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Quantum Trajectories and Autocorrelation Function in Semiconductor Microcavity

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We investigate the photon statistics of the field emitted from a semiconductor microcavity containing a quantum well within the quantum trajectory approach. A dynamical behavior of the autocorrelation function depending of the system parameters is discussed.

Keywords: Quantum optics, quantum fluctuations, quantum jumps.

1 Introduction

The nonclassical properties of the light emitted from cavities containing one or more two-level atoms coupled to a single mode of the electromagnetic field of an optical resonator have been investigated by several authors in the context of QED [4, 39].

Cavity QED systems are of current interest both for fundamental aspects and potential applications [40]. This field of fundamental research aims to obtain a better understanding of matter-radiance coupling. As for its applications, we hope to be able to manufacture optical devices with exceptional properties for quantum computing [1, 44, 48, 53]. Cavity QED are systems with high quality factor allowing the establishment of the strong coupling regime. Strong coupling regime is observed when the cavity mode is nearly resonant with a narrow optical transition of the active medium. If the coupling frequency corresponding to a single photon is larger than the relaxation frequencies of the medium and of the cavity, then the so-called vacuum Rabi splitting is observed [40]. The degeneracy between the cavity resonance and the medium is lifted and two lines appear in the reflection or in the transition spectrum of the system. A great deal of works have been done, both theoretically and experimentally, in order to gain a deep understanding of two nonclassical proprieties of the electromagnetic field emitted from Cavity QED systems. Namely, photon antibunching and squeezing. Photon antibunching is defined by a nonclassical form for the degree of second-order coherence [32]. Squeezing is defined by reduced quantum fluctuations, below those for vacuum state, in one quadrature phase amplitude of the field [31]. Beyond atomic physics cavity QED is now a subject of much interest in condensed matter physics [3, 20, 21]. Phenomena which are linked to quantum electrodynamics [25,41,42,50], such as Rabi splitting, have been observed in semiconductor microcavity [45, 51]. Recently, theoretical and experimental studies brought the proof of squeezing in Semi-conductor microcavities with quantum wells [13, 14, 22, 30, 34, 38, 47].

Squeezing and antibunching are complementary aspects of the non-classical field. The squeezing is linked to the wave aspect of light and reveals the subpoissonian variance of the electromagnetic field while antibunching is linked to the particle aspect of light and reveals the anticorrelation of pairs in photodetections [5, 6, 10]. Whereas the autocorrelation function, measuring the correlation between pairs of photodetection, was intensively studied in the atomic physics [5, 9, 46], few works examined it in the semiconductor cavity QED [11, 12, 14, 16, 18, 19, 23, 34, 37, 43]. In our previous paper [12], we have explored the dynamical behavior of the autocorrelation function in weakly pumped semiconductor cavity QED. This paper extends our previous study beyond the weak excitation regime, where an analytic expression can no more be derived, using the Monte Carlo wavefunction approach (known in atom physics as the quantum trajectory method) [5,52]. The quantum trajectory approach was developed initially in the field of atomic physics by Carmichael and his co-worker [2]. It describes the dissipative dynamics of an open quantum system using two complementary processes: A nonunitary time evolution by an effective non-Hermitian operator and quantum jumps whose outcomes are determined by random numbers [2,5,52]. In this paper, we apply this powerful numeric technique in order to explore in the strong coupling regime, the photon statistics of the emitted electromagnetic field from a cavity semiconductor microcavity containing quantum well. We simulate the autocorrelation function for different sets of parameters in order to show the effect of each system parameters.

2 Model

The considered system is a semiconductor microcavity made of a set of Bragg's mirrors. The internal two sides of the Bragg's mirrors are separated by a distance which is of the order of the wavelength λ. Inside the microcavity there is a quantum well localized in a position which corresponds to most of the electromagnetic field. This system is driven with a resonant Laser field.

We restrict the study to the case of semiconductors with two bands. The electromagnetic field can excite an electron of the valence band to the conduction band by creating a hole in the valence band. The electron and the hole interact by giving excitonic states, which are similar to the bound states of hydrogen atoms. The state 1S which is the low bound state (fundamental state) has the greatest oscillator strength. For this reason we take into account only this state for the exciton - photon interaction. The photonic and excitonic modes are quantified along the normal direction to the microcavity. Invariance by translation implies that the excitons with parallel wave vector $K_{//}$ can couple only with photons with equally parallel wave vectors $k_{//} = K_{//}$. This work deals only with the case of normal incident pumping mode irradiating the microcavity. This limits the excitation only to one cavity photonic mode with parallel wave vector $k_{//} = 0$. Furthermore the effects of the spins are neglected. In this case an effective Hamiltonian for the coupled exciton-photon system in the cavity can be written as [2, 7, 8, 11, 12, 16, 18, 19, 23, 24, 26, 36, 37, 43, 49, 52]

$$\begin{aligned} H = {} & \hbar\omega_{ph}\bar{a}^{+}\bar{a} + \hbar\omega_{exc}b^{+}b + \hbar g\left(\bar{a}^{+}b + b^{+}\bar{a}\right) + \hbar\alpha' b^{+}b^{+}bb \\ & - \hbar r' g'\left(b^{+}b^{+}b\bar{a} + h.c.\right) + \hbar(\varepsilon' e^{i\omega_L t}\bar{a}^{+} + h.c) + H_{rel}. \end{aligned} \tag{2.1}$$

The first two terms of the Hamiltonian correspond to the energies of free photon and exciton, where a and b are respectively the annihilation operators of photonic and excitonic modes verifying

$$\left[\bar{a}, \bar{a}^{+}\right] = 1 \qquad \text{and} \qquad \left[b, b^{+}\right] = 1. \tag{2.2}$$

$\omega_{exc}, \omega_{ph}$ are the frequencies of the photonic and excitonic modes of the cavity. The third term corresponds to the coupling between an exciton and a photon with g the coupling constant. The fourth term describes the exciton-exciton scattering due to Coulomb interaction. The fifth term represents the saturation of the interaction photon-exciton. The sixth term is the pump term from the laser outside the cavity, where $\varepsilon'$and ω_L are respectively the amplitude and the frequency of the pumping Laser. The last term H$_{rel}$ give rise to relaxation of the main exciton and photon modes. In the rotating frame with frequency ω_L and when the pumping laser, the cavity and the excitons are all in resonance ($\omega_L = \omega_{exc} = \omega_{ph}$), the Hamiltonian describing the total system can be written as

$$\begin{aligned} \bar{H} & = \hbar g'\left(\bar{a}^{+}b + b^{+}\bar{a}\right) + \hbar\alpha' b^{+}b^{+}bb - r\hbar g'\left(\bar{a}^{+}b^{+}bb + b^{+}b^{+}\bar{a}b\right)\hbar(\varepsilon\prime\bar{a}^{+} + \varepsilon'\bar{a}) + H_{rel} \\ & = H_I + H_{rel}. \end{aligned} \tag{2.3}$$

The parameter α', which represents the strength of the interaction between excitons, has the following expression [8]

$$\alpha' \simeq \frac{3E_{ex}a_{ex}}{S}, \tag{2.4}$$

where S is the quantization area, a_{ex} and E_{ex} represent respectively the two dimensional excitonic Bohr radius and Binding energy.

The non-linear term describing the saturation effects, the third term in the Hamiltonian, can be neglected. It is shown that it gives rise to small corrections as compared to the exciton-exciton scattering [49].

For simplicity, we transform the photonic operators $\bar{a}$ and $\bar{a}^+$respectively into $a = i\bar{a}$ and $a^+ = -i\bar{a}^+$, the commutation relation does not change $[a, a^+] = 1$, so that a and a^+ can be considered as creation and annihilation operators. We introduce a dimensionless 'normalized' time

$$t = \frac{\tau_{time}}{\tau_c}, \tag{2.5}$$

where τ_c is the round trip time in the cavity. We normalize all the characteristic constants of the system to $1/\tau_c$ as

$$g = g'\tau_c, \qquad \varepsilon = \varepsilon'\tau_c, \qquad \text{and} \qquad \alpha = \alpha'\tau_c. \tag{2.6}$$

Applying the standard methods of the quantum theory of damping [17, 35], where the thermal bath is supposed to be at $T = 0K$, the master equation is

$$\frac{\partial \rho}{\partial t} = -i\alpha \left[b^+b^+bb, \rho\right] + g\left[\left(a^+b - b^+a\right), \rho\right] + \varepsilon\left[\left(a^+ - a\right), \rho\right] + \frac{\partial \rho}{\partial t}\Big\rfloor_{diss}, \tag{2.7}$$

where $\partial\rho/\partial t\rfloor_{diss}$ represents the dissipation term associated with H_{rel}. It describes the dissipation due to the excitonic spontaneous emission rate $\gamma/2$ and to the cavity dissipation rate κ

$$\frac{\partial \rho}{\partial t}\rfloor_{diss} = \kappa\left(2a\rho a^+ + a^+a\rho + \rho a^+a\right) + \frac{\gamma}{2}\left(2b\rho b^+ + b^+b\rho + \rho b^+b\right). \tag{2.8}$$

3 Autocorrelation Function and Quantum Trajectories

We now turn our attention to the calculation of the autocorrelation function $g^{(2)}(t)$. Quantum mechanically, the autocorrelation function is defined as [15, 28, 29, 33]

$$g^{(2)}(\tau) = \frac{\langle a^+(0)\, a^+(\tau)\, a(\tau)\, a(0)\rangle}{\langle a^+a\rangle^2}, \tag{3.1}$$

where $g^{(2)}(\tau)$ is proportional to the probability of detecting one photon at time t and another one at time $t + \tau$, not necessarily emitted after the first one. In the weak excitation regime analytic expression has been derived [12]. Beyond this regime only numerical methods can be used in order to study the dynamical behavior of the autocorrelation function. We simulate this evolution using quantum trajectory approach. The quantum trajectory approach describes the dissipative dynamics of an open quantum system using two complementary processes: A nonunitary time evolution by an effective non-Hermitian operator and quantum jumps whose outcomes are determined by random numbers [2, 5, 52]. The non-Hermitian Hamiltonian can be directly derived from the master equation [5, 12, 33]. In our case the non-Hermitian Hamiltonian has the following expression

$$\widetilde{H} = H_I - i\hbar\kappa a^+a - i\hbar\frac{\gamma}{2}b^+b. \tag{3.2}$$

The conditioned wave function $|\Psi_c(t)\rangle$ satisfies a coherent evolution interrupted by instantaneous collapses at the time of excitonic dissipation or photonic emission. Between two collapses, the wave function evolution is governed by the Schrödinger equation with the non-Hermition Hamiltonian

$$\frac{d|\Psi_c(t)\rangle}{dt} = \frac{\widetilde{H}}{i\hbar}|\Psi_c(t)\rangle . \tag{3.3}$$

The wave function $|\Psi_c(t)\rangle$ can be expanded into a superposition of tensor product of pure excitonic and photonic states

$$|\Psi_c(t)\rangle = \sum_{l=0,m=0} A_{lm}|lm\rangle , \tag{3.4}$$

where $|lm\rangle$ is the state with l photons and m excitons in the cavity. In this coherent evolutive regime, the amplitudes A_{lm} satisfy for $l \neq 0$ and $m \neq 0$ the equations

$$\begin{aligned}
\frac{d}{dt}A_{lm} &= \varepsilon\left(\sqrt{l}A_{l-1,m} - \sqrt{l+1}A_{l+1,m}\right) \\
&\quad + g\left(\sqrt{l(m+1)}A_{l-1,m+1} - \sqrt{m(l+1)}A_{l+1,m-1}\right) \\
&\quad - im(m-1)\alpha A_{lm} - \left(\kappa l + \frac{m\gamma}{2}\right)A_{lm}, \\
\frac{d}{dt}A_{00} &= -\varepsilon A_{10}, \\
\frac{d}{dt}A_{0m} &= -\varepsilon A_{1,m} - g\sqrt{m}A_{1,m-1} - im(m-1)\alpha A_{0m} - \frac{m\gamma}{2}A_{0m}, \\
\frac{d}{dt}A_{l0} &= \varepsilon\left(\sqrt{l}A_{l-1,0} - \sqrt{l+1}A_{l+1,0}\right) + g\sqrt{l}A_{l-1,1} - \kappa l A_{l,0}.
\end{aligned} \tag{3.5}$$

It is worth noting that the conditioned wave function is unnormalized.

Photon emissions and exciton dissipations occur at random times at rate determined by the conditioned wave function $|\Psi_c(t)\rangle$. The photon emission occurs at rate

$$r_{ph} = 2\kappa\frac{\langle\Psi_c(t)|\, a^+a\,|\Psi_c(t)\rangle}{\sqrt{\langle\Psi_c(t)|\,\Psi_c(t)\rangle}} \tag{3.6}$$

and is accompanied by the wave function collapses

$$|\Psi_c(t)\rangle \to J_{ph}|\Psi_c(t)\rangle , \tag{3.7}$$

where $J_{ph} = \sqrt{2\kappa}a$ is the photonic jump operator.

The excitonic dissipations occur at the rate

$$r_{ex} = \gamma\frac{\langle\Psi_c(t)|\, b^+b\,|\Psi_c(t)\rangle}{\sqrt{\langle\Psi_c(t)|\,\Psi_c(t)\rangle}} \tag{3.8}$$

and is accompanied also by the wave function collapses

$$|\Psi_c(t)\rangle \to J_{ex}|\Psi_c(t)\rangle , \tag{3.9}$$

where $J_{ph} = \sqrt{\gamma} b$ represents the excitonic jump operator.

The autocorrelation function can be computed by the Quantum Monte Carlo simulation. In this case, the autocorrelation function is then written as a quotient of two ensemble time averages [27]

$$g^{(2)}(\tau) = \frac{\overline{\langle a^{+}(t_k)\, a^{+}(t_k+\tau)\, a(t_k+\tau)\, a(t_k)\rangle_c}}{\left(\overline{\langle a^{+}(t_l)\, a(t_l)\rangle_c}\right)^2}, \tag{3.10}$$

where t_k represent a set of times when photon emissions occur, t_l are the sample times which are chosen to avoid the intervals immediately after the jump times t_k. This ensures that both averages are taken in the steady state. Overbar denotes the average of an ensemble of sampling times t_k.

Figures 3.1-3.4 show the autocorrelation function as a function of the time delay for different sets of parameters. The nonlinear coefficient α, is evaluated from [24] to be 1.5×10^{-9} in the inverse round trip time for an active area of $0.1\ mm^2$.

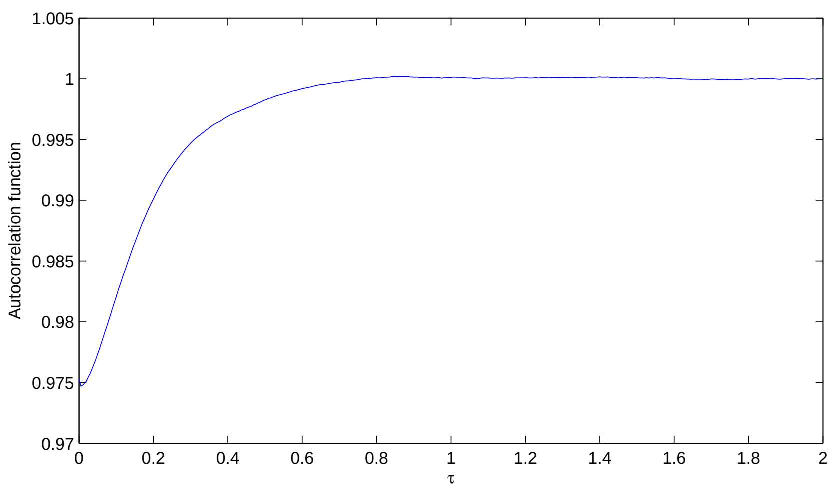

Figure 3.1: Autocorrelation function $g^{(2)}$ as a function of the delay τ with the photon-exciton coupling constant $g = 10$, laser pump amplitude $\varepsilon = 10$, exciton non-linearity $\alpha = 1.5 \times 10^{-9}$, cavity loss rate $\kappa = 0.5$ and excitonic dissipation rate $\gamma/2 = 0.5$.

Figures 3.1 and 3.2 correspond to strong coupling regime with two different values of excitonic dissipations $\gamma/2 = 0.5$ (Fig 3.1) and $\gamma/2 = 0.05$ (Fig 3.2). We have taken, for these plots, the exciton-photon coupling constant $g = 10$, the amplitude of the laser pump $\varepsilon = 10$ and the cavity dissipation rate $\kappa = 0.5$ in the unit of the inverse round trip time. The calculated photon average number and exciton average number inside the cavity are respectively 21.79 photons and $19, 57$ excitons for the first plot, 23.36 photons and 23.46 excitons for the second plot.

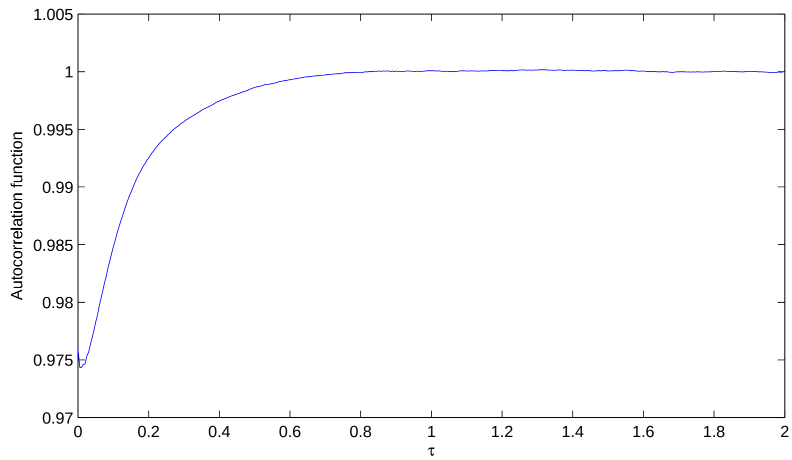

Figure 3.2: Autocorrelation function $g^{(2)}$ as a function of the delay τ with the photon-exciton coupling constant $g = 10$, laser pump amplitude $\varepsilon = 10$,exciton non-linearity $\alpha = 1.5 \times 10^{-9}$, cavity loss rate $\kappa = 0.5$ and excitonic dissipation rate $\gamma/2 = 0.05$.

Figure 3.3 describes the autocorrelation function dynamics in the moderately strong coupling regime with $g = 2$, $\varepsilon = 10$ and $\gamma/2 = \kappa = 0.5$. The calculated photon average number and exciton average number inside the cavity are respectively 23.77 photons and 19.94 excitons.

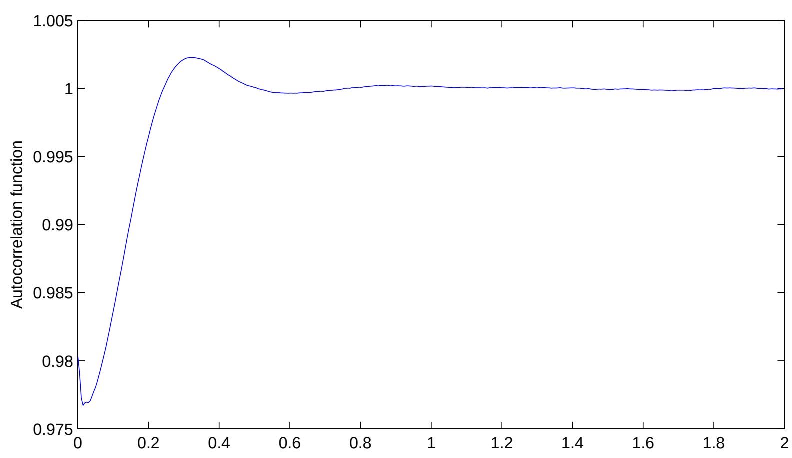

Figure 3.3: Autocorrelation function $g^{(2)}$ as a function of the delay τ with the photon-exciton coupling constant $g = 2$, laser pump amplitude $\varepsilon = 10$, exciton non-linearity $\alpha = 1.5 \times 10^{-9}$, cavity loss rate $\kappa = 0.5$ and excitonic dissipation rate $\gamma/2 = 0.5$.

Figure 3.4 corresponds to the variation of the autocorrelation function versus time delay for the moderate pumping regime $\varepsilon = 1$. This plot is computed with coupling constant $g = 10$, excitonic dissipation rate $\gamma/2 = 1$ and cavity dissipation rate $\kappa = 0.5$. The calculated average of the photon and exciton numbers are respectively 16.18 photons and 15.98 excitons.

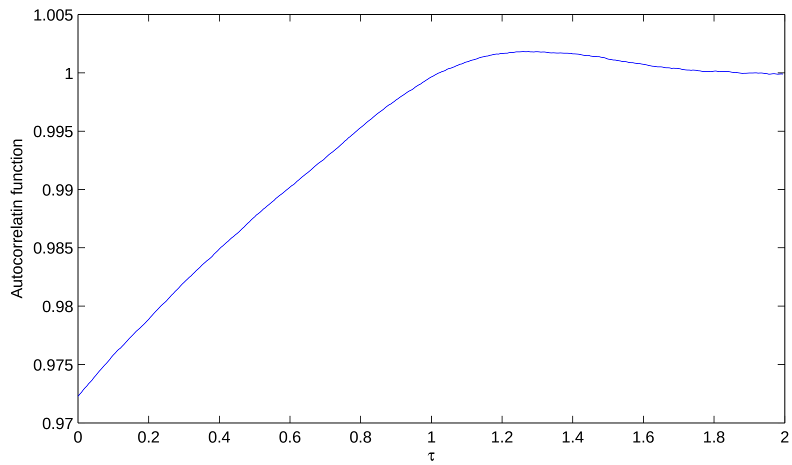

Figure 3.4: Autocorrelation function $g^{(2)}$ as a function of the delay τ with the photon-exciton coupling constant $g = 10$, laser pump amplitude $\varepsilon = 1$,exciton non-linearity $\alpha = 1.5 \times 10^{-9}$, cavity loss rate $\kappa = 0.5$ and excitonic dissipation rate $\gamma/2 = 0.5$.

From these simulations we can deduce that our system exhibits a small antibunching effect ($g^{(2)}(0) < 1$) in the strong coupling regime. This non classical effect is almost insensitive to the variation of the excitonic dissipation rate (See Fig 3.1 and Fig 3.2) compared to the other parameters variation. When the pump field amplitude ε decreases or the coupling constant g increases, the antibunching effect increases. This can be explained by the fact that by increasing the coupling constant g the effect of the non linear interaction (excitonic interaction) becomes more effective and induces more photon anticorrelations. By increasing the pump amplitude ε, the average number of photons inside the cavity, on one side, increases with high sensitivity compared to the other parameters variation, and on the other side the anticorrelation of photons increases with lower sensitivity. Since the autocorrelation function is a quotient of the photon correlation term to the square average of the mean average number of photons, the antibunching effect decreases with the increase of pump amplitude ε. Furthermore These simulations show that in the moderate coupling regime (Fig 3.3) and moderate pump amplitude (Fig 3.4), the system loses the anticorrelation behavior ($g^{(2)}(\tau) < 1$) in finite and small time delay.

4 Conclusion

In this work, we have studied the dynamical behavior of the autocorrelation function of the light emitted from driven optical semiconductor microcavity containing quantum well. The system is pumped with non-weak laser amplitude in the strong coupling regime. Using the quantum trajectory method we have analyzed the effect of the system parameters variation. We have shown that this system exhibits a small antibunching effect. By increasing the coupling photon-exciton or decreasing the pump amplitude, this non classical effect increases.

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