

The effect of roadway capacity expansion on facility sitting

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Abstract: Generally, the transportation planning problem involves with facility sitting, roadway selection, distribution route planning and some related problems, which are optimized based on given network topology. Recently, some of researches noticed that by changing the network topology, which corresponding to adding a decision whether new roadway construction between node-pairs in the given transportation network, is often more cost-effective than only sitting new facilities in given networks. Motivated by aforementioned topology changing result, this paper not only consider about new roadway construction, but also merge the roadway capacity expansion decision into transportation network design problem. The models generated from an integrated model for transportation network design and uncapacitated facility location problem. Then we merge the roadway capacity expansion part and facility capacity decision part into the original model. Mixed integrated programming and network transform mechanism for such models are proposed. Lastly, we measure the effect of roadway capacity expansion on facility sitting. The numerical testing with random generated data shows the feasible and effective of models. In conclusion part, the application in real case of proposed models and algorithms are identified.

Keywords: facility location, network design, roadway expansion, roadway construction, network flow

1 Introduction

A general facility location problem (FLP) is giving support to such questions: (a) which facility should be selected? (b) what is the service(distribution) routing so as to minimize the total costs? Drezner summarized the FLP as a problem involves a set of spatially distributed customers and a set of facilities to serve customer demands[1]. However, in realistic case, researchers and practitioners always encountered such a problem: in the case of transportation network planning, when making the facility locating decision, how to advisably selecting links for capacity expansions and construction synchronously in transportation networks[2][3][4][5]. Melkote and Daskin proposed an integrated model of facility location and transportation network design problem. The models and algorithms could solve the facility location and roadway construction synchronously but ignored the roadway capacity expansion part[6]. Some other scholars have also carried out relevance research to the problem Melkote and Daskin proposed, most of them focus on the algorithm improvement[7].

In this paper, based on comparison with Melkote and Daskin's study, we aim to show the effect of roadway capacity expansion on facility sitting. Since such problem is also belongs to TND category[8], we formal a improved TND model(ITND) and its corresponding super-network model, which can help us to make following decisions:

- (a) The location and design capacity decision of facilities.
- (b) The construction and expansion decision of existed or potential roadway in a transportation network.
- (c) Distribution routing decision.

The rest of this paper is organized as follows. The following section consisted with two parts, which are models for classic facility location problem and TND problem, and the ITND model for the TND problem with consideration of roadway capacity expansion. In Section 3, we give the supernetwork transform mechanism to the more realistic problem and employ the mix integrated programming to solve the transferred problem. The last section gives a conclusion of this paper, pointed out the insufficient part and future research direction.

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2 Model Formulation

2.1 Classic Facility Location and network design problem

First we introduce a classic example which given by Daskin in 1993[9]. As shown in Fig. 2.1, this problem involved with A to F six-node, each nodes stands for a candidate location place, and the links between each pair of nodes stands for the existed roadway, the dashed links stands for the roadway that do not currently exist, the related data are shown in the picture, this type of situation occurs in sparsely populated areas or in developing countries[10]. The six-node problem was first provided by Daskin for pure facility location problem, and been employed in various type of facility location problems, such as [11]. Melkote and Daskin has formulated a model for solving the integrated problem of facility location and network design, they also employed this six-node example to analysis if the model and algorithm they proposed is efficiency and effective[6].

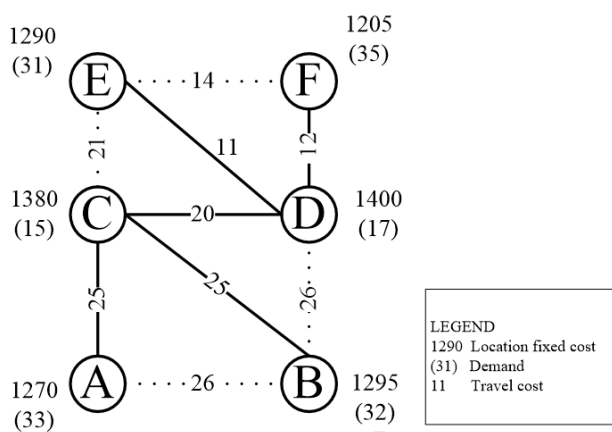


Fig. 1 A six-node network design problem

2.1.1 Pure facility location problem

The objective of this problem is to locate 2 facilities in the given network with minimum facility construction costs and distribution costs. The basic assumptions are: (1) Each node represents a demand point. (2)Facilities may only be located on the nodes of the network. (3) Only one facility may be located per node. (4) The network is a customer-to-server system, in which the demands themselves travel to the facilities to be served. (5) All travel costs are symmetric.

Daskin formulated the pure facility location problem as a classic UFLP (Uncapacitated fixed charge network

design) model, and solve the six-node problem with integer programming. The optimal solution is: the facilities are located at node C and F, facility which located at C serve the demand node A and B, facility which located at F serve the demand node D and E, the optimal total cost is 5127 units.

2.1.2 Integrated model of facility location and transportation network design

Melkote and Daskin formulated an integrated model of facility location and transportation network design on the basis of classic UFLP model.

The input parameters and data for the following models are:

- S Set of nodes
- L Set of undirected candidate links
- d_i demand at node i
- c_{ij} travel cost per unit flow on link (i, j)
- o_i fixed cost of constructing a facility at node i
- e_{ij} cost of constructing link (i, j)
- θ_{ij} unit cost of constructing a facility for operating the commodity
- W_i demand served by a facility at node i

$$z_i = \begin{cases} 1 & \text{if a facility is located at node } i \\ 0 & \text{otherwise} \end{cases}$$

$$x_{ij} = \begin{cases} 1 & \text{if link is constructed or utilized, where } i < j \\ 0 & \text{otherwise} \end{cases}$$

q_{ij}, q_{ji} flow of demands on link (i, j) in the i to j and j to i directions, respectively

To formulate the model as a pure network design problem, we should transform the actual flow into unit flow. With the definition of a commodity type set $\xi, \xi = \{1, 2, \dots, \xi\}$, we extent the network nodes set S to $S = (S, \xi)$. Then $c_{ij}^\xi = c_{ij}d_\xi$, and $q_{ij}^\xi = \frac{q_{ij}}{d_\xi}$. The model can write as:

$$\text{minimize } \sum_{i=1}^i \sum_{j=1}^j c_{ij}^\xi q_{ij}^\xi + \sum_{i=1}^i \sum_{j=1}^j c_{ij}^\xi x_{ij} + \sum_{i=1}^i \sum_{j=1}^j e_{ij} x_{ij} + \sum_{i=1}^i o_i z_i \tag{1}$$

$$\text{s.t. } z_i + \sum_{j=1}^j x_{ij} = 1, \forall j \in J \tag{2}$$

$$x_{\xi i} + \sum_{j=1}^j q_{ji}^\xi = \sum_{j=1}^j q_{ij}^\xi + W_i^\xi. \tag{3}$$

$$\sum_{j=1}^j q_{ji}^\xi = \sum_{j=1}^j q_{ij}^\xi + W_i^\xi \tag{4}$$

$$z_\xi + \sum_{i=1}^i W_i^\xi = 1 \tag{5}$$

$$q_{ij}^{\xi} \leq x_{ij} \tag{6}$$

$$W_i^{\xi} \leq z_i \tag{7}$$

$$x_{ij} + x_{ji} \leq 1 \tag{8}$$

$$W_i^{\xi} \geq 0, q_{ij}^{\xi} \in \{0, 1\}, z_i \in \{0, 1\}, x_{ij} \in \{0, 1\} \tag{9}$$

The objective minimizes the sum of facility location, transportation and link construction costs. Other details of this models are described by Melkote and Dakin. The optimal solution of this problem is: when the unit construction cost $u \in [0, 17.36]$, the facilities located at node A and F, construction the roadways between node E and F, and between node A and B. The facility F serve the demand node D and E, facility A serve the demand node B and C, the total cost is: $4320 + 40u$. And when $u \in [17.36, 21.69]$, the facility located at node A and D, construction the roadways between node D and B, facility A serve itself, and facility D serve all of the other demand nodes (except A), the total cost is: $4563 + 26u$.

It's obviously that when $u < 21.69$, the optimal total cost of the problem considering about transportation network design is less than the total cost of pure facility location problem. The proposed model can solve simple real-life religion planning problem, Melkote tests the model for the roadway networks and social service centers location planning of the State of Gujarat in western India. The optimal results support the government achieves the planning objective with minimum budget.

Although the model proposed by Melkote and Daskin has been proved its efficiency and effective for some real case planning problem, for supply chain network, the facilities involves are raw material plants or warehouse, manufacturing plants/warehouse and distribution center/retailers. We should take into account of the capacity of the supply chain network facilities, such as storage capacity, inventory capacity, and manufacturing capacity.

In our problem, the model not only solve the facility location and new roadway construction decision, but also help to make the decision of if we should expansion the existed roadway traffic capacity.

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help to make the decision of if we should expansion the existed roadway traffic capacity.

2.2 Improved model for integrated facility location, transportation network design problem

After above analysis, this section provide a model to solve the integrated facility location and transportation network design problem with consideration of both capacities of facilities and roadway construction or expansion.

2.2.1 New six-node problem

In order to show the difference between our model and proposed models, we improve the classic six-node problem as a new six-node problem. We describe the new six-node problem as:

(a) The decisions we should make in new six-node problem are: (1). where should we locate the facilities? (2). which roadways in the six-node network should be constructed? (3). which roadways in the six-node network should be expanded? (4). what amount should be the manufacturing capacity of each facility?

(b) What we should take into account are: (1). Minimize the total costs. (2). The capacity of facilities should satisfy the total demand of customers. (3). In the network planning, it is allowed that the roadways capacity less than the roadways flow quantities, which stands for that in this case the capacity of existed roadway should be expanded.

(c) Some assumptions of the network: (1). Each node in the origin network are both supply nodes and demand nodes. (2). We consider the total roadway traffic capacity between two demand nodes as the capacity of the corresponding links. (3). The flow of each link is the actual cargo transportation quantities between two nodes in a certain period. (4). The capacity of nodes: For the nodes which stand for the manufacturers, the capacity is the quantities of the plants production in a certain period. For which stand for the distribution centers or retailers, the capacity is the turnover quantities in a certain period. (5). The costs come with the links: transport cost, roadway construction cost and roadway expansion cost. Melkote has analyzed the relationship of roadway construction cost and transportation cost, and found the roadway construction cost is always directly proportional to the transportation cost. (6). The costs come with the nodes: the costs are facility fixed construction costs, and the variables operation costs.

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2.2.2 The supernetwork model

To formulate and solve an integrated model for new six-node problem and similarity problem, we add several supernodes and superlinks for facility location decision, roadway construction decision and roadway expansion decision.

(a) For all of the candidates location nodes, add a link from the origin nodes to the supernode, we label the node as node K; and label such superlinks set as $\bar{\bar{L}}$.

(b) For all of the candidates construction roadway links, although there is no physical roadway existed between the node-pairs, give superlinks of such node-pairs, we label such links set as \bar{L} .

(c) For all of the candidates expansion roadway links, first we create a super node to the nodes who adjacent to candidate expansion links, then add a superlink between each corresponding node-pairs, we label such links set as \bar{L} , add a superlink between each node and its corresponding supernode, these links are also grouped into set $\bar{\bar{L}}$.

Now we deal with the capacity, flow and relevant costs of the supernode and superlinks:

(a) The capacity of node K can be given as the total demand quantities of the entire network. The costs of such node can be set as 0.

(b) Since the actual meaning of capacity and flow with each superlinks in set \bar{L} which connected with node K are the capacity and turnover quantities of the

corresponding candidate location nodes. The costs involved with each superlinks are the costs come with the corresponding candidate location nodes which were already stated early in this paper.

(c) The capacity of links in set \bar{L} is the value that total network demand minus capacity of corresponding existed roadway link; for which in set $\bar{\bar{L}}$ is the value that total network demand minus total existed network capacities, as well as which in set $\bar{\bar{L}}$ corresponding to roadway expansion decision.

(d) Based on the proposed research, the construction costs for links in set \bar{L} is 0. Let represents the direct proportion coefficient of transportation cost and construction cost. The unit construction costs for links in set \bar{L} is $uc_{ij}, u > 0$. And let v_{ij} stands for the unit expansion cost, when the expected flow Eq is large than the existed roadway capacity q , we should expand the existed roadway, the expansion costs will be $(Eq - q)v_{ij}$.

Figure 2.2 shows the supernetwork for new six node problem. For a supernetwork model as the proposed analysis, we formulate the mathematic model as follows.

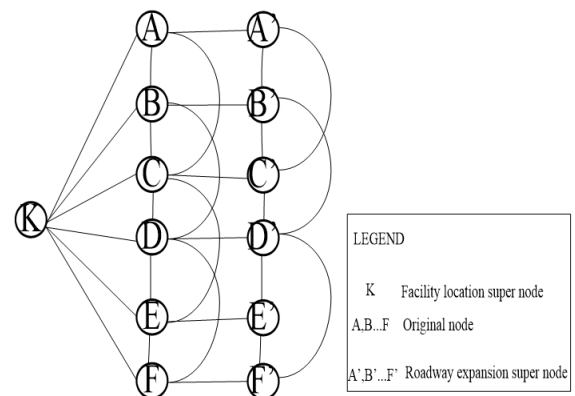


Fig. 2 Supernetwork for solving six-node problem

Since the new six-node problem is a “supply nodes also are demand nodes” network, we treat the flow conversation of such network as:

(a) The origin nodes of the flow:

$$q_i = \sum_{j=1}^j q_{ij}, i, j \in N, i \neq j, i \neq k.$$

(b) The transfer nodes:

$$\sum_{i=1}^i q_{ij} - W_i = \sum_{j=1}^j q_{ij}, i, j \in N, i \neq j, i \neq k.$$

(c) The destination node of the flow: $q_s = \sum_{i=1}^i q_{is}, i, k \in N$.

The capacity constraints of nodes and links are:

(a) Nodes: for the normal nodes, the capacity of the facility should be not less than the actual turnover

quantities, we may write: $W_i \geq \sum_{j=1}^j q_{ij}$. For the supernode K, its capacity should equals to the total demand of the network: $W_k = \sum_{i=1}^i \sum_{j=1}^j q_{ij}$.

(b) Links: For $(i, j) \in \bar{L}$, let f_{ij} stands for the existed capacity of each links, if the existed capacity of each links is not less than the actual flow, as $f_{ij} \geq q_{ij}$, the only cost involved in is transportation cost. Otherwise, the costs involved are transportation cost and expansion cost, we may write the expansion cost as $(f_{ij} - q_{ij})v_{ij}$. But for $(i, j) \in \bar{L}$, if $q_{ij} \geq 0$, the costs involved are construction cost and transportation cost: $(1 + u)c_{ij}$. And for the superlinks $(i, k) \in \bar{\bar{L}}$, the capacity constraint is: $W_i = f_{ik}$. The costs which involved is variables operation costs: $c_{ik} = \theta_i$.

In this improved model, we introduce a new variable Δq_{ij} to be the decision variable of if we expansion the roadways between two nodes.

Summarizing our discussion, the supernetwork model can be formulated as:

$$\begin{aligned} &\text{minimize } \sum_{i,j \in \bar{L}} q_{ij}c_{ij} + \sum_{i,j \in \bar{L}} (1+u)q_{ij}c_{ij} \\ &+ \sum_{i,k \in \bar{\bar{L}}} q_{ik}\theta_i + \sum_{i,j \in \bar{L}} \Delta q_{ij}v_{ij} + \sum_{i,j \in \bar{L}} o_i z_i \\ &\text{s.t. } q_i = \sum_{j=1}^j q_{ij}, i, j \in S, i \neq j, i \neq k \end{aligned} \quad (10)$$

$$\sum_{i=1}^i q_{ij} - W_i = \sum_{j=1}^j q_{ij}, i, j \in S, i \neq j, i \neq k \quad (11)$$

$$q_s = \sum_{i=1}^i q_{is}, i, k \in S \quad (12)$$

$$W_i \geq \sum_{j=1}^j q_{ij}, i, j \in S, i \neq j, i \neq k \quad (13)$$

$$W_k = \sum_{i=1}^i \sum_{j=1}^j q_{ij}, i, j \in S, i \neq j, i \neq k \quad (14)$$

$$W_i z_i \geq \sum_{i=1}^i \sum_{j=1}^j q_{ij}, i, j \in S, i \neq j, i \neq k \quad (15)$$

$$W_i = f_{ik}, i, j \in S, i \neq j, i \neq k \quad (16)$$

$$q_{ij} - f_{ij} \leq \Delta q_{ij}, i, j \in S, i \neq j, i \neq k \quad (17)$$

$$\Delta q_{ij} \geq 0, q_{ij} \geq 0, W_i \geq 0, \forall i, j \in S \quad (18)$$

$$z_i = (1, 0), y_{ij} = (1, 0) \quad (19)$$

The objective function minimizes the sum of transportation, roadways construction, facility location

and future operating, and roadway expansion costs. The Eq. (10)-(12) are the conservation of flow equation to the origin nodes, transfer nodes and destination node respectively, stating that the inbound flow to a node must equal the outbound flow from the node. The inbound flow consists of the total inbound demand plus the demand at the node, and the outbound flow is the total outbound demand plus the demand served at the node. Constraints (13) and (14) state the capacity constraints of normal nodes and supernode. Similarly, (15) forces the total designed facility capacity should satisfy the total network demand. Eq. (16) requires that the flow in the superlinks equal to the corresponding facility capacity. Constraint (17) keeps the new planning roadway capacity may not exceed the summary of roadway expansion capacity and its existed capacity. (18) and (19) are standard nonnegative and integrated constraints.

3 Numerical Experiment

So as to test and show the advantage to realistic cases of models we proposed in this paper. First, we start with giving some reasonable random data to the new six node problem we described in section 2.2. The random generation range is shown in Table 1.

Table 1 Test Data generation range

Test Data	Generation range	Unit
Customer demand	100-10,000	Item/period
Facility operation cost	60-20	Dollar/item
Existed roadway capacity	80-1,000	Item/period
Travel cost	0.8-2.2	Dollar/item
Facility construction cost	3,000-10,000	Dollar/item
Coefficient of roadway construction	2-22	-
Roadway expansion cost	3-20	Dollar/item
Numbers of candidate construction and expansion roadway	1-9	-

We generate 25 groups of data according to the range provided in Table 1, test the new six-node problem in Matlab 2010(a) by applied with improved intprog.m function [12]. The solving time of each group of data is conducted in less than one minute. The relation among roadway expansion cost, facility sitting numbers and fixed costs are shown in Fig. 3.

As described In Fig. 3., when the total roadway expansion cost increase, the facility sitting number decreased. When roadway capacity expansion satisfied to the total network customer demand, one facility may sufficient to serve all of the demand nodes in the network.

For the facility location fixed cost, it also tortuous decreased with the increase of roadway capacity expansion cost. It could be observed that when the roadway capacity expansion cost in certain range, the increase of roadway capacity expansion could bring to reduction of total network planning cost.

4 Conclusion and future directions

This paper formulated an integrated model for capacitated facility location and transportation network design problem with consideration of roadway capacity expansion. The model foundation is super network transform mechanism for original network. We applied the mixed integrated programming to solve such problem and show when the road capacity expansion cost expand, the total network cost could be reduced at certain range. Moreover, the proposed model for small size problem could be solved in small amount of computer time. With perspective of practical part, the model is more realistic to area transportation planning on account of most transportation planning problem are re-planning problem, which is planning among existed transportation network, has to deal with not only facility location, new roadway construction and existed roadway utilization problem but also face with existed roadway expansion decision. In the future research, we will test the sensitivity of each parameter of model. Some extension of such model could be conducted, such as with consideration of travel time constraints, uncertainty of customer demand, more efficient algorithms development.

Acknowledgement

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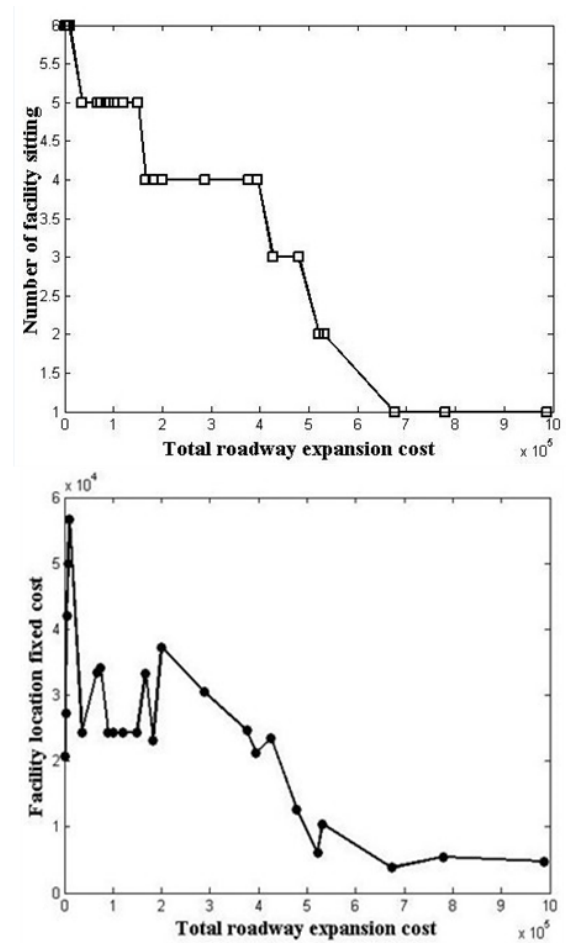


Fig. 3 The relationship of costs

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