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Mixed-Level Response Surface Designs via a Hybrid Genetic Algorithm

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Abstract: Response surface methodology is widely used for developing, improving and optimizing processes in various fields. In this paper, we present a general algorithmic method for constructing $2^{q_1}4^{q_2}$ mixed-level designs in order to explore and optimize response surfaces with respect to *D*-efficiency, where the predictor variables are at two and four equally spaced levels, by utilizing a hybrid genetic algorithm. Emphasis is given on various properties that arise from the implementation of the genetic algorithm, such as using genetic operators as local optimizers and the representation of the four levels of the design with a 2-bit Gray Code. We applied the genetic algorithm in several cases and the optimized mixed-level designs achieve good properties, thus demonstrating the efficiency of the proposed hybrid heuristic.

Keywords: Second-order response surface designs, Mixed-level designs, *D*-efficiency, Genetic algorithms, Optimization.
AMS Subject Classification: 62K20, 90C27.

1 Introduction

Response surface methodology is an effective tool for the exploration and the description of the relation between a response of interest and a number of experimental factors assumed affecting the response. In cases where an experimenter wishes to explore a quadratic relation between the response and the factors a design with three levels or more must be used. Factorial designs and fractional factorial designs in three or more levels are applied in such cases, while second-order designs with three-level factors have concentrated much interest. Box-Behnken ([2], [3]) designs are the most popular three-level designs used in a response surface framework. Central composite designs, introduced by Box and Wilson [6], are also employed when five levels for each involved factor are required. For a nice overview on second-order designs and response surfaces the interested reader can refer to the textbooks of Box and Draper [4], [5], of Khuri and Cornell [14] and of Myers and Montgomery [17].

Experimenters often come across with problems where different number of levels are assigned to the factors. These cases can be confronted by designing the experiment via mixed-level orthogonal arrays or near orthogonal arrays. Several authors have dealt with the construction and the properties of such designs ([16], [18], [20], [21]). However, the approach of a response surface problem using orthogonal arrays of this type is not always sufficient for exploring a second-order model. In this paper we focus on the algorithmic construction of mixed-level designs, where factors are in two and four levels (denoted by $2^{q_1}4^{q_2}$), having high estimation efficiency of the linear and quadratic components of the effects and the two-factors interactions. In the following sections we provide some preliminary concepts, describe our construction method and explore the properties of the new designs.

2 Design selection and evaluation

In a response surface framework, the objective is to model and optimize a response variable that is affected by several experimental factors. Since the real relation between the response and the factors is unknown, the first step is to find an

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approximation of the true functional relationship between them. The observed response y, can be written as a function of the exploratory variables $x_1, x_2, \ldots, x_k$, as follows:

$$y = f(x_1, x_2, \ldots, x_k) + \varepsilon,$$

where ε is a random error.

Generally, the first attempt is to approximate the shape of the response surface by fitting a first-order model to the response,

$$y = \beta_0 + \sum_{j=1}^{k} \beta_j x_j + \varepsilon, \tag{1}$$

where β_0, β_j, $j = 1, \ldots, k$ are unknown parameters and ε is a random error term. When the first-order model appears inadequate to describe the true relationship between the response and the predictor variables due to the existence of surface curvature, it is upgraded to a second-order model. In cases that mixed-level designs in two and four levels are used, model (2) is employed to explore a quadratic relation between the response variable y and k exploratory variables in total (q_1 variables in two levels and q_2 variables in four levels),

$$y = \beta_0 + \sum_{j=1}^{q_1} \beta_j x_j + \sum_{j=q_1+1}^{k} \beta_j x_j + \sum_{j=q_1+1}^{k} \beta_{jj} x_j^2 + \underbrace{\sum_{i=1}^{k} \sum_{j=1}^{k}}_{i<j} \beta_{ij} x_i x_j + \varepsilon \tag{2}$$

where β_0 corresponds to the general mean, $\beta_j, j = 1, \ldots, k$ are the coefficients of the first order effects, $\beta_{jj}, j = q_1 + 1, \ldots, k$ are the coefficients of the second order effects, $\beta_{ij}, i = 1, \ldots, k-1, j = i+1, \ldots, k$ correspond to the interactions between first order main effects, and ε is a random error term.

Furthermore, it is needed to have an estimation efficiency measure in order to evaluate and compare the available designs. Box and Draper [7] discussed as a measure of design efficiency the determinant maximization of the information matrix. In this paper we use D-efficiency introduced by Wang and Wu [19] for determining the overall efficiency for estimating a set of p effects

$$D_{eff} = |\mathbf{X}'\mathbf{X}|^{1/p}, \tag{3}$$

where $\mathbf{X}$ is the model matrix with its columns standardized, $\mathbf{X} = [x_0/||x_0||, x_1/||x_1||, \ldots, x_p/||x_p||]$, x_0 stands for the vector with all elements equal to 1, and x_i is the coefficient vector of the i-th effect, $i = 1, \ldots, p$. Since the columns of $\mathbf{X}$ are standardized, D-efficiency achieves its maximum value, which equals to 1, if and only if the x_i's are orthogonal to each other. More details can be found in [19]. In the case of a second-order (SOD) mixed-level design with k predictor variables in total (q_1 two-level factors and q_2 four-level factors), the matrix $\mathbf{X}$ consists of $p = \frac{k(k+1)+2(q_2+1)}{2}$ columns, corresponding to the coefficient vectors of constant, linear, quadratic and product terms defined as in Equation 2.

Also, we use the J_2 optimality criterion introduced in [21] for the evaluation of the new designs. Consider an $n \times k$ matrix $\mathbf{D} = [x_{ij}]$, and that weight $\omega_j > 0$ is assigned to the j-th column, which has s_j levels. The similarity between the i-th and r-th rows of $\mathbf{D}$ can be measured by the $\delta_{i,r}(\mathbf{D})$ value, given by:

$$\delta_{i,r}(\mathbf{D}) = \sum_{j=1}^{k} \omega_j \delta(x_{ij}, x_{rj}),$$

where $\delta(x,y) = 1$ if $x = y$ and 0 otherwise. If $\omega_j = 1$ for all j, then $\delta_{i,r}(\mathbf{D})$ is the number of coincidences between the i-th and r-th rows and

$$J_2(\mathbf{D}) = \sum_{1 \leqslant i \leqslant r \leqslant n} [\delta_{i,r}(\mathbf{D})]^2. \tag{4}$$

A design is J_2-optimal if it minimizes J_2. Xu [21] gave a lower bound for J_2 calculated as:

$$L(k) = 2^{-1} \left[\left(\sum_{j=1}^{k} n s_j^{-1} \omega_j \right)^2 + \left(\sum_{j=1}^{k} (s_j - 1)(n s_j^{-1} \omega_j)^2 \right) - n \left(\sum_{j=1}^{k} \omega_j \right)^2 \right]. \tag{5}$$

A design achieves the lower bound if and only if it is an orthogonal array. For more details see [21].

3 Construction via $(\mathbf{n/q_1, n/q_2})$-circulant generators

In this section we propose a new construction method of mixed-level $2^{q_1}4^{q_2}$ response surface designs from two circulant generators. Our goal is to construct a second-order design with n rows and q_1 two-level and q_2 four-level balanced columns. The $(n/q_1, n/q_2)$-circulant generated design will be a two-block circulant design.

Definition 1. *A two-block circulant design is a $2^{q_1}4^{q_2}$ design with n runs when the following conditions hold:*

1. $n \equiv 0 \pmod 4$,
2. *n/q_1 and n/q_2 are positive integers,*
3. *the 2^{q_1} and 4^{q_2} blocks are generated by n/q_1 and n/q_2 shifts, respectively.*

For the construction of a $(n/q_1, n/q_2)$-circulant design, the two generators are column vectors of length n and comprise the first and the (q_1+1)-*th* column of the $n \times (q_1+q_2)$ design matrix. Each of the 2^{q_1} columns of this matrix is obtained from each previous column by moving the last n/q_1 elements to the first positions and cyclicly permutating the other elements downwards. Similarly, the rest 4^{q_2} columns are constructed by n/q_2 shifts.

Remark: The choice of balanced generators ensures the construction of a $(n/q_1, n/q_2)$-circulant balanced design.

The next example illustrates the construction of a $2^3 4^2$ mixed-level design in 24 runs. First, consider the following two generators which are used for the construction of a design matrix with three balanced columns in two levels, coded as -1 and $+1$, and two balanced columns in four levels, coded as -3, -1, 1, 3:

$g_1 =$	-1	-1	-1	-1	1	-1	1	1	1	1	-1	1	-1	-1	1	1	-1	-1	-1	1	1	1	1	-1
$g_2 =$	1	-3	-3	-3	-3	3	3	3	-3	-1	3	-3	3	1	-3	3	1	-1	-1	3	3	1	-1	-3

The first generator g_1 is the first column of the design matrix. The second column is obtained from the first by moving the last 8 elements to the first positions and cyclicly permutating the other elements downwards. The third column is produced in a similar way from the second column. The fourth column of the design is the second generator g_2, while the fifth column occurs by moving the last 12 elements of the fourth column to the first positions and cyclicly permutating the other elements downwards. So, we obtain the following design matrix in the transpose form:

-1	-1	-1	-1	1	-1	1	1	1	1	-1	1	-1	-1	1	1	-1	-1	-1	1	1	1	1	-1
-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	1	-1	1	1	1	1	-1	1	-1	-1	1	1
1	1	-1	1	-1	-1	1	1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	1	-1	1	1
1	-3	-3	-3	-3	3	3	3	-3	-1	3	-3	3	1	-3	3	1	-1	-1	3	3	1	-1	-3
3	1	-3	3	1	-1	-1	3	3	1	-1	-3	1	-3	-3	-3	-3	3	3	3	-3	-1	3	-3

Next, we create the two columns that correspond to the quadratic effects of the two four-level factors, and the 10 columns corresponding to the two-factor interactions between the five main effects. The produced design requires only 6 more runs than the saturated model and can estimate efficiently a second-order response surface model with 18 unknown parameters since it has 90.24% D-efficiency.

4 Optimization of response surface designs by means of genetic algorithms

Genetic algorithms form a powerful metaheuristic that mimicks processes from the Theory of Evolution to establish search algorithms by defining algorithmic analogues of biological concepts such as reproduction, crossover and mutation. Genetic algorithms were introduced in 1970 by John Holland [13] aiming to design an artificial system having properties similar to natural systems. In this paper, we assume some basic familiarity with Genetic Algorithm concepts. The concepts necessary for a description of the Genetic Algorithm (GA) can be found in Goldberg [12], in Stefanie Forrest's article [11] and in the Handbook of Genetic Algorithms edited by Davis [8].

GAs are attractive because of their robustness and flexibility in terms of a computer implementation and, mathematically, they do not require a differentiable objective function thereby reducing the chance of reporting local optima. Some earlier attempts utilizing a GA in the construction of response surface designs have been given by Drain et al. [10]. However, this approach, while promising, lacked of an efficient coding of the chromosomes i.e. the number of the experimental runs forming the design. In particular, the authors proposed utilizing and constructing the whole design; thus restricting the GA to evolve in finding optimal response surface designs in several cases. A successful reduction in terms of computational complexity of an efficient representation of the candidate design, has been proposed in [15]. In this application, the authors integrated as a core ingredient of the GA the use of sequential juxtaposition of suitable generators forming block circulant matrices.

4.1 A hybrid genetic algorithm for response surface designs

Chromosomes representation The two circulant generators considered in the case of a $2^{q_1}4^{q_2}$ response surface design will be represented by binary vectors of total length $3n$ bits when constructing a $n\times(q_1+q_2)$ design matrix. In particular the first n bits of the binary vector represent the 2^{q_1} block of the design, while we restrain the rest $2n$ bits to represent the 4^{q_2} block. This construction is valid when we represent a two-block circulant design. In the case of mixed level response surface designs, the genes constitute of four possible values representing the two and four levels of the designs. This encoding process add to the compactness of the GA, since a small amount of storage is needed for a $n\times(q_1+q_2)$ design matrix to be represented by the GA. In particular for a $2^{q_1}4^{q_2}$ design having a $n\times(q_1+q_2)$ design matrix it is required $3n$ bits to be reserved in memory. In contrast, if we had to represent the whole design matrix we would have to reserve $n(q_1+q_2)$ bits. Therefore the space complexity is reduced from $\mathcal{O}(n^2)$ to $\mathcal{O}(n)$ since $q_1+q_2<n$ in a $2^{q_1}4^{q_2}$ response surface design.

Chromosomes encoding and decoding A suitable encoding to binary variables was needed since the genetic operators behave better in binary arithmetic (Goldberg, [12]). The answer to this vital question found in the field of Combinatorics and Computer Science in terms of representing a 2-bit Gray Code, $GC_2=\{00,01,11,10\}$ when considering the 4^{q_2} block of the design. For more details, on Gray Codes we refer the interested reader to Carla [9]. More precisely, we mapped each level of the 4^{q_2} block to a codeword of the 2-bit Gray Code, i.e. $\{-3,-1,1,3\}\rightarrow\{00,01,11,10\}$, and each level of the 2^{q_1} block to binary values $\{-1,1\}\rightarrow\{1,0\}$, thus transforming the problem on its binary equivalent which allowed us to carry on with the next stages of utilizing a GA.

4.2 Tuning GA parameters

Initial population consists of random chromosomes. We found it useful to generate these chromosomes by retrieving samples of binary sequences of length $3n$ retrieved from the uniform distribution. The option to generate random binary sequences of length n and $2n$ which correspond to the 2^{q_1} and 4^{q_2} block of the design was discarded because the probability of these sequences to be balanced, i.e. having the same number of zeros and ones (corresponding to balanced designs), is smaller than the probability of the initial sequences of length $3n$ to be balanced. A sample chromosome of the aforementioned construction for a response surface design with 8 runs is the concatenation of the following binary strings $01101100\mid 1001110110001100$ where the first 8 bits corresponds to the 2^{q_1} block and the rest 16 bits to the 4^{q_2} block of the design, respectively.

An objective function for response surface designs The crucial choice of the objective function (OF) subject to be optimized arise naturally from the theoretical framework of D-efficient designs. D-efficiency is the most widely known and used optimality criterion. As stated in Xu (2002), J_2 and A_2 optimality criteria are good surrogates for the D_{eff} criterion. We have computed the J_2 optimality criterion for our designs and its corresponding lower bound, which we achieve for several cases as it can be seen at Table 1. Moreover, the last optimality criteria perform a smaller discrimination between the designs thus making them a less robust choice choice for OF. The genetic algorithm attempts in both cases to maximize the value of D-efficiency with respect to its upper bound which is equal to 1, in the orthogonal case. Due to the theoretical background and statistical justifications, given below, when a value of D-efficiency was detected in the range of $[0.65,1.00]$ we considered we have found an acceptable (feasible) solution. The theoretical background is that when $q_1\geq 2$ and $q_2\geq 2$ then it is known that OA exist when $n\equiv 0 \pmod{16}$. Therefore, since we have run our algorithm for a wide variety of cases for n and the ones that OA do not exist, in these cases we have $D_{eff}<1$. Moreover, a statistical justification for accepting values of D_{eff} within the aforementioned range is that when considering the statistical model defined in equation (2), a value of $D_{eff}\geq 0.65$ provides at least a moderate safeguard against multicollinearity, which effects the estimates of the model parameters.
We are now able to describe the three genetic operators of reproduction, crossover and mutation as specifically have been applied by the genetic algorithm we have used.

Crossover We defined the basic genetic operation, crossover, that splits a pair of binary integers at a random position and combines the head of one with the tail of the other and vice versa. We implement an additional variant of crossover where the crossover point is fixed to the $n+1$ position of a chromosome of length $3n$. In this variant we exchange the first n positions of the parents and keep the rest chromosome unchanged. In this way, we model the phenomenon of two response surface designs having a 2^{q_1} block of good quality with respect to D-efficiency. Moreover, the selection of parents that will contribute to this mutual exchange of 2^{q_1} blocks is not limited to elitist solutions, but was allowed to inferior solutions to contribute to the procedure because we observed that a chromosome with low overall D-efficiency may have a 2^{q_1} block that is of superior quality when it is mixed with other candidate 4^{q_2} blocks.

Mutation Additional operations, such as inverting a section of the binary representation (inversion) or randomly changing the state (0 or 1) of individual bits (mutation), also transform the population. We used the mutation operator as a local optimizer where it was taken into account that only an even number of bits was changed until a balanced solution was found. From the produced list of neighbors of the original solution the optimizer discards solutions that differ in odd bit positions or even same bits, i.e. “00” or “11”, because the produced solution is not balanced. From the set of produced solutions we keep the one with the best fitness, and apply the same procedure of mutation to a portion of the population according to a probability retrieved from the uniform distribution.

Selection and reproduction Before each such cycle (generation), population members are selected on the basis of their fitness (the value of the objective function for that solution) to be the “parents” of the new generation. This operator acts by imposing a minimum degree of performance of the objective function in every generation. We implemented a ranking selection procedure where the solutions are ordered according to their objective value in descending order until the desired fixed number of parents is obtained. The rest of the population is selected through hybridization of proportional selection where inferior solutions have less chance of being selected, but are not excluded because even not very good chromosomes can contain useful genes that should remain available for recombination. Ranking selection first ranks the population and then every chromosome receives fitness from this ranking. The worst will have fitness 1, second worst 2 etc. and the best will have fitness N (where is N is the number of chromosomes in population).

Termination condition of the genetic algorithm was set a predefined number of evolved generations. This number of generations was proportional to the size of the response surface design that the genetic algorithm was searching for in each case. Thus the GA required only a few generations to find a small sized optimal response surface design, while a larger design required additional generations to be evolved. We note that, the time complexity of the algorithm was relatively small compared to exhaustive search algorithms.
We give below a description of our hybrid GA in pseudo-code form in the case of mixed-level RSD’s.

Algorithm 1 Hybrid GA for mixed-level response surface designs

```
function HGA2MRSD(n, q1, q2, N)
Require: n, q1, q2 > 0    ▷ Input runs n, number of q1 two-level and q2 four-level balanced columns, and N maximum iterations
    gen ← 1                                              ▷ Reset number of generations
    init_pop ← GENERATEINITIALPOPULATION(n)              ▷ Generate sequences of length 3n
    EVALUATE(OF(init_pop))                               ▷ Evaluate initial population
    max_gen ← N
    pop ← init_pop
    while gen ≤ max_gen do
        chrom ← ENCODE(n, q1, q2, pop)
        chrom(gen) ← SELECT((n, q1, q2, chrom))(gen − 1)
        RANKING(chrom(gen))
        CROSSOVER(chrom(gen))
        MUTATION(chrom(gen))
        repeat
            PROPORTIONALSELECT(chrom(gen))
        until chrom(gen) == chrom
        pop ← DECODE(chrom)
        EVALUATE(OF(pop))
        if OF(pop) ∈ [0.65, 1.00] then
            report acceptable design found
        end if
        gen ← gen + 1
    end while
end function
```

5 Results

Table 1 summarizes the best results occurred from the proposed constructive method for designs with run size between 8 and 64 and 2-8 factors. The first column corresponds to the number of runs n of the design, q_1 and q_2 denote the number of two-level and four-level factors respectively, k is the total number of factors and p is the total number of parameters with respect to model 2. The calculated values of the designs estimation efficiency according to relation 3 are listed in the sixth column. The values of the J_2 criterion (equation 4) and the lower bound $L(k)$ (equation 5) are reported in the next two columns respectively, while the fraction $L(k)/J_2$, as an efficiency measure of the designs, is given in the last column. The designs are available by the authors on request.

Table 1: Results of the proposed constructive method.

n	q_1	q_2	k	p	D-eff	J_2	$L(k)$	$L(k)/J_2$	n	q_1	q_2	k	p	D-eff	J_2	$L(k)$	$L(k)/J_2$
8	1	1	2	5	100,00%	16	16	100,00%	44	2	4	6	26	92,18%	4680	4290	91,67%
8	2	1	3	8	91,97%	36	36	100,00%	44	4	2	6	24	93,71%	6714	6589	98,14%
12	1	2	3	9	92,38%	78	63	80,77%	48	1	6	7	35	74,40%	5642	5016	88,90%
12	2	1	3	8	92,27%	118	108	91,53%	48	2	3	5	19	96,75%	4267	4152	97,30%
16	1	2	3	9	96,21%	144	136	94,44%	48	2	4	6	26	94,65%	5488	5184	94,46%
16	2	1	3	8	100,00%	216	216	100,00 %	48	2	6	8	43	72,70%	8008	7536	94,11%
16	2	2	4	13	90,99%	304	272	89,47%	48	3	2	5	18	97,67%	5358	5304	98,99%
20	1	2	3	9	97,76%	266	235	88,35%	48	3	3	6	25	96,19%	6627	6480	97,78%
20	1	4	5	20	66,22%	502	400	79,68%	48	3	4	7	33	85,39%	7980	7752	97,14%
20	2	2	4	13	93,85%	504	465	92,26%	48	4	2	6	24	95,15%	8144	7920	97,25%
20	4	1	5	17	74,78%	1107	1000	90,33%	48	4	3	7	32	83,24%	9448	9336	98,81%
24	1	3	4	14	91,14%	544	492	90,44%	48	4	4	8	41	75,79%	11500	10848	94,33%
24	1	4	5	20	79,37%	824	636	77,18%	48	6	1	7	30	88,23%	13146	12936	98,40%
24	2	2	4	13	96,65%	772	708	91,71%	48	6	2	8	39	76,38%	15040	14736	97,98%
24	2	3	5	19	90,82%	988	888	89,88%	52	2	4	6	26	95,82%	6436	6162	95,74%
24	3	1	4	12	98,48%	996	960	96,39%	52	4	4	8	41	82,84%	13860	12870	92,86%
24	3	2	5	18	90,24%	1278	1176	92,02%	56	1	7	8	44	70,06%	10061	8596	85,44%
24	4	1	5	17	87,12%	1545	1500	97,09%	56	2	7	9	53	68,23%	13720	12432	90,61%
28	1	4	5	20	81,07%	1342	924	68,85%	56	4	1	5	17	98,76%	9161	9100	99,33%
28	2	2	4	13	97,55%	1040	1001	96,25%	56	4	2	6	24	97,04%	11138	10948	98,29%
28	4	1	5	17	93,43%	2138	2100	98,22%	56	4	4	8	41	87,49%	15672	15064	96,12%
28	4	2	6	24	78,88%	2654	2485	93,63%	56	7	1	8	38	86,78%	23676	23296	98,39%
32	1	4	5	20	85,48%	1388	1264	91,07%	56	7	2	9	48	70,89%	26542	26152	98,53%
32	2	4	6	26	88,12%	2388	2112	88,44%	56	8	1	9	47	75,23%	29700	29484	99,27%
32	4	1	5	17	92,70%	2825	2800	99,12%	60	1	5	6	27	88,16%	7410	6570	88,66%
32	4	2	6	24	86,86%	3402	3328	97,82%	60	1	6	7	35	81,43%	8928	8205	91,90%
36	1	3	4	14	94,71%	1393	1251	89,81%	60	2	5	7	34	90,68%	11219	10230	91,18%
36	1	4	5	20	86,13%	1818	1656	91,09%	60	2	6	8	43	86,92%	13020	12255	94,12%
36	2	3	5	19	93,22%	2458	2223	90,44%	60	3	4	7	33	90,53%	12904	12480	96,71%
36	2	4	6	26	89,77%	3036	2754	90,71%	60	3	5	8	42	82,21%	15761	14730	93,46%
36	3	2	5	18	92,96%	3022	2871	95,00%	60	3	6	9	52	76,44%	18366	17145	93,35%
36	3	3	6	25	91,65%	3681	3483	94,62%	60	4	3	7	32	89,47%	15640	14955	95,62%
36	3	4	7	33	68,48%	4706	4140	87,97%	60	4	4	8	41	88,01%	17940	17430	97,16%
36	4	2	6	24	89,71%	4374	4293	98,15%	60	4	5	9	51	70,84%	21389	20070	93,83%
36	4	3	7	32	68,29%	5629	5031	89,38%	60	5	1	6	23	98,38%	15224	15120	99,32%
36	6	1	7	30	81,58%	7226	7056	97,65%	60	5	2	7	31	91,00%	18176	17655	97,13%
40	1	4	5	20	90,32%	2400	2100	87,50%	60	5	3	8	40	82,07%	20784	20355	97,94%
40	1	5	6	27	80,39%	2883	2680	92,96%	60	5	4	9	50	67,94%	23928	23220	97,04%
40	2	4	6	26	92,82%	3840	3480	90,63%	60	6	1	7	30	92,81%	20802	20580	98,93%
40	2	5	7	34	78,22%	4811	4220	87,72%	60	6	2	8	39	86,25%	23718	23505	99,10%
40	4	2	6	24	92,06%	5592	5380	96,21%	60	6	3	9	49	78,96%	27108	26595	98,11%
40	5	1	6	23	92,23%	6597	6480	98,23%	64	4	4	8	41	89,55%	20936	19968	95,38%
40	5	2	7	31	81,27%	7854	7520	95,75%	64	8	1	9	47	83,43%	39229	38880	99,11%

6 Conlcusion

Our main concern was to construct efficient designs with run size near to the saturated case. In order to provide designs with higher estimation efficiency an increase to the number of total runs was required. The proposed method enabled us to construct designs with economical run size and high efficiency for the estimation of a second order model. This fact is the most significant advantage of these new designs over mixed-level orthogonal arrays available in the literature which, in many cases, are inadequate to estimate a second-order model. The new mixed-level designs can be very practical in design of experiments when a response surface model should be fitted.

The approach to construct optimal response surface designs by means of optimization is of current interest [10,15]. Our efforts were concentrated on the maximization of the value of D-efficiency via a hybrid heuristic search, because the D_{eff} criterion is one of the most well-known criteria for comparing response surface designs. The flexibility of genetic algorithms allows different objective functions to be optimized. Therefore, if another criterion (i.e. J_2-optimality) was under consideration our hybrid GA could be applied in a similar manner. However, we would have to evaluate a number of parameters for the success of such an application. Due to the randomness of the genetic algorithms a different approach may behave better or worse. Genetic algorithms appeared to be a successful and promising approach for the construction of D-optimal response surface designs since their compactness of encoding allowed us to use OF information (not derivatives) and probabilistic transition rules (not deterministic). Furthermore, the encoding process of the chromosomes to generators significantly restrained the space complexity, thus we were able to represent large response surface designs with a few amount of storage space. A conceptual comparison of several optimization algorithms can be found in [1] where their respective advantages and disadvantages are explained, in detail.

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