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Guaranteed Cost Control for Nonlinear Singular System with Time Delays Using T-S Fuzzy Model

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Abstract: Singular system is a natural representation of dynamical systems. Guaranteed cost fuzzy control problem for a class of nonlinear singular system is addressed. The nonlinear singular system contains time-varying and norm-bounded uncertainties simultaneously in the matrices of states, delayed states and control inputs. Nonlinear singular system is formulated in the framework of Takagi-Sugeno (T-S) fuzzy system. Parallel-distributed compensation (PDC) scheme is equipped to design T-S fuzzy controller. Sufficient conditions are stated to guarantee both the stabilization and close-loop system's performance requirements. The conditions are obtained via linear matrix inequalities (LMIs) techniques, which can be solved through convex optimization method efficiently. Finally, numerical example demonstrates the usage of the proposed results.

Keywords: Nonlinear singular system, guaranteed cost control, Takagi-Sugeno (T-S) fuzzy system, linear matrix inequality (LMI), time-delay system

1 Introduction

The problem of stabilizing singular systems has been widely studied during past decades because of its practical interest [1,2]. The singular system is also referred to as singular system, implicit system, generalized state-space system, differential-algebraic system and semi-state system [1], and has a tighter representation for a wider class of systems for representing real independent parametric perturbations in comparison with traditional state-space model. On the other hand, dynamical systems with time delays constitute the basic mathematical models of real phenomena, and are very common in various industrial fields, for instance, in chemical processes, communication network, transportation systems, environmental systems, and power systems. And, time delays also appear in actuator and state measurements. Since time delays frequently cause serious deterioration of the performance and even stability of the system, various topics has been addressed over the last decades [3,4,5,6]. Due to the difficulties of constructing Lyapunov function and the complexity of the existence and uniqueness of the solution, there still remain some difficulties in tackling the nonlinear singular systems with time delays.

On the other hand, fuzzy logic approach [7,8] has proven to be an efficient method to represent complex nonlinear systems by fuzzy sets and fuzzy reasoning. The rule-based structure of fuzzy controller allows the designer to implement a complex controller design within an intuitively straightforward framework. Among various fuzzy modeling methods, Takagi-Sugeno (T-S) fuzzy model [7] is one of the most popular frameworks. It is based on a fuzzy partition of input space. In each fuzzy subspace, a linear input-output relation is formed. The output of fuzzy reasoning is given by the aggregation of the values inferred by some implications that are applied to an input. Generally speaking, there are two kinds of fuzzy logic controller. One is model-free fuzzy controller [8], and the other is model-based fuzzy controller. The former depends on heuristic knowledge from experts, and is featured by difficulties in guaranteeing the stability and control performance of the closed-loop system. Therefore, the latter has attracted a lot of attention from researchers during the last decade [9,10,11].

T-S fuzzy logic controller design using parallel-distributed compensation (PDC) scheme had been proposed and developed in [9]. Fuzzy model-based controller can combine the merits of both fuzzy controller and conventional linear theory, and furthermore guarantee

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stability in the sense of Lyapunov and control performance theoretically. Moreover, linear matrix inequality (LMI) techniques [12] also make model-based fuzzy controller design more convenient. However, when the controller is design for a real plant, it is also desirable to design a controller that not only makes the closed-loop system stable but also guarantees an adequate level of performance [13]. There exists voluminous literature on the subject of stabilization of linear singular system with time delays [2,3,6]. Especially, the robust stability analysis of uncertain discrete-time singular fuzzy systems was discussed in [10]. However, guaranteed cost controller for nonlinear singular system with time delays is still an open problem for the researchers.

This paper deals with guaranteed cost fuzzy controller design for a class of nonlinear singular fuzzy system with time-delay. The uncertainty is assumed to be norm-bounded and appears in the state matrices of current states, delayed states as well as input matrix. First, the nonlinear singular system is described by T-S fuzzy model. The performance index considered in the paper is an integral quadratic cost function as the regulator problem. Then, the sufficient conditions for guaranteed cost fuzzy controller are presented through PDC scheme. And the conditions are reduced to a set of LMIs, which can nowadays resort to some popular commercial software. Finally, numerical example is given to illustrate the effectiveness of the controller design.

This paper is organized as follows. T-S fuzzy singular system with time delays is constructed in Section 2. Some use lemmas are presented in Section 3. Section 4 deals with guaranteed cost fuzzy controller. Section 5 illustrates the results on a numerical example. Finally, in Section 6, concluding remarks end the paper.

2 Problem Statement

Recent studies in [9] had shown that fuzzy model is a universal approximator of any smooth nonlinear systems having a first order that is differentiable. The appeal of a linear T-S model is that it renders itself naturally to Lyapunov based system analysis and design techniques. The following T-S fuzzy singular system is constructed to approximate the nonlinear singular with time delay and uncertainties.

$$\begin{aligned}
&\text{Plant Rule } i: \\
&\text{IF } z_1(t) \text{ is } M_{i1} \text{ and}, \cdots, \text{ and } z_p \text{ is } M_{ip}, \\
&\text{THEN } E\dot{x}(t) = (A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di}) \\
&\qquad\qquad \times x(t-d) + (B_i + \Delta B_i)u(t) \\
&\qquad x(t) = \varphi(t), t \in [-d, 0], \\
&\qquad i = 1, 2, \cdots, r,
\end{aligned} \tag{1}$$

where $z(t) = \{z_1(t), z_2(t), \cdots, z_q(t)\}$ denote the variables of premise part, A_i, $A_{di} \in \Re^{n\times n}$, $B_i \in \Re^{n\times m}$, $x(t) = [x_1(t)\ x_2(t)\ \cdots\ x_n(t)]^{\mathrm{T}}$ denotes state vector, $u(t)$ denotes control input vector, and M_{il} denotes fuzzy sets, and r denotes the number of IF-THEN rules. E is singular matrix with $\mathrm{rank}(E) = q \le n$. $d > 0$ is time delay, φ are continuous vector-valued initial functions. ΔA_i, $\Delta A_{di} \in \Re^{n\times n}$, $\Delta B_i \in \Re^{n\times m}$ represent the system's uncertainty matrices and satisfy Assumption 1.

Assumption 1*Uncertainty matrices ΔA_i, ΔB_i and ΔA_{di} are norm-bounded, and have the following structures*

$$\left[\,\Delta A_i\ \Delta B_i\ \Delta A_{di}\,\right] = D_i F_i(t) \left[\,E_{1i}\ E_{2i}\ E_{di}\,\right], \tag{2}$$

where D_i, E_{i1} and E_{i2} are constant real matrices of appropriate dimensions, and $F_i(t) \in \Re^{i\times j}$ is unknown matrix-valued functions with Lebesgue-measurable elements, may be time-varying and satisfies

$$F_i^{\mathrm{T}}(t)F_i(t) \le I, \tag{3}$$

where I is the identity matrix with appropriate dimensions. Uncertainties ΔA_i, ΔB_i and ΔA_{di} are said to be admissible if both (2) and (3) hold. This form has been widely used to deal with time-varying uncertainties.

By using the fuzzy inference method with a singleton fuzzifier, product inference, and center average defuzzifiers, the final output of T-S fuzzy model is obtained as

$$\begin{aligned} E\dot{x}(t) = \sum_{i=1}^{r} \mu_i(z(t))[&(A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di}) \\ &\times x(t-d) + (B_i + \Delta B_i)u(t)] \end{aligned}, \tag{4}$$

where

$$\mu_i(z(t)) = \omega_i(z(t)) \Big/ \sum_{i=1}^{r} \omega_i(z(t)),$$

$$\omega_i(z(t)) = \prod_{j=1}^{p} M_{ij}(z(t)),$$

and $M_{ij}(z(t))$ denotes the degree of membership of $z(t)$ on M_{ij}. The degree of membership satisfies

$$\sum_{i=1}^{r} \omega_i(z(t)) > 0,\ \ \omega_i(z(t)) \ge 0,\ \ i = 1,\ 2,\ \cdots,\ r.$$

Note that for all t, there exists

$$\sum_{i=1}^{r} \mu_i(z(t)) = 1,\ \ \mu_i(z(t)) \ge 0,\ \ i = 1,\ 2,\ \cdots,\ r,$$

where $\mu_i(z(t))$ can be taken as the weights of normalized IF-THEN rules.

As for PDC scheme, fuzzy controller and fuzzy model (4) possess the same premises. Then, if we suppose that all the system's states are observable, the i-th controller rule can be expressed by

$$\begin{aligned}&\text{Controller Rule } i:\\&\text{IF } z_1(t) \text{ is } M_{i1} \text{ and}, \cdots, \text{and } z_p \text{ is } M_{ip},\\&\text{THEN } u(t)=K_i x(t),\ i=1,\ 2,\ \cdots,\ r,\end{aligned}\tag{5}$$

where $u(t)$ is the local control, and K_i is the local feedback gains.

At the consequent part, fuzzy control rules have linear state feedback gain. It has been proved that the controller using the PDC scheme is an approximator for any nonlinear state feedback controller [9]. The overall fuzzy controller can be represented as follows

$$u(t)=\sum_{i=1}^{r}\mu_i(z(t))K_i x(t).\tag{6}$$

Therefore, the fuzzy controller is to design local feedback gain K_is. Then, the combinations of (4) and (6) results in the overall closed-loop fuzzy system

$$\begin{aligned}E\dot{x}(t)=&\sum_{i=1}^{r}\mu_i(z(t))\{[(A_i+\Delta A_i)+(B_i+\Delta B_i)K_i]\\&\times x(t)+(A_{di}+\Delta A_{di})x(t-d)\},\\x(t)=&\varphi(t),\ t\in[-d,\ 0].\end{aligned}\tag{7}$$

In the following, we introduce some definitions and useful properties for the system (7).

Definition 1. *A pencil $sE-\sum_{i=1}^{r}\mu_i(z(t))A_i$ (or pair (E - $\sum_{i=1}^{r}\mu_i(z(t))A_i$) is regular, if $\det(sE-\sum_{i=1}^{r}\mu_i(z(t))A_i)$ is not identically zero.*

Fuzzy singular system (7) has no impulsive mode (or impulse free) if and only if $\operatorname{rank}(E) = \operatorname{degdet}(sE - \sum_{i=1}^{r}\mu_i(z(t))A_i)$.

The notations $\det(\cdot)$, $\operatorname{rank}(\cdot)$ and $\deg(\cdot)$ denote determinant, rank and degree of a matrix, respectively. The property of regularity guarantees the existence and uniqueness of solution for any specified initial condition. The condition of impulse free ensures that singular system has no infinite poles.

The performance cost function associated with the system (1) is given by

$$J=\int_0^{\infty}[x^T(t)Qx(t)+u^{\mathrm{T}}(t)Ru(t)]\mathrm{d}t,\tag{8}$$

where positive semi-definite $Q\in\Re^{n\times n}$ and positive definite $R\in\Re^{m\times m}$ are constant weight matrices.

The definition of guaranteed cost fuzzy controller for nonlinear singular system with time delays (1) is given as follows:

Definition 2. *Consider the delayed fuzzy singular system (1). The fuzzy controller (6) is called guaranteed cost fuzzy control law for the singular system with time delays (4), if there exist fuzzy controller (6) and a positive scalar J^* such that*

(i) fuzzy singular closed-loop system (7) is regular, impulse free and asymptotically stable,

(ii) the cost function (8) is bounded, i.e. J^*, where J^* is the upper bound of the performance cost function (8).

The objective of this paper is to give the sufficient conditions of guaranteed cost fuzzy controller (6) for the fuzzy system with time delays (1) based on LMI technique.

3 Mathematical Preliminaries

Before proceeding with the research on stability conditions for the closed-loop fuzzy singular system with time delays (7), some useful lemmas are introduced first.

Lemma 1. *[4] Let A,D,E,F and P be real matrices of appropriate dimensions with $P>0$ and F satisfying $F^{\mathrm{T}}F\le I$. Then we have the following*

(i) For any scalar $\varepsilon>0$,

$$DFE+(DFE)^{\mathrm{T}}\le\varepsilon^{-1}DD^{\mathrm{T}}+\varepsilon E^{\mathrm{T}}E.\tag{9}$$

(ii) For any scalar $\varepsilon>0$, such that $\varepsilon I-EPE^{\mathrm{T}}>0$

$$\begin{aligned}(A+DEF)P(A+DFE)^{\mathrm{T}}\le&APA^{\mathrm{T}}+\varepsilon DD^{\mathrm{T}}\\&+APE^{\mathrm{T}}(\varepsilon I-EPE^{\mathrm{T}})^{-1}EPA^{\mathrm{T}},\end{aligned}\tag{10}$$

or equivalently

$$\begin{aligned}(A+DEF)P(A+DFE)^{\mathrm{T}}\le&\varepsilon DD^{\mathrm{T}}\\&+A(P^{-1}-\varepsilon^{-1}E^{\mathrm{T}}E)^{-1}A^{\mathrm{T}},\end{aligned}\tag{11}$$

(iii) For any scalar $\varepsilon>0$ such that $I-\varepsilon E^{\mathrm{T}}E>0$,

$$\begin{aligned}(A+DEF)(A+DEF)^{\mathrm{T}}\le&\varepsilon^{-1}DD^{\mathrm{T}}\\&+A(I-\varepsilon E^{\mathrm{T}}E)^{-1}A^{\mathrm{T}},\end{aligned}\tag{12}$$

$$AD+(AD)^{\mathrm{T}}\le\varepsilon APA^{\mathrm{T}}+\varepsilon^{-1}D^{\mathrm{T}}P^{-1}D.\tag{13}$$

Lemma 2. *[12] Let $F:V\to S^n$ be an affine function which is partitioned according to*

$$F(x)=\begin{bmatrix}F_{11}(x) & F_{12}(x)\\ F_{21}(x) & F_{22}(x)\end{bmatrix},\tag{14}$$

where $F_{11}(x)$ is square. Then $F(x)>0$ if and only if

$$\begin{cases}F_{11}(x)>0,\\F_{22}(x)-F_{12}(x)F_{11}^{-1}(x)F_{21}(x)>0.\end{cases}\tag{15}$$

Lemma 3.[14] *if* $\int_{-d}^{0}\varphi(\tau)\varphi^{\mathrm{T}}(\tau)\mathrm{d}t = WW^{\mathrm{T}}$, *then for any* $X = X^{\mathrm{T}} > 0$, *the following formulae*

$$\int_{-d}^{0}\varphi(\tau)X\varphi^{\mathrm{T}}(\tau)\mathrm{d}t = \mathrm{tr}(W^{\mathrm{T}}XW), \tag{16}$$

holds, where $\mathrm{tr}(\cdot)$ *denote the trace of* $(\cdot)$.

4 Main Results

Now we are in a position to present the main results in this paper.

Theorem 1. *As for fuzzy system (1) and performance cost function (8), the fuzzy closed-loop system with time delays (7) is asymptotically stable and the controller (6) is state-feedback guaranteed cost fuzzy controller for the system (1), if there exist feedback gains* K_i*s and symmetric positive definite matrix* P, μ, *and* M *such that*

$$E^{\mathrm{T}}P = PE \geq 0, \tag{17}$$

$$\begin{aligned}&\tilde{\Phi}_{ii} + PA_{\mathrm{d}i}(R_1 - \varepsilon_{1i}^{-1}E_{\mathrm{d}i}^{\mathrm{T}}E_{\mathrm{d}i})^{-1}A_{\mathrm{d}i}^{\mathrm{T}}P + \varepsilon_{2i}^{-1}(E_{1i}\\ &+ E_{2i}K_i)^{\mathrm{T}}(E_{1i} + E_{2i}K_i) + Q + K_i^{\mathrm{T}}RK_i < 0,\end{aligned} \tag{18}$$

$$\begin{aligned}&\tilde{\Phi}_{ij} + \varepsilon_{2ij}^{-1}(E_{1i} + E_{2i}K_j)^{\mathrm{T}}(E_{1i} + E_{2i}K_j) + \varepsilon_{3ij}(E_{1j}\\ &+E_{2j}K_i)^{\mathrm{T}}(E_{1j} + E_{2j}K_i) + PA_{\mathrm{d}i}(R_1 - \varepsilon_{3i}^{-1}E_{\mathrm{d}i}^{\mathrm{T}}E_{\mathrm{d}i})^{-1}\\ &\times A_{\mathrm{d}i}^{\mathrm{T}}P + PA_{\mathrm{d}j}(R_1 - \varepsilon_{\mathrm{d}j}^{-1}E_{\mathrm{d}j}^{\mathrm{T}}E_{\mathrm{d}j})^{-1}A_{\mathrm{d}j}^{\mathrm{T}}P\\ &+ 2Q + \varepsilon_{1ij}K_i^{\mathrm{T}}K_i + \varepsilon_{1ij}^{-1}K_j^{\mathrm{T}}RRK_j < 0,\end{aligned} \tag{19}$$

$$\begin{bmatrix} -\mu & \varphi^{\mathrm{T}}(0) \\ \varphi(0) & -P^{-1}\end{bmatrix} < 0, \tag{20}$$

$$\begin{bmatrix} -M & W^{\mathrm{T}} \\ W & -R_1^{-1}\end{bmatrix} < 0, \tag{21}$$

where

$$\begin{aligned}\tilde{\Phi}_{ii} &= A_i^{\mathrm{T}}P + PA_i + K_i^{\mathrm{T}}B_i^{\mathrm{T}}P + PB_iK_i + R_1\\ &\quad + (\varepsilon_{1i}{+}\varepsilon_{2i}PD_iD_i^{\mathrm{T}}P),\\ \tilde{\Phi}_{ij} &= A_i^{\mathrm{T}}P + PA_i + K_j^{\mathrm{T}}B_i^{\mathrm{T}}P + PB_iK_j + A_j^{\mathrm{T}}P\\ &\quad + PA_j + K_i^{\mathrm{T}}B_j^{\mathrm{T}}P + PB_jK_i + 2R_1 + (\varepsilon_{2ij}\\ &\quad + \varepsilon_{2i})PD_iD_i^{\mathrm{T}}P + (\varepsilon_{3ij} + \varepsilon_{3i})PD_jD_j^{\mathrm{T}}P,\\ &\int_{-d}^{0}\varphi^{\mathrm{T}}(\tau)\varphi(\tau)\mathrm{d}\tau = WW^{\mathrm{T}},\end{aligned}$$

where $1 \leq i < j \leq r$, $\varepsilon_{\alpha i}$ $(\alpha = 1,2,3)$ *and* $\varepsilon_{\alpha ij}$ *are arbitrary positive scalars, and * denotes the transposed element in the symmetric position.*

And also for any admissible uncertainties, the performance cost function (8) of fuzzy closed-loop system (7) satisfies

$$J \leq J^{*} = \min_{\mu,P,R_1,K_i,M}(\mu + \mathrm{tr}(M)). \tag{22}$$

Proof. Firstly, we define a Lyapunov functional candidate as follows

$$V(x(t)) = x^{\mathrm{T}}(t)Px(t) + \int_{t-d}^{t}x^{\mathrm{T}}(\tau)R_1x(\tau)\mathrm{d}\tau, \tag{23}$$

where P is a time-invariant, symmetric positive definite matrix. Then, by using the inequality (17), the time derivative of $V(x(t))$ is given by

$$\begin{aligned}\dot{V}(x(t)) &= \dot{x}^{\mathrm{T}}(t)EPx(t) + x^{\mathrm{T}}(t)EP\dot{x}(t) + [x^{\mathrm{T}}(s)R_1x(s)]_{t-d}^{t}\\ &= \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(z(t))\mu_j(z(t))\{x^{\mathrm{T}}(t)[P(A_i + \Delta A_i)\\ &\quad + P(B_i + \Delta B_i)K_j + (A_i^{\mathrm{T}} + \Delta A_i^{\mathrm{T}})P + K_j^{\mathrm{T}}(B^{\mathrm{T}}\\ &\quad + \Delta B_i^{\mathrm{T}})P]x(t) + x^{\mathrm{T}}(t)P(A_{\mathrm{d}i} + \Delta A_{\mathrm{d}i})x(t-d)\\ &\quad + x^{\mathrm{T}}(t-d)(A_{\mathrm{d}i}^{\mathrm{T}} + \Delta A_{\mathrm{d}i}^{\mathrm{T}})Px(t)\}x(t) + x^{\mathrm{T}}(t)R_1x(t)\\ &\quad - x^{\mathrm{T}}(t-d)R_1x(t-d).\end{aligned}$$

By use of Lemma 1, some manipulations will result in

$$\begin{aligned}\dot{V}(x(t)) &= \sum_{i=1}^{r}\mu_i^2(z(t))\{x^{\mathrm{T}}(t)[Q + K_i^{\mathrm{T}}RK_i + PA_i + P\Delta A_i\\ &\quad + PB_iK_i + A_i^{\mathrm{T}}P + \Delta A_i^{\mathrm{T}}P + K_i^{\mathrm{T}}B_i^{\mathrm{T}}P + K_i^{\mathrm{T}}\Delta B_i^{\mathrm{T}}P]x(t)\\ &\quad + x^{\mathrm{T}}(t)P(A_{\mathrm{d}i} + \Delta A_{\mathrm{d}i})x(t-d) + x^{\mathrm{T}}(t-d)(A_{\mathrm{d}i}^{\mathrm{T}} + \Delta A_{\mathrm{d}i}^{\mathrm{T}})\\ &\quad \times Px(t) - x^{\mathrm{T}}(t)Qx(t) - x^{\mathrm{T}}(t)K_i^{\mathrm{T}}RK_ix(t)\}\\ &\quad + \sum_{i<j}^{r}\mu_i(z(t))\mu_j(z(t))\{[x^{\mathrm{T}}(t)(2Q + K_i^{\mathrm{T}}RK_j + K_j^{\mathrm{T}}RK_i\\ &\quad + A_i^{\mathrm{T}}P + K_j^{\mathrm{T}}\Delta B_i^{\mathrm{T}}P + \Delta A_i^{\mathrm{T}}P + K_j^{\mathrm{T}}\Delta B_i^{\mathrm{T}}P + PA_i\\ &\quad + PB_iK_j + P\Delta A_i + P\Delta B_i^{\mathrm{T}}K_j + A_j^{\mathrm{T}}P + K_i^{\mathrm{T}}B_j^{\mathrm{T}}P\\ &\quad + \Delta A_j^{\mathrm{T}}P + K_i^{\mathrm{T}}\Delta B_j^{\mathrm{T}}P + PA_j + PB_jK_i + P\Delta A_j\\ &\quad + P\Delta B_jK_i - 2Q - K_i^{\mathrm{T}}RK_j - K_j^{\mathrm{T}}RK_i]x(t)\\ &\quad + x^{\mathrm{T}}(t)P(A_{\mathrm{d}i} + \Delta A_{\mathrm{d}i})x(t-d) + x^{\mathrm{T}}(t-d)(A_{\mathrm{d}i}^{\mathrm{T}}\\ &\quad + \Delta A_{\mathrm{d}i}^{\mathrm{T}})Px(t) + x^{\mathrm{T}}(t)P(A_{\mathrm{d}j} + \Delta A_{\mathrm{d}j})x(t-d)\\ &\quad + x^{T}(t-d)(A_{\mathrm{d}j}^{\mathrm{T}} + \Delta A_{\mathrm{d}j}^{\mathrm{T}})Px(t) + x^{\mathrm{T}}R_1x(t)\\ &\quad - x^{T}(t-d)R_1x(t-d)\}.\end{aligned}$$

Note that the following inequalities hold

$$\begin{aligned}&x^{\mathrm{T}}(t)Qx(t) + u^{\mathrm{T}}(t)Ru(t)\\ &\quad = \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i^2(z(t))\mu_j^2(z(t))[x^{\mathrm{T}}(t)Qx(t)\\ &\qquad + x^{\mathrm{T}}(t)K_i^{\mathrm{T}}RK_jx(t)]\\ &\quad = \sum_{i=1}^{r}\mu_i^2(z(t))[x^{\mathrm{T}}(t)Qx(t) + x^{\mathrm{T}}(t)K_i^{\mathrm{T}}RK_ix(t)]\end{aligned}$$

$$+\sum_{i<j}^{r}\mu_i(z(t))\mu_j(z(t))[x^{\mathrm{T}}(t)K_i^{\mathrm{T}}RK_jx(t)$$
$$+x^{\mathrm{T}}(t)K_j^{\mathrm{T}}RK_ix(t)+2x^{\mathrm{T}}(t)Qx(t)]$$
$$\leq\sum_{i=1}^{r}\mu_i^2(z(t))x^{\mathrm{T}}(t)(Q+K_i^{\mathrm{T}}RK_i)x(t)$$
$$+\sum_{i<j}^{r}\mu_i(z(t))\mu_j(z(t))x^{\mathrm{T}}(t)[2Q+\varepsilon_{1ij}K_i^{\mathrm{T}}K_i$$
$$+\varepsilon_{1ij}^{-1}K_j^{\mathrm{T}}RRK_j]x(t).$$

Then, from Lemma 1 and Assumption 1, we obtain

$$\dot V(x(t))\leq\sum_{i=1}^{r}\mu_i^2(z(t))\{[x^{\mathrm{T}}(t)Qx(t)+x^{\mathrm{T}}(t)K_i^{\mathrm{T}}RK_ix(t)$$
$$+x(t)(A^{\mathrm{T}}P+PB_iK_i+PA_i+K_i^{\mathrm{T}}B_i^{\mathrm{T}}P)x(t)+(\varepsilon_{1i}$$
$$+\varepsilon_{2i})x^{\mathrm{T}}(t)PD_iD_i^{\mathrm{T}}Px(t)+\varepsilon_{2i}^{-1}x^{\mathrm{T}}(t)(E_{1i}+E_{2i}K_i)^{\mathrm{T}}$$
$$\times(E_{1i}+E_{2i}K_i)x(t)+x^{\mathrm{T}}(t)PA_{\mathrm{d}i}(R_1-\varepsilon_i^{-1}E_{\mathrm{d}i}^{\mathrm{T}}E_{\mathrm{d}i})^{-1}$$
$$\times A_{\mathrm{d}i}^{\mathrm{T}}Px(t)-x^{\mathrm{T}}(t)R_1x(t)]-x^{\mathrm{T}}(t)K_i^{\mathrm{T}}RK_ix(t)$$
$$-x^{\mathrm{T}}(t)Qx(t)\}+\sum_{i<j}^{r}\mu_i(z(t))\mu_j(z(t))\{[2x^{\mathrm{T}}(t)Qx(t)$$
$$+\varepsilon_{1ij}x^{\mathrm{T}}(t)K_i^{\mathrm{T}}K_ix(t)+\varepsilon_{1ij}^{-1}x^{\mathrm{T}}(t)K_j^{\mathrm{T}}RRK_jx(t)$$
$$+x^{\mathrm{T}}(t)(A_i^{\mathrm{T}}P+PA_i+K_j^{\mathrm{T}}B_i^{\mathrm{T}}P+PB_iK_j+A_j^{\mathrm{T}}P$$
$$+PA_j+K_i^{\mathrm{T}}B_j^{\mathrm{T}}P+PB_jK_i)x(t)+(\varepsilon_{2ij}+\varepsilon_{3i})$$
$$\times x^{\mathrm{T}}(t)PD_iD_i^{\mathrm{T}}Px(t)+(\varepsilon_{3ij}+\varepsilon_{3i})x^{\mathrm{T}}(t)PD_jD_j^{\mathrm{T}}Px(t)$$
$$+\varepsilon_{2ij}^{-1}x^{\mathrm{T}}(t)(E_{1i}+E_{2i}K_j)^{\mathrm{T}}(E_{1i}+E_{2i}K_j)x(t)$$
$$+\varepsilon_{3ij}^{-1}x^{\mathrm{T}}(t)(E_{1j}+E_{2j}K_i)^{\mathrm{T}}(E_{1j}+E_{2j}K_i)x(t)$$
$$+x^{\mathrm{T}}(t)R_1x(t)]-2x^{\mathrm{T}}(t)Qx(t)-x^{\mathrm{T}}(t)K_j^{\mathrm{T}}RK_ix(t)$$
$$-x^{\mathrm{T}}(t)K_i^{\mathrm{T}}RK_jx(t)\}.$$

Because of the inequalities (18) and (19), the above formulae can be rewritten as follows

$$\dot V(x(t))=-x^{\mathrm{T}}(t)\left(Q+\left(\sum_{i=1}^{r}\mu_i(z(t))K_i\right)^{\mathrm{T}}\times R\left(\sum_{i=1}^{r}\mu_i(z(t))K_i\right)\right)x(t)<0. \tag{24}$$

Therefore, the fuzzy closed-loop singular system (7) is asymptotically stable. Then, integrating both sides of (24) from $t=0$ to $t=T$ yields

$$V(x(T))-V(x(0))<\int_0^T(-x^{\mathrm{T}}(t)Qx(t)$$
$$-\left(\sum_{i=1}^{r}\mu_i(z(t))(K_ix(t))^{\mathrm{T}}\right)R\left(\sum_{i=1}^{r}\mu_i(z(t))(K_ix(t))\right)\mathrm{d}t.$$

Considering that the closed-loop singular system (7) is asymptotically stable, we have $x(\infty)\to 0$ and

$$J=\int_0^T\left(x^{\mathrm{T}}(t)Qx(t)+u^{\mathrm{T}}(t)Ru(t)\right)\mathrm{d}t$$
$$\leq\varphi^{\mathrm{T}}(0)P\varphi(0)+\int_{-d}^{0}\varphi^{\mathrm{T}}(\tau)R_1\varphi(\tau)\mathrm{d}\tau=\bar J^*.$$

From Lemma 2, (20) is equivalent to

$$\varphi^T(0)P\varphi(0)<\mu. \tag{25}$$

By use of Lemma 3, we obtain

$$\int_{-d}^{0}\varphi^{\mathrm{T}}(\tau)R_1\varphi(\tau)\mathrm{d}\tau=\mathrm{tr}(W^{\mathrm{T}}R_1W). \tag{26}$$

Because (21) is equivalent to $W^TR_1W<M$, we get

$$\int_{-d}^{0}\varphi^{\mathrm{T}}(\tau)R_1\varphi(\tau)\mathrm{d}\tau=\mathrm{tr}(M). \tag{27}$$

By (25), (26) and (27), we obtain

$$\bar J^*\leq J^*=\min_{\mu,P,R_1,K_i,M}(\mu+\mathrm{tr}(M)). \tag{28}$$

According to Definition 2, (6) is guaranteed cost fuzzy controller for the singular system with time delays (4). Then, J^* is the corresponding upper bound of the performance cost function (8).

The search for the common matrix P and K_is nowadays can resort to some efficient numerical methods in terms of LMIs. However, the conditions are not jointly convex in K_is and P in Theorem 1. Hence, Theorem 2 is proposed, in which the LMIs are tractable.

Theorem 2.*The controller (6) is guaranteed cost fuzzy controller for the closed-loop T-S fuzzy singular system with time delays (7), if there exist matrices M_is, symmetric positive definite matrix N and U, μ and M such that the LMIs*

$$NE^{\mathrm{T}}=EN>0, \tag{29}$$

$$\begin{bmatrix}-\mu & \varphi^{\mathrm{T}}(0)\\ \varphi(0) & -N\end{bmatrix}<0, \tag{30}$$

$$\begin{bmatrix}-M & W^{\mathrm{T}} & I\\ W & 0 & 0\\ I & 0 & U\end{bmatrix}<0, \tag{31}$$

and (32), (33) hold, where "" denotes generically each of its symmetric blocks*

$$\Phi_{ii}=NA_i^{\mathrm{T}}+A_iN+M_i^{\mathrm{T}}B_i^{\mathrm{T}}+(\varepsilon_{1i}+\varepsilon_{2i})D_iD_i^{\mathrm{T}}+U,$$

$$\Phi_{ij}=NA_i^{\mathrm{T}}+A_iN+M_j^{\mathrm{T}}B_i^{\mathrm{T}}+A_jN+M_iB_j^{\mathrm{T}}+B_jM_i$$

$$\begin{bmatrix} \Phi_{ii} & * & * & * & * & * \\ E_{1i}N+E_{2i}M_i & -\varepsilon_{2i}I & * & * & * & * \\ NA_{di}^{\mathrm{T}} & 0 & -U & * & * & * \\ N & 0 & 0 & -Q^{-1} & * & * \\ M_i & 0 & 0 & 0 & -R^{-1} & * \\ 0 & 0 & E_{di}N & 0 & 0 & -\varepsilon_{1i}I \end{bmatrix} < 0, \quad (32)$$

$$\begin{bmatrix} \Phi_{ij} & * & * & * & * & * & * & * & * & * \\ E_{1i}N+E_{2i}M_j & -\varepsilon_{1ij}I & * & * & * & * & * & * & * & * \\ E_{1j}N+E_{2j}M_i & 0 & -\varepsilon_{3ij}I & * & * & * & * & * & * & * \\ NA_{di}^{\mathrm{T}} & 0 & 0 & -U & * & * & * & * & * & * \\ NA_{dj}^{\mathrm{T}} & 0 & 0 & 0 & -U & * & * & * & * & * \\ N & 0 & 0 & 0 & 0 & -1/2Q^{-1} & * & * & * & * \\ M_i & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{1ij}I & * & * & * \\ RM_j & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{1ij}^{-1}I & * & * \\ 0 & 0 & 0 & E_{di}N & 0 & 0 & 0 & 0 & -\varepsilon_{3i}I & * \\ 0 & 0 & 0 & 0 & E_{dj}N & 0 & 0 & 0 & 0 & -\varepsilon_{3j}I \end{bmatrix} < 0, \quad (33)$$

$$+(\varepsilon_{2ij}+\varepsilon_{2i})D_iD_i^{\mathrm{T}}+(\varepsilon_{3ij}+\varepsilon_{3j})D_jD_j^{\mathrm{T}}+2U.$$

Furthermore, feedback gain K_i and symmetric positive definite matrix P are obtained by

$$P=N^{-1},\ K_i=M_iN^{-1},\ R_1=N^{-1}UN^{-1}. \quad (34)$$

And for any admissible uncertainties, the performance cost function (8) of fuzzy closed-loop system (7) satisfies

$$J \le J^* = \min_{\mu,N,U,M_i,M}(\mu+\mathrm{tr}(M)). \quad (35)$$

*Proof.*By use of Schur complement in Lemma 2, we obtain (36) and (37). Multiply (36) and (37) with

$$\Pi_1=\mathrm{diag}(P^{-1},I,P^{-1},I,I,I), \quad (38)$$

and

$$\Pi_2=\mathrm{diag}(P^{-1},I,I,P^{-1},P^{-1},I,I,I,I,I), \quad (39)$$

both left and right side, respectively. Let $N=P^{-1}$, $M_i=K_iP^{-1}$ and $U=P^{-1}R_1P^{-1}$. Then, the LMIs (32) and (33) can be obtained. Similarly, multiplying (17) with P^{-1} both left and right side will result in LMI (29). Similarly, LMIs (30) and (31) can also be obtained. So far, the LMI (29)-(33) can be solved by convex optimization method.

5 Numerical Example

To demonstrate the effectiveness of our method, we consider nonlinear time-delay system approximated by the following IF-THEN fuzzy rules

Plant Rule 1: IF $x_1(t)$ is P,

THEN $E\dot{x}(t)=(A_1+\Delta A_1)x(t)+(A_{d1}+\Delta A_{d1})x(t-d)$

$+(B_1+\Delta B_1)u(t)$,

Plant Rule 2: IF $x_1(t)$ is N,

$E\dot{x}(t)=(A_2+\Delta A_2)x(t)+(A_{d2}+\Delta A_{d2})x(t-d)$

$+(B_2+\Delta B_2)u(t)$,

where the membership functions of ‘P’, ‘N’ are given as follows

$$M_1(x_1(t))=1-\frac{1}{1+\exp(-2x_1)}, \quad (40)$$

$$M_1(x_1(t))=1-M_1(x_1(t)). \quad (41)$$

Matrices $\Delta A_i(t), \Delta A_{di}(t)$ and $\Delta B_i(t)$ are assumed to have the form of (2). Then, the relevant matrices in T-S fuzzy model are given as follows

$$E=\begin{bmatrix}1&0\\0&0\end{bmatrix},\ A_1=\begin{bmatrix}0&1\\-1&-2\end{bmatrix},\ A_{d1}=\begin{bmatrix}0&0\\0.2&0.1\end{bmatrix},$$

$$B_1=\begin{bmatrix}0\\0.1\end{bmatrix},\ A_2=\begin{bmatrix}0&1\\-2&-2\end{bmatrix},\ A_{d2}=\begin{bmatrix}0&0\\0.1&0.5\end{bmatrix},$$

$$B_2=\begin{bmatrix}0\\0.1\end{bmatrix},\ D_1=\begin{bmatrix}0.1\\0.2\end{bmatrix},\ D_2=\begin{bmatrix}0.1\\0.5\end{bmatrix},$$

$$E_{11}^{\mathrm{T}}=E_{12}^{\mathrm{T}}=\begin{bmatrix}1\\0\end{bmatrix},\ E_{d1}=E_{d2}=\begin{bmatrix}0.1&0\end{bmatrix},$$

$$\varphi(t)=\begin{bmatrix}\exp(t+1)&0\end{bmatrix}^{\mathrm{T}},\ E_{21}=0.3,\ E_{22}=0.2,$$

$$F_1=F_2=\sin(t),\ d=1.$$

Choose the scalar coefficients $\varepsilon_{\alpha i}(\alpha=1,2,3)=\varepsilon_{\alpha ij}=1$. By using Matlab LMI Control Toolbox [15], the

$$
\begin{bmatrix}
\tilde{\Phi}_{ii} & * & * & * & * & * \\
E_{1i}+E_{2i}K_i & -\varepsilon_{2i}I & * & * & * & * \\
A_{di}^{\mathrm{T}} & 0 & -R_1 & * & * & * \\
I & 0 & 0 & -Q^{-1} & * & * \\
K_i & 0 & 0 & 0 & -R^{-1} & * \\
0 & 0 & E_{di} & 0 & 0 & -\varepsilon_{1i}I
\end{bmatrix} < 0, \tag{36}
$$

$$
\begin{bmatrix}
\tilde{\Phi}_{ij} & * & * & * & * & * & * & * & * & * \\
E_{1i}+E_{2i}K_j & -\varepsilon_{1ij}I & * & * & * & * & * & * & * & * \\
E_{1j}+E_{2j}K_i & 0 & -\varepsilon_{3ij}I & * & * & * & * & * & * & * \\
A_{di}^{\mathrm{T}}P & 0 & 0 & -R_1 & * & * & * & * & * & * \\
A_{dj}^{\mathrm{T}}P & 0 & 0 & 0 & -R_1 & * & * & * & * & * \\
I & 0 & 0 & 0 & 0 & -1/2Q^{-1} & * & * & * & * \\
K_i & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{1ij}I & * & * & * \\
RK_j & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{1ij}^{-1}I & * & * \\
0 & 0 & 0 & E_{di} & 0 & 0 & 0 & 0 & -\varepsilon_{3i}I & * \\
0 & 0 & 0 & 0 & E_{dj} & 0 & 0 & 0 & 0 & -\varepsilon_{3j}I
\end{bmatrix} < 0, \tag{37}
$$

positive definite matrices P, R_1 and feedback gain K_is can be obtained as follows

$$
P = \begin{bmatrix} 5.0910 & 2.4098 \\ 2.4098 & 2.2416 \end{bmatrix}, \; R = \begin{bmatrix} 1.1587 & 0.7717 \\ 0.7717 & 1.3059 \end{bmatrix},
$$

$$
K_1^{\mathrm{T}} = \begin{bmatrix} -0.4847 \\ -0.5798 \end{bmatrix}, \; K_2^{\mathrm{T}} = \begin{bmatrix} -1.0992 \\ -1.0867 \end{bmatrix}.
$$

Then, the corresponding upper bound of performance index function can be obtained as $J^* = 78.9363$.

6 Conclusion

In this paper, guaranteed cost fuzzy controller design has been addressed for a class of nonlinear singular system with time delays through the fuzzy interpolation of a series of linear systems. The controller is reduced to the solution of a set of LMIs, which make the design much more convenient. Furthermore, an example has shown that the effectiveness of the proposed T-S fuzzy logic controller.

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