Applied Mathematics & Information Sciences Letters
An International Journal

http://dx.doi.org/10.18576/amisl/130101

Bounds for Covering Radius in Specific Classes of Codes over Finite Rings R

P. Chella Pandian [1]

[1]Department of Mathematics, Vivekananda College(Autonomous), Tiruvedakam West-625 234, Madurai, Tamil Nadu, India.

Received: 25 Oct. 2024, Revised: 25 Nov. 2024, Accepted: 25 Dec. 2024
Published online: 1 Jan. 2025

Abstract: This paper derives bounds for the covering radius of repetition codes for specific classes of codes with different weights and already discrete types of repetition codes $R = Z_3 + uZ_3 = \{a + ub : a, b \in Z_3\}, u^2 = 0$.

Keywords: Finite ring, Code, Covering radius, Generator matrix, Different weight.

1 Introduction

In coding theory for the last five decades, many researchers have been interested in codes over finite rings and the special types of rings $\mathbb{Z}_{2n}$, where $2n$ is a characteristic of the ring.

The best well known that codes can be constructed by cyclic codes and gray map over a finite ring $\mathbb{Z}_4$ for binary non-linear code [13] and many research articles have indicated codes over a finite ring $\mathbb{Z}_4$ received much attention [1,3,4]. Coding theory to find the maximum error-correcting capability of codes by using the most essentially parameter of covering radius of a code.

In binary code, [2,8,9], the researchers are studying for covering radius of linear codes and nonlinear codes can be obtained from codes over a finite ring $\mathbb{Z}_4$ via the gray mapping. In [5,6], the author finds bounds of covering radius with different types of repetition codes with respect to various weights, by use this, so found to the another structure of ring in this paper.

In this correspondence, to determine the Repetition Codes of some classes of blocks with a cover radius of the codes on the finite commutative chain ring $R = \{a + ub : a, b \in Z_3\}$ of the integer modulo 3 assigned to different weight.

2 Preliminaries

Let R be a finite commutative chain ring, where $R = Z_3 + uZ_3 = \{a + ub : a, b \in Z_3\} = \{0, 1, 2, u, u_1, u_2, u_3, u_4, u_5\}$, with $u^2 = 0, u_1 = 1 + u, u_2 = 2 + u, u_3 = 2u, u_4 = 1 + 2u, u_5 = 2 + 2u$ and $Z_3 = \{0, 1, 2\}$ with characteristic 3. If $C \subseteq R$, then C is said to be a *code*. A code C is called the *linear code*, if the ring R is an R-submodule of R^n, where n is the length of a code. The elements of the code $C \subseteq R$ are called a codeword of C.

A Gray Map $g : (Z_3 + uZ_3) \to Z_3^2$ as

$$g(0) = 00, g(1) = 01, g(2) = 02,$$

$$g(u) = 10, g(1 + u) = 11, g(2 + u) = 12,$$

$$g(2u) = 20, g(1 + 2u) = 21, g(2 + 2u) = 22,$$

then the Gray map $g_1 : (Z_3 + uZ_3)^n \to Z_3^{2n}$ is defined by $g_1(x) = (g(x_1), g(x_2), \cdots, g(x_n))$, where $x = (x_1, x_2, \cdots, x_n)$ in [12].

In [7], Let $x \in R$. The Bachoc weight of x is defined as

$$w_B(x) = \begin{cases} 0 \text{ if } x = 0 \\ 1 \text{ if } x = 1, 2, u_1, u_2, u_4, u_5, \\ 3 \text{ if } x = u, u_3. \end{cases}$$

Let $x_i \in R_{,i=0 \text{ to } n-1}$ be the codeword of Bachoc weight of x_i is defined as $\sum_{i=0}^{n-1} w_B(x_i)$. If $c_1, c_2 \in C$, be any two distinct codewords of Bachoc distance is defined as $d_B(C) = \{d_B(c_1, c_2) | c_1 - c_2 \neq 0, \text{ and } c_1, c_2 \in C\}$.

The minimum Bachoc weight of C is

$$d_B(C) = \min\{d_B(c_1, c_2) | c_1 - c_2 \neq 0, \text{ and } c_1, c_2 \in C\}.$$

* Corresponding author e-mail: pchellapandian@vivekanandacollege.ac.in

In C is a *linear code*, thus

$$d_B(C) = \min\{w_B(c) | c \neq 0 \in C\}.$$

Therefore, the minimum distance is equal to the minimum weight. That is,

$$d_B(c_1, c_2) = w_B(c_1 - c_2).$$

Let $C \subseteq R^n$ is a linear code, where n is a length of code, the number of codewords N and the minimum Bachoc distance d_B, is said to be an (n, N, d_B) code in R.

In C is a linear code of length n over R. Define, $d_B(z, C) = \min\{d_B(z, c) | \forall c \in C\}$, for any $z \in R^n$, here $d_B(z, C)$ be the Bachoc distance between z and C.

*Example 1.*Let $x = 1\ u\ u_1\ u_3\ u_5 \in R$. Then, the Bachoc weight of x is $w_B(x) = w_B(1) + w_B(u) + w_B(u_1) + w_B(u_3) + w_B(u_5) = 1+3+1+3+1 = 9$.

In Gray weight, let $x = (x_1, x_2, \cdots, x_n)$ be a codeword of codes over R is defined as follows

$$w_G(x) = \begin{cases} 0 \text{ if } x = 0 \\ 1 \text{ if } x = 1, 2, u \text{ and } u_3 \\ 2 \text{ if otherwise} \end{cases}$$

in [12]. Use this, found to covering radius of code.

The parameters of Gray weight code are an (n, N, d_G). In the Gray distance(weight), let $c_1, c_2 \in R^n$ be any two different codewords is defined as $d_G(c_1, c_2) = wt_G(c_1 - c_2)$. Let C be a linear code of length n over R. Then $d_G(z, C) = \min\{d_B(z, c) | \forall c \in C\}$, for any $z \in R^n$.

Let $x = (x_1, x_2, \cdots, x_n)$ be a codeword of codes over R and use ref.[14], the Lee weight of x as given

$$w_L(x) = \begin{cases} 0 \text{ if } x = 0 \\ 1 \text{ if } x = 1, u_5 \\ 2 \text{ if } x = 2, u_4 \\ 3 \text{ if } x = u, u_1, u_2, u_3. \end{cases}$$

The parameters of the Lee weight code are (n, N, d_L). In Lee the distance weight between the codewords c_1 and $c_2 \in R^n$ is defined as $d_L(c_1, c_2) = wt_L(c_1 - c_2)$ and also $d_L(z, C) = \min\{d_L(z, c) | \forall c \in C\}$, for any $z \in R^n$.

The Chinese Euclidean weight of x is

$$w_{CE}(x) = \begin{cases} 0 \text{ if } x = 0 \\ 1 \text{ if } x = 1, u_5 \\ 2 \text{ if } x = 2, u_4 \\ 3 \text{ if } x = u, u_3 \\ 4 \text{ if } x = u_1, u_2 \end{cases}$$

in [11], where $x = (x_1, x_2, \cdots, x_n)$ be a codeword of codes over R.

In C is a linear code with Chinese Euclidean weight, is an (n, N, d_{CE}) code. Define, $d_{CE}(c_1, c_2) = wt_{CE}(c_1 - c_2)$, where $c_1, c_2 \in R^n$ and $d_{CE}(z, C) = \min\{d_{CE}(z, c) | \forall c \in C\}$, for any $z \in R^n$.

In Homogeneous weight, if $x = (x_1, x_2, \cdots, x_n)$ be a codeword in R and use in[13], the Homogeneous weight of x is

$$w_H(x) = \begin{cases} 0 \text{ if } x = 0 \\ 3 \text{ if } x = u, u_3, \\ 2 \text{ if otherwise.} \end{cases}$$

In C is a linear code with homogeneous weight, is a (n, N, d_H) code. Define $d_H(c_1, c_2) = wt_H(c_1 - c_2)$, where $c_1, c_2 \in R^n$ and $d_H(z, C) = \min\{d_H(z, c) | \forall c \in C\}$, for any $z \in R^n$.

*Example 2.*Let $x = 1\ u\ u_1\ u_3\ u_5 \in R$. Then, $w_G(x) = w_G(1) + w_G(u) + w_G(u_1) + w_G(u_3) + w_G(u_5)$. From the Gray weight, so $w_G(x) = 1+1+2+1+2 = 7$. Similarly to the other weight.

So,

Code	$w_G(x)$	$w_L(x)$	$w_{CE}(x)$	$w_H(x)$
x	7	13	12	12

3 Repetition code with Covering radius of code in R

Let d be the distance of a code C in R with assigned to alternate weight, such as Bachoc weight, Gray weight, Lee weight, Chinese Euclidean weight and Homogeneous weight. The *covering radius* of a code C is

$$r_d(C) = \max_{v \in R^n} \left\{ \min_{c \in C} \{d(v, c)\} \right\},$$

where C is a code and $r_d(C)$ is a covering radius of the code C.

In $F_q = \{0, 1, \beta_2, \cdots, \beta_{q-1}\}$ is a finite field. Let C be a q-ary *repetition code* C over F_q. That is $C = \{\bar{\beta} = (\beta\beta\cdots\beta) | \beta \in F_q\}$ and the repetition code C is an $[n, 1, n]$ code. Therefore, the covering radius of the code C is $\lfloor \frac{n(q-1)}{q} \rfloor$ in [10].

Let C be a block repetition code of size n, the parameter of C is an $[n(q-1), 1, n(q-1)]$ be a generated by $G = [\overbrace{11\cdots1}^{n}\overbrace{\beta_2\beta_2\cdots\beta_2}^{n}\cdots\overbrace{\beta_{q-1}\beta_{q-1}\cdots\beta_{q-1}}^{n}]$. By above, thus the covering radius of the code C is $\lfloor \frac{n(q-1)^2}{q} \rfloor$, since it will be equivalent to a repetition code of length $(q-1)n$.

A linear code $C \subseteq R$ is said to be a Generator matrix(G), if the basis elements in a row of matrix.

Consider the repetition code over R, there are two type of repetition codes of length n viz.

1.Type I-(A Generator matrix(G_I) with unit element in R and its generated by the code C_I)
2.Type II-(A Generator matrix(G_{II}) with zero divisor element in R and its generated by the code C_{II})

Type	Generator Matrix	Parameters
Type I	$G.M.T\ I$	$[n,1,n], d_i = n$
Type II	$G.M.T\ II$	$(n,3,3n), d_j = 3n$

here, $G.M.TI = [\overbrace{1\cdots1}^{n}], [\overbrace{2\cdots2}^{n}], [\overbrace{u_1\cdots u_1}^{n}], [\overbrace{u_2\cdots u_2}^{n}], [\overbrace{u_4\cdots u_4}^{n}], [\overbrace{u_5\cdots u_5}^{n}]$ and $G.M.TII = [\overbrace{uu\cdots u}^{n}], [\overbrace{u_3u_3\cdots u_3}^{n}], [\overbrace{uu_3\ uu_3\cdots uu_3}^{n}][\overbrace{u_3u\ u_3u\cdots u_3u}^{n}]$

Theorem 1. *Let $G_{I(II)}$ be a generator matrix with unit element(zero divisor element) in R and it is generated by the code $C_{I(II)}$, then*

1. $r_B(C_I) = \frac{4n}{3}$,
2. $\frac{3n}{2} \le r_B(C_{II}) \le 2n$, *where $r_B(C_{I(II)})$ is a covering radius of $C_{I(II)}$ with bachoc distance(weight).*

Proof. Let $x \in R^n$ with η_0 times $0'$s, η_1 times $1'$s, η_2 times $2'$s, η_3 times u's, η_4 times u_1's, η_5 times u_2's, η_6 times u_3's, η_7 times u_4's, η_8 times u_5's in x and $\sum_i \eta_i = n$ and the code $c_i \in \{\beta(C_I)|\beta \in R^n\}$, where $i = 0$ to 8. Then

$$\begin{aligned} d_B(x,c_0) &= wt_B(x - 00\cdots0) \\ &= 0\eta_0 + 1\eta_1 + 2\eta_2 + u\eta_3 + u_1\eta_4 + u_2\eta_5 + u_3\eta_6 \\ &\quad + u_4\eta_7 + u_5\eta_8 \\ &= \eta_1 + \eta_2 + 3\eta_3 + \eta_4 + \eta_5 + 3\eta_6 + \eta_7 + \eta_8 \\ d_B(x,c_0) &= n - \eta_0 + 2\eta_3 + 2\eta_6. \end{aligned}$$

Similarly,
$d_B(x,c_1) = n - \eta_1 + 2\eta_4 + 2\eta_7$, $d_B(x,c_2) = n - \eta_2 + 2\eta_5 + 2\eta_8$,
$d_B(x,c_3) = n - \eta_3 + 2\eta_6 + 2\eta_0$, $d_B(x,c_4) = n - \eta_4 + 2\eta_7 + 2\eta_1$,
$d_B(x,c_5) = n - \eta_5 + 2\eta_8 + 2\eta_2$, $d_B(x,c_6) = n - \eta_6 + 2\eta_0 + 2\eta_3$,
$d_B(x,c_7) = n - \eta_7 + 2\eta_1 + 2\eta_4$, $d_B(x,c_8) = n - \eta_8 + 2\eta_2 + 2\eta_5$.

Therefore, $d_B(x,C_I) = \min\{d_B(x,c_i)|i = 0 \text{ to } 8\} \le \frac{4n}{3}$. Thus, $r_B(C_I) \le \frac{4n}{3}$.

Now, let us take $y = \overbrace{00\cdots0}^{b}\overbrace{11\cdots1}^{b}\overbrace{22\cdots2}^{b}\overbrace{uu\cdots u}^{b}\overbrace{u_1u_1\cdots u_1}^{b}\overbrace{u_2u_2\cdots u_2}^{b}\overbrace{u_3u_3\cdots u_3}^{b}\overbrace{u_4u_4\cdots u_4}^{b}\overbrace{u_5u_5\cdots u_5}^{n-8b} \in R^n$, where $b = \lfloor\frac{n}{9}\rfloor$. So, $d_B(y,c_i) = 12b$, $i = 0$ to 8. Thus, $r_B(C_I) \ge \min\{d_B(y,c_i)|\ i = 0 \text{ to } 8\} \ge \frac{4n}{3}$ and hence, $r_B(C_I) = \frac{4n}{3}$.

Let $x = \overbrace{uu\cdots u}^{\frac{n}{2}}\overbrace{000\cdots0}^{\frac{n}{2}} \in R^n$. The code $C_{II} = \{\beta(uu\cdots u) \mid \beta \in R^n\}$, that is $C_{II} = \{00\cdots0, uu\cdots u, u_3u_3\cdots u_3\}$ is generated by Type-II. Thus, $r_B(C_{II}) \ge \frac{3n}{2}$.

If $x \in R^n$ be any codeword and take x has η_0 links $0'$s, η_1 links $1'$s, η_2 links $2'$s, η_3 links u's, η_4 links u_1's, η_5 links u_2's, η_6 links u_3's, η_7 links u_4's η_8 links u_5's, with $\sum_i \eta_i = n$, where $i = 0$ to 8. Then, $r_B(C_{II}) \le 2n$.

Theorem 2. *Prove that the following*

1. $r_G(C_I) = \frac{4n}{3}, r_G(C_{II}) = n$,
2. $r_L(C_I) = 2n = r_L(C_{II})$,
3. $r_{CE}(C_I) = \frac{20n}{9}, \frac{3n}{2} \le r_{CE}(C_{II}) \le 2n$ *and*
4. $r_H(C_I) = 2n, \frac{3n}{2} \le r_H(C_{II}) \le 2n$.

Proof. The methods of proof is pursue Theorem 1, by using the generator matrix is G_I and G_{II} with different weight, such as $w_G(x), w_L(x), w_{CE}(x)$ and $w_H(x)$.

4 Block repetition code for the same size of length(n)

Let $G_1 = [\overbrace{11\cdots1}^{n}\overbrace{uu\cdots u}^{n}]$ be the generated matrix of two block repetition codes of length is n and it is denoted $BRep^{2n}$ code and its parameters of $BRep^{2n}$ code is an $[2n,1,3n,3n,3n,3n,3n]$.

Theorem 3. *Prove the following*

1. $r_B(BRep^{2n}) = \frac{8n}{3}$,
2. $r_G(BRep^{2n}) = \frac{7n}{3}$,
3. $r_L(BRep^{2n}) = 4n$,
4. $r_{CE}(BRep^{2n}) = \frac{38n}{9}$ *and*
5. $\frac{7n}{2} \le r_H(BRep^{2n}) \le 4n$.

Proof. A generator matrix G_1 and [8] and use to theorem 1, then

$$r_B(BRep^{2n}) \ge \frac{8n}{3}. \tag{1}$$

Let $y = (z_1 \mid z_2) \in R^{2n}$, where $z_1, z_2 \in R^{2n}$. Let us take in z_1, η_0 appears $0'$s, η_1 appears $1'$s, η_2 appears $2'$s, η_3 appears u's, η_4 appears u_1's, η_5 appears u_2's, η_6 appears u_3's, η_7 appears u_4's, η_8 appears u_5's and z_2, η_0 appears $0'$s, η_1 appears $1'$s, η_2 appears $2'$s, η_3 appears u's η_4 appears u_1's, η_5 appears u_2's, η_6 appears u_3's, η_7 appears u_4's, η_8 appears u_5's, with $\sum_j r_j = \sum_j s_j = n$ and $c_j \in \{\alpha(G_1)|\alpha \in R^{2n}\}, j = 0$ to 8.

Then, $d_B(y, BRep^{2n}) = \min\{d_B(y,c_j)|j = 0 \text{ to } 8\}$ is less than or equal to $\frac{4n}{3} + \frac{4n}{3} = \frac{8n}{3}$. Thus, $d_B(y, BRep^{2n}) \le \frac{8n}{3}$. Hence,

$$r_B(BRep^{2n}) \le \frac{8n}{3} \tag{2}$$

By (1) and (2), thus

$$r_B(BRep^{2n}) = \frac{8n}{3}.$$

The remaining Proof of the Theorem 3 is follows from first part and so, done.

Corollary 1. *Let*

$$G_I = [\overbrace{11\cdots1}^{n}\overbrace{22\cdots2}^{n}\overbrace{u_1u_1\cdots u_1}^{n}\overbrace{u_2u_2\cdots u_2}^{n}\overbrace{u_4u_4\cdots u_4}^{n}\overbrace{u_5u_5\cdots u_5}^{n}] \quad (3)$$

is a generator matrix with unit element in R. Then

1. $r_B(BRep^{6n}) = 8n$,
2. $r_G(BRep^{6n}) = 8n$,
3. $r_L(BRep^{6n}) = 12n$,
4. $r_{CE}(BRep^{6n}) = \frac{40n}{3}$ *and*
5. $r_H(BRep^{6n}) = 12n$.

Proof. In (3) and use to Theorem 1, 2 and 3.

Corollary 2. *Let*

$$G_{II} = [\overbrace{uu\cdots u}^{n}\overbrace{u_3u_3\cdots u_3}^{n}] \quad (4)$$

is a generator matrix with zero divisor element in R. Prove the following

1. $3n \leq r_B(BRep^{2n}) \leq 4n$,
2. $r_G(BRep^{2n}) = 2n$,
3. $r_L(BRep^{2n}) = 4n$,
4. $3n \leq r_{CE}(BRep^{2n}) \leq 4n$ *and*
5. $3n \leq r_H(BRep^{2n}) \leq 4n$.

Proof. In (4) is apply to Theorem 1, 2 and 3.

5 Bolck repetition code with different size of the length (l_1, l_2)

Let

$$G = [\overbrace{11\cdots1}^{l_1}\overbrace{uu\cdots u}^{l_2}] \quad (5)$$

be the generated matrix for the two different blocks of size l_1 and l_2 length of the repetition code $BRep^{l_1+l_2}$ and its parameters of $BRep^{l_1+l_2}$ code is an $[l_1+l_2, 1, \min\{3l_1, l_1+3l_2\}, \min\{l_1, l_1+l_2\}, \min\{3l_1, l_1+3l_2\}, \min\{3l_1, l_1+3l_2\}, \min\{3l_1, 2l_1+3l_2\}]$

Theorem 4. *Show that*

1. $r_B(BRep^{l_1+l_2}) = \frac{4(l_1+l_2)}{3}$,
2. $r_G(BRep^{l_1+l_2}) = \frac{4(l_1+l_2)}{3}$,
3. $r_L(BRep^{l_1+l_2}) = 2(l_1+l_2)$,
4. $r_{CE}(BRep^{l_1+l_2}) = \frac{20l_1}{9} + 2l_2$ *and*
5. $r_H(BRep^{l_1+l_2}) = 2(l_1+l_2)$.

Proof. A generator matrix (5), use to Theorem 3 and apply the two different size of length(l_1, l_2).

Corollary 3. *Let*

$$G_{II} = [\overbrace{uu\cdots u}^{l_1}\overbrace{u_3u_3\cdots u_3}^{l_2}] \quad (6)$$

is a generator matrix with zero divisor element and different length in R. Find the following

1. $\frac{4(l_1+l_2)}{3} \leq r_B(BRep^{l_1+l_2}) \leq 2(l_1+l_2)$,
2. $r_G(BRep^{l_1+l_2}) = l_1+l_2$,
3. $r_L(BRep^{l_1+l_2}) = 2(l_1+l_2)$,
4. $\frac{3(l_1+l_2)}{2} \leq r_{CE}(BRep^{l_1+l_2}) \leq 2(l_1+l_2)$ *and*
5. $\frac{3(l_1+l_2)}{2} \leq r_H(BRep^{l_1+l_2}) \leq 2(l_1+l_2)$.

Proof. In (6) with different length is applied to Theorem 4.

Corollary 4. *Let*

$$G_I = [\overbrace{11\cdots1}^{l_1}\overbrace{22\cdots2}^{l_2}\overbrace{u_1u_1\cdots u_1}^{l_3}\overbrace{u_2u_2\cdots u_2}^{l_4}\overbrace{u_4u_4\cdots u_4}^{l_5}\overbrace{u_5u_5\cdots u_5}^{l_6}]. \quad (7)$$

be a generator matrix with unit element and different length in R. Then

1. $r_B(BRep^{\sum l_i}) = \frac{4(\sum l_i)}{3}$,
2. $r_G(BRep^{\sum l_i}) = \frac{4(\sum l_i)}{3}$,
3. $r_L(BRep^{\sum l_i}) = 2(\sum l_i)$,
4. $r_{CE}(BRep^{\sum l_i}) = \frac{20(\sum l_i)}{9}$ *and*
5. $r_H(BRep^{\sum l_i}) = 2(\sum l_i)$, *where* $i = 1,2,3,4,5,6$.

Proof. In (7) with different length is applied to Theorem 4.

6 Conclusion

This work is for finite ring with nine elements and estimation of lower bound and upper bound for the covering radius of repetition codes for specific classes of codes with different weights and also discrete types of repetition codes are determined, use this covering radius reduce to the error in communication channel of all electronic field, in particularly information theory, then got to the petter message.

References

[1] T. Aoki, P. Gaborit, M. Harada, M. Ozeki and P. Solé, *On the covering radius of $\mathbb{Z}_4$ codes and their lattices,* IEEE Trans. Inform. Theory, vol. 45, no. 6, pp. 2162-2168(1999).

[2] E. Bardellotto and F. Fabris, *Binary List Decoding Beyond Covering Radius,* Journal of Information and Optimization Sciences, 35(5-6), 561-570(2014).

[3] A. Bonnecaze, P. Solé , C. Bachoc and B. Mourrain, *Type II codes over $\mathbb{Z}_4$,* IEEE Trans. Inform. Theory, 43, 969-976(1997).

[4] A. Bonnecaze and P. Udaya, *Cyclic Codes and Self-Dual Codes over $F_2 + uF_2$,* IEEE Trans. Inform. Theory, 45(4), 1250-1254(1999).

[5] P. ChellaPandian, *On Covering Radius of Codes over $R = \mathbb{Z}_2 + u\mathbb{Z}_2$, where $u^2 = 0$, using chinese euclidean distance,* Journal of Discrete Mathematics, Algorithms and Applications, Vol. 9, No. 2, pp. 1750017(1-8)(2017).

[6] P. ChellaPandian, *On Covering Radius of Codes Over $R = \mathbb{Z}_2 + u\mathbb{Z}_2$, where $u^2 = 0$, Using Bachoc Distance,* International Journal of Mathematics And its Applications, Vol. 5, no. 4-C, pp.277-282(2017).

[7] Christine Bachoc, *Applications of Coding Theory to the Construction of Modular Lattices,* Journal of combinatorial theory, Series A 78, 92 119(1997).
[8] D. G. Cohen, G. M. Karpovsky, F. H. Mattson and R. J. Schatz , *Covering radius-Survey and recent results,* IEEE Trans. Inform. Theory, vol.31, no. 3, pp. 328-343(1985).
[9] C. Cohen, A. Lobstein and A. J. N. Sloane, *Further Results on the Covering Radius of Codes,* IEEE Trans. Inform. Theory, vol. 32, no. 5, pp. 680-694(1986).
[10] C. Durairajan, *On Covering Codes and Covering Radius of Some Optimal Codes,* Ph.D. Thesis, Department of Mathematics, IIT Kanpur, (1996).
[11] K. M. Gupta, G. D. Glynn and T. Aaron Gulliver, *On Senary Simplex Codes.* In International Symposium, On Applied Algebra, Algebraic Algorithms and Error-Correcting Codes (Springer, Berlin Heidelberg), pp. 112–121(2001).
[12] T. Gulliver and M. Harada, *Codes Over* $F_3 + uF_3$ *and Improvements to the Bounds on Ternary Linear Codes,* University of Canterbury and Yamagata University, New Zealand and Japan, June(1998).
[13] R. A. Hammons, P. V. Kumar, A. R. Calderbank and P. Sol'e, *The* Z_4*-linearity of kerdock, preparata, gothals and related codes,* IEEE Trans. Inform. Theory, 40, 301-319(1994).
[14] B. Yildiz and Odemis Ozger, *A generalization of the lee weight to* $\mathbb{Z}_{p^k}$, TWMS J. App. Eng. Math., Vol.2, No.2, pp.145-153(2012).

P. Chella Pandian received his M. Phil. Mathematics from Madurai Kamaraj University, Madurai, Tamilnad, India and Ph. D. Mathematics from Bharathidasan University, Tiruchirappalli, Tamilnadu, India. Currently, he is working as an Assistant Professor in the Department of Mathematics, Vivekananda College(Autonomous), Madurai, Tamilnadu, India. His research interests are in the areas of Algebraic Coding Theory(Algebra). He has published more than 25 research articles in reputed national/international journals. He is a reviewer and editor of mathematical journals. He has supervised 20 Master theses and 10 M. Phil. theses.