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Stabilization of discrete-time switched linear singular systems via a stochastic approach

Bo Men[1,2] *and Qingling Zhang*[1] *and Guoliang Wang*[1,3] *and Juan Zhou*[1]

[1]Institute of System Sciences, Northeastern University, Liaoning, China, 110004
[2]School of Mathematics and Systems Science, Shenyang Normal University, Liaoning, China, 110034
[3]School of Information and Control Engineering Liaoning Shihua University, Liaoning, China, 113001

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Abstract: A stochastic stabilization problem for a class of discrete-time switched linear singular system is considered in this paper. By using the Bernoulli variable, a kind of stochastic controller is developed, which ensures the discrete-time switched linear singular system stochastically stable. Based on a linear matrix inequality technology, a sufficient existence condition for such new controller is proposed, which bridges the gain-scheduled and gain-common controller design methods. Finally, an example is provided to demonstrate the effectiveness of the proposed approach.

Keywords: stochastic stabilization, discrete-time switched linear singular systems, linear matrix inequalities

1 Introduction

Switched systems belong to a special class of hybrid control systems, which consist of a finite number of subsystems and a logical rule that orchestrates switching between these subsystems. And singular systems (also referred to as descriptor systems, implicit systems, generalized state-space systems, differential-algebraic systems) have comprehensive practical background, great progress have been made in the theory and its applications since 1970s. Both of them have attracted much attention in recent years because of their ability to capture the dynamical behaviour of many natural phenomena [1]-[6].The stabilization problem is one of the most important control problems. It consists of designing a controller which guarantees that the closed-loop system will be stable and has some desired specifications. This problem has attracted a lot of researchers and many results have been reported in the literature either for the deterministic systems and the stochastic ones [7]-[10].

On the other hand, switched linear singular (SLS) systems are an important class of switched systems, which arises from, for example, electrical networks and economic systems [11-14, 21, 22]. This kind of systems is switching among a set of singular systems. In recent years, more and more attention has been paid to SLS systems due to their theoretical and practical significant. However, to the best of the authors' knowledge, few results exist for the class of discrete-time SLS systems. There are two reasons that account for this situation. One is singular systems are difficult to tackle since stability, regularity and causality should be considered at the same time [16]; the other is switching between several discrete-time singular systems makes the problem more complicated. Ref. [19] and [21] studied the reachable and observability problems respectively. Moreover, Ref. [16, 17, 20, 22-25] studied the stability problem. Ref. [11, 18, 22] gave methods to design controllers for SLS systems and the controllers designed there are all gain-scheduled controllers (where different controllers are designed for each operating point and controllers are switched when the operating condition change).

It is very known that for discrete-time SLS systems, the traditional controller design methods are generally classified into two categories: a gain-common (GC) controller and a gain-scheduled (GS) controller. A GC controller is to design a common gain for each subsystem and a GS controller is to design different controllers for each subsystem. But as we know criteria obtained through the GS controllers are less conservative than the ones obtained by the GC controller because it's difficult to find the common gain to satisfy the different subsystems.

* Corresponding author e-mail: carlo.bianca@polito.it

However in many practical applications, such as network control systems, the signal is transmitted through an unreliable network and suffers induced time-delay and packet drop. Based on this fact, it is claimed that the GS controller is very ideal, which has the application scope limited. As a result, it is seen that both of them are two extreme design methods. Motivated by the above investigations, we will consider the stabilization problems for discrete-time SLS systems via a stochastic approach, which will make a bridge between them.

In this paper, the stabilization problem of discrete-time SLS systems via exploiting a stochastic controller is studied, where the resulting closed-loop system is stochastically stable. For discrete-time SLS systems, it is known that the switching property and singular derivative matrix will lead to a strong coupling between the switching Lyapunov matrix and common controller, which make the controller design complicated. In order to deal with such problems, sufficient condition for the existence of a kind of stochastic controller (SC) is obtained as strict linear matrix inequalities (LMIs). Compared with traditionally GS and GC controllers, the accessible probability of mode is taken into consideration in the presented design method, whose advantages are also illustrated by an example.

Notation: R^n denotes the n-dimensional Euclidean space, and $R^{m\times n}$ is the set of all $m\times n$ real matrices. $\varepsilon(\cdot)$ is the expectation operator with respect to some probability measure. $N(X)$ denotes the right zero subspace of X for a given vector or matrix X .We use $'*'$ as an ellipsis for the terms induced by symmetry in symmetric block matrices and $diag\{\cdots\}$ in a block-diagonal matrix, and $(M)^* \triangleq M+M^T$.

2 Problem Formulation

Consider a class of discrete-time SLS systems described as

$$E_i x(k+1) = A_i x(k) + B_i u_i(k) \tag{1}$$

where $i:\{0,1,\cdots\}\to \Lambda=\{1,2,\cdots,N\}$ is the switching law; $x(k)\in R^n$ is the state vector, $u_i(k)\in R^m$ is the control input. Matrix $E_i\in R^{n\times n}$ may be singular, which is assumed to be $rank(E_i)=r\le n$. A_i and B_i are known matrices of compatible dimensions.

For discrete-time SLS systems, the traditional controller design methods are generally classified into two categories:

$$u_i(k)=K_i x(t),\, u_i(k)=Kx(t)$$

where both of them are two extreme design methods. In this paper, a stochastic controller is developed as follows:

$$u(k)=\alpha(k)K_i x(k)+(1-\alpha(k))Kx(k) \tag{2}$$

where K_i and K are controller gains to be determined and $\alpha(k)$ is an indicator function satisfying the Bernoulli process and is described as

$$\alpha(k)=\begin{cases}1 & \text{if the subsystem is activated successfully}\\ 0 & \text{otherwise}\end{cases} \tag{3}$$

Then, we have

$$\begin{aligned} Pr\{\alpha(k)=1\}=\varepsilon(\alpha(t))=\alpha\\ Pr\{\alpha(k)=0\}=1-\alpha \end{aligned} \tag{4}$$

Moreover, it can be readily verified that

$$\varepsilon((1-\alpha(k)))=1-\alpha \tag{5}$$

$$\varepsilon((\alpha(k)-\alpha))=0 \tag{6}$$

$$\varepsilon((1-\alpha(k))^2)=(1-\alpha)^2 \tag{7}$$

Remark 1

In this paper, Bernoulli variable $\alpha(k)$ is introduced to express the activated probability of the subsystem of discrete-time SLS systems available to controller operation. It is the first time that $\alpha(k)$ is used for the stabilization problem in discrete time SLS systems. Contrary to other papers such as in [29,30] where $\alpha(k)$ was used, the problem is more complicated for discrete-time SLS systems, because the model in this paper has multiple subsystems. In addition, controller (2) is less conservative than GC, which also has more application scope than GS and whose superiorities are illustrated by a numerical example.

Remark 2

Compared with the traditional controller design methods, controller (2) is more advantageous. Because the GC controller design method finds a common controller for all subsystems, the solvable solution set is smaller than the one generated by (2). When the mode is accessible with some probability and there is no solution to a GC controller, we may still get an effective controller of form (2). In this sense, it is said that the GC controller design method is an overdesign and is more conservative.

Applying controller (2) to system (1) results in the following discrete-time closed-loop switched singular system of the form

$$E_i x(k+1)=\tilde{A}x(k)+(\alpha(k)-\alpha)\hat{A}x(k) \tag{8}$$

where

$$\begin{aligned} \tilde{A}_i=\{A_i+B_i[(1-\alpha(k))K+\alpha K_i]\}\\ \hat{A}_i=B_iK_i \end{aligned} \tag{9}$$

let

$$\overline{A}_i=\{A_i+B_i[(1-\alpha)K+\alpha K_i]\} \tag{10}$$

Definition 1. The set of finite or countable time and active subsets is called a switching sequence, that is $\{(\tau_0,i_0),(\tau_1,i_1),\cdots,(\tau_s,i_s)\}$ and $\tau_0<\tau_1<\cdots<\tau_s<\infty$,$i_j\in(1,2,\cdots,N)$, $j=1,2,\cdots,n$.

Definition 2. Consider the system (8)

1)For a given $i \in \Lambda$, the pair $(E_i, \tilde{A}_i)$ is said to be regular if there exists a constant scalar $s \in C$ such that $det(sE_i - \tilde{A}_i) \neq 0$.
The discrete-time SLS system (8) is said to be regular if every pair $(E_i, \tilde{A}_i), i \in \Lambda$ is regular.

2)For a given $i \in \Lambda$, the pair $(E_i, \tilde{A}_i)$ is said to be causal if there exists a constant scalar $s \in C$ such that $deg(det(sE_i - \tilde{A}_i)) = rank(E_i)$.
The discrete-time SLS system (8) is said to be causal if every pair $(E_i, \tilde{A}_i), i \in \Lambda$ is causal.

Definition 3. Consider the system (8)

1) Discrete-time SLS system (8) is said to be stochastically stable if there exist symmetric matrices $V_i > 0$ and correspondent switching law, such that

$$\varepsilon(\Delta V_i(E_i x(k))) = V_i[E_i x(k+1)] - V_i[E_i x(k)] < 0$$

2) Discrete-time SLS (8) is said to be stochastically admissible if it is regular, causal and stochastically stable.

Assumption1 [22]. For any $i \in \Lambda$, $N(E_i)$ is the same.

3 Stochastically Stabilization Analysis and Design

Theorem 1.

Suppose Assumption 1 holds, then the resulting closed-loop system (8) via given controller (2) is stochastically admissible for arbitrary switching laws if there exist symmetric matrices $V_i > 0$, such that the following coupled inequalities hold for all $i \in \Lambda$.

$$E_i^T V_i E_i \geq 0 \tag{11}$$

$$\bar{A}_i^T V_i \bar{A}_i - E_i^T V_i E_i \tag{12}$$

Proof:

First, we show that discrete-time SLS (8) is regular and causal. It is known by reference [22] that there always exist nonsingular matrices M_i and N, such that

$$M_i E_i N = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, M_i \bar{A}_i N = \begin{bmatrix} \hat{A}_{i1} & \hat{A}_{i2} \\ \hat{A}_{i3} & \hat{A}_{i4} \end{bmatrix}$$

$$M_i^{-T} V_i M_i^{-1} = \begin{bmatrix} \hat{V}_{i1} & \hat{V}_{i2} \\ \hat{V}_{i2}^T & \hat{V}_{i4} \end{bmatrix} \tag{13}$$

From (11), we have $V_{i1} \geq 0$ .Similarly, by pre-multiplying and post-multiplying (12) by N^T and N , respectively, one concludes that

$$\begin{bmatrix} * & * \\ * & \hat{A}_{i2}^T V_{i1} \hat{A}_{i2} + H + H^T \end{bmatrix} < 0 \tag{14}$$

where

$$H = \hat{A}_{i2}^T V_{i2} \hat{A}_{i4} + \frac{1}{2} \hat{A}_{i4}^T V_{i4} \hat{A}_{i4} \tag{15}$$

where $*$ denotes the terms that are not used in (15). Taking into account (14) and $V_{i1} \geq 0$, it is obtained

$$H + H^T < 0 \tag{16}$$

which implies that $\hat{A}_{i4}^T$ is nonsingular. Then, for each $i \in \Lambda$, pair (E_i, A_i) is regular and causal.
Because inequality (12) holds, the following inequality is satisfied

$$\begin{aligned} \varepsilon(\Delta V_i) = \{ & x^T(k) A_i^T V_i A_i x(k) \\ & + (1-\alpha) x^T(k) A_i^T V_i B_i K x(k) \\ & + \alpha x^T(k) A_i^T V_i B_i K_i x(k) \\ & + (1-\alpha) x^T(k) K^T B_i^T V_i A_i x(k) \\ & + (1-\alpha)^2 x^T(k) K^T B_i^T V_i B_i K x(k) \\ & + \alpha(1-\alpha) x^T(k) K^T B_i^T V_i B_i K_i x(k) \\ & + \alpha x^T(k) K_i^T B_i^T V_i A_i x(k) \\ & + \alpha(1-\alpha) x^T(k) K_i^T B_i^T V_i B_i K x(k) \\ & + \alpha^2 x^T(k) K_i^T B_i^T V_i B_i K_i x(k) \\ & - x^T(k) E_i^T V_i E_i x(k) < 0 \end{aligned} \tag{17}$$

That is

$$\begin{aligned} & \Delta V_i = x^T(k+1) E_i^T V_i E_i x(k+1) \\ & - x^T(k) E_i^T V_i E_i x(k) \\ & = \left\{ \begin{array}{c} x^T(k)\{A_i^T + [(1-\alpha(k))K^T + \alpha K_i^T] B_i^T\} \\ +(\alpha(k)-\alpha) x^T(k) K_i^T B_i^T \end{array} \right\} V_i \\ & \left\{ \begin{array}{c} \{A_i + B_i[(1-\alpha(k))K + \alpha K_i] x(k)\} \\ +(\alpha(k)-\alpha) B_i K_i x(k) \end{array} \right\} \\ & - x^T(k) E_i^T V_i E_i x(k) < 0 \end{aligned} \tag{18}$$

Then system (8) via given controller (2) is stochastically admissible.

Remark 3

Via giving controller (2) before hand, theorem 2 gives a sufficient condition for stabilization of discrete-time SLS system (8). Because E_i is singular, and $E_i^T V_i E_i$ is positive semi-definite. Then the inequality (12) can only be transformed to a non-strict LMI inequality, therefore we can't obtain the controller. So we apply congruent transformation method to get solve the problem.

Theorem 2.

Suppose Assumption 1 holds, then the resulting closed-loop system (8) via given controller (2) is stochastically admissible for arbitrary switching laws if there exist matrix G and symmetric matrices $V_i > 0$, such that the following LMIs hold for all $i \in \Lambda$.

$$E_i^T V_i E_i \geq 0 \tag{19}$$

following

$$\begin{bmatrix} \begin{array}{c} (A_i G)^* - (1-\alpha)(B_i Y)^* \\ -\alpha(B_i Y_i)^* - (G)^* \end{array} & \sharp & \sharp & \sharp \\ \begin{array}{c} A_i G - (1-\alpha) B_i Y \\ -\alpha B_i Y_i - G^T \end{array} & -2(G)^* & \sharp & \sharp \\ 0 & G + I & V_i - 2I & \sharp \\ G + I & 0 & 0 & -E_i^T V_i E_i - 2I \end{bmatrix} < 0 \tag{20}$$

Then the desired controller gain of form (2) is given as

$$K_i = -Y_iG^{-1}, K = -YG^{-1} \tag{21}$$

Proof:

Taking into account (21), it is obtained by (20) that

$$\begin{bmatrix} \begin{array}{c}\{A_iG + B_i[(1-\alpha)(-YG^{-1}) + \alpha(-Y_iG^{-1})]G\} \\ +G^T\{A_i + B_i[(1-\alpha)(-YG^{-1}) \\ +\alpha(-Y_iG^{-1})]\}^T - G^T - G\}\end{array} & \sharp & \sharp & \sharp \\ \begin{array}{c}\{A_i + B_i[(1-\alpha)(-YG^{-1}) \\ +\alpha(-Y_iG^{-1})]\}G - G^T\end{array} & -2G-2G^T & \sharp & \sharp \\ 0 & G+I & V_i - 2I & \sharp \\ G+I & 0 & 0 & -E_i^TV_iE_i - 2I \end{bmatrix} \tag{22}$$

$$= \begin{bmatrix} \bar{A}_iG + G_i^T\bar{A}_i^T - G^T - G & \sharp & \sharp & \sharp \\ \bar{A}_iG - G^T & -2G-2G^T & \sharp & \sharp \\ 0 & G+I & V_i-2I & \sharp \\ G+I & 0 & 0 & -E_i^TV_iE_i - 2I \end{bmatrix} < 0 \tag{23}$$

Let

$$\begin{bmatrix} \bar{A}_iG + G_i^T\bar{A}_i^T - G^T - G & \sharp & \sharp & \sharp & \sharp & \sharp \\ \bar{A}_iG - G^T & -2G-2G^T & \sharp & \sharp & \sharp & \sharp \\ 0 & G+I & V_i-2I & \sharp & \sharp & \sharp \\ 0 & 0 & 0 & -I & \sharp & \sharp \\ 0 & 0 & 0 & 0 & -I & \sharp \\ G+I & 0 & 0 & 0 & 0 & -E_i^TV_iE_i - 2I \end{bmatrix} \tag{24}$$

then, it is concluded that

$$\Omega_1(k) = Z_1^TZ_2^TZ_3^T\Omega_4(k)Z_3Z_2Z_1 < 0 \tag{25}$$

$$\Omega_1 = \begin{bmatrix} \bar{A}_i^TV_i\bar{A}_i - E_i^TV_iE_i & \bar{A}_i^TV_i & 0 \\ V_i\bar{A}_i & -I & 0 \\ 0 & 0 & -I \end{bmatrix} < 0 \tag{26}$$

where

$$Z_1 = \begin{bmatrix} G^{-1} & 0 & 0 \\ G^{-1}\bar{A}_i & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, Z_2 = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & G & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}$$

$$Z_3 = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \\ G & 0 & 0 & 0 & 0 \end{bmatrix}$$

So we get

$$\bar{A}_i^TV_i\bar{A}_i - E_i^TV_iE_i < 0 \tag{27}$$

Then it implies that Theorem 1 holds, which means that closed-loop system (8) via controller (2) is stochastically admissible.

Corollary 1

Suppose Assumption 1 holds, then resulting closed-loop system (8) via an GC $u_i(k) = Kx(t)$ is stochastically admissible for arbitrary switching laws if there exist matrix G and symmetric matrices $V_i > 0$, such that the following LMIs hold for all $i \in \Lambda$.

$$E_i^TV_iE_i \geq 0 \tag{28}$$

$$\begin{bmatrix} (A_iG)^* - (B_iY)^* - (G)^* & \sharp & \sharp & \sharp \\ A_iG - B_iY - G^T & -(2G)^* & \sharp & \sharp \\ 0 & G+I & V_i-2I & \sharp \\ G+I & 0 & 0 & -E_i^TV_iE_i - 2I \end{bmatrix} < 0 \tag{29}$$

Then a desired controller gain of form (2) is given as

$$K = -YG^{-1} \tag{30}$$

Corollary 2

Suppose Assumption 1 holds, then resulting closed-loop system (8) via an GS $u_i(k) = K_ix(t)$ is stochastically admissible for arbitrary switching laws if there exist matrix G and symmetric matrices $V_i > 0$, such that the following LMIs hold for all $i \in \Lambda$.

$$E_i^TV_iE_i \geq 0 \tag{31}$$

$$\begin{bmatrix} (A_iG)^* - (B_iY_i)^* - (G)^* & \sharp & \sharp & \sharp \\ A_iG - B_iY_i - G^T & -(2G)^* & \sharp & \sharp \\ 0 & G+I & V_i-2I & \sharp \\ G+I & 0 & 0 & -E_i^TV_iE_i - 2I \end{bmatrix} < 0 \tag{32}$$

Then a desired controller gain of form (2) is given as

$$K_i = -Y_iG^{-1} \tag{33}$$

Remark 4
It is noticed that the criteria obtained in this paper are related to the stochastically stabilization problem of discrete-time SLS systems. However, because of system derivative matrix E_i satisfying $rank(E_i) = r \leq n$, the results of normal switching systems can also be obtained easily via similar methods.

4 Numerical Examples

Example: Consider a discrete-time SLS system of form (1) with

$$E_1 = \begin{bmatrix} 0.5 & 0.6 \\ 1 & 1.2 \end{bmatrix}, E_2 = \begin{bmatrix} 1 & 0.6 \\ 1 & 1.2 \end{bmatrix}, A_1 = \begin{bmatrix} 1.8 & 2 \\ 0.15 & 1 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 1 & 2 \\ 0.15 & 1 \end{bmatrix}, B_1 = \begin{bmatrix} -1 & 0.5 \\ 0.2 & 5 \end{bmatrix}, B_2 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

By Corollary 1, it is known that there is no solution to a GC. It means that one cannot design a stabilizing controller without any mode information. On the other hand, by Corollary 2, it is obtained that

$$G = \begin{bmatrix} 0.4274 & -0.0143 \\ 0.0716 & 0.4268 \end{bmatrix}, Y_1 = \begin{bmatrix} -0.4610 & -0.7415 \\ 0.0485 & 0.0292 \end{bmatrix}$$
$$Y_2 = \begin{bmatrix} -0.1433 & -0.7676 \\ 0.1501 & -0.0021 \end{bmatrix} V_1 = \begin{bmatrix} 0.5522 & -0.0521 \\ -0.0521 & 0.5034 \end{bmatrix}$$
$$V_2 = \begin{bmatrix} 0.5247 & -0.0409 \\ -0.0409 & 0.6045 \end{bmatrix}$$

Then, one has the controller gains of form (2) as

$$K_1 = \begin{bmatrix} -0.7832 & -1.7636 \\ 0.1014 & 0.0718 \end{bmatrix}, K_2 = \begin{bmatrix} -0.0338 & -1.7996 \\ 0.3501 & 0.0068 \end{bmatrix}$$

For such desired controller, it is said that it needs the system mode available on-line. But in many practical applications, such as network control systems, the signal is transmitted through an unreliable network and suffers induced time-delay and packet drop. Based on this fact, it is claimed that the afore-referred controller is very ideal, which has the application scope limited. By the method proposed in this paper, one can design a stochastic controller whose operation mode signal is not necessary. That is if $\alpha = 0.8$, based on Theorem 2, we have the controller gains of form (2) as

$$K_1 = \begin{bmatrix} 0.0174 & 1.8166 \\ -0.1388 & 0.1554 \end{bmatrix}, K_2 = \begin{bmatrix} -0.9278 & 1.8623 \\ -0.3839 & 0.2476 \end{bmatrix}$$
$$K = \begin{bmatrix} 3.8016 & 1.8951 \\ 0.1166 & -0.9876 \end{bmatrix}$$

Then we get a closed-loop system of the form (8) with

$$E_1 = \begin{bmatrix} 0.5 & 0.6 \\ 1 & 1.2 \end{bmatrix}, E_2 = \begin{bmatrix} 1 & 0.6 \\ 1 & 1.2 \end{bmatrix}$$
$$\tilde{A}_1 = \begin{bmatrix} 1.0653 & 2.9755 \\ 0.3076 & 0.2125 \end{bmatrix}, \tilde{A}_2 = \begin{bmatrix} -0.5026 & 3.1108 \\ -0.1338 & 1.0006 \end{bmatrix}$$

By theorem 1 in reference [29], it is known that there exist

$$V_1 = \begin{bmatrix} 0.5536 & -0.0541 \\ -0.0541 & 0.5039 \end{bmatrix}, V_2 = \begin{bmatrix} 0.5269 & -0.0476 \\ -0.0476 & 0.5913 \end{bmatrix}$$

such that the closed-loop system is stable.

From this example, it is concluded that the operation mode signal of the desired controller can suffer 20% loss.

5 Conclusion

In this paper, we have investigated the stochastically stabilization problem for a class of discrete-time SLS systems. Instead of a GC Lyapunov function method, a new kind of design method referred to be a CS controller is proposed, which bridges the following two extreme cases: GS and GC controller design methods. Sufficient criteria for CS controller are given in terms of strict LMIs. Finally, the utility of the developed theories are illustrated by a numerical example.

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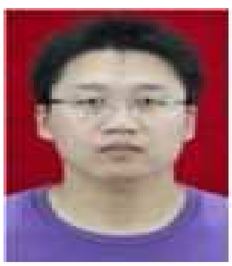
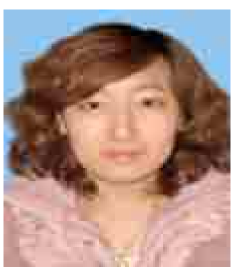
Bo Men received the MS degree in mathematics from the Institute of System Sciences, Northeastern University in 2006. She is currently a lecture in Shenyang Normal University. Her research interests are in the areas of switched systems, singular systems.

Qingling Zhang is a professor at Northeastern University, Shenyang, China. He received his BS and MS degrees from the Mathematics Department and his Ph.D. degree from the Automatic Control Department of Northeastern University, in 1982, 1986, and 1995, respectively. His main research interests are descriptor systems, biological control, Network control, dissipative control. He has published eight books and more than 260 papers about control theory and applications. Professor Zhang received 14 prizes from central and local governments for his research. He has also received the Golden Scholarship from Australia in 2000.



Guo-LiangWang received his Ph.D. degree from Northeastern University in 2010. He is now an associated professor with the School of Information and Control Engineering, Liaoning Shihua University. His research interests are Markovian jump systems and singular systems.



Juan Zhou received the MS degree from Jilin University in 2006. She is currently a lecture in Northeastern University. Her research interests are in the areas of singular systems.