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Adaptive Control Based on Incremental Hierarchical Sliding Mode for Overhead Crane Systems

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Abstract: Incremental hierarchical sliding mode control (IHSMC) methodology involves $2n-2$ sign switches of controller parameters for an underactuated system with n subsystems. Too many sign switches trouble parameter tuning. This paper presents an adaptive control design approach based on IHSMC methodology for overhead crane systems with 2 subsystems with only 1 sign switch. The system stability is proven by Barbalat's lemma and Lasalle's invariance principle in the sense of Lyapunov theory. Simulation results illustrate the feasibility of the presented method by transport control of overhead crane systems.

Keywords: adaptive control, sliding mode control, hierarchical structure, overhead cranes

1 Introduction

Overhead crane systems are usually employed to move materials horizontally in industries because they can move loads far beyond the normal capability of a human. But their performance may be constrained by the fact that their loads are free to swing with a pendulum-type motion. In practice, operating an overhead crane by manual is hard to resist the pendulum-type motion that is harmful for industry safety. Automation of operation is desirable because high positioning accuracy, small swing angle, and short transportation time are required [1]. As far as transport control of overhead crane systems is concerned, the objective is to transport the loads to the required position as fast and as accurately as possible without free swings. Many control approaches concerning the control problem have been reported in recent years, i.e., fuzzy control [1,4], adaptive control [3], feedback linearizing control [8], wave-based robust control [5], sliding mode control [2,6,7], *ect*. Other reports about this topic can be found in [13] and [14].

Incremental hierarchical sliding mode control (IHSMC) [2] is a methodology to solve control problems of a class of underactuated systems with n subsystems and 1 control input. For an underactuated system with n subsystems, the incremental sliding surfaces of IHSMC are with $2n-1$ layers. To ensure the stability of all the surfaces, signs of parameters of the sliding surfaces need to be switched except the first layer (theorem 1 in [2]). But too many sign switches may trouble parameter tuning. In nature, overhead crane systems belong to underactuated systems with 2 subsystems and 1 input. A crane system in [2] was utilized as a benchmark to verify the feasibility of IHSMC. But the signs of two sliding-surface parameters had to be switched for the stability of this system. In this paper, we investigate an adaptive control based on IHSMC for overhead crane systems with only 1 sign switch.

The remainder of this work is organized as follows. Section 2 describes the control design. The stability analysis is presented in section 3. Section 4 shows the simulation results. Conclusion is drawn in section 5 at last.

2 Adaptive Control Design Based on IHSMC for Overhead Crane Systems

2.1 Dynamic Model

Fig. 1 illustrates structure of an overhead crane system. This system consists of the trolley subsystem and the load

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subsystem. We assume there is on static friction, the rope is inflexible, the rope mass is ignored, and the load is regarded as a material particle. Using Lagrange's method, its motion equations can be derived as

$$(m+M)\ddot{x}+mL(\ddot{\theta}\cos\theta-\dot{\theta}^2\sin\theta)=f \tag{1}$$

$$\ddot{x}\cos\theta+L\ddot{\theta}+g\sin\theta=0 \tag{2}$$

here M is trolley mass, m is load mass, L is rope length, g is gravitational acceleration, θ is swing angle of the load with respect to the vertical line, x is trolley position with respect to the origin x, f is control force applied to the trolley.

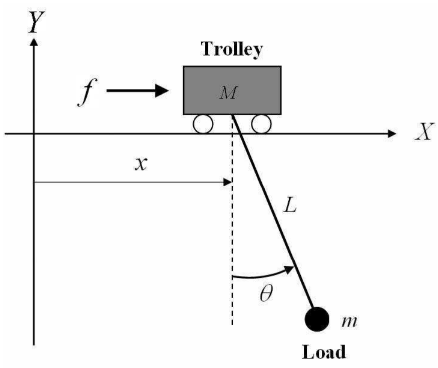

Fig. 1: Structure of an overhead crane system

The motion equations (1) and (2) can be transformed to the state space expression as

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=f_1(\mathbf{x})+b_1(\mathbf{x})\cdot u\\ \dot{x}_3=x_4\\ \dot{x}_4=f_2(\mathbf{x})+b_2(\mathbf{x})\cdot u\end{cases} \tag{3}$$

Here $\mathbf{x}=[x_1,\ x_2,\ x_3,\ x_4]^T$, $x_1=x$; $x_3=\theta$; x_2 is trolley velocity; x_4 is angular velocity of the load; $u=f$ is the control input; f_i and b_i $(i=1,2)$ can be written as

$$f_1=\frac{m\cdot L\cdot x_4^2\cdot\sin x_3+m\cdot g\sin x_3\cdot\cos x_3}{M+m\cdot\sin^2 x_3}$$

$$b_1=\frac{1}{M+m\cdot\sin^2 x_3}$$

$$f_2=-\frac{(m+M)\cdot g\cdot\sin x_3+m\cdot L\cdot x_4^2\cdot\sin x_3\cdot\cos x_3}{(M+m\cdot\sin^2 x_3)\cdot L}$$

$$b_2=-\frac{\cos x_3}{(M+m\cdot\sin^2 x_3)\cdot L}$$

2.2 IHSMC-Based Adaptive Control Design

By means of the methodology of IHSMC, the incremental sliding surfaces of this overhead crane system are designed as

$$\begin{aligned}s_1&=c_1x_1+x_2\\ s_2&=c_2x_3+s_1\\ s_3&=c_3x_4+s_2\end{aligned} \tag{4}$$

where c_1 is a positive constant, c_2 is constant, and c_3 is defined as a time-varying parameter. Due to the derivative relation between x_1 and x_2 in (3), we define c_1 is positive on the aspect of the system stability.

Based on the methodology of equivalent control of variable structure control [9], the SMC law usually includes two parts: switching control and equivalent control. Here we still adopt it and define the total control law u of the adaptive IHSMC as

$$u=u_{eq}+u_{sw} \tag{5}$$

where u_{eq} is the equivalent control and u_{sw} is the switching control.

In order to ensure (5) make the last layer sliding surface s_3 asymptotically stable, a Lyapunov function is defined as

$$V(t)=\frac{1}{2}s_3^2 \tag{6}$$

Differentiating V with respect to time t in (6) yields

$$\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}s_3}\frac{\mathrm{d}s_3}{\mathrm{d}t}=s_3\dot{s}_3=s_3\frac{\mathrm{d}(c_3x_4+s_2)}{\mathrm{d}t} \tag{7}$$

Since c_3 is a time-varying parameter, its adaptive law can be derived from (7) in the sense of Lyapunov. So we have

$$\frac{\mathrm{d}V}{\mathrm{d}t}=s_3(\dot{c}_3x_4+c_3\dot{x}_4+\dot{s}_2) \tag{8}$$

Substituting (3), (4), and (5) into (8) yields

$$\begin{aligned}\frac{\mathrm{d}V}{\mathrm{d}t}&=s_3[\dot{c}_3x_4+c_3(f_2+b_2u)+(c_2\dot{x}_3+\dot{s}_1)]\\ &=s_3[\dot{c}_3x_4+c_3f_2+c_2x_4+f_1+c_1x_2+(c_3b_2+b_1)u]\\ &=s_3[\dot{c}_3x_4+c_3f_2+c_2x_4+f_1+c_1x_2+\\ &\quad(c_3b_2+b_1)(u_{eq}+u_{sw})]\end{aligned} \tag{9}$$

In order to have the stability of the third layer sliding surface, let

$$\begin{cases}c_3f_2+c_2x_4+c_1x_2+(c_3b_2+b_1)u_{eq}=0\\ \kappa s_3+\eta sgn(s_3)+(c_3b_2+b_1)u_{sw}=0\\ \dot{c}_3x_4+f_1=0\end{cases} \tag{10}$$

Here κ and η are positive constants, $sgn(\cdot)$ is sign function. Substituting (10) into (9), we have

$$\frac{\mathrm{d}V}{\mathrm{d}t}=-\kappa s_3^2-\eta|s_3|\le 0 \tag{11}$$

which means the sliding motion of the third layer sliding surface s_3 occurs at t_f. From (10), the equivalent control law, the switching control law and the adaptive law of c_3 of the presented adaptive IHSMC are gotten as

$$\begin{cases} u_{eq} = \dfrac{-c_3 f_2 - c_2 x_4 - c_1 x_2}{c_3 b_2 + b_1} \\ u_{sw} = \dfrac{-k s_3 - \eta sgn(s_3)}{c_3 b_2 + b_1} \\ \dot{c}_3 = -\dfrac{f_1 x_4}{||x_4||^2 + \delta} \end{cases} \tag{12}$$

Here δ is a small positive constant to avoid the expression is singular when x_4 is equal to zero.

3 Stability Analysis

In this section, we shall prove that only 1 sign switch of the controller parameters is able to make this control system possess the asymptotic stability.

Theorem 3.1. Consider an overhead crane systems (3) under the IHSMC-based adaptive control law (12). Then the third-layer sliding surface s_3 and the second-layer sliding surface s_2 are asymptotically stable.

Proof. Integrating both sides of (11) yields

$$\begin{aligned} \int_0^t \mathrm{d}V &= \int_0^t s_3[-k s_3 - \eta sgn(s_3)]\ \mathrm{d}t \\ V(t) - V(0) &= \int_0^t (-k s_3^2 - \eta |S_3|)\ \mathrm{d}t \end{aligned} \tag{13}$$

So we can obtain

$$V(0) = V(t) + \int_0^t (k s_3^2 + \eta |s_3|)\ \mathrm{d}t \geq \int_0^t (k s_3^2 + \eta |s_3|)\ \mathrm{d}t \tag{14}$$

From (14), (15) becomes

$$\lim_{t\to\infty} \int_0^t (k s_3^2 + \eta |s_3|)\ \mathrm{d}t < \infty \tag{15}$$

From (15), there exist

$$\lim_{t\to\infty} \int_0^t k s_3^2\ \mathrm{d}t < \infty \tag{16}$$

$$\lim_{t\to\infty} \int_0^t \eta |s_3|\ \mathrm{d}t < \infty \tag{17}$$

We can conclude that

$$s_3 \in \mathscr{L}_2 \tag{18}$$

$$s_3 \in \mathscr{L}_1 \tag{19}$$

From (6) and (14), we can get

$$\frac{1}{2} s_3^2 = V(t) = V(0) - \int_0^t (k s_3^2 + \eta |s_3|)\ \mathrm{d}t < V(0) < \infty \tag{20}$$

here (20) means

$$s_3 \in \mathscr{L}_\infty \tag{21}$$

From (11), we have

$$\frac{\mathrm{d}V}{\mathrm{d}t} = s_3 \frac{\mathrm{d}s_3}{\mathrm{d}t} = -k s_3^2 - \eta |s_3| < \infty \tag{22}$$

so we have

$$\dot{s}_3 \in \mathscr{L}_\infty \tag{23}$$

(24) can be drawn from (18), (21) and (23) on account of Barbalat's lemma [10], i.e., *the third layer sliding surface s_3 is of asymptotic stability.*

$$\lim_{t\to\infty} s_3 = 0 \tag{24}$$

Define a set

$$\mathbb{S}_c = \left\{ s_3 \in \mathbb{R}^2 \mid \frac{\mathrm{d}V}{\mathrm{d}t} \leq c,\ c > 0 \right\} \tag{25}$$

Since $\frac{\mathrm{d}V}{\mathrm{d}t} \leq 0$ in (11), we know $\mathbb{S}_c$ is positively invariant and compact. By LaSalle's principle [11], s_3 approaches the largest invariant set in

$$\mathbb{S} = \left\{ s_3 \in \mathbb{S}_c \mid \frac{\mathrm{d}V}{\mathrm{d}t} = 0 \right\} \tag{26}$$

Since the sliding mode of the third-layer surface s_3 takes place at t_f, this control system does not contain any discontinuous term in time interval $[t_f,\ \infty)$, and becomes an autonomous one. As a result, we have

$$\begin{aligned} \mathbb{S} &= \{ s_3 \mid s_3 = 0 \cap \dot{s}_3 = 0 \} \\ &= \{ x_4,\ s_2 \mid c_3 x_4 + s_2 = 0 \cap (c_3 x_4)' + \dot{s}_2 = 0 \} \end{aligned} \tag{27}$$

Assume s_2 and x_4 do not converge to the origin by the axes x_4 and s_2 as $t \to \infty$. Then, s_3 would converge to a point of the sliding surface s_3 on phase plane by x_4 versus s_2 except the origin. This case contradicts the fact that $\lim_{t\to\infty} s_3 = 0$. So the assumption is false. From proof by contradiction, we have both s_2 and x_4 do converge to the origin rather than other points on the phase plane. Moreover, we already know $\mathbb{S}$ is attracting, so the largest invariant set in $\mathbb{S}$ contains no sets other than the coordinate origin. On account of Lasalle's invariance principle, we have

$$\lim_{t\to\infty} s_2 = 0 \qquad \lim_{t\to\infty} x_4 = 0 \tag{28}$$

i.e. *s_2 and x_4 are asymptotically stable.* □

Theorem 3.2. Consider an overhead crane systems (3) under the IHSMC-based adaptive control law (12). Then the sliding surface s_1 is asymptotically stable if (29) is satisfied.

$$c_2 = \begin{cases} c_2 & if\ \ x_3 \cdot s_1 \geq 0 \\ -c_2 & if\ \ x_3 \cdot s_1 < 0 \end{cases} \tag{29}$$

Proof. Substituting $s_2 = c_2 \cdot x_3 + s_1$ into (28), we have

$$\lim_{t\to\infty} s_2 = \lim_{t\to\infty}(c_2 \cdot x_3 + s_1) = 0 \tag{30}$$

Since $\lim_{t\to\infty} x_4 = 0$ and $\dot{x}_3 = x_4$, we have $\lim_{t\to\infty} x_3 = const.$. So (30) becomes

$$\lim_{t\to\infty} s_1 = -\lim_{t\to\infty}(c_2 \cdot x_3) = const. \tag{31}$$

If (29) is satisfied, there exisits

$$sgn(c_2 \cdot x_3 \cdot s_1) \geq 0 \tag{32}$$

From (31) and (32), this constant in (31) is zero rather than others. Thus, we can draw

$$\lim_{t\to\infty} x_3 = 0 \qquad \lim_{t\to\infty} s_1 = 0 \tag{33}$$

(33) means both x_3 and s_1 are asymptotically stable if (29) is satisfied. □

Comment 1: Substituting $x_2 = \dot{x}_1$ into $\lim_{t\to\infty} s_1 = 0$ in (33), we have

$$\lim_{t\to\infty}(c_1 \cdot x_1 + \dot{x}_1) = 0$$

This means the two system states x_1 and x_2 are locally exponentially stable.

Comment 2: As proven in theorems 1 and 2, all the sliding surfaces are asymptotically stable. There also exists tiny difference among them. From (11), the sliding mode of s_3 takes place in finite time because of the existence of the discontinuous switching control in (5). But the sliding modes of s_2 and s_1 are just asymptotically reachable or are reachable in infinite time because their reachability cannot be ensured by (11). Although (29) is discontinuous, it cannot ensure the reachability of s_1 and s_2 under the Lyapunov function (6). This may inspire one to explore a new Lyapunov function and to deduce a novel switching control law, making all the sliding surfaces possess the reachability.

Comment 3: From our design process, c_1 and c_2 have each individual function, i.e., c_2 switches its sign to get a stable s_1, c_1 is positive to guarantee x_1 and x_2 are locally exponentially stable. The function of the adaptive control law of c_3 is to make the sliding mode of s_3 reachable as soon as possible.

Comment 4: The same incremental sliding surface in [2] was presented. But 2 sign switches of the controller parameters in [2] were needed to guarantee the stability of the sliding surfaces. Although such the method can predigest the system stability analysis [2], it troubles parameter tuning. Whereas, only one sign switch of our controller parameters can ensure the stability of the entire sliding surfaces.

4 Simulation Results

In this section, the validity of the IHSMC-based adaptive control is demonstrated by the transport control problem of an overhead crane system. The physical parameters of the overhead crane system are determined as [12] $M = 37.32\ kg$, $m = 5\ kg$ and $L = 1.05\ m$. The parameters of the adaptive IHSMC law is selected as $\delta = 0.01$, $c_1 = 3.98$, $c_2 = 0.25$, $k = 1.20$ and $\eta = 0.06$ after trial and error. The initial value of c_3 is selected as $c_3^0 = 0.80$. The initial state vector $\mathbf{x}^0$ and the desired state vector $\mathbf{x}^d$ are $[2,\ 0,\ 0,\ 0]^T$ and $[0,\ 0,\ 0,\ 0]^T$, respectively.

The simulation results in Fig. 2 demonstrate all the system states and the control input. As proven, all the states are asymptotically stable. From Fig. 2a the system states can achieve the control objective from $\mathbf{x}^0$ to $\mathbf{x}^d$ at about 4.6 s. Especially, there is no overshoot of the state variable x_1, this means the trolley could directly arrive at the desired position with no oscillation. This trait is important for transport control of overhead crane systems in industries. The incremental sliding surfaces, the adaptive process of c_3, and the switch process of c_2 are displayed in Fig. 3. As proven in theorem 1 and 2, all the surfaces possess the asymptotic stability under the IHSMC-based adaptive control law with only 1 sign switch of the controller parameter c_2. Phase plane plots of s_3, s_2, and s_1 are illustrated in Fig. 4. As pointed out in comment 2, only the sliding mode of s_3 is reachable in finite time, yet the sliding modes of s_2 and s_1 is asymptotically reachable.

5 Conclusions

This paper has proposed an IHSMC-based adaptive control approach for the transport control problem of overhead crane systems, belonging to underactuated systems extensively used in industries. The system stability is analyzed by Barbalat's lemma and Lasalle's invariance principle in the sense of Lyapunov. The presented method with only 1 sign switch of the controller parameters can achieve transport control of overhead crane systems. Simulation results show the feasibility of the presented IHSMC-based adaptive control approach.

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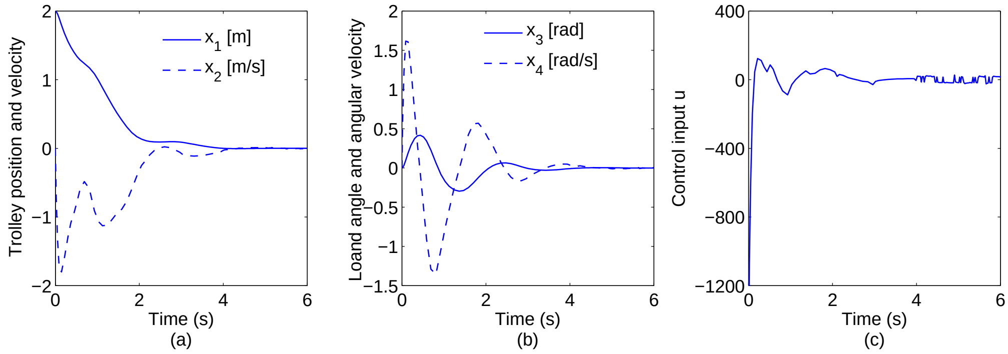

Fig. 2: Simulation results. (a). Trolley position x_1 and velocity x_2, (b). Load angle x_3 and angular velocity x_4, (c). Control input u.

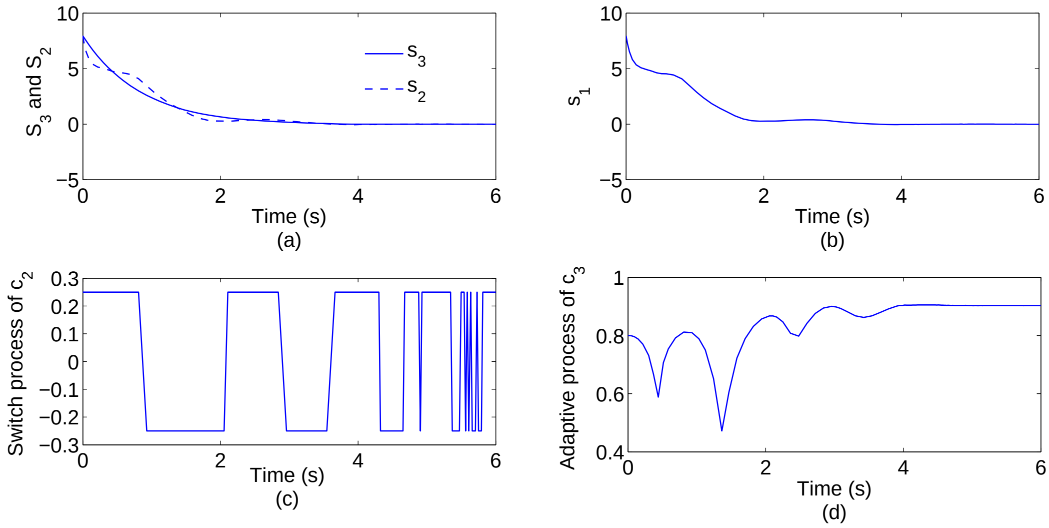

Fig. 3: Simulation results. (a). the third layer sliding surfaces s_3 and the second layer sliding surfaces s_2, (b). the first layer sliding surface s_1, (c). Switch process of c_3, (d). Adaptive process of c_2.

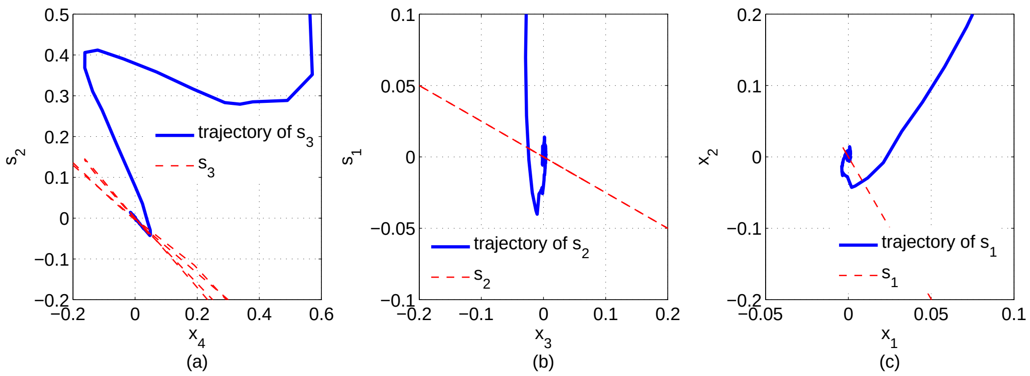

Fig. 4: Phase plane plots. (a). the third layer sliding surfaces s_3, (b). the second layer sliding surfaces s_2, (c). the first layer sliding surface s_1.

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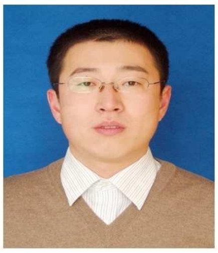

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