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# Coherent Risk Measure Based on Relative Entropy

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**Abstract:** This article proposes a new coherent risk measure called iso-entropic risk measure, which is based on relative entropy under the theory framework of Artzner et al.(1999). It is pointed that this measure is just the negative expectation of the risk portfolio position under the probability measure through Esscher transformation. This iso-entropic risk measure is not a 0-1 risk measure and very smooth in contrast with another important coherent risk measure $AV@R$ (*Average Value at Risk*). And it is a little larger than $AV@R$ at the same level, namely it is has more prudence. So it maybe a better coherent risk measure.

**Keywords:** Risk Management, Iso-entropic Risk Measure, Coherent Risk Measure, Relative Entropy, Calculus of Variations.

## 1. Introduction

How to measure the risk of the uncertainty in the future value of a position is one of the basic tasks in finance. The most well-known and widely used in practice methods to this task are variance and subsequent $V@R$(*Value at Risk*). In the finance context the standard deviation of continuous growth rates usually is called volatility. However, both of them have serious drawbacks. One important drawback for variance is that it is not monotonic: a better gamble, i.e., a gamble with higher gains and lower losses, may well have a higher variance and thus be wrongly viewed as having a higher riskiness. About $V@R$, it takes into account only the quantile of the distribution without caring about what is happening to the left and to the right of the quantile. And it is concerned only with the probability of the loss and does not care about the size of the loss. However, it is obvious that the size of loss should be taken into account(Cherny and Madan, 2008)[6]. Further criticism of variance and $V@R$ can be found in Artzner et al.(1997)[2] as well as in numerous discussions in financial journals.

At the final of last century, a new very promising method to quantify risk was proposed in the landmark paper by Artzner, Delbaen, Eber and Heath(1999)[3]. They introduced the notion of coherent risk measure, and gave the axioms for the measure. And later, the coherent risk measure was extended to the class of convex risk measures in[11, 14, 15]. Since their seminal work, the theory of coherent risk measures has rapidly been evolving; it already occupies a considerable part of the modern financial mathematics. Some of these papers are [1, 4, 5, 7, 8, 10, 11, 14, 16–18, 20–22, 30]. Excellent reviews on the theory of coherent risk measure are given in [12]. Recently, the most fashionable coherent risk measure is $AV@R$ (also called Conditional Value at Risk, Expected Shortfall,or Tail Value at Risk). As compared to $V@R$, it measures not only the probability of loss but its severity as well. Kusuoka (2001)[20] proved that $AV@R$ is the smallest law invariant coherent risk measure that dominates $V@R$. It is seemly that $AV@R$ might be the most important subclass of coherent risk measures. However, its disadvantage is that it depends only on the tail of the distribution, i.e. it is a 0-1 risk measure, so it is not smooth[6].

Our article here propose a new coherent risk measure based on relative entropy, which is obtained under the the theory framework of coherent risk measure from Artzner et al.(1999)[3]. We call this new risk measure iso-entropic risk measure. It is pointed that this risk measure is not 0-1 risk measure, so it is a smooth one. And, we prove that at the same level, the iso-entropic risk measure is more large than $AV@R$ for the same position or portfolio.

The remainder of the article is organized as follows. Section 2 give a brief introduction for monetary, convex and coherent risk measures; Section 3 propose the iso-entropic risk measure based on relative entropy. Section 4

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compare our iso-entropic risk measure to several important risk measures in detail. And at last, Section 5 concludes.

## 2. Acceptance set, monetary, convex, and coherent risk measures

Here, We introduce some concepts related to coherent risk measures. More detail see Föllmer and Schied(2008)[13] and Artzner et al.(1999)[3].

In financial theory, the uncertainty of value for a position (a asset or a portfolio) in the future is usually described by a random variable $X:\Omega\to\mathrm{R}$ on a probability space $(\Omega,\mathcal{F},P)$, where $\Omega$ is a fixed set of scenarios. For instance, $X$ can be the (discounted) value of the portfolio or some economic capital. The goal of risk measure is to determine a number $\rho(X)$ that quantifies the risk and can serve as a capital requirement, i.e., as the minimal amount of capital which, if added to the position and invested in a risk-free manner, makes the position acceptable. The following axiomatic approach to such risk measures was initiated in the coherent case by[2,3] and later extended to the class of convex risk measures in [11,14,15]. In the sequel, $\mathcal{G}$ denotes a given linear space of functions $X:\Omega\to\mathrm{R}$ containing the constants. Let $\mathcal{G}$ be the set of all risks, that is the set of all real valued functions on $\Omega$.

As Artzner et al.[3] point that a first, crude but crucial, measurement of the risk of a position will be whether its future value belongs or does not belong to the subset of acceptable risks, as decided by a investor or a supervisor. For an unacceptable risk (i.e. a position with an unacceptable future value) one remedy may be to alter the position. Another remedy is to look for some commonly accepted instruments which, added to the current position, make its future value become acceptable to the investor/supervisor. The current cost of getting enough of this or these instrument(s) is a good candidate for a measure of risk of the initially unacceptable position. Based on this, a series of definitions are given as follows.

**Definition 2.1.** A *measure of risk* $\rho$ is a mapping from $\mathcal{G}$ into $\mathrm{R}$.

**Definition 2.2.** An *acceptance set*: We call $\mathcal{A}$ a set of final values, expressed in currency, are accepted by one investor/supervisor.

It must be pointed that there are different acceptance sets for different investors/supervisors because they are heterogeneous when faced with risk assets. There is a correspondence between acceptance sets and measures of risk.

**Definition 2.3.** *Risk measure associated to an acceptance set*: the risk measure associated to the acceptance set $\mathcal{A}$ is the mapping from $\mathcal{G}$ into $\mathrm{R}$ denoted by $\rho_{\mathcal{A}}$ and defined by

$$\rho_{\mathcal{A}}(X)=\inf\{m\in\mathrm{R}\,|m+X\in\mathcal{A}\}.$$

The risk measure is the smallest amount of units of date 0 money which invested in the admissible asset, must be added at date 0 to the planned future net worth $X$ to make it acceptable. Note that we work with discounted quantities; cf.[3,19] for a discussion of forward risk measures and interest rate ambiguity.

**Definition 2.4.** *Acceptance set associated to a risk measure*: the acceptance set associated to a risk measure $\rho$is the set denoted by $\mathcal{A}_\rho$ and defined by

$$\mathcal{A}_\rho=\inf\{X\in\mathcal{G}\,|\rho(X)\le 0\}.$$

**Definition 2.5.** A measure of risk $\rho$is called a *monetary risk measure* if $\rho(0)$ is finite and if $\rho$ satisfies the following conditions for all $X,Y\in\mathcal{G}$.

*Monotonicity*: If $X\le Y$, then $\rho(X)\ge\rho(Y)$.

*Translation invariance*: If $c\in\mathrm{R}$, then $\rho(X+c)=\rho(X)-c$.

The financial meaning of monotonicity is clear: the downside risk of a position is reduced if the payoff profile is increased. Translation invariance is also called cash invariance. This is motivated by the interpretation of $\rho(X)$ as a capital requirement, i.e., $\rho(X)$ is the amount which should be raised in order to make $X$ acceptable from the point of view of a investor/supervisor, as Definition 2.3. Thus, if the risk-free amount $c$ is appropriately added to the position or to the economic capital, then the capital requirement is reduced by the same amount.

**Definition 2.6.** A monetary risk measure $\rho$ is called a *convex risk measure* if $\rho$ satisfies the following conditions

*Convexity*: $\rho(\lambda X+(1-\lambda)Y)\le\lambda\rho(X)+(1-\lambda)\rho(Y)$, for $0\le\lambda\le 1$.

The axiom of convexity gives a precise meaning to the idea that diversification should not increase the risk.

**Definition 2.7.** A convex risk measure $\rho$ is called a *coherent risk measure* if $\rho$ satisfies the following conditions

*Positive Homogeneity*: if $\lambda\ge 0$, then $\rho(\lambda X)=\lambda\rho(X)$.

Under the assumption of positive homogeneity, the convexity of a monetary risk measure is equivalent to

*Subadditivity*: $\rho(X+Y)\le\rho(X)+\rho(Y)$.

So, a coherent risk measure must satisfies four axioms: monotonicity, translation invariance, positive homogeneity and convexity or subadditivity.

## 3. Coherent risk based on relative entropy

In this setction, one new coherent risk measure is proposed based on the given relative entropy.

Suppose now that $\mathcal{G}$ consists of measureable functions on $(\Omega,\mathcal{F})$. According to the basic representation theorem proved by Artzner, Delbaen, Eber, and Heath (1999)[3] for a finite $\Omega$ and by Delbaen (2002)[8] in the general case, any coherent risk measure $\rho$ admits a representation of the form

$$\rho(X)=-\inf_{Q\in\mathcal{D}}E_Q[X]to11.51 \tag{1}$$

with a certain set $\mathcal{D}$ of probability measures absolutely continuous with respect to $P$. Here, we apply relative entropy to define the set $\mathcal{D}$ of probability measures.

$$\mathcal{D}=\{Q:H(Q\,|P)=H\} \tag{2}$$

Where $H(Q|P) = E\left[\frac{dQ}{dP}\log\frac{dQ}{dP}\right]$ is relative entropy of $Q \ll P$. Relative entropy is also called Kullback–Leibler divergence or information divergence. From Jensen's inequality, we know that $H(Q|P) \geq 0$. Here, we can interpret the meaning for the formula (1), (2): the probability $P$ is the observation, and $Q$ may be the true one which generates $X$, but the investor/supervisor don't knows $Q$ well, she only knows the 'distance' from $P$ to $Q$, namely Kullback–Leibler divergence here. Kullback–Leibler divergence is just a pseudo-distance, because $H(Q|P) \neq H(P|Q)$. Now, the investor/supervisor try to find the worst expectation of a position (a asset or a portfolio) $X$ given the divergence $H(Q|P) = H$ from $P$ to $Q$. Because the set $\mathcal{D}$ of probability measures is induced under the same relative entropy, so we call it iso-entropy induction set of probability measures.

Apparently, this is a functional extremum problem with equality constraints. Denote $dQ = q(x)\,dx$ and $dP = p(x)\,dx$, rewrite the problem (1), (2) as follows:

$$\begin{aligned} &\rho(X) = -\inf_{q(x)} \int_{\mathcal{G}} xq(x)\,dx \\ &s.t.\ H = \int_{\mathcal{G}} q(x)\log\frac{q(x)}{p(x)}dx \\ &\qquad 1 = \int_{\mathcal{G}} q(x)\,dx \end{aligned} \tag{3}$$

Where the second constraint must be satisfied naturally.

Now, we use calculus of variations to solve the problem. Write functional with Lagrange multipliers as follows:

$$\begin{aligned} &\mathcal{J}(q(x)) = \int_{\mathcal{G}} xq(x)\,dx - m_1\int_{\mathcal{G}} q(x)\,dx \\ &-m_2\int_{\mathcal{G}} q(x)\log\frac{q(x)}{p(x)}dx \end{aligned} \tag{4}$$

In which $m_1, m_2$ are Lagrange multipliers. The calculus of variations of functional $\mathcal{J}$ is

$$\begin{aligned} &\delta\mathcal{J}(q(x)) = \frac{\partial\mathcal{J}(q(x)+\alpha\delta q(x))}{\partial\alpha}|_{\alpha=0} \\ &= \int_{\mathcal{G}}\left(x - m_1 - m_2 - m_2\log\frac{q(x)}{p(x)}\right)\delta q(x)\,dx \end{aligned} \tag{5}$$

According to Lemma of calculus of variations, functional $\mathcal{J}$ gets extremum at $q_0(x)$, then $\delta\mathcal{J}(q_0(x)) = 0$, and so we get

$$x - m_1 - m_2 - m_2\log\frac{q_0(x)}{p(x)} = 0 \tag{6}$$

Utilizing unitary condition of probability(the second constraint), we get

$$q_0(x) = p(x)\frac{e^{-mx}}{E[e^{-mx}]} \tag{7}$$

Where $m = -1/m_2$. And $E[\bullet] = E_P[\bullet]$, the subscript is omit in the sequel. We can see the fomula (7) is just a Esscher transformation of $p(x)$[9].

Then, applying iso-relative-entropy condition(the first constraint), $m$ is determined. Denote $f(m) = H(Q_0|P) - H$. We need to find $m$ to satisfy $f(m) = 0$. About zero-point for function $f(m)$, we have theorem as follows.

**Theorem 3.1.** If $H = 0$,then $f(m)$ has only one zero-point which is at $m = 0$; and if $H > 0$,then $f(m)$ have two zero-points which are at $m \in (0,\infty)$ and $m \in (-\infty, 0)$, respectively.

The brief proofs is as follows:

$$\begin{aligned} &f(m) = \int_{\mathcal{G}} q_0(x)\log\frac{q_0(x)}{p(x)}dx - H \\ &= E\left[\frac{\left(-mx-\log E\left[e^{-mx}\right]\right)e^{-mx}}{E[e^{-mx}]}\right] - H \\ &= E[g(m,x)] - H \end{aligned}$$

$$\begin{aligned} &\frac{\partial g}{\partial m} = \frac{q_0(x)}{p(x)}\Big(mx^2 + x\log E\left[e^{-mx}\right] \\ &-x - \left(1 - mx + \log E\left[e^{-mx}\right]\right)\frac{E\left[xe^{-mx}\right]}{E[e^{-mx}]}\Big) \\ &= \frac{q_0(x)}{p(x)}\Big(mx^2 + x\log E\left[e^{-mx}\right] - x \\ &-\left(1 - mx + \log E\left[e^{-mx}\right]\right)E_{Q_0}[x]\Big) \\ &\frac{df(m)}{dm} = E\left[\frac{\partial g}{\partial m}\right] = m\left(E_{Q_0}\left[x^2\right] - \left(E_{Q_0}[x]\right)^2\right) \\ &= m\sigma_{Q_0}^2 \end{aligned}$$

Because of $\sigma_{Q_0}^2 > 0$, so there is only one extremum point for function $f(m)$,and further it is minimum point, which is at $m = 0$. Because of the minmum of $f(m)$is $f(0) = -H \leq 0$, so we can get the theorem. The proof is completed.

From theorem 3.1., we have following corollary.

**Corollary 3.2.** If $H = 0$,then functional $\mathcal{J}$ has only one extremum which is at $q_0(x) = p(x)$; and if $H > 0$,then $\mathcal{J}$ have two extremums which are $q_0(x) = p(x)\frac{e^{-mx}}{E[e^{-mx}]}$ at $m \in (0,\infty)$ and $m \in (-\infty, 0)$, respectively.

So we have the following proposition about coherent risk measure based on relative entropy.

**Proposition 3.3.** Given the relative entropy $H(Q|P) = H$, the coherent risk measure has the form

$$\rho(X) = -E_{Q_0}[X] = -\frac{E\left[Xe^{-mX}\right]}{E\left[e^{-mX}\right]} \tag{8}$$

In which $m$ satisfy $f(m) = 0$, and $m \geq 0$.

The reason for $m \geq 0$ is because of $\frac{d\left(E_{Q_0}[X]\right)}{dm} = -\sigma_{Q_0}^2$.

And from iso-relative-entropy condition:

$E\left[\frac{\left(-mx-\log E\left[e^{-mx}\right]\right)e^{-mx}}{E[e^{-mx}]}\right] = H,$

the coherent risk measure can take another form

$$\rho(X) = \frac{H + \log E\left[e^{-mX}\right]}{m} \tag{9}$$

In which, $m$ is determined by $H$ uniquely. For convenience, we call it iso-entropic risk measure for our new coherent risk measure, and denote $\rho^{ie}(X)$.

In order to apply our iso-entropic risk measure similar like $V@R$(Value at Risk) or *AV@R* (Average Value at Risk), we discuss relative entropy further. Entropy can describe the uncertainty, and relative entropy describe the 'distance' or divergence of two uncertaintys. Confidence level has the similar meaning. So we denote $H = \log \frac{1}{\lambda}, \lambda \in (0,1]$ . Then we can get iso-entropic risk measure at confidence level $\lambda$:

$$\rho_\lambda^{ie}(X) = \frac{\log \frac{1}{\lambda} + \log E\left[e^{-mX}\right]}{m} \tag{10}$$

In which, $m$ is determined by $\lambda$ uniquely.

Here, $\lambda$ serves as the risk aversion parameter. We have $\rho_\lambda^{ie}(X) \underset{\lambda \downarrow 0}{\rightarrow} -essinf_\omega\, X(\omega)\, and \rho_1^{ie}(X) = -E[X]$.

When $X$ has Gaussian distribution, $X \sim N(\mu, \sigma)$, then $m = \sqrt{2H}/\sigma$. So we have iso-entropic risk measure for Gaussian distribution:

$$\rho_\lambda^{ie}(X) = \sigma\sqrt{2H} - \mu \tag{11}$$

Where $\mu = E[X], \sigma^2 = E\left[(X-\mu)^2\right]$.

## 4. Comparison for several risk measures

In this section,we compare our coherent risk measure based on relative entropy with other several important risk measures.

The first one is $V@R$(*Value at Risk*), which is the most fashionable one now. $V@R$ at level $\lambda \in (0,1]$, defined for $X$ on a probability space $(\Omega, \mathcal{F}, P)$ is

$$V@R_\lambda(X) = \inf\{m \in \mathrm{R}\,|P[X+m<0] \le \lambda\} \tag{12}$$

$V@R$ satisfies monotonicity, translation invariance and positive homogeneity, but not subadditivity, so it is just a monetary risk measure, not a convex one.

The second risk measure is *AV@R* (*Average Value at Risk*). At level $\lambda \in (0,1]$, *AV@R* is defined as

$$AV@R_\lambda(X) = \frac{1}{\lambda}\int_0^\lambda V@R_\alpha(X)d\alpha \tag{13}$$

*AV@R* is also called *Conditional Value at Risk*, *Expected Shortfall*,or *Tail Value at Risk*. According to Föllmer and Schied(2008)[13], there is another definition for *AV@R:*

$$AV@R_\lambda(X) = -\inf_{Q\in\mathcal{D}} E_Q[X]$$
$$\text{for } \mathcal{D} = \left\{Q : \frac{dQ}{dP} \le \frac{1}{\lambda}\right\} \tag{14}$$

So it gets

$$AV@R_\lambda(X) = \frac{1}{\lambda}E[-X\,|X \le z_\lambda(X)],$$
$$z_\lambda(X) = \inf[x : F(x) \ge \lambda] \tag{15}$$

Apparently, *AV@R* is coherent risk measure according to the basic representation theorem proved by Artzner, Delbaen, Eber, and Heath (1999)[3]. It satisfies four axioms: monotonicity, translation invariance, positive homogeneity and subadditivity. From formula (15), we know that it depends only on the tail of the distribution, i.e. it is a 0-1 risk measure, so it is not smooth. In contrast with this, our coherent risk measure based on relative entropy $\rho_\lambda^{ie}(X) = \frac{\log\frac{1}{\lambda} + \log E\left[e^{-mX}\right]}{m}$ depends on the whole dietribution, it is very smooth. Concerning to *V@R*, *AV@R* and our risk measure $\rho_\lambda^{ie}(X)$, we have the following theorem.

**Theorem 4.1.** At same level $\lambda \in (0,1]$, the following formula exists

$$\rho_\lambda^{ie}(X) \ge AV@R_\lambda(X) \ge V@R_\lambda(X) \tag{16}$$

The brief proofs is as follows:

$AV@R_\lambda(X) \ge V@R_\lambda(X)$ is apparent. Let us see why $\rho_\lambda^{ie}(X) \ge AV@R_\lambda(X)$.

Pay attention that the solution of $Q$ foro optimum problem (14) satisfies $dQ_\lambda/dP = \frac{1}{\lambda}1_{X \le z_\lambda(X)}$. And the relative entropy from $P$ to $Q_\lambda$ is:

$$H(Q_\lambda\,|P) = E\left[\frac{dQ_\lambda}{dP}\log\frac{dQ_\lambda}{dP}\right] = \log\frac{1}{\lambda} = H$$

So, $Q_\lambda$is one element of $\mathcal{D} = \left\{Q : H(Q\,|P) = H = \log\frac{1}{\lambda}\right\}$. However, from the result of section 3, we know that the optimization for $-\inf_{Q\in\mathcal{D}} E_Q[X]$ ,

so we get $\rho_\lambda^{ie}(X) \ge AV@R_\lambda(X)$. The proof is completed.

In fact, for arbitrary $dQ_\lambda/dP = \frac{1}{\lambda}1_{X\in(a,b)}$, if $E\left[1_{X\in(a,b)}\right] = \lambda$, then $H(Q_\lambda\,|P) = E\left[\frac{dQ_\lambda}{dP}\log\frac{dQ_\lambda}{dP}\right] = \log\frac{1}{\lambda} = H.$

Figure 1. is the illustration of these three risk measures supposed that $X$ has Gaussian distribution, $X \sim N(\mu, \sigma)$. Pay attention that when $\lambda \to 0$, all the three measures $\underset{\lambda\downarrow 0}{\rightarrow} -essinf_\omega X(\omega)$, but when $\lambda = 1$, $V@R_\lambda(X) \underset{\lambda\downarrow 1}{\rightarrow} -esssup_\omega X(\omega)$ $\rho_1^{ie}(X) = AV@R_1(X) = -E[X]$.

The last risk measure is entropic risk measure, defined by

$$\rho^{ent}(X) = \frac{\log E\left[e^{-\theta X}\right]}{\theta},\ \theta¿0 \tag{17}$$

The entropic risk measure satisfies three axioms: monotonicity, translation invariance, and convexity but not for positive homogeneity, so it is just a convex risk measure, not a coherent one. Compare it with our iso-entropic risk measure $\rho_\lambda^{ie}(X)$ in formula (9), we will find that they are very analogous in form. In fact, they are diffierent. The parameter $\theta$ in entropic risk measure $\rho^{ent}(X)$ is free, but parameter $m$ in our iso-entropic risk measure $\rho_\lambda^{ie}(X)$ is not free, it is defined by $H$ and $X$, namely $m = m(H, X)$.

If $X$ has Gaussian distribution, $X \sim N(\mu, \sigma)$, then entropic risk measure has the form: $\rho^{ent}(X) = \frac{\theta}{2}\sigma^2 - \mu$, $\theta$¿0. It is interesting to mention here that the iso-entropic risk measure used in this paper is different from the quantum entropy which has been used to measure the entanglement between two or more parties [24-30].

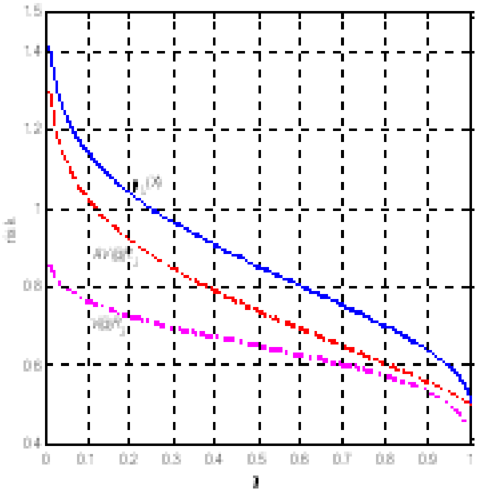


**Figure 1** An Illustration of three risk measures under differnt level of $\lambda$ $X \sim N(\mu, \sigma)$, $\sigma = 0.3$, $\mu = -0.5$, from top to bottom: $\rho_{\lambda}^{ie}(X)$, $AV@R_{\lambda}(X)$, $V@R_{\lambda}(X)$, respectively.

## 5. Conclusion

This article proposes a new coherent risk measure called iso-entropic risk measure. Comparing with several current important risk measures, it turns out that this new risk measure has advantage over the others. It is coherent, it is a smooth measure, it is a more prudent risk measure than *AV@R*. So it maybe a better coherent risk measure in the future financial market in my opinion. But, this needs to be verified in many ways, including pure theoretical and empirical work. In particular, the problem of capital allocation, the problem of pricing and hedging, optimal portfolio choice and equilibrium and so on, must be studied under our new coherent measure in practice.


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