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# Conformable General Double Transform and Its Applications for Solving Conformable Partial Differential Equations

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**Abstract:** In this paper, we introduce a new conformable transform known as conformable general double transform. We presented its essential properties and proved some useful results such as the double convolution theorem and derivative properties. Furthermore, we apply the proposed conformable general double transform to solve some conformable partial differential equations such as heat, wave, Advection- diffusion and telegraph equations. The results demonstrate the strength and efficiency of the presented method in solving various problems in the fields of physics and engineering compared with other methods.

**Keywords:** General double transform; conformable partial fractional differential equations; conformable general double transform; fractional heat equation; fractional Telegraph equation.

## 1. Introduction

Partial differential equations have recently emerged as crucial for simulating a variety of real-world applications in engineering and science, including mathematical biology, fluid dynamics, optics, electrical circuits, and quantum physics [1,2]. Several definitions of fractional derivatives and integrals have been discussed in the literature, including those by Riesz, Weyl, Riemann-Liouville, Caputo, Hadamard, and others. Numerous strange characteristics of these fractional derivatives, such as the fact that not all of them adhere to the product rules, quotient, and others, result in a variety of problems in engineering and physics applications. The authors Khalil et al. [3] proposed an interesting definition known as the conformable fractional derivative that meets most traditional characteristics of derivatives.

Recently, Laplace transform has widely been applied to solve PDEs [4,5]. In addition, researchers have contributed extensions to the original Laplace transform such as Fourier [6], Sumudu [7], Elzaki [8], Gamar [9], Kamal [10], ARA [11], Jafari [12], Mellin [13], among many other transforms.

Moreover, many researchers and mathematicians have developed new methods to obtain solutions of conformable partial differential equations, such as the conformable double Laplace transform by Ozkan and Abdeljawad et al [14-17], [18-20] the conformable double Sumudu transform proposed by Abdeljawad and Mohamed et al, the conformable double Laplace- Sumudu transform [21-23] by Ahmed and Honggang et al, [24] Abd Elmohmoud et al solved nonlinear fractional Burgers' equations by the conformable double Elzaki transform, and Hamza et al used the conformable double ARA transform to solve Regular and Singular conformable fractional coupled Burgers' equations [25], and the conformable triple Laplace transform solved two-dimensional nonlinear Telegraph equations [26] by Deresse, and the conformable triple Sumudu transform was proposed by Gharib et al to solve regular and single dimensional identical burger equation [27].

In the current study, we introduce a novel concept of conformable double transform in two dimensional spaces. It is called the conformable general double transform. Recognizing that the conformable general double transform has additional properties, the problem of the study was to propose a general conformable transform that would generate many conformable transforms by changing the values of its constituent functions, and apply it in solving differential equations to explain some real phenomena, for example, vibration equations in physical systems and analysis of fluid flow in channels and pipes, as well as in modeling population growth and reducing the spread of diseases, as well as in analyzing climate models and predicting weather conditions to reduce natural disasters.

After presenting the definition of the conformable general double transform for function of two variables in the positive quadrant plane, we proved the basic properties concern the existing conditions theorem. Furthermore, we provided the conformable double general transform of some known functions. Later on, we establish new results relative to the partial differential derivatives. Finally, we use this

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new transform to solve some first-order and second-order partial differential equations.

## 2. Basic Definitions and Theorems for General Double Integral Transform

This section includes the basic properties and definitions of the general double transform.

**Definition 2.1.** [28,29] Let $g(x,y)$ be an integrable function of two variables $x, y > 0$.Then the general double transform denoted by $\psi_D(u,v)$ is defined as

$$\mathcal{G}[g(x,y);(u,v)] = \psi_D(u,v) = \eta(u)\rho(v)\int_0^\infty\int_0^\infty g(x,y)\, e^{-(\varphi(u)x+w(v)y)}dxdy. \tag{1}$$

where $\varphi(u)$ and $w(v)$ are the transform functions for $x$ and $y$ respectively.

And,

$$\mathcal{G}^{-1}[\psi_D(u,v)] = g(x,y) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{1}{\eta(u)}e^{\varphi(u)x}du\,\frac{1}{2\pi i}\int_{b-i\infty}^{b+i\infty}\frac{1}{\rho(v)}e^{w(v)y}\,dv. \tag{2}$$

where $a$ and $b$ are real constants.

Recognizing that this transform it has additional properties, namely, it can generate many double integral transforms by changing the values of $\eta(u)$, $\rho(v)$, $\varphi(u)$ and $w(v)$.

**Definition 2.2.** [28,29] If $g(x,y)$ defined on $[0,X]\times[0,Y]$ satisfies the following condition

$$|g(x,y)| \le Ke^{\alpha x+\delta y}, \qquad \exists K > 0, \qquad \forall x > X$$

and $\forall y > Y$ . Then, $g(x,y)$ is called a function of exponential orders $\alpha$ and $\delta$ as $x, y \to \infty$ .

**Theorem 2.1.** [28] The existence condition of general double transform of the continuous function $g(x,y)$ defined on $[0,X]\times[0,Y]$ is to be of exponential orders $\alpha$ and $\delta$,for $\mathrm{Re}[\varphi(u)] > \alpha$ and $\mathrm{Re}[w(v)] > \delta$.

**Proof of Theorem 2.1.** From the definition of general double transform, we get

$$|\psi_D(u,v)| = \left|\eta(u)\rho(v)\int_0^\infty\int_0^\infty g(x,y)\, e^{-(\varphi(u)x+w(v)y)}dxdy\right|$$
$$\le \eta(u)\rho(v)\int_0^\infty\int_0^\infty |g(x,y)|\, e^{-(\varphi(u)x+w(v)y)}dxdy$$
$$\le K\,\eta(u)\rho(v)\int_0^\infty e^{-(\varphi(u)-\alpha)x}dx\int_0^\infty e^{-(w(v)-\delta)y}dy$$
$$= \frac{\eta(u)\rho(v)\,K}{(\varphi(u)-\alpha)(w(v)-\delta)}.$$

where $\mathrm{Re}[\varphi(u)] > \alpha$ and $\mathrm{Re}[w(v)] > \delta$.

**Definition 2.3.** [28] The convolution of $g(x,y)$ and $h(x,y)$ is denoted by $(g ** h)(x,y)$ defined by:

$$(g ** h)(x,y) = \int_0^x\int_0^y g(x-\alpha, y-\delta)h(\alpha,\delta)\,d\alpha d\delta. \tag{3}$$

**Theorem 2.2.** [28] Let $\mathcal{G}[g(x,y)] = \psi_D(u,v)$. Then,

$$\mathcal{G}[g(x-\alpha,y-\delta)H(x-\alpha,y-\delta)] = e^{-\varphi(u)\alpha-w(v)\delta}\psi_D(u,v). \tag{4}$$

where $H(x-\alpha,y-\delta)$ denotes the unit step function defined by

$$H(x-\alpha,y-\delta) = \begin{cases}1, x>\alpha, y>\delta.\\ 0, Otherwise.\end{cases}$$

**Theorem 2.3. (General Double Convolution Theorem)** [28,30]. If $\mathcal{G}[g(x,y)] = \psi_D(u,v)$ and $\mathcal{G}[h(x,y)] = \mu_D(u,v)$ then

$$\mathcal{G}[(g ** h)(x,y)] = \frac{1}{\eta(u)\rho(v)}\psi_D(u,v)\mu_D(u,v). \tag{5}$$

In the following, we state some basic properties of general double transform

Assume that $\psi_D(u,v) = \mathcal{G}[g(x,y)]$ and $\mu_D(u,v) = \mathcal{G}[h(x,y)]$ and $a, b \in \mathcal{R}$.Then

$$\mathcal{G}[a\,g(x,y) + b\,h(x,y)] = a\,\mathcal{G}[g(x,y)] + b\,\mathcal{G}[h(x,y)]. \tag{6}$$

$$\mathcal{G}^{-1}[a\,\psi_D(u,v) + b\,\mu_D(u,v)] = a\,\mathcal{G}^{-1}[\psi_D(u,v)] + b\,\mathcal{G}^{-1}[\mu_D(u,v)]. \tag{7}$$

$$\mathcal{G}[x^a y^b] = \eta(u)\rho(v)\frac{\Gamma(a+1)\Gamma(b+1)}{\varphi(u)^{a+1}\,w(v)^{b+1}}, \quad a,b > 0. \tag{8}$$

$$\mathcal{G}[e^{ax+by}] = \frac{\eta(u)\rho(v)}{(\varphi(u)-a)(w(v)-b)}, \quad \varphi(u) > a, w(v) > b. \tag{9}$$

$$\mathcal{G}[\cos(ax)\cos(by)] = \frac{\eta(u)\varphi(u)}{\varphi^2(u)+a^2}\frac{\rho(v)w(v)}{w^2(v)+b^2}, a,b \in \mathcal{R}. \quad (10)$$

$$\mathcal{G}[\sin(ax)\sin(by)] = \frac{a\,\eta(u)}{\varphi^2(u)+a^2}\frac{b\rho(v)}{w^2(v)+b^2}, a,b \in \mathcal{R}. \quad (11)$$

$$\mathcal{G}\left[\frac{\partial^n h(x,y)}{\partial x^n}\right] = \varphi^n(u)\mu_D(u,v) - \eta(u)\sum_{s=1}^{n-1}\varphi^{n-1-s}(u)\frac{d^s G_I(0,v)}{dx^s}, \; n \in \mathbb{N}. \quad (12)$$

Where $G_I(0,v)$ is be the general integral transform of $h(0,y)$[13].

## 3. Conformable Fractional Derivatives (CFD)

In this section we introduce the conformable general double transform using the following definitions:

**Definition 3.1.** [25,31] Let $h\left(\frac{x^r}{r},\frac{y^t}{t}\right): \mathcal{R}\times(0,\infty)\to\mathcal{R}$ , then:

i. The conformable space fractional partial derivative of $h\left(\frac{x^r}{r},\frac{y^t}{t}\right)$ order $r$ is defined as

$$\frac{\partial^r}{\partial x^r}h\left(\frac{x^r}{r},\frac{y^t}{t}\right) = \lim_{\beta\to 0}\frac{h\left(\frac{x^r}{r}+\beta x^{1-r},\frac{y^t}{t}\right)-h\left(\frac{x^r}{r},\frac{y^t}{t}\right)}{\beta}, \frac{x^r}{r},\frac{y^t}{t}>0, 0<r,t\le 1. \quad (13)$$

ii. The conformable space fractional partial derivative of $h\left(\frac{x^r}{r},\frac{y^t}{t}\right)$ order $t$ is defined as

$$\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},\frac{y^t}{t}\right) = \lim_{\xi\to 0}\frac{h\left(\frac{x^r}{r},\frac{y^t}{t}+\xi y^{1-t}\right)-h\left(\frac{x^r}{r},\frac{y^t}{t}\right)}{\xi}, \frac{x^r}{r},\frac{y^t}{t}>0, 0<r,t\le 1. \quad (14)$$

### 3.1 Conformable Fractional Derivatives of Some Basic Functions

In the following arguments, we introduce conformable fractional derivatives for some basic functions.

- Let $h\left(\frac{x^r}{r},\frac{y^t}{t}\right) = \frac{x^r}{r}\frac{y^t}{t}$, then:

$$\frac{\partial^r}{\partial x^r}\left(\frac{x^r}{r}\frac{y^t}{t}\right) = \frac{y^t}{t}, \quad \frac{\partial^t}{\partial y^t}\left(\frac{x^r}{r}\frac{y^t}{t}\right) = \frac{x^r}{r}.$$

- Let $h\left(\frac{x^r}{r},\frac{y^t}{t}\right) = \left(\frac{x^r}{r}\right)^n$, then:

$$\frac{\partial^r}{\partial x^r}\left(\frac{x^r}{r}\right)^n = n\left(\frac{x^r}{r}\right)^{n-1}, \quad \frac{\partial^t}{\partial y^t}\left(\frac{x^r}{r}\right)^n = 0.$$

- Let $h\left(\frac{x^r}{r},\frac{y^t}{t}\right) = \left(\frac{x^r}{r}\right)^n\left(\frac{y^t}{t}\right)^m$, then:

$$\frac{\partial^r}{\partial x^r}\left(\left(\frac{x^r}{r}\right)^n\left(\frac{y^t}{t}\right)^m\right) = n\left(\frac{x^r}{r}\right)^{n-1}\left(\frac{y^t}{t}\right)^m, \quad \frac{\partial^t}{\partial y^t}\left(\left(\frac{x^r}{r}\right)^n\left(\frac{y^t}{t}\right)^m\right) = m\left(\frac{x^r}{r}\right)^n\left(\frac{y^t}{t}\right)^{m-1}.$$

- Let $h\left(\frac{x^r}{r},\frac{y^t}{t}\right) = \sin\lambda\left(\frac{x^r}{r}\right)\sin\beta\left(\frac{y^t}{t}\right)$, then:

$$\frac{\partial^r}{\partial x^r}\left(\sin\lambda\left(\frac{x^r}{r}\right)\sin\beta\left(\frac{y^t}{t}\right)\right) = \lambda\cos\lambda\left(\frac{x^r}{r}\right)\sin\beta\left(\frac{y^t}{t}\right),$$

$$\frac{\partial^t}{\partial y^t}\left(\sin\lambda\left(\frac{x^r}{r}\right)\sin\beta\left(\frac{y^t}{t}\right)\right) = \beta\sin\lambda\left(\frac{x^r}{r}\right)\cos\beta\left(\frac{y^t}{t}\right).$$

- Let $h\left(\frac{x^r}{r},\frac{y^t}{t}\right) = e^{\lambda\frac{x^r}{r}+\beta\frac{y^t}{t}}$, then:

$$\frac{\partial^r}{\partial x^r}\left(e^{\lambda\frac{x^r}{r}+\beta\frac{y^t}{t}}\right) = \lambda e^{\lambda\frac{x^r}{r}+\beta\frac{y^t}{t}}, \quad \frac{\partial^t}{\partial y^t}\left(e^{\lambda\frac{x^r}{r}+\beta\frac{y^t}{t}}\right) = \beta e^{\lambda\frac{x^r}{r}+\beta\frac{y^t}{t}}.$$

**Property 3.1**. Let $r,t\in(0,1]$ and $h(x,t)$ be differentiable function of order $r$ and $t$ at the point points $x,y>0$ respectively, then

$$\frac{\partial^r}{\partial x^r}h\left(\frac{x^r}{r},\frac{y^t}{t}\right) = x^{1-r}\frac{\partial}{\partial x}h\left(\frac{x^r}{r},\frac{y^t}{t}\right),$$

$$\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},\frac{y^t}{t}\right) = y^{1-t}\frac{\partial}{\partial y}h\left(\frac{x^r}{r},\frac{y^t}{t}\right).$$

**Proof of Property 3.1**. Using definition **3.1** and putting $j = \beta x^{1-r}$ ,we get

$$\frac{\partial^r}{\partial x^r}h\left(\frac{x^r}{r},\frac{y^t}{t}\right) = \lim_{j\to 0}\frac{h\left(\frac{x^r}{r}+j,\frac{y^t}{t}\right)-\left(\frac{x^r}{r},\frac{y^t}{t}\right)}{jx^{r-1}} = x^{1-r}\lim_{j\to 0}\frac{h\left(\frac{x^r}{r}+j,\frac{y^t}{t}\right)-\left(\frac{x^r}{r},\frac{y^t}{t}\right)}{j}.$$

Therefore,

$$\frac{\partial^r}{\partial x^r}h\left(\frac{x^r}{r},\frac{y^t}{t}\right) = x^{1-r}\frac{\partial}{\partial x}h\left(\frac{x^r}{r},\frac{y^t}{t}\right).$$

Similarly, we can easily prove that

$$\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},\frac{y^t}{t}\right) = y^{1-t}\frac{\partial}{\partial y}h\left(\frac{x^r}{r},\frac{y^t}{t}\right).$$

## 4. Conformable General Double Transform (CGDT)

Let's in this section prove that some properties and theorems of the conformable general double transform.

**Definition 4.1.** Let $h\left(\frac{x^r}{r},\frac{y^t}{t}\right)$ be a piecewise continuous function on the interval $[0,\infty)\times[0,\infty)$ having exponential order. Consider for some $a, b \in \mathcal{R}$, $\sup_{\frac{x^r}{r}, \frac{y^t}{t} > 0} \frac{\left|h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)\right|}{e^{a\frac{x^r}{r} + b\frac{y^t}{t}}}$. Thus, under these conditions the conformable general double transform is given by

$$\mathcal{G}_x^r \mathcal{G}_y^t \left[h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)\right] = \mu_{\mathcal{D}_{r.t}}(u, v)$$

$$= \eta(u)\rho(v) \int_0^\infty \int_0^\infty e^{-\varphi(u)\frac{x^r}{r} - w(v)\frac{y^t}{t}} \left[h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)\right] x^{r-1} y^{t-1} dx\, dy. \quad (15)$$

Where $u, v \in \mathbb{C}$, $0 < r, t \le 1$ and the integrals are by means of conformable fractional with respect to $\frac{x^r}{r}$ and $\frac{y^t}{t}$ respectively.

### 4.1. The CGDT in Relation to Other Conformable Double Transforms

In this section, we discuss the relationship between the new conformable general double transform and some familiar conformable double transforms such as the Laplace [14-17] , Sumudu [18,20] , Elzaki [24] , Laplace-Sumudu [21-23] transforms and ARA transform [25].

- If $\eta(u) = \rho(v) = 1$ and $\varphi(u) = u, w(v) = v$, then these new conformable turns into the conformable double Laplace transform.

$$\mathcal{G}_x^r \mathcal{G}_y^t \left[h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)\right] = \int_0^\infty \int_0^\infty e^{-u\frac{x^r}{r} - v\frac{y^t}{t}} \left[h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)\right] x^{r-1} y^{t-1} dx\, dy.$$

- If $\eta(u) = \varphi(u) = \frac{1}{u}$ and $\rho(v) = w(v) = \frac{1}{v}$, then the new conformable transform turns into the conformable double Sumudu transform.

$$\mathcal{G}_x^r \mathcal{G}_y^t \left[h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)\right] = \frac{1}{uv} \int_0^\infty \int_0^\infty e^{-\frac{1}{u}\frac{x^r}{r} - \frac{1}{v}\frac{y^t}{t}} \left[h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)\right] x^{r-1} y^{t-1} dx\, dy.$$

- If $\eta(u) = u, \rho(v) = v$ and $\varphi(u) = \frac{1}{u}, w(v) = \frac{1}{v}$, then the new transform turns into the conformable double Elzaki transform.

$$\mathcal{G}_x^r \mathcal{G}_y^t \left[h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)\right] = uv \int_0^\infty \int_0^\infty e^{-\frac{1}{u}\frac{x^r}{r} - \frac{1}{v}\frac{y^t}{t}} \left[h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)\right] x^{r-1} y^{t-1} dx\, dy.$$

- If $\eta(u) = 1, \varphi(u) = u$ and $\rho(v) = w(v) = \frac{1}{v}$, then the new transform turns into the conformable double Laplace-Sumudu transform.

$$\mathcal{G}_x^r \mathcal{G}_y^t \left[h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)\right] = \frac{1}{v} \int_0^\infty \int_0^\infty e^{-u\frac{x^r}{r} - \frac{1}{v}\frac{y^t}{t}} \left[h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)\right] x^{r-1} y^{t-1} dx\, dy.$$

### 4.2. Some Properties and Theorems of Conformable Double General Transform

In this section, we proceed to prove some basic properties and theorems such as existence, conformable general double convolution theorems and conformable fractional derivatives.

**Property 4.1.** Let $h\left(\frac{x^r}{r}, \frac{y^t}{t}\right) = f\left(\frac{x^r}{r}\right) g\left(\frac{y^t}{t}\right), x > 0, t > 0$. Then

$$\mathcal{G}_x^r \mathcal{G}_y^t \left[f\left(\frac{x^r}{r}\right) g\left(\frac{y^t}{t}\right)\right] = \mathcal{G}_x \mathcal{G}_y [f(x) g(y)]. \quad (16)$$

**Proof of Property 4.1*:*** Using the definition of CGDT, we get

$$\mathcal{G}_x^r \mathcal{G}_y^t \left[f\left(\frac{x^r}{r}\right) g\left(\frac{y^t}{t}\right)\right] = \eta(u)\rho(v) \int_0^\infty \int_0^\infty e^{-\varphi(u)\frac{x^r}{r} - w(v)\frac{y^t}{t}} \left[f\left(\frac{x^r}{r}\right) g\left(\frac{y^t}{t}\right)\right] x^{r-1} y^{t-1} dx\, dy. \quad (17)$$

Substituting:

$\varrho = \frac{x^r}{r}$, $\varsigma = \frac{y^t}{t}$, $d\varrho = x^{r-1} dx$ and $d\varsigma = y^{t-1} dy$ in Eq. (17) and simplifying, we obtain

$$\mathcal{G}_x^r \mathcal{G}_y^t \left[f\left(\frac{x^r}{r}\right) g\left(\frac{y^t}{t}\right)\right] = \eta(u)\rho(v) \int_0^\infty \int_0^\infty e^{-\varphi(u)\varrho} e^{-w(v)\varsigma} [f(\varrho) g(\varsigma)]\, d\varrho\, d\varsigma = \mathcal{G}_x \mathcal{G}_y [f(x) g(y)].$$

Where $\mathcal{G}_x$ and $\mathcal{G}_y$ the general double transform of $f(x)$ and $g(y)$ respectively.

### 4.3. Conformable General Double Transform of Some Certain Functions

In the following arguments, we introduce the conformable general double transform for some basic functions.

i. Let $h\left(\frac{x^r}{r}, \frac{y^t}{t}\right) = 1$, then:

$$\mathcal{G}_x^r \mathcal{G}_y^t [1] = \eta(u)\rho(v) \int_0^\infty \int_0^\infty e^{-\varphi(u)\frac{x^r}{r} - w(v)\frac{y^t}{t}} x^{r-1} y^{t-1} dx\, dy.$$

From **Property 4.1** and Eq.(8),we get

$$\mathcal{G}_x^r\mathcal{G}_y^t[1] = \mathcal{G}_x\mathcal{G}_y[1] = \frac{\eta(u)\rho(v)}{\varphi(u)w(v)}.$$

ii. Let $h\left(\frac{x^r}{r},\frac{y^t}{t}\right) = \left(\frac{x^r}{r}\right)^n\left(\frac{y^t}{t}\right)^m$ ,then:

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[\left(\frac{x^r}{r}\right)^n\left(\frac{y^t}{t}\right)^m\right] = \eta(u)\rho(v)\int_0^\infty\int_0^\infty e^{-\varphi(u)\frac{x^r}{r}-w(v)\frac{y^t}{t}}\left[\left(\frac{x^r}{r}\right)^n\left(\frac{y^t}{t}\right)^m\right]x^{r-1}y^{t-1}dxdy.$$

From **Property 4.1** and Eq.(8),we get

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[\left(\frac{x^r}{r}\right)^n\left(\frac{y^t}{t}\right)^m\right] = \mathcal{G}_x\mathcal{G}_y[x^n y^m] = \frac{\eta(u)\rho(v)\Gamma(n+1)\Gamma(m+1)}{\varphi(u)^{n+1}\,w(v)^{m+1}}$$

iii. Let $h\left(\frac{x^r}{r},\frac{y^t}{t}\right) = e^{\lambda\frac{x^r}{r}+\beta\frac{y^t}{t}}$ ,then:

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[e^{\lambda\frac{x^r}{r}+\beta\frac{y^t}{t}}\right] = \eta(u)\rho(v)\int_0^\infty\int_0^\infty e^{-\varphi(u)\frac{x^r}{r}-w(v)\frac{y^t}{t}}\left[e^{\lambda\frac{x^r}{r}+\beta\frac{y^t}{t}}\right]x^{r-1}y^{t-1}dx\,dy.$$

From **Property 4.1** and Eq.(9),we get

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[e^{\lambda\frac{x^r}{r}+\beta\frac{y^t}{t}}\right] = \mathcal{G}_x\mathcal{G}_y\left[e^{\lambda x+\beta t}\right] = \frac{\eta(u)\rho(v)}{(\varphi(u)-\lambda)(w(v)-\beta)}.$$

iv. Let $h\left(\frac{x^r}{r},\frac{y^t}{t}\right) = \sin\lambda\left(\frac{x^r}{r}\right)\sin\beta\left(\frac{y^t}{t}\right)$ ,then:

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[\sin\lambda\left(\frac{x^r}{r}\right)\sin\beta\left(\frac{y^t}{t}\right)\right]$$

$$= \eta(u)\rho(v)\int_0^\infty\int_0^\infty e^{-\varphi(u)\frac{x^r}{r}-w(v)\frac{y^t}{t}}\left[\sin\lambda\left(\frac{x^r}{r}\right)\sin\beta\left(\frac{y^t}{t}\right)\right]x^{r-1}y^{t-1}dx\,dy.$$

From **Property 4.1** and Eq. (11), we get

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[\sin\lambda\left(\frac{x^r}{r}\right)\sin\beta\left(\frac{y^t}{t}\right)\right] = \mathcal{G}_x\mathcal{G}_y[\sin\lambda x\sin\beta y] = \frac{\lambda\beta\,\eta(u)\rho(v)}{(\varphi^2(u)+\lambda^2)(w^2(v)+\beta^2)}.$$

**Theorem 4.1. (The Convolution Theorem for Conformable General Double Transform).** If $\mathcal{G}[g(x,y)] = \psi_{\mathcal{D}}(u,v)$ and $\mathcal{G}[h(x,y)] = \mu_{\mathcal{D}}(u,v)$ then both exist for $u > 0$ and $v > 0$, then:

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[(g**h)\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right] = \frac{1}{\eta(u)\rho(v)}\psi_{\mathcal{D}_{r,t}}(u,v)\mu_{\mathcal{D}_{r,t}}(u,v). \quad (18)$$

*where $**$ denotes the convolution of the conformable functions.*

**Proof of Theorem 4.1.** By using the definition of CGDT, Lemma (4.1) and Theorem (9) in [28], one can easily show the proof

**4.3 Existence Condition for the conformable double General Transform**

If $h\left(\frac{x^r}{r},\frac{y^t}{t}\right)$ is an exponential order $a$ and $b$ as $\frac{x^r}{r}\to\infty, \frac{y^t}{t}\to\infty$, if there exists a positive constant $E>0$, such that for all $x > X$, $y > Y$ and $\left|h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right| \le E\,e^{a\frac{x^r}{r}+b\frac{y^t}{t}}$.Then it is easy to get:

$h\left(\frac{x^r}{r},\frac{y^t}{t}\right) = O\left(e^{a\frac{x^r}{r}+b\frac{y^t}{t}}\right)$ as $\frac{x^r}{r}\to\infty$ and $\frac{y^t}{t}\to\infty.$

Thus, the function $h\left(\frac{x^r}{r},\frac{y^t}{t}\right)$ is called an exponential order as $\frac{x^r}{r}\to\infty$ and $\frac{y^t}{t}\to\infty.$

**Theorem 4.2.** If $h\left(\frac{x^r}{r},\frac{y^t}{t}\right)$ is a continuous function on the region $(0,X)\times(0,T)$ of exponential orders $a$ and $b$ , then the conformable general double transform of $h\left(\frac{x^r}{r},\frac{y^t}{t}\right)$ exists for all $Re[\varphi(u)] > a$ and $Re[w(v)] > b$.

**Proof of Theorem 4.2.** Using the definition of the CGDT of $h\left(\frac{x^r}{r},\frac{y^t}{t}\right)$, we have

$$\left|\mu_{\mathcal{D}_{r,t}}(u,v)\right| = \left|\eta(u)\rho(v)\int_0^\infty\int_0^\infty e^{-\varphi(u)\frac{x^r}{r}-w(v)\frac{y^t}{t}}\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]x^{r-1}y^{t-1}dx\,dy\right|,$$

$$\le \eta(u)\rho(v)\int_0^\infty\int_0^\infty e^{-\varphi(u)\frac{x^r}{r}-w(v)\frac{y^t}{t}}\left|h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right|x^{r-1}y^{t-1}dx\,dy,$$

$$\le E\,\eta(u)\rho(v)\int_0^\infty\int_0^\infty e^{-\varphi(u)\frac{x^r}{r}-w(v)\frac{y^t}{t}}\left(e^{a\frac{x^r}{r}+b\frac{y^t}{t}}\right)x^{r-1}y^{t-1}dx\,dy,$$

$$\le E\,\eta(u)\rho(v)\int_0^\infty\int_0^\infty e^{-(\varphi(u)-a)\frac{x^r}{r}-(w(v)-b)\frac{y^t}{t}}x^{r-1}y^{t-1}dx\,dy,$$

$$= \frac{E\,\eta(u)\rho(v)}{(\varphi(u)-a)(w(v)-b)}.$$

For $Re[\varphi(u)] > a$ and $Re[w(v)] > b$.

**Theorem 4.3.** Let $\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]$ exist, then the conformable general double transform of the functions can be represented as follows:

i. $\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{x^r}{r}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]=-\frac{\eta(u)}{\varphi'(u)}\frac{\partial}{\partial u}\left(\frac{1}{\eta(u)}\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]\right).$

ii. $\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{y^t}{t}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]=-\frac{\rho(v)}{w'(v)}\frac{\partial}{\partial v}\left(\frac{1}{\rho(v)}\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]\right).$

**Proof of Theorem 4.3.**

**Proof of (i).** Using the definition of CGDT of $h\left(\frac{x^r}{r},\frac{y^t}{t}\right)$ and differentiating the both sides with respect to $u$ in Eq.(15), we have

$$\frac{\partial}{\partial u}\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]=\rho(v)\int_0^\infty e^{-w(v)\frac{y^t}{t}}y^{t-1}dy\left(\int_0^\infty\frac{\partial}{\partial u}\left(\eta(u)\,e^{-\varphi(u)\frac{x^r}{r}}\right)h\left(\frac{x^r}{r},\frac{y^t}{t}\right)x^{r-1}dx\right). \quad (19)$$

Calculating the partial derivative of second integral, we can get

$$\frac{\partial}{\partial u}\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]=\rho(v)\int_0^\infty e^{-w(v)\frac{y^t}{t}}y^{t-1}dy\int_0^\infty\left(-\eta(u)\varphi'(u)\frac{x^r}{r}+\eta'(u)\right)e^{-\varphi(u)\frac{x^r}{r}}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)x^{r-1}dx. \quad (20)$$

Substituting Eq. (20) into Eq. (19), we obtain

$$\begin{aligned}\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{x^r}{r}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]&=\frac{1}{\varphi'(u)}\left(-\frac{\partial}{\partial u}\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]+\frac{\eta'(u)}{\eta(u)}\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]\right)\\&=-\frac{\eta(u)}{\varphi'(u)}\frac{\partial}{\partial u}\left(\frac{1}{\eta(u)}\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]\right).\end{aligned} \quad (21)$$

Similarly, we can easily prove that:

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{y^t}{t}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]=-\frac{\rho(v)}{w'(v)}\frac{\partial}{\partial v}\left(\frac{1}{\rho(v)}\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]\right).$$

We conclude that the above results can be expanded as follows:

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[\left(\frac{x^r}{r}\right)^n h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]=(-1)^n\frac{\partial^n}{\partial u^n}\left(\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]\right). \quad (22)$$

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[\left(\frac{y^t}{t}\right)^n h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]=(-1)^n\frac{\partial^n}{\partial v^n}\left(\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]\right). \quad (23)$$

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[\left(\frac{x^r}{r}\right)^n\left(\frac{\partial^r}{\partial x^r}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right)\right]=(-1)^n\frac{\partial^n}{\partial u^n}\left(\mathcal{G}_x^r\mathcal{G}_y^t\left[\left(\frac{\partial^r}{\partial x^r}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right)\right]\right). \quad (24)$$

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[\left(\frac{y^t}{t}\right)^n\left(\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right)\right]=(-1)^n\frac{\partial^n}{\partial v^n}\left(\mathcal{G}_x^r\mathcal{G}_y^t\left[\left(\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right)\right]\right). \quad (25)$$

**Theorem 4.4**. Let Let $\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]$ exist, then the conformable general double transform of the conformable fractional derivatives of order $r$ and $t$ for $0<r,t\le 1$ can be represented as follows

i. $$\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{\partial^r}{\partial x^r}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]=\varphi(u)\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]-\eta(u)\mathcal{G}_y^t\left[h\left(0,\frac{y^t}{t}\right)\right].$$

ii. $$\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{\partial^{2r}}{\partial x^{2r}}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]=\varphi^2(u)\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]-\varphi(u)\eta(u)\mathcal{G}_y^t\left[h\left(0,\frac{y^t}{t}\right)\right]-\eta(u)\,\mathcal{G}_y^t\left[\frac{\partial^r}{\partial x^r}h\left(0,\frac{y^t}{t}\right)\right].$$

iii. $$\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]=w(v)\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]-\rho(v)\,\mathcal{G}_x^r\left[h\left(\frac{x^r}{r},0\right)\right].$$

iv. $$\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{\partial^{2t}}{\partial y^{2t}}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]=w^2(v)\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]-w(v)\rho(v)\,\mathcal{G}_x^r\left[h\left(\frac{x^r}{r},0\right)\right]-\rho(v)\mathcal{G}_x^r\left[\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},0\right)\right].$$

**Proof of Theorem 4.4.**

i. Using the definition of CGDT for $\frac{\partial^r}{\partial x^r}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)$, we have

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{\partial^r}{\partial x^r}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]=\eta(u)\rho(v)\int_0^\infty\int_0^\infty e^{-\varphi(u)\frac{x^r}{r}-w(v)\frac{y^t}{t}}\left[\frac{\partial^r}{\partial x^r}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]x^{r-1}y^{t-1}dx\,dy. \quad (26)$$

Eq.(26) can be written as

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{\partial^r}{\partial x^r}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right] = \rho(v)\int_0^\infty e^{-w(v)\frac{y^t}{t}}y^{t-1}\left(\eta(u)\int_0^\infty e^{-\varphi(u)\frac{x^r}{r}}\left(\frac{\partial^r}{\partial x^r}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right)x^{r-1}dx\right)dy. \tag{27}$$

Applying **Property 3.1**, then Eq. (27) becomes

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{\partial^r}{\partial x^r}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right] = \rho(v)\int_0^\infty e^{-w(v)\frac{y^t}{t}}y^{t-1}\left\{\eta(u)\int_0^\infty e^{-\varphi(u)\frac{x^r}{r}}\frac{\partial}{\partial x}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)dx\right\}dy. \tag{28}$$

Thus, the integral inside curly bracket given by

$$\eta(u)\int_0^\infty e^{-\varphi(u)\frac{x^r}{r}}\frac{\partial}{\partial x}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)dx = \eta(u)\left(-h\left(0,\frac{y^t}{t}\right)+\varphi(u)\int_0^\infty e^{-\varphi(u)\frac{x^r}{r}}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)x^{r-1}dx\right). \tag{29}$$

Substituting Eq. (29) into Eq. (28), we obtain

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{\partial^r}{\partial x^r}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right] = \varphi(u)\left(\eta(u)\rho(v)\int_0^\infty\int_0^\infty e^{-\varphi(u)\frac{x^r}{r}-w(v)\frac{y^t}{t}}\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]x^{r-1}y^{t-1}dxdy\right) - \eta(u)\rho(v)\int_0^\infty e^{-w(v)\frac{y^t}{t}}\left[h\left(0,\frac{y^t}{t}\right)\right]y^{t-1}dy.$$

Therefore,

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{\partial^r}{\partial x^r}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right] = \varphi(u)\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right] - \eta(u)\mathcal{G}_y^t\left[h\left(0,\frac{y^t}{t}\right)\right].$$

In the same manner, the CGDT of $\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)$, $\frac{\partial^{2r}}{\partial x^{2r}}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)$ and $\frac{\partial^{2t}}{\partial y^{2t}}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)$ can be obtained.

We note that the above results can be extended as follows:

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{\partial^{nr}}{\partial x^{nr}}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right] = \varphi^n(u)\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right] - \eta(u)\sum_{j=1}^{n-1}\varphi^{n-j-1}(u)\left(\mathcal{G}_y^t\left[\frac{\partial^{jr}}{\partial x^{jr}}h\left(0,\frac{y^t}{t}\right)\right]\right). \tag{30}$$

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{\partial^{nt}}{\partial y^{nt}}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right] = w^n(v)\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right] - \rho(v)\sum_{\ell=1}^{n-1}w^{n-\ell-1}(v)\left(\mathcal{G}_x^r\left[\frac{\partial^{\ell t}}{\partial y^{\ell t}}h\left(\frac{x^r}{r},0\right)\right]\right). \tag{31}$$

The proof of Eqs.(30) and (31) can be obtained by induction.

*The below theorem establishes the CGDT of the functions* $\frac{x^r}{r}\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)$ and $\frac{y^t}{t}\frac{\partial^r}{\partial x^r}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)$.

**Theorem 4.5.** Let $\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]$ exist, then

i. $\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{x^r}{r}\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right] = -\frac{\eta(u)w(v)}{\varphi'(u)}\frac{\partial}{\partial u}\left(\frac{1}{\eta(u)}\left[\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]\right]\right) + \frac{\eta(u)\rho(v)}{\varphi'(u)}\frac{d}{du}\left(\mathcal{G}_x^r\left[h\left(\frac{x^r}{r},0\right)\right]\right).$

ii. $\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{y^t}{t}\frac{\partial^r}{\partial x^r}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right] = -\frac{\rho(v)\varphi(u)}{w'(v)}\frac{\partial}{\partial v}\left(\frac{1}{\rho(v)}\left[\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]\right]\right) + \frac{\eta(u)\rho(v)}{w'(v)}\frac{d}{dv}\left(\mathcal{G}_y^t\left[h\left(0,\frac{y^t}{t}\right)\right]\right).$

**Proof of Theorem 4.5.** Using the definition of CGDT of $\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)$, we get

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right] = \eta(u)\rho(v)\int_0^\infty\int_0^\infty e^{-\varphi(u)\frac{x^r}{r}-w(v)\frac{y^t}{t}}\left[\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]x^{r-1}y^{t-1}dx\,dy. \tag{32}$$

By differentiating the both sides with respect to $u$ in Eq.(32), we have

$$\frac{\partial}{\partial u}\left(\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]\right) = \rho(v)\int_0^\infty e^{-w(v)\frac{y^t}{t}}\left(\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right)y^{t-1}dy\left\{\int_0^\infty\frac{\partial}{\partial u}\left(\eta(u)e^{-\varphi(u)\frac{x^r}{r}}\right)x^{r-1}dx\right\}. \tag{33}$$

we calculate the partial derivative in second integral as follows

$$\int_0^\infty\frac{\partial}{\partial u}\left(\eta(u)e^{-\varphi(u)\frac{x^r}{r}}\right)x^{r-1}dx = \eta'(u)\int_0^\infty e^{-\varphi(u)\frac{x^r}{r}}x^{r-1}dx - \eta(u)\varphi'(u)\int_0^\infty\frac{x^r}{r}e^{-\varphi(u)\frac{x^r}{r}}x^{r-1}dx. \tag{34}$$

Substituting Eq. (34) into Eq. (33), we get

$$\frac{\partial}{\partial u}\left(\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]\right)$$
$$=\rho(v)\eta'(u)\int_0^\infty\int_0^\infty e^{-\varphi(u)\frac{x^r}{r}-w(v)\frac{y^t}{t}}\left[\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]x^{r-1}y^{t-1}dxdy$$
$$-\rho(v)\eta(u)\varphi'(u)\int_0^\infty\int_0^\infty e^{-\varphi(u)\frac{x^r}{r}-w(v)\frac{y^t}{t}}\left[\frac{x^r}{r}\left(\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right)\right]x^{r-1}y^{t-1}dxdy$$

And,

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{x^r}{r}\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]$$
$$=-\frac{1}{\varphi'(u)}\left\{\frac{\partial}{\partial u}\left[\mathcal{G}_x^r\mathcal{G}_y^t\left(\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right)\right]+\frac{\eta'(u)}{\eta(u)}\left(\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]\right)\right\}$$

Thus,

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{x^r}{r}\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]=-\frac{\eta(u)}{\varphi'(u)}\frac{\partial}{\partial u}\left(\frac{1}{\eta(u)}\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]\right)$$

Using **Theorem 4.4**, we have

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{x^r}{r}\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]$$
$$=-\frac{\eta(u)w(v)}{\varphi'(u)}\frac{\partial}{\partial u}\left(\frac{1}{\eta(u)}\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]\right)+\frac{\eta(u)\rho(v)}{\varphi'(u)}\frac{d}{du}\left(\mathcal{G}_x^r\left[h\left(\frac{x^r}{r},0\right)\right]\right)$$

In a similar manner, one can prove that

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{y^t}{t}\frac{\partial^r}{\partial x^r}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]$$
$$=-\frac{\varphi(u)\rho(v)}{w'(v)}\frac{\partial}{\partial v}\left(\frac{1}{\rho(v)}\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]\right)+\frac{\eta(u)\rho(v)}{w'(v)}\frac{d}{dv}\left(\mathcal{G}_y^t\left[h\left(0,\frac{y^t}{t}\right)\right]\right)$$

## 5. Applications

In this section, we find the solutions of conformable fractional heat equation, conformable fractional wave equation, conformable fractional Advection- diffusion equation, conformable fractional telegraph equation and boundary value problem of conformable fractional partial differential equation using conformable general double transform. We note that if $r=t=1$ in the following problems, one can obtain the solutions of some problems, which was studied in [17,18,20,32,33]:

**Example 5.1.**

Consider the following homogeneous conformable fractional heat equation of the form

$$\frac{\partial^{2r}}{\partial x^{2r}}\left(h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right)=\frac{\partial^t}{\partial y^t}\left(h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right). \quad (35)$$

subject to

$$h\left(\frac{x^r}{r},0\right)=e^{\frac{x^r}{r}},\quad h\left(0,\frac{y^t}{t}\right)=e^{\frac{y^t}{t}},\quad \frac{\partial^r}{\partial x^r}h\left(0,\frac{y^t}{t}\right)=e^{\frac{y^t}{t}}. \quad (36)$$

**Solution:**

By applying the conformable general double transform for both sides of Eq. (35), we have

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{\partial^{2r}}{\partial x^{2r}}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]=\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]. \quad (37)$$

By partial derivative properties of CGDT, we get

$$\varphi^2(u)\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]-\varphi(u)\eta(u)\mathcal{G}_y^t\left[h\left(0,\frac{y^t}{t}\right)\right]-\eta(u)\,\mathcal{G}_y^t\frac{\partial^r}{\partial x^r}\left[h\left(0,\frac{y^t}{t}\right)\right]$$
$$=w(v)\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]-\rho(v)\,\mathcal{G}_x^r\left[h\left(\frac{x^r}{r},0\right)\right] \quad (38)$$

Substituting

$$\mathcal{G}_y^t\left[h\left(0,\frac{y^t}{t}\right)\right]=\frac{\rho(v)}{w(v)-1},\left(\mathcal{G}_y^t\frac{\partial^r}{\partial x^r}\left[h\left(0,\frac{y^t}{t}\right)\right]\right)=\frac{\rho(v)}{w(v)-1},\mathcal{G}_x^r\left[h\left(\frac{x^r}{r},0\right)\right]=\frac{\eta(u)}{\varphi(u)-1}.$$

in Eq. (38) and rearranging the terms, we have

$$(\varphi^2(u)-w(v))\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]=\frac{\rho(v)\eta(u)\varphi(u)+\eta(u)\rho(v)}{w(v)-1}-\frac{\eta(u)\rho(v)}{\varphi(u)-1}. \quad (39)$$

By simplifying, we obtain

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]=\frac{\eta(u)\rho(v)}{(\varphi(u)-1)(w(v)-1)}.$$

Thus, we can get the solution as

$$h\left(\frac{x^r}{r},\frac{y^t}{t}\right)=e^{\frac{x^r}{r}+\frac{y^t}{t}}. \quad (40)$$

for $r=t=1$, the exact solution becomes

$$h(x,y)=e^{x+y}.$$

This result is consistent with the result obtained in [20] double conformable Sumudu transform.

The following figures (Figure 1 and Figure 2 ) illustrates the 3D graph of the exact and approximate solutions of Example 5.1 at $r=t=1$ and Figure 3, we sketch the approximate solutions of $h\left(\frac{x^r}{r},\frac{y^t}{t}\right)$ for Eq. (40) at $r=t=1,0.99,0.98,0.97$ in 2D.

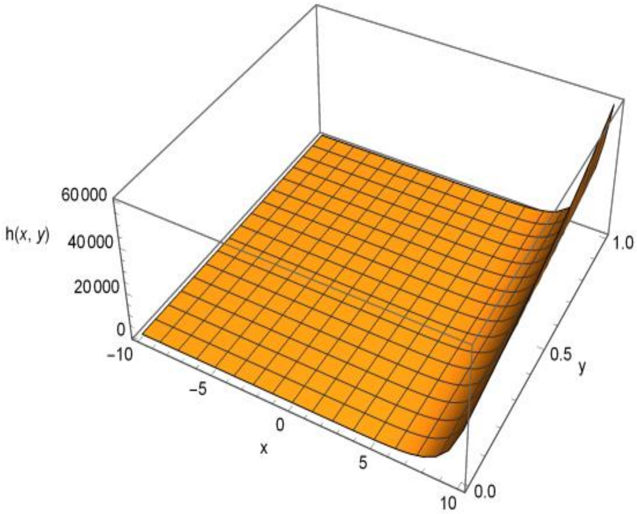


**Fig. 1:** Plots of the 3D exact solutions of Eq .(40).

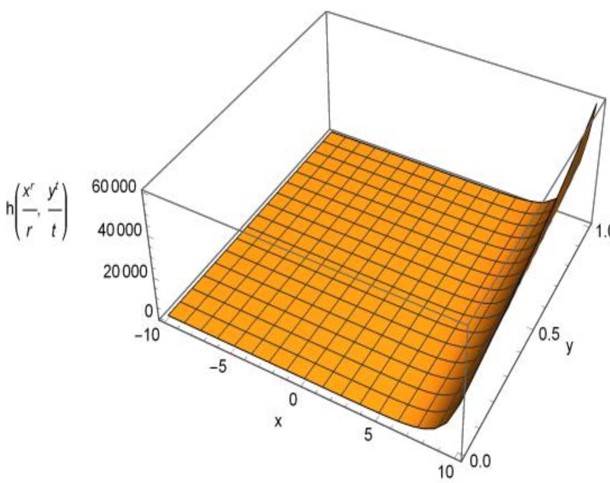


**Fig. 2:** Plots of the 3D approximate solutions of Eq .(40) obtained using the CGDT.

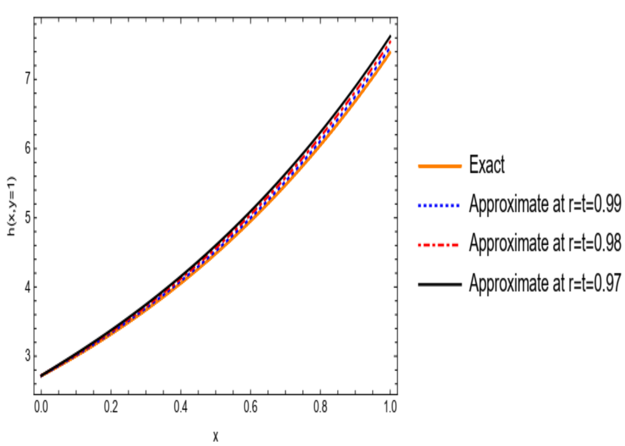


**Fig. 3:** The exact and CGDT method solution of Eq. (40) at $y = 1$ and $r = t = 1, 0.99, 0.98, 0.97$.

Moreover, in **Table 1** we present the absolute error of $h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)$ obtained by exact and CGDT at different values of $r, t$ and $y$.

**Table 1:** The absolute error of $h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)$ obtained by exact and CGDT at different values of $r, t$ and $y$.

| $x$ | $y$ | $r$ | $t$ | $Exact$ | $CGDT$ | $\lvert h_{Exact} - h_{CGDT} \rvert$ |
|---|---|---|---|---|---|---|
| 1 | 0.1 | 0.99 | 0.99 | 1.2214 | 1.22965 | 0.00825 |
| 1 | 0.2 | 0.99 | 0.99 | 1.34986 | 1.3616 | 0.01174 |
| 1 | 0.3 | 0.99 | 0.99 | 1.49182 | 1.50691 | 0.01509 |
| 1 | 0.1 | 0.98 | 0.98 | 1.2214 | 1.23825 | 0.01685 |
| 1 | 0.2 | 0.98 | 0.98 | 1.34986 | 1.37384 | 0.02398 |
| 1 | 0.3 | 0.98 | 0.98 | 1.49182 | 1.52262 | 0.0308 |
| 1 | 0.1 | 0.97 | 0.97 | 1.2214 | 1.24724 | 0.02584 |
| 1 | 0.2 | 0.97 | 0.97 | 1.34986 | 1.38659 | 0.03673 |
| 1 | 0.3 | 0.97 | 0.97 | 1.49182 | 1.53898 | 0.04716 |

**Example 5.2.**

Consider the following homogeneous conformable fractional wave equation of the form

$$\frac{\partial^{2r}}{\partial x^{2r}}\left(h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)\right) = \frac{\partial^{2t}}{\partial y^{2t}}\left(h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)\right). \tag{41}$$

subject to

$$h\left(\frac{x^r}{r}, 0\right) = e^{\frac{x^r}{r}}, \quad \frac{\partial^t}{\partial y^t} h\left(\frac{x^r}{r}, 0\right) = e^{\frac{x^r}{r}}, \quad h\left(0, \frac{y^t}{t}\right) = e^{\frac{y^t}{t}}, \quad \frac{\partial^r}{\partial x^r} h\left(0, \frac{y^t}{t}\right) = e^{\frac{y^t}{t}}. \tag{42}$$

**Solution:**

By applying the conformable general double transform for both sides of Eq. (41), we have

$$\mathcal{G}_x^r \mathcal{G}_y^t \left[\frac{\partial^{2r}}{\partial x^{2r}} h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)\right] = \mathcal{G}_x^r \mathcal{G}_y^t \left[\frac{\partial^{2t}}{\partial y^{2t}} h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)\right]. \tag{43}$$

By partial derivative properties of CGDT, we get

$$\varphi^2(u)\mathcal{G}_x^r \mathcal{G}_y^t \left[h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)\right] - \varphi(u)\eta(u)\mathcal{G}_y^t \left[h\left(0, \frac{y^t}{t}\right)\right] - \eta(u)\, \mathcal{G}_y^t \left[\frac{\partial^r}{\partial x^r} h\left(0, \frac{y^t}{t}\right)\right]$$
$$= w^2(v)\mathcal{G}_x^r \mathcal{G}_y^t \left[h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)\right] - w(v)\rho(v)\, \mathcal{G}_x^r \left[h\left(\frac{x^r}{r}, 0\right)\right] - \rho(v)\mathcal{G}_x^r \left[\frac{\partial^t}{\partial y^t} h\left(\frac{x^r}{r}, 0\right)\right]. \tag{44}$$

Substituting

$$\mathcal{G}_x^r \left[h\left(\frac{x^r}{r}, 0\right)\right] = \frac{\eta(u)}{\varphi(u) - 1}, \quad \mathcal{G}_x^r \left[\frac{\partial^t}{\partial y^t} h\left(\frac{x^r}{r}, 0\right)\right] = \frac{\eta(u)}{\varphi(u) - 1},$$

$$\mathcal{G}_y^t\left[h\left(0,\frac{y^t}{t}\right)\right]=\frac{\rho(v)}{w(v)-1},\qquad \mathcal{G}_y^t\frac{\partial^r}{\partial x^r}\left[h\left(0,\frac{y^t}{t}\right)\right]=\frac{\rho(v)}{w(v)-1}.$$

in Eq. (44) and rearranging the terms, we have

$$(\varphi^2(u)-w^2(v))\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]$$
$$=\frac{\rho(v)\eta(u)\varphi(u)+\eta(u)\rho(v)}{w(v)-1}-\frac{\left(\eta(u)\rho(v)w(v)+\eta(u)\rho(v)\right)}{\varphi(u)-1}.$$

By simplifying, we obtain

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]=\frac{\eta(u)\rho(v)}{(\varphi(u)-1)(w(v)-1)}.$$

Thus, we can get the solution as

$$h\left(\frac{x^r}{r},\frac{y^t}{t}\right)=e^{\frac{x^r}{r}+\frac{y^t}{t}}. \tag{45}$$

for $r=t=1$, the exact solution becomes

$$h(x,y)=e^{x+y}.$$

This result is consistent with the result obtained in [20] double conformable Sumudu transform.

The following figures (Figure 4 and Figure 5 ) illustrates the 3D graph of the exact and approximate solutions of Example 5.2 at $r=t=1$ and Figure 6, we sketch the approximate solutions of $h\left(\frac{x^r}{r},\frac{y^t}{t}\right)$ for Eq. (45) at $r=t=1,0.99,0.98,0.97$ in 2D.

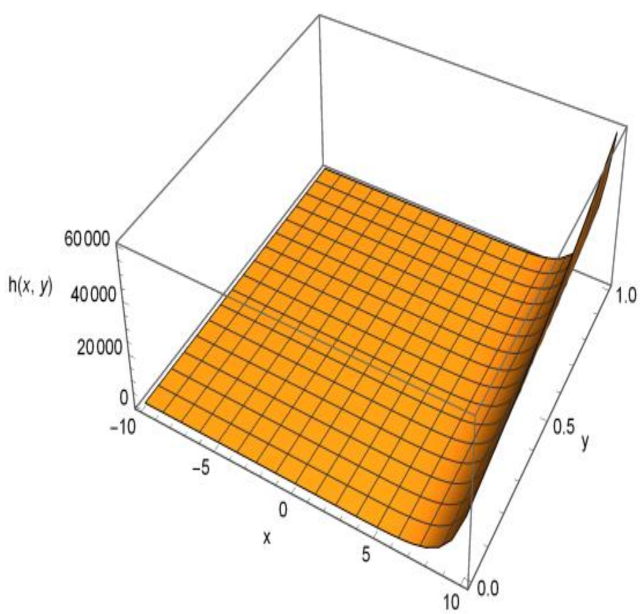


**Fig. 4:** Plots of the 3D exact solutions of Eq .(45).

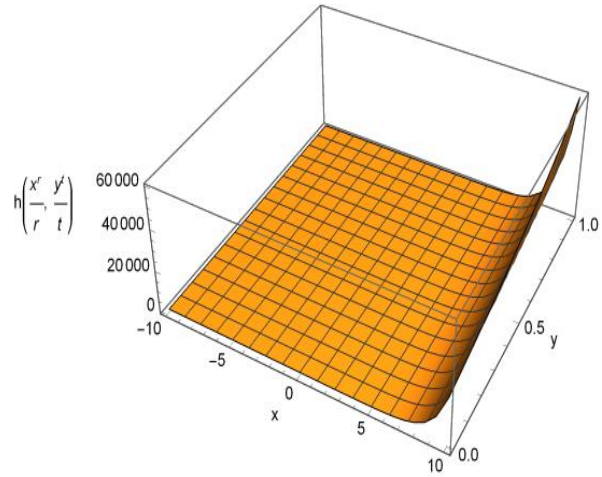


**Fig. 5:** Plots of the 3D approximate solutions of Eq .(45) obtained using the CGDT.

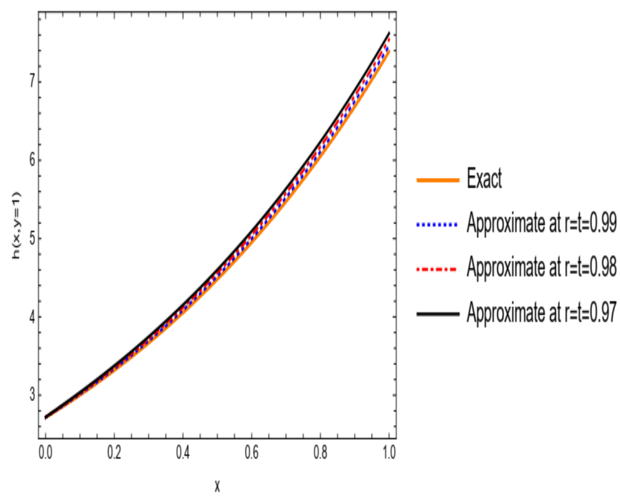


**Fig. 6:** The exact and CGDT method solution of Eq. (45) at $y=1$ and $r=t=1,0.99,0.98,0.97$.

Moreover, in **Table 2** we present the absolute error of $h\left(\frac{x^r}{r},\frac{y^t}{t}\right)$ obtained by exact and CGDT at different values of $r,t$ and $y$.

**Table 2:** The absolute error of $h\left(\frac{x^r}{r},\frac{y^t}{t}\right)$ obtained by exact and CGDT at different values of $r,t$ and $y$.

| $x$ | $y$ | $r$ | $t$ | $Exact$ | $CGDT$ | $\lvert h_{Exact}-h_{CGDT}\rvert$ |
|---|---|---|---|---|---|---|
| 1 | 0.1 | 0.99 | 0.99 | 1.2214 | 1.22965 | 0.00825 |
| 1 | 0.2 | 0.99 | 0.99 | 1.34986 | 1.3616 | 0.01174 |
| 1 | 0.3 | 0.99 | 0.99 | 1.49182 | 1.50691 | 0.01509 |
| 1 | 0.1 | 0.98 | 0.98 | 1.2214 | 1.23825 | 0.01685 |
| 1 | 0.2 | 0.98 | 0.98 | 1.34986 | 1.37384 | 0.02398 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 0.3 | 0.98 | 0.98 | 1.49182 | 1.52262 | 0.0308 |
| 1 | 0.1 | 0.97 | 0.97 | 1.2214 | 1.24724 | 0.02584 |
| 1 | 0.2 | 0.97 | 0.97 | 1.34986 | 1.38659 | 0.03673 |
| 1 | 0.3 | 0.97 | 0.97 | 1.49182 | 1.53898 | 0.04716 |

**Example 5.3.**

Consider the following conformable fractional Advection-diffusion equation of the form

$$\frac{\partial^t}{\partial y^t}\left(h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right)-\frac{\partial^{2r}}{\partial x^{2r}}\left(h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right)+\frac{\partial^r}{\partial x^r}\left(h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right)=0. \tag{46}$$

subject to

$$h\left(\frac{x^r}{r},0\right)=e^{\frac{x^r}{r}}-\frac{x^r}{r} \quad , \quad h\left(0,\frac{y^t}{t}\right)=1+\frac{y^t}{t} \quad , \quad \frac{\partial^r}{\partial x^r}h\left(0,\frac{y^t}{t}\right)=0. \tag{47}$$

Solution:

By applying the conformable general double transform for both sides of Eq. (46), we have

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{\partial^t}{\partial y^t}\left(h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right)-\frac{\partial^{2r}}{\partial x^{2r}}\left(h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right)+\frac{\partial^r}{\partial x^r}\left(h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right)\right]=0. \tag{48}$$

Using the conformable convolution theorem and partial derivative properties of CGDT, we get

$$w(v)\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]-\rho(v)\,\mathcal{G}_x^r\left[h\left(\frac{x^r}{r},0\right)\right]-\varphi^2(u)\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]$$
$$+\varphi(u)\eta(u)\mathcal{G}_y^t\left[h\left(0,\frac{y^t}{t}\right)\right]+\eta(u)\,\mathcal{G}_y^t\left[\frac{\partial^r}{\partial x^r}h\left(0,\frac{y^t}{t}\right)\right]+\varphi(u)\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]$$
$$-\eta(u)\mathcal{G}_y^t\left[h\left(0,\frac{y^t}{t}\right)\right]=0. \tag{49}$$

Substituting

$$\mathcal{G}_y^t\left[h\left(0,\frac{y^t}{t}\right)\right]=\rho(v)\left(\frac{1}{w(v)}+\frac{1}{w^2(v)}\right), \qquad \mathcal{G}_y^t\left[\frac{\partial^r}{\partial x^r}h\left(0,\frac{y^t}{t}\right)\right]=0,$$

$$\mathcal{G}_x^r\left[h\left(\frac{x^r}{r},0\right)\right]=\eta(u)\left(\frac{1}{\varphi(u)-1}-\frac{1}{\varphi^2(u)}\right).$$

in Eq. (49) and rearranging the terms, we have

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]=\frac{\rho(v)\,\eta(u)\left(\frac{1}{\varphi(u)-1}-\frac{1}{\varphi^2(u)}\right)-\varphi(u)\eta(u)\rho(v)\left(\frac{1}{w(v)}+\frac{1}{w^2(v)}\right)+\eta(u)\rho(v)\left(\frac{1}{w(v)}+\frac{1}{w^2(v)}\right)}{\left(w(v)-\varphi^2(u)+\varphi(u)\right)}.$$

By simplifying, we obtain

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]=\frac{\eta(u)\rho(v)}{w(v)(\varphi(u)-1)}-\frac{\eta(u)\rho(v)}{\varphi^2(u)w(v)}+\frac{\eta(u)\rho(v)}{\varphi(u)w^2(v)}.$$

Thus, we can get the exact solution as

$$h\left(\frac{x^r}{r},\frac{y^t}{t}\right)=e^{\frac{x^r}{r}}-\frac{x^r}{r}+\frac{y^t}{t}. \tag{50}$$

for $r=t=1$, the exact solution becomes

$$h(x,y)=e^x-x+y.$$

This result is consistent with the result obtained in [32] by Adomian decomposition method.

The following figures (Figure 7 and Figure 8) illustrates the 3D graph of the exact and approximate solutions of Example 5.3 at $r=t=1$ and Figure 9, we sketch the approximate solutions of $h\left(\frac{x^r}{r},\frac{y^t}{t}\right)$ for Eq. (50) at $r=t=1, 0.99, 0.98, 0.97$ in 2D.

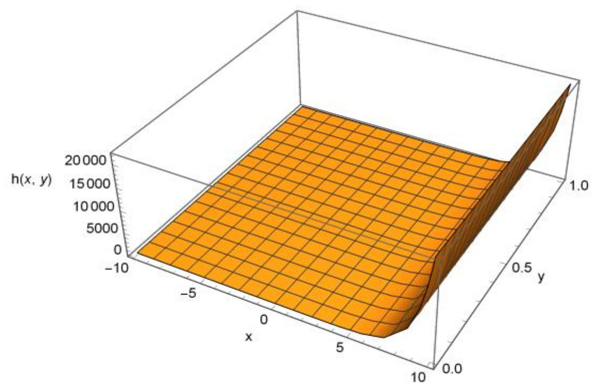


**Fig. 7:** Plots of the 3D exact solutions of Eq .(50).

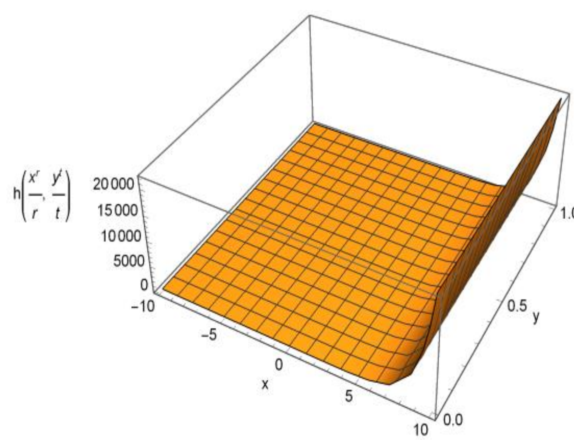


**Fig. 8:** Plots of the 3D approximate solutions of Eq .(50) obtained using the CGDT.

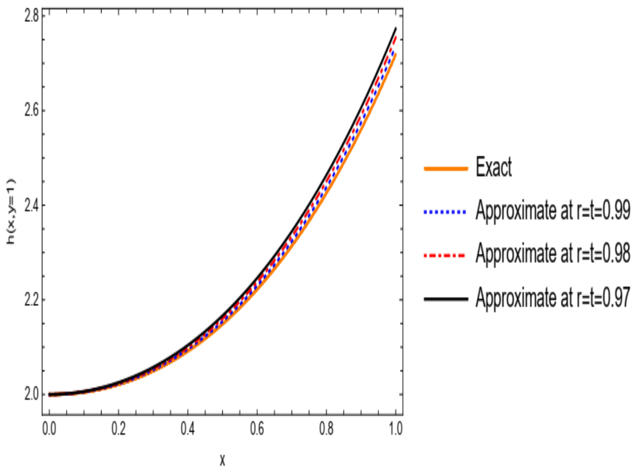


**Fig. 9:** The exact and CGDT method solution of Eq. (50) at $y = 1$ and $r = t = 1, 0.99, 0.98, 0.97$.

Moreover, in **Table 3** we present the absolute error of $h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)$ obtained by exact and CGDT at different values of $r, t$ and $y$.

**Table 3:** The absolute error of $h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)$ obtained by exact and CGDT at different values of $r, t$ and $y$.

| $x$ | $y$ | $r$ | $t$ | $Exact$ | $CGDT$ | $\lvert h_{Exact} - h_{CGDT} \rvert$ |
|---|---|---|---|---|---|---|
| 1 | 0.1 | 0.99 | 0.99 | 1.10517 | 1.10889 | 0.00372 |
| 1 | 0.2 | 0.99 | 0.99 | 1.1214 | 1.12596 | 0.00456 |
| 1 | 0.3 | 0.99 | 0.99 | 1.14986 | 1.15560 | 0.00574 |
| 1 | 0.1 | 0.98 | 0.98 | 1.10517 | 1.11277 | 0.00760 |
| 1 | 0.2 | 0.98 | 0.98 | 1.1214 | 1.13071 | 0.00931 |
| 1 | 0.3 | 0.98 | 0.98 | 1.14986 | 1.16159 | 0.01173 |
| 1 | 0.1 | 0.97 | 0.97 | 1.10517 | 1.1168 | 0.01163 |
| 1 | 0.2 | 0.97 | 0.97 | 1.12140 | 1.13566 | 0.01426 |
| 1 | 0.3 | 0.97 | 0.97 | 1.14986 | 1.16784 | 0.01798 |

**Example 5.4.**

Consider the following conformable fractional telegraph equation of the form

$$\frac{\partial^{2r}}{\partial x^{2r}}\left(h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right)-\frac{\partial^{2t}}{\partial y^{2t}}\left(h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right)-\frac{\partial^{t}}{\partial y^{t}}\left(h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right)-h\left(\frac{x^r}{r},\frac{y^t}{t}\right)=1-\left(\frac{x^r}{r}\right)^2-\frac{y^t}{t}. \tag{51}$$

subject to

$$h\left(\frac{x^r}{r},0\right)=\left(\frac{x^r}{r}\right)^2, \quad h\left(0,\frac{y^t}{t}\right)=\frac{y^t}{t}, \quad \frac{\partial^r}{\partial x^r}h\left(0,\frac{y^t}{t}\right)=0, \quad \frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},0\right)=1. \tag{52}$$

**Solution:**

By applying the conformable general double transform for both sides of Eq. (51), we have

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[\frac{\partial^{2r}}{\partial x^{2r}}\left(h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right)-\frac{\partial^{2t}}{\partial y^{2t}}\left(h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right)-\frac{\partial^{t}}{\partial y^{t}}\left(h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right)-h\left(\frac{x^r}{r},\frac{y^t}{t}\right)-1+\left(\frac{x^r}{r}\right)^2+\frac{y^t}{t}\right]=0. \tag{53}$$

Using the conformable convolution theorem and partial derivative properties of CGDT, we get

$$\begin{aligned}&\varphi^2(u)\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]-\varphi(u)\eta(u)\mathcal{G}_y^t\left[h\left(0,\frac{y^t}{t}\right)\right]-\eta(u)\,\mathcal{G}_y^t\left[\frac{\partial^r}{\partial x^r}h\left(0,\frac{y^t}{t}\right)\right]\\&\quad-w^2(v)\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]+w(v)\rho(v)\,\mathcal{G}_x^r\left[h\left(\frac{x^r}{r},0\right)\right]+\rho(v)\mathcal{G}_x^r\left[\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},0\right)\right]\\&\quad-w(v)\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]+\rho(v)\,\mathcal{G}_x^r\left[h\left(\frac{x^r}{r},0\right)\right]-\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]\\&\quad=\frac{\eta(u)\rho(v)}{w(v)\varphi(u)}-\frac{2\eta(u)\rho(v)}{\varphi^3(u)w(v)}-\frac{\eta(u)\rho(v)}{\varphi(u)w^2(v)}.\end{aligned} \tag{54}$$

Substituting

$$\mathcal{G}_y^t\left[h\left(0,\frac{y^t}{t}\right)\right]=\frac{\rho(v)}{w^2(v)}, \qquad \mathcal{G}_y^t\left[\frac{\partial^r}{\partial x^r}h\left(0,\frac{y^t}{t}\right)\right]=0,$$

$$\mathcal{G}_x^r\left[h\left(\frac{x^r}{r},0\right)\right]=\frac{2\eta(u)}{\varphi^3(u)}, \quad \mathcal{G}_x^r\left[\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},0\right)\right]=\frac{\eta(u)}{\varphi(u)}$$

in Eq. (54) and rearranging the terms, we have

$$\begin{aligned}&(\varphi^2(u)-w^2(v)-w(v)-1)\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]\\&\quad=\frac{\eta(u)\rho(v)}{w(v)\varphi(u)}-\frac{2\eta(u)\rho(v)}{\varphi^3(u)w(v)}-\frac{\eta(u)\rho(v)}{\varphi(u)w^2(v)}+\frac{\varphi(u)\eta(u)\rho(v)}{w^2(v)}-\frac{2\eta(u)w(v)\rho(v)}{\varphi^3(u)}\\&\quad-\frac{\eta(u\rho(v))}{\varphi(u)}-\frac{2\eta(u)\rho(v)}{\varphi^3(u)}.\end{aligned}$$

By simplifying, we obtain

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]=\frac{2\eta(u)\rho(v)}{\varphi^3(u)w(v)}+\frac{\eta(u)\rho(v)}{\varphi(u)w^2(v)}.$$

Thus, we can get the exact solution as

$$h\left(\frac{x^r}{r},\frac{y^t}{t}\right)=\left(\frac{x^r}{r}\right)^2+\frac{y^t}{t}. \tag{55}$$

for $r = t = 1$, the exact solution becomes

$$h(x, y) = x^2 + y.$$

This result is consistent with the result obtained in [33] by combination of Laplace transform and variational iteration method.

The following figures (Figure 10 and Figure 11 ) illustrates the 3D graph of the exact and approximate solutions of Example 5.4 at $r = t = 1$ and Figure 12, we sketch the approximate solutions of $h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)$ for Eq. (55) at $r = t = 1, 0.99, 0.98, 0.97$ in 2D.

**Fig. 10:** Plots of the 3D exact solutions of Eq .(55).

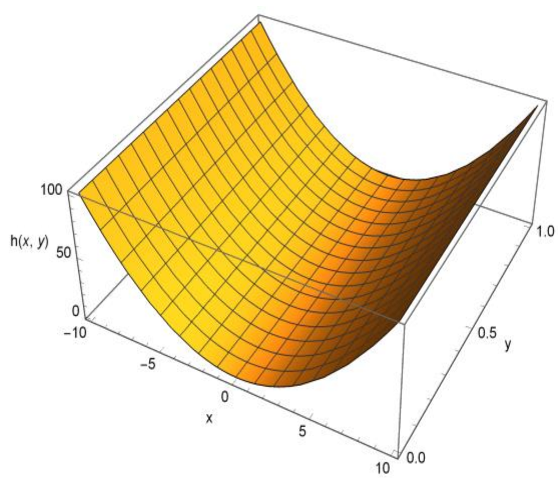


**Fig. 11:** Plots of the 3D approximate solutions of Eq .(55) obtained using the CGDT.

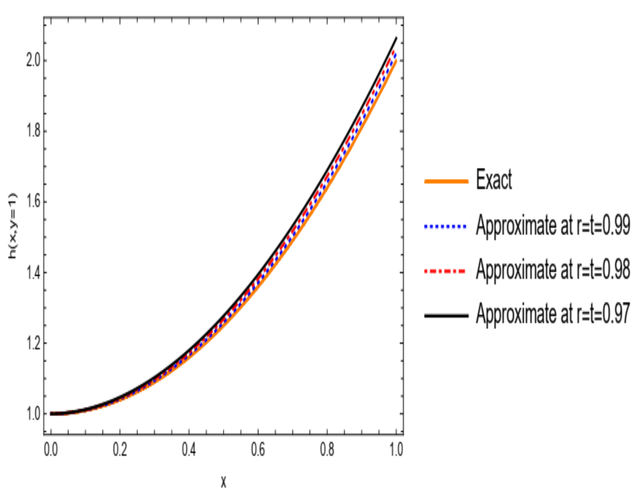


**Fig. 12:** The exact and CGDT method solution of Eq. (55) at $y = 1$ and $r = t = 1, 0.99, 0.98, 0.97$.

Moreover, in **Table 4** we present the absolute error of $h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)$ obtained by exact and CGDT at different values of $r, t$ and $y$.

**Table 4:** The absolute error of $h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)$ obtained by exact and CGDT at different values of $r, t$ and $y$

| $x$ | $y$ | $r$ | $t$ | $Exact$ | $CGDT$ | $\lvert h_{Exact} - h_{CGDT}\rvert$ |
|---|---|---|---|---|---|---|
| 1 | 0.1 | 0.99 | 0.99 | 0.11 | 0.114047 | 0.004047 |
| 1 | 0.2 | 0.99 | 0.99 | 0.14 | 0.145510 | 0.005510 |
| 1 | 0.3 | 0.99 | 0.99 | 0.19 | 0.197428 | 0.007428 |
| 1 | 0.1 | 0.98 | 0.98 | 0.11 | 0.118267 | 0.008267 |
| 1 | 0.2 | 0.98 | 0.98 | 0.14 | 0.151269 | 0.011269 |
| 1 | 0.3 | 0.98 | 0.98 | 0.19 | 0.205184 | 0.015184 |
| 1 | 0.1 | 0.97 | 0.97 | 0.11 | 0.122669 | 0.012669 |
| 1 | 0.2 | 0.97 | 0.97 | 0.14 | 0.157288 | 0.017288 |
| 1 | 0.3 | 0.97 | 0.97 | 0.19 | 0.213285 | 0.023285 |

**Example 5.5.**

Consider the following boundary value problem of conformable fractional partial differential equation

$$\frac{\partial^{2r}}{\partial x^{2r}}\left(h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)\right) - \frac{\partial^{2t}}{\partial y^{2t}}\left(h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)\right) = \sin\left(\pi \frac{x^r}{r}\right), \quad 0 < \frac{x^r}{r} < 1, \frac{y^t}{t} > 0. \tag{56}$$

subject to

$$h\left(\frac{x^r}{r}, 0\right) = 0 \quad , \quad h\left(0, \frac{y^t}{t}\right) = 0 \ , \ h\left(1, \frac{y^t}{t}\right) = 0, \quad \frac{\partial^t}{\partial y^t} h\left(\frac{x^r}{r}, 0\right) = 0. \tag{57}$$

**Solution:**

By applying the conformable general double transform for both sides of Eq. (56), we have

$$\mathcal{G}_x^r \mathcal{G}_y^t \left[\frac{\partial^{2r}}{\partial x^{2r}}\left(h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)\right) - \frac{\partial^{2t}}{\partial y^{2t}}\left(h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)\right)\right] = \mathcal{G}_x^r \mathcal{G}_y^t \left[\sin\left(\pi \frac{x^r}{r}\right)\right]. \tag{58}$$

Using the conformable convolution theorem and partial derivative properties of CGDT, we get

$$\begin{aligned} &\varphi^2(u)\mathcal{G}_x^r \mathcal{G}_y^t \left[h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)\right] - \varphi(u)\eta(u)\mathcal{G}_y^t \left[h\left(0, \frac{y^t}{t}\right)\right] - \eta(u)\, \mathcal{G}_y^t \left[\frac{\partial^r}{\partial x^r} h\left(0, \frac{y^t}{t}\right)\right] \\ &\quad - w^2(v)\mathcal{G}_x^r \mathcal{G}_y^t \left[h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)\right] + w(v)\rho(v)\, \mathcal{G}_x^r \left[h\left(\frac{x^r}{r}, 0\right)\right] + \rho(v)\mathcal{G}_x^r \left[\frac{\partial^t}{\partial y^t} h\left(\frac{x^r}{r}, 0\right)\right] \\ &= \frac{\rho(v)}{w(v)} \frac{\pi \eta(u)}{\varphi^2(u) + \pi^2}. \end{aligned} \tag{59}$$

Substituting

$$\mathcal{G}_y^t\left[h\left(0,\frac{y^t}{t}\right)\right]=0,\quad ,\mathcal{G}_x^r\left[h\left(\frac{x^r}{r},0\right)\right]=0,\quad \mathcal{G}_x^r\left[\frac{\partial^t}{\partial y^t}h\left(\frac{x^r}{r},0\right)\right]=0.$$

in Eq. (59) and rearranging the terms, we have

$$\left(\varphi^2(u)-w^2(v)\right)\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]-\eta(u)\,\mathcal{G}_y^t\left[\frac{\partial^r}{\partial x^r}h\left(0,\frac{y^t}{t}\right)\right]=\frac{\rho(v)}{w(v)}\frac{\pi\eta(u)}{\varphi^2(u)+\pi^2}.$$

By simplifying, we obtain

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]$$
$$=\frac{1}{\left(\varphi^2(u)-w^2(v)\right)}\,\mathcal{G}_y^t\left[\frac{\partial^r}{\partial x^r}h\left(0,\frac{y^t}{t}\right)\right]+\frac{\rho(v)\pi\eta(u)}{w(v)\left(\varphi^2(u)-w^2(v)\right)\left(\varphi^2(u)+\pi^2\right)}. \tag{60}$$

Eq. (60) can be written in the form

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]=\frac{1}{\left(\varphi^2(u)-w^2(v)\right)}\,\mathcal{G}_y^t\left[\frac{\partial^r}{\partial x^r}h\left(0,\frac{y^t}{t}\right)\right]+\frac{\rho(v)\pi\eta(u)(\varphi^2(u)+\pi^2)}{w(v)\left(\varphi^2(u)-w^2(v)\right)(\varphi^2(u)+\pi^2)\varphi^2(u)+\pi^2}.$$

Or

$$\mathcal{G}_x^r\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]=$$
$$\frac{1}{2w(v)}\left\{\frac{1}{w(v)-\varphi(u)}+\frac{1}{w(v)+\varphi(u)}\right\}\left\{\mathcal{G}_y^t\left[\frac{\partial^r}{\partial x^r}h\left(0,\frac{y^t}{t}\right)\right]+\frac{\rho(v)(w^2(v)+\pi^2)-\rho(v)w^2(v)}{\pi w(v)(w^2(v)+\pi^2)}\right\}+\left\{\frac{\rho(v)w^2(v)-\rho(v)(w^2(v)+\pi^2)}{\pi^2w(v)(w^2(v)+\pi^2)}\right\}\frac{\eta(u)\pi}{(\varphi^2(u)+\pi^2)}.$$

Taking the inverse general transform with respect to $u$, we obtain

$$\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]=\frac{1}{2w(v)}\left\{e^{w(v)\frac{x^r}{r}}+e^{-w(v)\frac{x^r}{r}}\right\}\left\{\mathcal{G}_y^t\left[\frac{\partial^r}{\partial x^r}h\left(0,\frac{y^t}{t}\right)\right]+\frac{1}{\pi}\left\{\frac{\rho(v)}{w(v)}-\frac{\rho(v)w(v)}{w^2(v)+\pi^2}\right\}\right\}+\frac{1}{\pi^2}\left\{\frac{\rho(v)w(v)}{w^2(v)+\pi^2}-\frac{\rho(v)}{w(v)}\right\}\sin\left(\pi\frac{x^r}{r}\right).$$

As $\frac{x^r}{r}\to 1$, then $h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\to 0$, thus

$$\mathcal{G}_y^t\left[\frac{\partial^r}{\partial x^r}h\left(0,\frac{y^t}{t}\right)\right]=-\frac{1}{\pi}\left\{\frac{\rho(v)}{w(v)}-\frac{\rho(v)w(v)}{w^2(v)+\pi^2}\right\}.$$

Thus,

$$\mathcal{G}_y^t\left[h\left(\frac{x^r}{r},\frac{y^t}{t}\right)\right]=\frac{1}{\pi^2}\sin\left(\pi\frac{x^r}{r}\right)\left\{\frac{\rho(v)w(v)}{w^2(v)+\pi^2}-\frac{\rho(v)}{w(v)}\right\}.$$

Taking the inverse general transform with respect to $v$, we obtain

$$h\left(\frac{x^r}{r},\frac{y^t}{t}\right)=\frac{1}{\pi^2}\sin\left(\pi\frac{x^r}{r}\right)\left\{\cos\left(\pi\frac{y^t}{t}\right)-1\right\}. \tag{61}$$

for $r=t=1$, the exact solution becomes

$$h(x,y)=\frac{1}{\pi^2}\sin(\pi x)\left\{\cos(\pi y)-1\right\}.$$

This result is consistent with the results obtained in [17,18] by the conformable double Laplace transform and the conformable double Sumudu transform.

The following figures (Figure 13 and Figure 14 ) illustrates the 3D graph of the exact and approximate solutions of Example 5.5 at $r=t=1$ and Figure 15, we sketch the approximate solutions of $h\left(\frac{x^r}{r},\frac{y^t}{t}\right)$ for Eq. (61) at $r=t=1,0.99,0.98,0.97$ in 2D.

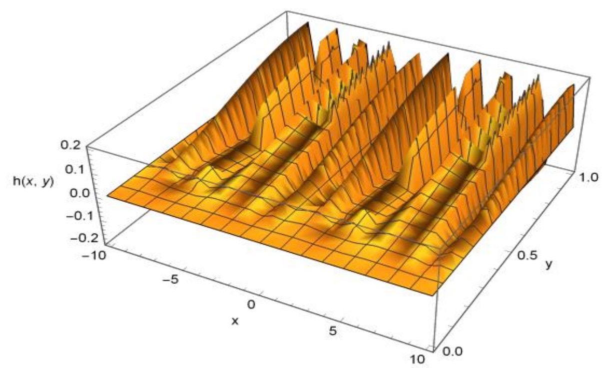


**Fig. 13:** Plots of the 3D exact solutions of Eq .(61).

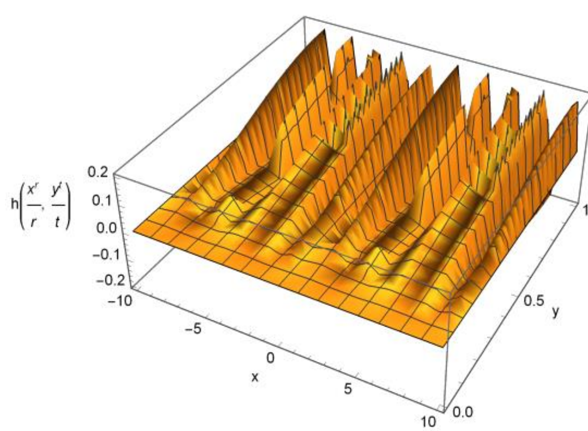


**Fig. 14:** Plots of the 3D approximate solutions of Eq .(61) obtained using the CGDT.

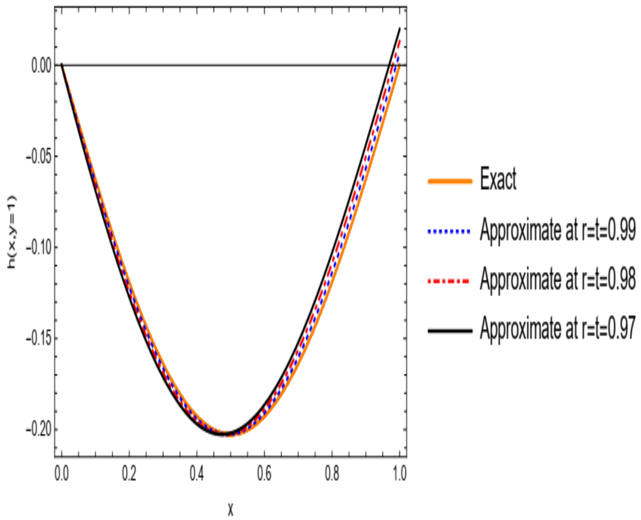


**Fig. 15:** The exact and CGDT method solution of Eq. (61) at $y = 1$ and $r = t = 1, 0.99, 0.98, 0.97$.

Moreover, in **Table 5** we present the absolute error of $h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)$ obtained by exact and CGDT at different values of $r, t$ and $y$.

**Table 5:** The absolute error of $h\left(\frac{x^r}{r}, \frac{y^t}{t}\right)$ obtained by exact and CGDT at different values of $r, t$ and $y$.

| $x$ | $y$ | $r$ | $t$ | $Exact$ | $CGDT$ | $\lvert h_{Exact} - h_{CGDT} \rvert$ |
|---|---|---|---|---|---|---|
| 1 | 0.1 | 0.99 | 0.99 | -0.0015324 | -0.0016894 | 0.0001569 |
| 1 | 0.2 | 0.99 | 0.99 | -0.0029148 | -0.0031832 | 0.0002684 |
| 1 | 0.3 | 0.99 | 0.99 | -0.0040119 | -0.0043484 | 0.0003365 |
| 1 | 0.1 | 0.98 | 0.98 | -0.0015324 | -0.0018628 | 0.0003304 |
| 1 | 0.2 | 0.98 | 0.98 | -0.0029148 | -0.0034766 | 0.0005618 |
| 1 | 0.3 | 0.98 | 0.98 | -0.0040119 | -0.0047126 | 0.0007007 |
| 1 | 0.1 | 0.97 | 0.97 | -0.0015324 | -0.0020544 | 0.0005220 |
| 1 | 0.2 | 0.97 | 0.97 | -0.0029148 | -0.0037971 | 0.0008823 |
| 1 | 0.3 | 0.97 | 0.97 | -0.0040119 | -0.0051066 | 0.0010947 |

All the above figures and tables of selected examples are obtained using Mathematica software 13.

## 6. Results and Discussion

In this section, we discuss the accuracy and applicability of the proposed method by comparing the approximate and exact solutions using tables and graphs. Figures 1, 2, 4, 5, 7, 8, 10, 11, 13 and 14 present the 3D plot solutions of Examples 1-5 obtained by the CGDT method and compared with the exact solutions at $r = t = 1$. These figures show that the approximate solutions obtained by the present method are almost identical to the exact solutions. Figures 3, 6, 9, 12 and 15 present comparisons between the linear plots of the approximate solutions from the proposed method and the exact solutions of Examples 1-5 for the value of $x$ and different values of the fractional degree $r$ and .$t$ From our view of the figures, we conclude that the numerical solutions are very close to the exact solutions when $r, t \to 1$. While tables from 1 to 5 provide us with a comparative study between the exact and approximate solutions for each example in terms of the absolute error at $x = 1, y = 0.1, 0.2, 0.3$ and $r = t = 0.99, 0.98, 0.97$ as is apparent from the tables and figures provided. Accordingly, we have confirmed that the solution using our current method converges quickly towards the exact solution

## 7. Conclusions

Inspired by the new general double transform in two dimensions, we introduced a novel approach called a conformable general double transform in two dimensional spaces. This new transform collects and implies the known conformable double Laplace transforms into the positive quadrant plane. Practically, we proved some essential properties related to the presented conformable general double transform such as partial derivative properties. To establish the effectiveness of this conformable, we apply it to solve some partial differential equations subject to appropriate conditions. It is very important to investigate the convergence behavior of the conformable general double transform. Furthermore, it should be pointed out that this novel transform can be combined with any numerical iteration methods in order to solve nonlinear PDEs and PDEs with variables coefficients. Furthermore, in the future I will investigate the possibility of applying this approach to other complex systems and fuzzy differential equations.

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