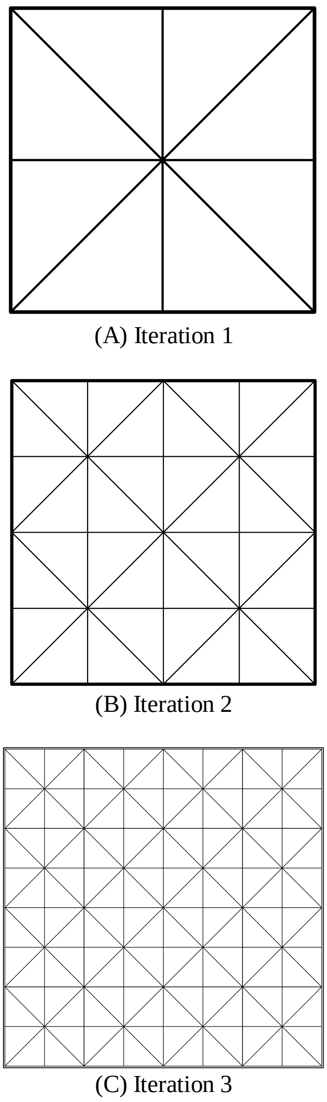


Figure 3.1: The basic type of space-filling curve proposed in this paper.

$\Omega = \{z_{ij} \mid r_{ij} < r_{ik}, 1 \le i \le n,$ j and k are corresponding axles for city $i\}$ (3.1)

Then, if the total length of SFC is one unit, we may obtain the corresponding value of each z_{ij}.in the SFC, say θ_{ij}. Finally, following the order of values of θ_{ij}, from the smallest to the largest, we may obtain the tour of cities. Fig. 3.3(B) and Fig. 3.3(C) illustrate the sequence of SFC with iteration 2 and iteration 3.

3.3 Example

Consider the example with ten cities shown in Fig. 3.4. Their corresponding coordinates in [0,1]×[0,1] are shown in Table 3.1. If the new SFC with iteration 1 is used, then the projected points (marked in red) are shown in Fig. 3.4(A) and their corresponding values of θ are shown in Fig. 3.4(C) and Table 3.1. Finally, rank the values of all θ in Fig. 3.4(C), we obtain the tour for these ten cities as A→B→E→F→D→G→I→J→H→C→A in Fig. 3.4(B).

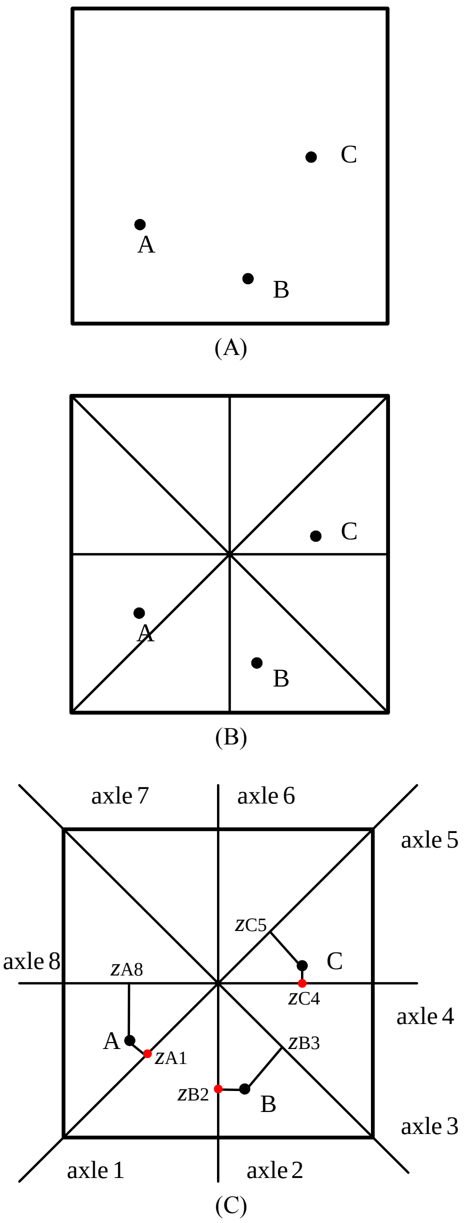


Figure 3.2: The type of space-filling curve used in this paper

3.4 Neighborhood Exchange Method

In this paper, to improve the solutions of the SFC method, we use the simple 2-point neighborhood exchange method (NEM). That is, if the sequence of tour is A→B→C by the SFC method, then we may update this sequence with the best sequence among the following three B→A→C, C →B→A and A→C→B if there is any one better than the tour A→B→C.

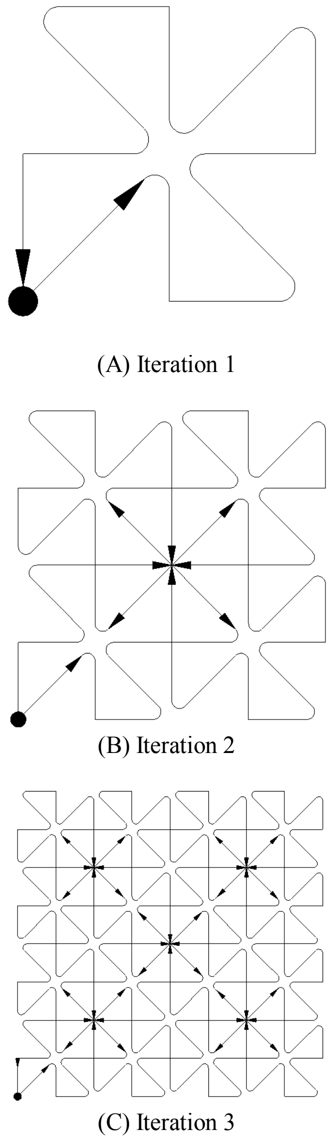


Figure 3.3: The tour of the new SFC used in this paper. (• the start and end point)

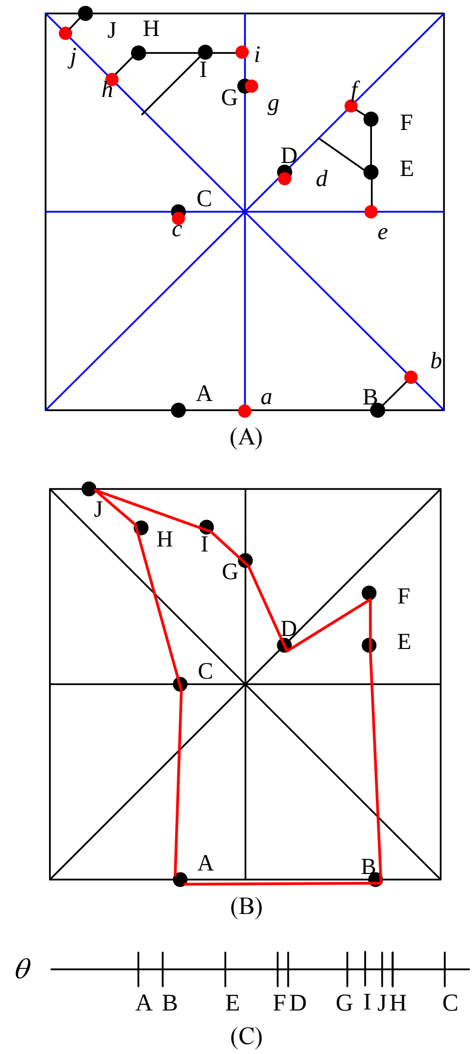


Figure 3.4: (A) The projected points and θ for the 10 points (iteration=1). (B) The tour. (C) The corresponding values of θ.

Table 3.1: The coordinates and θ for the ten points

City	X-axle	Y-axle	θ
A	0.3	0.0	0.2500
B	0.8	0.0	0.2646
C	0.3	0.5	0.9172
D	0.6	0.6	0.5586
E	0.8	0.6	0.4172
F	0.8	0.7	0.5366
G	0.5	0.8	0.7086
H	0.2	0.9	0.7720
I	0.4	0.9	0.7293
J	0.1	1.0	0.7573

4 Numerical Results

4.1 Test Problems

Two parts of test problems are experimented in this paper.

Part I: 100 random problems. Each random problem has 100 cities with coordinates randomly generated in [0,1]×[0,1]. Numerical results of this part are reported in Table 4.1.

Part II: 14 benchmark problems with cities vary from 76 to 1002 in the TSPLIB [3]. Numerical results of this part are reported in Table 4.2.

For comparison, we use the new SFC based method and the typical “square” SFC based method by Platzman and Bartholdi [4] to solve each test problem. In addition, we also apply the neighborhood exchange method to improve the solutions by these two SFC based methods.

Table 4.1: Numerical results of test problems (Part I). (*I*=iteration)

I	Value	New SFC(1)	Square SFC(2)	New SFC + NEM(3)	Square SFC + NEM(4)
5	No. of wins(5)	76	24	78	22
	mean CPU	0.1044	0.3054	1.1461	1.3531
	Mean length	9.6308	9.7819	9.2309	9.3646
6	No. of wins	58	42	58	42
	mean CPU	0.1421	0.6270	1.1936	1.6817
	Mean length	9.6096	9.6392	9.2274	9.2556
7	No. of wins	45	55	50	50
	mean CPU	0.1904	1.2686	1.2502	2.3317
	Mean length	9.6075	9.5901	9.2273	9.2136
8	No. of wins	33	67	40	60
	mean CPU	0.2408	2.5590	1.3086	3.6306
	Mean length	9.6074	9.5648	9.2273	9.1866
9	No. of wins	26	74	39	61
	mean CPU	0.3008	5.1580	1.3763	6.2390
	Mean length	9.6074	9.5564	9.2273	9.1812
10	No. of wins	25	75	38	62
	mean CPU	0.3662	10.39954	1.44977	11.48742
	Mean length	9.6074	9.5550	9.2273	9.1799

(1) New SFC: the proposed space-filling curve method.
(2) Square SFC: the space-filling curve method used by Platzman and Bartholdi [4].
(3) New SFC+NEM: the proposed SFC method with the use of neighborhood exchange method.
(4) Square SFC+NEM: the SFC method used by Platzman and Bartholdi [4] with the use of neighborhood exchange method.
(5) No. of wins: the number of test problems that the new SFC based method performs better than the square SFC based method.

Table 4.2: Numerical results of test problems (Part II). (*I*=iteration)

Method Compare	New SFC > Square SFC (1)						New SFC+NEM > Square SFC+NEM(2)					
Problem	*I*=5	*I*=6	*I*=7	*I*=8	*I*=9	*I*=10	*I*=5	*I*=6	*I*=7	*I*=8	*I*=9	*I*=10
Pr76	0	0	0	0	0	0	1	1	1	1	1	1
Pr107	0	0	0	0	0	0	0	0	0	0	0	0
Pr124	0	0	0	0	0	0	0	0	0	0	0	0
Pr136	1	1	1	1	1	1	1	0	0	0	0	0
Pr144	0	0	0	0	0	0	1	0	0	0	0	0
Pr152	1	1	1	0	0	0	1	1	1	1	1	1
Pr226	1	1	1	0	0	0	1	1	1	0	0	0
Pr264	1	0	0	0	0	0	1	1	1	0	0	0
Pr299	1	1	1	1	1	1	1	1	1	1	1	1
Pr439	1	0	0	0	0	0	1	1	1	1	1	1
Pr1002	1	1	1	1	0	0	1	1	1	1	0	0
Rd100	1	1	1	1	1	1	1	1	1	1	1	1
Rd400	1	1	0	0	0	0	1	1	1	1	1	1
Bier127	1	1	1	1	1	1	1	1	1	1	1	1
Total	10	8	7	5	4	4	12	10	10	8	7	7
Win (%)	0.71	0.57	0.50	0.36	0.29	0.289	0.86	0.71	0.71	0.57	0.50	0.50

(1) The proposed space-filling curve method performs better than (or tie) the space-filling curve method used by Platzman and Bartholdi [4]. 1=Yes, 0=No..
(2) The proposed SFC method with the use of neighborhood exchange method performs better than (or tie) the space-filling curve method used by Platzman and Bartholdi [4] with the use of neighborhood exchange method. 1=Yes, 0=No.

4.2 Numerical Results

Part I: From Table 4.1, we observe that:

(a) The new SFC based method performs better than the typical square SFC based method when the number of SFC recursive iterations is 5 or 6, while their performance is similar when the number of SFC recursive iteration is 7. However, the new SFC based method performs worse to the typical square SFC based method when the number of recursive iterations is 8, 9, or 10.

(b) With the use of NEM, the new SFC based method performs better than the typical square SFC based method when the number of recursive iterations is 5 and 6, while their performance is similar when the number of SFC recursive iteration is 7.

(c) The CPU time by the new SFC based method is less than that of the typical square SFC based method when the number of re-cursive iteration is 5, 6, 7, 8, 9, or 10.

Part II: From Table 4.2, we observe that:

(a) The new SFC based method performs better than the typical square SFC based method when the number of recursive iterations is 5 or 6, while their performance is similar when the number of SFC recursive iteration is 7. However, the new SFC based method performs worse to the typical square SFC based method when the number of recursive iterations is 8, 9, or 10.

(b) With the use of NEM, the new SFC based method performs better than the typical square SFC based method when the number of recursive iterations is 5, 6, 7 or 8, while their performance is similar when the number of SFC recursive iteration is 9 or 10.

5 Conclusions

In this paper:

(a) We have developed a new SFC based method for the TSPs and have applied it for solving 100 random problems and 14 benchmark problems.

(b) We have compared the numerical results of the new SFC based method with those of the typical square SFC based method.

(c) Limited numerical results have shown that the proposed new SFC based method performs better and faster than the typical square new SFC based method when few CPU time is available, i.e., the real-time environment.

(d) The solutions by the new SFC based method can be used as the initial solutions for most of artificial intelligence methods.

In the future, one may develop the other SFC based methods and derive their theoretical results for the TSPs. In addition, one may explore the other applications for the space-filling curves.

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Yi-Chih Hsieh received his Ph.D. degree in Industrial Engineering from The University of Iowa, USA, and his research interests include optimisation, operations research, and applications of programming and artificial intelligence. He is now a professor at the Department of Industrial Management, National Formosa University, Taiwan.

Peng-Sheng You received a Ph.D. degree in Management Science and Engineering from University of Tsukuba, Japan, and his research interests include inventory management and supply chain management. He is now a professor at the Graduate Institute of Marketing and Logistics/Transportation, National Chiayi University, Taiwan.