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Error Minimization Algorithm Using Barycentric Coordinates for Wireless Positioning Systems

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Abstract: This paper presents Barycentric Coordinates using Closed-Points (BCCP) algorithm to minimize positioning error occurred by AP arrangement form. Triangular AP arrangement form and straight line AP arrangement form were used to include measurement error and conduct simulation in which the presented algorithm and least-square method was used for simulation performance analysis. As result of simulation performance analysis, there was no big difference in BCCP algorithm and least-square method in triangular AP arrangement form. However, least-square method showed an average of 48.8m in straight line AP arrangement form while BCCP algorithm showed 4.28m error.

Keywords: Ranging; Positioning; Trilateration; Access point arrangment; TOA; RSSI

1 Introduction

Necessity on accurate indoor/outdoor positioning is being demanded in medical, military, and firefighting fields that study on indoor/outdoor positioning is being actively conducted in several research groups [1–7]. In particular, positioning algorithms using wireless communication systems are one of the current priority research areas [8]. Typical ranging methods for measuring distances between AP and mobile devices using wireless communication systems include the time of arrival (TOA) method based on the packet data arrival times and the received signal strength indication (RSSI) method based on the attenuation rates of the received signal strengths over propagation distances [9, 10]. Generally, trilateration using measured distance between 3 AP and mobile node is used to determine the location of mobile nodes. Therefore, the location of mobile nodes can be found by applying the distance information between AP and mobile node, which is measured by TOA or RSSI method, in trilateration algorithm. The trilateration algorithm is widely used in positioning systems and algorithms, such as the dynamic triangulation method using wireless pyroelectric infrared sensors and the auto-localization algorithm using trilateration for mobile positioning [11, 12]. The least-square method is one of the typical positioning methods. The least-square method uses the equations made from the coordinates of the AP arrangements and the distances between the AP and the mobile devices. The least-square method finds the locations that have the minimum solutions among the sums of the square of the error values made in the results of every single equation [13].

For trilateration using distance between AP and mobile nodes, a minimum of 3 AP is needed. Practically, measured distance errors can occur due to signal noise, interference, multipath, imprecise clock, and etc [14, 15]. Moreover, When geometric method is used for positioning during trilateration using at least 3 AP, error by AP arrangement form can occur [16]. This AP arrangement form can cause restrictions by applied field or installed space. Therefore, there is necessity of improvement by analyzing positioning error considering measured distance errors by both physical and geometrical factors. So, this study presents Barycentric Coordinates using Closed-Points (BCCP) algorithm as a plan to minimize error factors.. The positioning error values have been analyzed for the two typical forms of AP arrangements considering various ranging error values. The first form of the AP arrangement is the triangular shape, where a mobile device makes a triangle with the points formed by the AP arrangement. The second form is the straight line shape, where a mobile device is appropriately leaved from the straight line. Also,

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the analysis results of the proposed BCCP algorithm are compared with those of the least-square method for performance analysis of the BCCP algorithms.

This paper discusses on results of positioning error improvement by AP arrangement form in the following context. The least-square was studies in chapter II, procedure of BCCP algorithm was discussed in chapter III, and simulation comparison and analysis results of presented BCCP algorithm and least-square are described in chapter IV. Lastly, the results conclude this study.

2 Related Work

The ranges that are the basis for TOA and RSSI are subject to various causes of imprecision, among them noise, channel interference, multipath interference, and imprecise clocks [14, 15]. The positioning accuracy can be improved by incorporating into the location process a larger number of fixed stations than the minimum required for unambiguous location estimation. A two-dimensional layout is made of three fixed terminals labeled AP1, AP2, and AP3, with known coordinates and a mobile device, the location of which is to be determined. If the true location could be measured exactly, the coordinates of the mobile device would be at the intersection point of the circles formed with the fixed stations at the centers, with the radii equal to the distances to the mobile device. However, the actual ranging values, designated D1, D2, and D3, are not exact, the circles do not cross at one point, and it is necessary to define a criterion for deciding on the estimated location coordinates. The equations of the three circles defined by the base station locations AP1(x1, y1) through AP3(x3, y3) and the measured distances to the mobile device, D1 through D3, respectively, are as follows:

$$
\begin{aligned}
(x_1-x_m)^2+(y_1-y_m)^2 &= D_1^2\\
(x_2-x_m)^2+(y_2-y_m)^2 &= D_2^2\\
(x_3-x_m)^2+(y_3-y_m)^2 &= D_3^2
\end{aligned}
\tag{1}
$$

We describe a method for estimating the location of the mobile device using a least-squares error criterion. Using the least-square method, the location estimate has the coordinates xm, ym that minimize the function F:

$$F=\sum_{i=1}^{M}\left(\sqrt{(x_i-x_m)^2+(y_i-y_m)^2}-D_i\right)^2 \tag{2}$$

where M=3 in our example. The coordinates xm, ym that minimize the nonlinear Equation 2 can be found using an iterative algorithm based on a Taylor series expansion or gradient descent [19–21]. Such a method may be too time consuming and inconvenient to implement in many applications. An alternative approach described below gives a closed form solution to the estimation problem. It works by first creating a set of linear equations from Equation 1. Expanding the factors on the left side of Equation 1 and subtracting equations AP2 and AP3 from AP1 gives the following new set of M 1 equations:

$$
\begin{aligned}
(x_1-x_2)x_m+(y_1-y_2)y_m &= \frac{1}{2}\left(x_1^2-x_2^2+y_1^2-y_2^2+D_2^2-D_2^2\right)\\
(x_1-x_3)x_m+(y_1-y_3)y_m &= \frac{1}{2}\left(x_1^2-x_3^2+y_1^2-y_3^2+D_3^2-D_2^2\right)
\end{aligned}
\tag{3}
$$

Equation 3 is an overdetermined set of linear equations in x, y. It can be expressed in matrix form as:

$$
\begin{gathered}
A\cdot P_m=b\\
A=\begin{pmatrix} x_1-x_2 & y_1-y_2\\ x_1-x_3 & y_1-y_3\end{pmatrix}\\
b=\frac{1}{2}\cdot\begin{pmatrix} x_1^2-x_2^2+y_1^2-y_2^2+D_2^2-D_1^2\\ x_1^2-x_3^2+y_1^2-y_3^2+D_3^2-D_1^2\end{pmatrix}\\
P_m=\binom{x_m}{y_m}
\end{gathered}
\tag{4}
$$

The closed form least-square solution to Equation 3 is [13, 17]:

$$p_m=\begin{pmatrix}x_m\\ y_m\end{pmatrix}=\left(A^{\tau}.A\right)^{-1}.A^{\tau}.b \tag{5}$$

3 BCCP Algorithm

A. Closed-point decision The proposed BCCP algorithm consists of two main elements: the closed-point (CP) decision method and the barycentric coordinate decision method. This section first introduces the CP decision method. The distance (Di) between the target mobile device and the i-th AP (APi) among the AP used for positioning can be measured by the ranging algorithms, such as the TOA or RSSI method. After that, circles with radii of Di can be generated. Due to the ranging error elements, Di is generally longer than the real distance. Thus, there are two intersection points made by the two circles. The coordinates of the intersection points made by two arbitrary circles among the formed circles can be calculated. The calculation method for the coordinates of the intersection points(IP) using AP1, AP2, D1, and D2 is as follows.

$$
\begin{aligned}
(x_1-x_{ip})^2+(y_1-y_{ip})^2 &= D_1^2\\
(x_2-x_{ip})^2+(y_2-y_{ip})^2 &= D_2^2
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
x_{ip} &= x_1+D_1\cos\theta\\
x_{ip} &= y_1+D_1\sin\theta
\end{aligned}
\tag{7}
$$

Equation 6 is the equation of the circle using the coordinates of the center of AP1 (x1, y1) and AP2 (x2, y2) and the distances D1 and D2. The intersection points of the circles can be solved using Equation 7. The parameter in Equation 7 is the angle between the two intersection points, and this can be derived using Equation 8

$$\theta = \tan^{-1}\left(\frac{y_2-y_1}{x_2-x_1}\right) \pm \cos^{-1}\left(\frac{D_1^2-D_2^2+(x_2-x_1)+(y_2-y_1)^2}{2D_1\cdot\sqrt{(x_2-x1)^2+(y_2-y_1)^2}}\right) \tag{8}$$

Therefore, the intersection points of the two circles are calculated by substituting Equation 8 into Equation 7. All of the possible intersection points for three circles are shown in Figure 1.

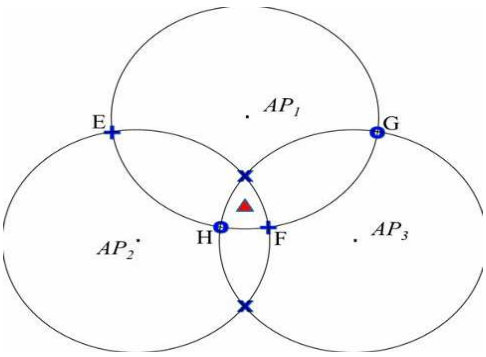


Fig. 1 All of the intersection points of AP1&AP2, AP2&AP3 and AP1&AP3

The red triangle marker indicates the location of the mobile device, and the three AP are arranged at the center of each of the three circles formed from the distances D1, D2, and D3. The intersection points of the two circles with the center points of AP1 and AP2 are indicated by the marker +, the intersection points of the two circles with the center points of AP1 and AP3 are indicated by o, and the intersection points of the two circles with the center points of AP2 and AP3 are indicated by x as presented in Figure 1. Next, look at the procedure to select a closed-point such as +, o, or x for 6 intersection points. The + coordinates are denoted by E and F, and the o coordinates are denoted by G and H in Figure 1. It is assumed that E is (xE, yE), F is (xF, yF), G is (xG, yG), and H is (xH, yH). The nearest intersection points are chosen by calculating the distances between E& G, E& H, F& G, and F& H.

$$\begin{aligned}
(x_B-x_G)^2+(y_B-y_G)^2 &= d_1\\
(x_B-x_H)^2+(y_B-y_H)^2 &= d_2\\
(x_F-x_G)^2+(y_F-y_G)^2 &= d_3\\
(x_F-x_H)^2+(y_F-y_H)^2 &= d_4
\end{aligned} \tag{9}$$

To find the smallest value, they are compared using the obtained values for d1, d2, d3, and d4. Assuming that the smallest value is d4, F and H are selected as the Closed-points. Using the same procedure, we can find the Closed-point of x, + and x, o. B. Barycentric coordinates calculation A triangle is formed with the vertices as the Closed-point found by the procedure described in the previous section and is presented in Figure 2.

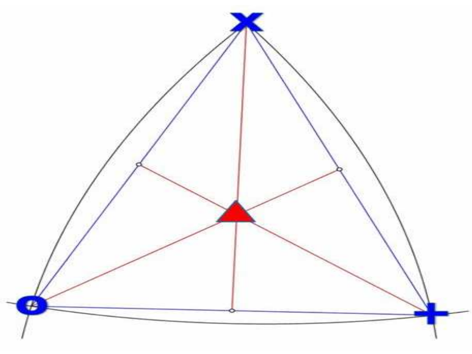


Fig. 2 Barycentric coordinates of the triangle are the intersection point of segments connected from three vertices to the center point of the opposite side of each vertex point

The barycentric coordinates of this triangle are the coordinates of the mobile device for positioning. The barycentric coordinates of the triangle are the intersection point of the segments connecting the three vertices to the center points of the opposite sides of each vertex point, as shown in Figure 2. Assuming that x is (CPx1, CPy1), o is (CPx2, CPy2), and + is (CPx3, CPy3) for the Closed-point found in Figure 2, the Barycentric coordinates could be calculated as shown in the equation below.

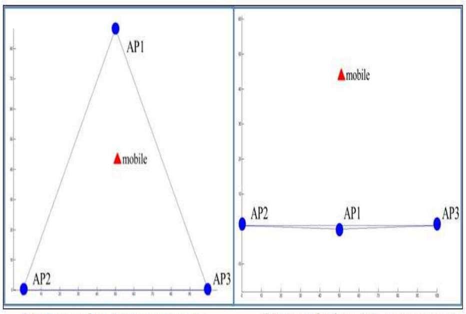
$$\begin{aligned}
x_{BC} &= \frac{1}{3}\cdot\sum_{i=1}^{3} CPx_i\\
y_{BC} &= \frac{1}{3}\cdot\sum_{i=1}^{3} CPy_i
\end{aligned} \tag{10}$$

As a result, the Barycentric coordinates are the location coordinates of the mobile device as calculated using Equation 10.

4 Simulation

A. Simulation conditions

To confirm the utility of the BCCP algorithm introduced above, an analysis is performed of the positioning errors as affected by the forms of the AP arrangements. The proposed BCCP algorithms are compared with the least-square method for the general trilateration method. The coordinates about the triangular AP arrangement are AP1 (50, 86), AP2 (0, 0), and AP3 (100, 0), and the coordinates of the straight line AP arrangement are AP1 (50, 0), AP2 (0, 1), and AP3 (100, 1). Also, consider the general situation in which the coordinates of the mobile device are arbitrary (50, 43). Here, the coordinates of the AP for the straight-line arrangement case are not perfectly aligned. The reason is that all of the values of the y-axis in the matrix equation of the least-square method become zero, and the equation cannot be calculated when the AP arrangements are perfectly aligned [10].



(a) Triangular AP arrangement (b) Straight line AP arrangement

Fig. 3 The forms of the AP arrangements

TOA which depends on packet propagation time is used for the ranging method of AP and mobile nodes. Because the transmission velocity of a wireless packet is 300,000 kilo meter per second which is the speed of light, to have 30 centimeter accuracy we set 300 MHz Clock with 1ns resolution. We calculated a distance between AP and mobile node, and inserted clock error of 0 to 10 ticks, 10 to 20 ticks and 0 to 20 ticks into the distance, and then derived the ranging error of 0 to 3 m, 3 to 6 m, and 0 to 6 m. We performed simulations of three ranging error 100 times individually by using MATLAB.

B. Simulation results

We performed simulations 100 times based on the measurement error of three conditions, 0 to 3 m, 3 to 6 m and 0 to 6 m.

$$
\begin{aligned}
error_{distance,BCCP} &= \sqrt{(x_M - x_{BC})^2 + (y_M - y_{BC})^2} \\
error_{distance,LS} &= \sqrt{(x_M - x_{IS})^2 + (y_M - y_{IS})^2}
\end{aligned} \tag{11}
$$

The Equation 11 calculates the distance between coordinates of a mobile node and coordinates calculated by the least-square method and BCCP algorithm.

$$
mean = \frac{1}{n}\sum_{i=1}^{n} error_{distance} \tag{12}
$$

$$
Std = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(error_{distance} - mean)^2} \tag{13}
$$

The results in three conditions simulated 100 times each show an average and a standard deviation(Std) calculated by the Equation 12 and 13, as shown in following table.

Table 1 TRIANGULAR AP ARRANGEMENT SIMULATION RESULT

Method	0~3 meter Ranging error		3~6 meter Ranging error		0~6 meter Ranging error	
	Error(m) Mean	Error(m) Standard deviation	Error(m) Mean	Error(m) Standard deviation	Error(m) Mean	Error(m) Standard deviation
LS	1.73	1.02	3.44	1.76	5.83	2.09
BCCP	1.67	0.86	3.68	2.03	4.83	1.57

Table 2 STRAIGHT LINE AP ARRANGEMENT SIMULATION RESULT

Method	0~3meter Ranging error		3~6m Ranging error		0~6m Ranging error	
	Error(m) Mean	Error(m) Standard deviation	Error(m) Mean	Error(m) Standard deviation	Error(m) Mean	Error(m) Standard deviation
LS	30.93	266.14	37.59	30.66	80.87	63.63
BCCP	2.96	1.44	4.74	2.96	5.13	2.06

C. Analysis of simulation result

The least-square simulation results shows an average of 3.66m and a standard deviation of 1.63m over 300 simulations for ranging errors of 0 3m, 3 6m, and 0 6m with a triangular AP arrangement. The BCCP algorithm simulation results show an average of 3.39m and a standard deviation of 1.49m. The simulation results of the least-square method and the BCCP algorithm show a negligible difference. The least-square simulation results show an average of 48.8m and a standard deviation of 40.15m over 300 simulations for ranging errors of 0 3m, 3 6m, and 0 6m with straight line AP arrangement. The BCCP algorithm simulation results show an average of 4.28m and a standard deviation of 2.15m. As shown in the least-square results, a big average error is identified. That is because of the characteristics of the least-square where a minimum sum of the square of error is derived. For better understanding, we show the details in the Figure 4.

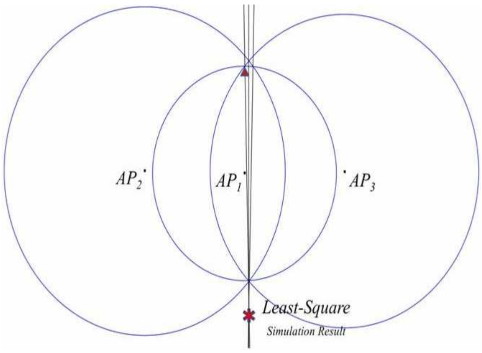


Fig. 4 The forms of the AP arrangements

In case of straight line AP arrangement, the least-square is difficult to being used for indoor-positioning. But positioning method using BCCP algorithm shows results less than the distance error. In other words, the proposed algorithm based on the barycenter of neighboring points is much less affected by locations of APs than the least-square method. From the analysis results, the positioning method applying the proposed BCCP algorithm has positioning error values that are smaller than the initial ranging error sets. Therefore, it is confirmed that the proposed BCCP algorithm is weakly influenced by the forms of the AP arrangements when compared with the results of the least-square method.

5 Conclusion

This paper suggests the barycentric coordinates using the closed-points (BCCP) algorithm to minimize the positioning error values. The positioning error values have been analyzed for the two typical forms of AP arrangements while considering the various ranging error values. The analysis results of the proposed BCCP algorithm are compared with those of the LS method for performance analysis of the BCCP algorithms. In the case of the triangular AP arrangement, the positioning errors have similar values for the two positioning methods applying the least-square method and the BCCP algorithm. In the case of the straight line AP arrangement, the positioning results have an average distance error of 48.8 m and a standard deviation of 40.15 m for the least-square method and an average distance error of 4.28 m and a standard deviation of 2.15 m are obtained for the positioning using the BCCP algorithm. Thus, it is clearly confirmed that the suggested BCCP algorithm is weakly influenced by the forms of the AP arrangements when compared to the results of the least-square method. Our methodology will provide more precise location detection when used with GPS and wireless communications either outdoors or indoors. The suggested positioning method using the BCCP algorithm can be used for various applications, such as indoor/outdoor positioning systems, military applications, and firefighting equipment.

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