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Definite Integral involving Bessel Function

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Abstract: In this paper we have developed certain definite integral involving Bessel Function and Hypergeometric function. The results appears here are new.

Keywords: Bessel Function, Hypergeometric Function, Pochhammer symbol.

1 Introduction

Yurry A. Brychkov[Brychkov p.199(4.7.4.1)] has derived the below formula

$$\int_0^1 \tau \, \log\tau \, J_0(a\tau)\, d\tau = -\frac{1}{a^2}\left[J_0(a)-1\right] \tag{1.1}$$

First kind Bessel function is denoted by $J_\tau(y)$, and defined as

$$J_\tau(y) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!\,\Gamma(k+\tau+1)}\left(\frac{y}{2}\right)^{2k+\tau} \tag{1.2}$$

where $\Gamma(\varpi)$ is the gamma function.
The first kind of modified Bessel function is defined as

$$I_\omega(z) = \iota^{-\omega} J_\omega(\iota z) = \sum_{i=0}^{\infty} \frac{1}{i!\,\Gamma(i+\omega+1)}\left(\frac{z}{2}\right)^{2i+\omega} \tag{1.3}$$

Generalized hypergeometric function ${}_pF_q(a_1,...,a_p;b_1,...,b_q;z)$ is a function which can be described in the form of a hypergeometric series, i.e., a series for which the ratio of consecutive terms can be written

$$\frac{c_{m+1}}{c_m} = \frac{P(m)}{Q(m)} = \frac{(m+a_1)(m+a_2)...(m+a_p)}{(m+b_1)(m+b_2)...(m+b_q)(m+1)}\,z. \tag{1.4}$$

Where $m+1$ in the denominator is present for documentary causes of notation[Koepf p.12(2.9)], and the developing generalized hypergeometric function is written as

$${}_pF_q\left[\begin{matrix} a_1,a_2,\cdots,a_p\,; \\ b_1,b_2,\cdots,b_q\,; \end{matrix}\; z\right] = \sum_{m=0}^{\infty} \frac{(a_1)_m(a_2)_m\cdots(a_p)_m z^m}{(b_1)_m(b_2)_m\cdots(b_q)_m m!} \tag{1.5}$$

where the parameters $b_1,b_2,\cdots,b_q$ are positive integers.

The ${}_pF_q$ series converges for all finite z if $p \le q$, converges for $|\, z \,| < 1$ if $p = q+1$, diverges for all z , $z \neq 0$ if $p > q+1$[Luke p.156(3)].

The function ${}_2F_1(a,b;c;z)$ corresponding to $p = 2, q = 1$, is the first hypergeometric function to be studied (and, in general, arises the most frequently in physical problems), and so is frequently known as "the" hypergeometric equation or, more explicitly, Gauss's hypergeometric function [Gauss p.123-162].

In mathematics, the falling factorial or Pochhammer symbol (sometimes called the descending factorial,falling sequential product, or lower factorial) is defined as the polynomial[Steffensen p.8]

$$(p)_n = p(p-1)(p-2)\cdots(p-n+1) = \prod_{k=1}^{n}(p-k+1) = \prod_{k=0}^{n-1}(p-k) \tag{1.6}$$

2 Main Formulae of the Integration

$$\int_0^1 q^2 \, \log q \, J_1(aq)\, dq + = \frac{2\,J_0(a) + a\,J_1(a) - 2}{a^3} \tag{2.1}$$

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$$\int_0^1 q^3 \log q\, J_1(aq)\, dq+ = -\frac{1}{50}\, a\, {}_2F_3\left(\frac{5}{2},\frac{5}{2};2,\frac{7}{2},\frac{7}{2};-\frac{a^2}{4}\right) \quad (2.2)$$

$$\int_0^1 q^4 \log q\, J_1(aq)\, dq+ = \frac{2(3a^2-8)J_0(a)+a(a^2-20)J_1(a)+16}{a^5} \quad (2.3)$$

$$\int_0^1 q^5 \log q\, J_1(aq)\, dq+ = -\frac{1}{98}\, a\, {}_2F_3\left(\frac{7}{2},\frac{7}{2};2,\frac{9}{2},\frac{9}{2};-\frac{a^2}{4}\right) \quad (2.4)$$

$$\int_0^1 q^6 \log q\, J_1(aq)\, dq+ = \frac{a^5J_5(a)+8(a^2+32)aJ_1(a)-2(a^2-8)(a^2+24)J_0(a)-384}{a^7} \quad (2.5)$$

$$\int_0^1 q^7 \log q\, J_1(aq)\, dq+ = -\frac{1}{162}\, a\, {}_2F_3\left(\frac{9}{2},\frac{9}{2};2,\frac{11}{2},\frac{11}{2};-\frac{a^2}{4}\right) \quad (2.6)$$

$$\int_0^1 q^8 \log q\, J_1(aq)\, dq+ = \frac{1}{a^9}\left[2(7a^6-408a^4+6720a^2-9216)J_0(a)+ +a(a^6-132a^4+4416a^2-36096)J_1(a)+18432\right] \quad (2.7)$$

$$\int_0^1 q^9 \log q\, J_1(aq)\, dq+ = -\frac{1}{242}\, a\, {}_2F_3\left(\frac{11}{2},\frac{11}{2};2,\frac{13}{2},\frac{13}{2};-\frac{a^2}{4}\right) \quad (2.8)$$

$$\int_0^1 q^{10} \log q\, J_1(aq)\, dq+ = \frac{1}{a^{11}}\left[2(9a^8-992a^6+43008a^4-620544a^2+737280)J_0(a)+ +a(a^8-224a^6+15744a^4-436224a^2+3219456)J_1(a)-1474560\right] \quad (2.9)$$

$$\int_0^1 q^{20} \log q\, J_1(aq)\, dq+ = \frac{1}{20a}\left[{}_1F_2\left(10;1,11;-\frac{a^2}{4}\right)-{}_2F_3\left(10,10;1,11,11;-\frac{a^2}{4}\right)\right] \quad (2.10)$$

$$\int_0^1 q^{30} \log q\, J_1(aq)\, dq+ = -\frac{1}{2048}\, a\, {}_2F_3\left(16,16;2,17,17;-\frac{a^2}{4}\right) \quad (2.11)$$

$$\int_0^1 q^{32} \log q\, J_1(aq)\, dq+ = -\frac{1}{2312}\, a\, {}_2F_3\left(17,17;2,18,18;-\frac{a^2}{4}\right) \quad (2.12)$$

$$\int_0^1 q^{33} \log q\, J_1(aq)\, dq+ = -\frac{1}{2450}\, a\, {}_2F_3\left(\frac{35}{2},\frac{35}{2};2,\frac{37}{2},\frac{37}{2};-\frac{a^2}{4}\right) \quad (2.13)$$

$$\int_0^1 q^{36} \log q\, J_1(aq)\, dq+ = -\frac{1}{2888}\, a\, {}_2F_3\left(19,19;2,20,20;-\frac{a^2}{4}\right) \quad (2.14)$$

$$\int_0^1 q^{39} \log q\, J_1(aq)\, dq+ = -\frac{1}{3362}\, a\, {}_2F_3\left(\frac{41}{2},\frac{41}{2};2,\frac{43}{2},\frac{43}{2};-\frac{a^2}{4}\right) \quad (2.15)$$

$$\int_0^1 q^2 \log q\, J_2(aq)\, dq = = -\frac{1}{200}\, a^2\, {}_2F_3\left(\frac{5}{2},\frac{5}{2};3,\frac{7}{2},\frac{7}{2};-\frac{a^2}{4}\right) \quad (2.16)$$

$$\int_0^1 q^3 \log q\, J_2(aq)\, dq = = \frac{-(a^2-8)J_0(a)+6aJ_1(a)-8}{a^4} \quad (2.17)$$

$$\int_0^1 q^4 \log q\, J_2(aq)\, dq = = -\frac{1}{392}\, a^2\, {}_2F_3\left(\frac{7}{2},\frac{7}{2};3,\frac{9}{2},\frac{9}{2};-\frac{a^2}{4}\right) \quad (2.18)$$

$$\int_0^1 q^5 \log q\, J_2(aq)\, dq+ = \frac{2a(5a^2-68)J_1(a)-(a^4-44a^2+96)J_0(a)+96}{a^6} \quad (2.19)$$

$$\int_0^1 q^6 \log q\, J_2(aq)\, dq+ = -\frac{1}{648}\, a^2\, {}_2F_3\left(\frac{9}{2},\frac{9}{2};3,\frac{11}{2},\frac{11}{2};-\frac{a^2}{4}\right) \quad (2.20)$$

$$\int_0^1 q^7 \log q\, J_2(aq)\, dq+ = \frac{1}{a^8}\left[a^6J_6(a)-8(a^2-8)(5a^2+48)J_0(a)- -4(a^4-40a^2-416)a\,J_1(a)-3070\right] \quad (2.21)$$

$$\int_0^1 q^8 \log q\, J_2(aq)\, dq =$$

$$= -\frac{1}{968}\, a^2 \, {}_2F_3\left(\frac{11}{2}, \frac{11}{2}; 3, \frac{13}{2}, \frac{13}{2}; -\frac{a^2}{4}\right) \quad (2.22)$$

$$\int_0^1 q^{15} \, \log q \, J_2(aq)\, dq =$$

$$= -\frac{1}{7a^2}\left[-15\, {}_1F_2\left(7; 1, 8; -\frac{a^2}{4}\right) + \right.$$

$$\left. +8\, {}_2F_3\left(7, 7; 1, 8, 8; -\frac{a^2}{4}\right) + 7J_0(a)\right] \quad (2.23)$$

$$\int_0^1 q^{20} \, \log q \, J_2(aq)\, dq =$$

$$= -\frac{1}{4232}\, a^2 \, {}_2F_3\left(\frac{23}{2}, \frac{23}{2}; 3, \frac{25}{2}, \frac{25}{2}; -\frac{a^2}{4}\right) \quad (2.24)$$

$$\int_0^1 q^{15} \, \log q \, J_2(aq)\, dq =$$

$$= -\frac{1}{12a^2}\left[-25\, {}_1F_2\left(12; 1, 13; -\frac{a^2}{4}\right) + \right.$$

$$\left. +13\, {}_2F_3\left(12, 12; 1, 13, 13; -\frac{a^2}{4}\right) + 12J_0(a)\right] \quad (2.25)$$

$$\int_0^1 q^{32} \, \log q \, J_2(aq)\, dq =$$

$$= -\frac{1}{9800}\, a^2 \, {}_2F_3\left(\frac{35}{2}, \frac{35}{2}; 3, \frac{37}{2}, \frac{37}{2}; -\frac{a^2}{4}\right) \quad (2.26)$$

$$\int_0^1 q^{42} \, \log q \, J_2(aq)\, dq =$$

$$= -\frac{1}{16200}\, a^2 \, {}_2F_3\left(\frac{45}{2}, \frac{45}{2}; 3, \frac{47}{2}, \frac{47}{2}; -\frac{a^2}{4}\right) \quad (2.27)$$

$$\int_0^1 q^{87} \, \log q \, J_2(aq)\, dq +$$

$$= -\frac{1}{64800}\, a^2 \, {}_2F_3\left(45, 45; 3, 46, 46; -\frac{a^2}{4}\right) \quad (2.28)$$

$$\int_0^1 q^{100} \, \log q \, J_2(aq)\, dq =$$

$$= -\frac{1}{84872}\, a^2 \, {}_2F_3\left(\frac{103}{2}, \frac{103}{2}; 3, \frac{105}{2}, \frac{105}{2}; -\frac{a^2}{4}\right) \quad (2.29)$$

$$\int_0^1 q^{1} \, \log q \, J_3(aq)\, dq =$$

$$= -\frac{1}{1200}\, a^3 \, {}_2F_3\left(\frac{5}{2}, \frac{5}{2}; 4, \frac{7}{2}, \frac{7}{2}; -\frac{a^2}{4}\right) \quad (2.30)$$

$$\int_0^1 q^{2} \, \log q \, J_3(aq)\, dq +$$

$$= -\frac{1}{a^3}\left[a^2\, {}_2F_3\left(1, 1; 2, 2, 2; -\frac{a^2}{4}\right) + a\, J_1(a) + 6\, J_0(a) - 6\right] \quad (2.31)$$

$$\int_0^1 q^{3} \, \log q \, J_3(aq)\, dq =$$

$$= -\frac{1}{2352}\, a^3 \, {}_2F_3\left(\frac{7}{2}, \frac{7}{2}; 4, \frac{9}{2}, \frac{9}{2}; -\frac{a^2}{4}\right) \quad (2.32)$$

$$\int_0^1 q^{8} \, \log q \, J_3(aq)\, dq =$$

$$= \frac{1}{a^9}\left[a^7\, J_7(a) + 4(-25a^4 + \right.$$

$$+688a^2 + 3008)a\, J_1(a) + 2(3a^6 -$$

$$\left. -344a^4 + 832a^2 + 15360)J_0(a) - 30720\right] \quad (2.33)$$

$$\int_0^1 q^{14} \, \log q \, J_3(aq)\, dq +$$

$$= \frac{1}{a^{15}}\left[-2(15a^{12} - 4264a^{10} + 583040a^8 - 43038720a^6 + \right.$$

$$+1549025280a^4 - 19823984640a^2 +$$

$$+19818086400)J_0(a) - a(a^{12} - 584a^{10} + 112160a^8 -$$

$$-11479046a^6 + 619683840a^4 - 14869463040a^2 +$$

$$\left. +99114024960)J_1(a) + 39636172800)\right] \quad (2.34)$$

$$\int_0^1 q^{19} \, \log q \, J_3(aq)\, dq =$$

$$= -\frac{1}{25392}\, a^3 \, {}_2F_3\left(\frac{23}{2}, \frac{23}{2}; 4, \frac{25}{2}, \frac{25}{2}; -\frac{a^2}{4}\right) \quad (2.35)$$

$$\int_0^1 q^{29} \, \log q \, J_3(aq)\, dq =$$

$$= -\frac{1}{52272}\, a^3 \, {}_2F_3\left(\frac{33}{2}, \frac{33}{2}; 4, \frac{35}{2}, \frac{35}{2}; -\frac{a^2}{4}\right) \quad (2.36)$$

$$\int_0^1 q^{67} \, \log q \, J_3(aq)\, dq =$$

$$= -\frac{1}{241968}\, a^3 \, {}_2F_3\left(\frac{71}{2}, \frac{71}{2}; 4, \frac{73}{2}, \frac{73}{2}; -\frac{a^2}{4}\right) \quad (2.37)$$

$$\int_0^1 q^{89} \, \log q \, J_3(aq)\, dq =$$

$$= -\frac{1}{415152}\, a^3 \, {}_2F_3\left(\frac{93}{2}, \frac{93}{2}; 4, \frac{95}{2}, \frac{95}{2}; -\frac{a^2}{4}\right) \quad (2.38)$$

$$\int_0^1 q^{103} \, \log q \, J_3(aq)\, dq =$$

$$= -\frac{1}{549552}\, a^3 \, {}_2F_3\left(\frac{107}{2}, \frac{107}{2}; 4, \frac{109}{2}, \frac{109}{2}; -\frac{a^2}{4}\right) \quad (2.39)$$

3 Derivation of the Integration

Derivation of Equation(2.1)

$$\int_0^1 q^2 \log q\, J_1(aq)\, dq+$$

$$= \Big[\frac{2\, J_0(a\sqrt{q^2}) + a\sqrt{q^2}\, J_1(a\sqrt{q^2}) - 2}{a^3} +$$

$$+\frac{1}{4} aq^4 \log q\, {}_0\tilde{F}_1\left(;3;-\frac{a^2q^2}{4}\right)\Big]_0^1$$

$$= \left[\frac{2\, J_0(a) + a\, J_1(a) - 2}{a^3} - \frac{2\, J_0(0)}{a^3}\right] =$$

$$= \frac{2\, J_0(a) + a\, J_1(a) - 2}{a^3}$$

Derivation of Equation(2.2)

$$\int_0^1 q^3 \log q\, J_1(aq)\, dq =$$

$$= \Big[-\frac{1}{50} a\, q^5 \Big\{ {}_2F_3\left(\frac{5}{2},\frac{5}{2};2,\frac{7}{2},\frac{7}{2};-\frac{a^2q^2}{4}\right) -$$

$$-5 \log q\, {}_1F_2\left(\frac{5}{2};2,\frac{7}{2};-\frac{a^2q^2}{4}\right)\Big\}\Big]_0^1$$

$$= \left[-\frac{1}{50} a\left\{ {}_2F_3\left(\frac{5}{2},\frac{5}{2};2,\frac{7}{2},\frac{7}{2};-\frac{a^2}{4}\right)\right\} + 0\right] =$$

$$= -\frac{1}{50}\, a\, {}_2F_3\left(\frac{5}{2},\frac{5}{2};2,\frac{7}{2},\frac{7}{2};-\frac{a^2}{4}\right)$$

Derivation of Equation(2.4)

$$\int_0^1 q^5 \log q\, J_1(aq)\, dq =$$

$$= \Big[-\frac{1}{98} a\, q^7 \Big\{ {}_2F_3\left(\frac{7}{2},\frac{7}{2};2,\frac{9}{2},\frac{9}{2};-\frac{a^2q^2}{4}\right) -$$

$$-7 \log q\, {}_1F_2\left(\frac{7}{2};2,\frac{9}{2};-\frac{a^2q^2}{4}\right)\Big\}\Big]_0^1$$

$$= \left[-\frac{1}{98} a\left\{ {}_2F_3\left(\frac{7}{2},\frac{7}{2};2,\frac{9}{2},\frac{9}{2};-\frac{a^2}{4}\right)\right\} + 0\right] =$$

$$= -\frac{1}{98} a\, {}_2F_3\left(\frac{7}{2},\frac{7}{2};2,\frac{9}{2},\frac{9}{2};-\frac{a^2}{4}\right)$$

Derivation of Equation(2.5)

$$\int_0^1 q^6 \log q\, J_1(aq)\, dq =$$

$$= \Big[\frac{1}{a^3}\Big(\frac{384\, J_0(aq)}{a^4} - \frac{384}{a^4} +$$

$$+\frac{192q\, J_1(aq)}{a^3} + \frac{48q^2\, J_2(aq)}{a^2} -$$

$$-q^4 \log q\big((a^2q^2 - 24)J_4(aq) + 2aq\, J_5(aq)\big) +$$

$$+\frac{1}{32} a^6 q^{10}\, {}_0F_1\Big(;6;-\frac{1}{4}a^2q^2\Big) -$$

$$-2q^4\, J_4(aq) + \frac{8q^3\, J_3(aq)}{a}\Big)\Big]_0^1$$

$$= \Big[\frac{1}{a^3}\Big(\frac{384\, J_0(a)}{a^4} - \frac{384}{a^4} + \frac{192\, J_1(a)}{a^3} + \frac{48q^2\, J_2(a)}{a^2} -$$

$$-1 \log 1\big((a^2 - 24)J_4(a) + 2a\, J_5(a)\big) +$$

$$+\frac{1}{32} a^6\, {}_0F_1\Big(;6;-\frac{1}{4}a^2\Big) - 2\, J_4(a) + \frac{8\, J_3(a)}{a}\Big)\Big] -$$

$$-\left[\frac{1}{a^3}\Big(\frac{384\, J_0(0)}{a^4} - \frac{384}{a^4}\Big)\right]$$

$$= \frac{1}{a^7}\big(a^5 J_5(a) + 8(a^2+32)aJ_1(a) -$$

$$-2(a^2-8)(a^2+24)J_0(a) - 384\big)$$

Derivation of Equation(2.11)

$$\int_0^1 q^{30} \log q\, J_1(aq)\, dq+$$

$$= -\frac{1}{2048}\Big[aq^{32}\Big({}_2F_3(16,16;2,17,17;-\frac{1}{4}a^2q^2) -$$

$$-32 \log q\, {}_1F_2(16;2,17;-\frac{1}{4}a^2q^2)\Big)\Big]_0^1$$

$$= -\frac{1}{2048}\Big[a\Big({}_2F_3(16,16;2,17,17;-\frac{1}{4}a^2)\Big)\Big]$$

$$= -\frac{1}{2048}\, a\, {}_2F_3\left(16,16;2,17,17;-\frac{a^2}{4}\right)$$

Derivation of Equation(2.15)

$$\int_0^1 q^{39} \log q\, J_1(aq)\, dq =$$

$$= -\frac{1}{3362}\Big[aq^{41}\Big({}_2F_3(\frac{41}{2},\frac{41}{2};2,\frac{43}{2},\frac{43}{2};-\frac{1}{4}a^2q^2) -$$

$$-41 \log q\, {}_1F_2(\frac{41}{2};2,\frac{43}{2};-\frac{1}{4}a^2q^2)\Big)\Big]_0^1$$

$$= -\frac{1}{3362}\Big[a\Big({}_2F_3(\frac{41}{2},\frac{41}{2};2,\frac{43}{2},\frac{43}{2};-\frac{1}{4}a^2)\Big)\Big]$$

$$= -\frac{1}{3362}\, a\, {}_2F_3\left(\frac{41}{2},\frac{41}{2};2,\frac{43}{2},\frac{43}{2};-\frac{a^2}{4}\right)$$

By applying same method other results can be derived.

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