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Collatz Conjectures Proof by Number Theory

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Abstract: In this paper, I discuss about different features of numbers, and with these features and using of powers of numbers (2) and (3) with a new method in number theory, I prove that the Collatz conjecture is true for all numbers. Collatz conjecture was proposed by Lothar Collatz in 1937. The famous mathematician Paul Erdos said about the Collatz conjecture, "Mathematics may not be ready for such problems." Despite several efforts in this field, This dangerous problem remains unsolved. Unfortunately, there are no mathematical models and formulas for the Collatz conjecture that I can use to prove this problem. Therefore, The scope of my study in this field has been limited. So I prove this conjecture with my own achievements all along this article. In this paper, I prove the accuracy of this conjecture with a new approach to number theory. Categorize numbers in groups and using different and beautiful features in numbers, Finding a location of the next numbers which are constructed by odd numbers, Using powers of the number (2) and number (3) as the foundation of my work, New method to show the same cycle of numbers in this problem, I introduce and innovative method and distinct functionality to demonstrate this conjecture. Proof of the Collatz conjecture and several achievements in this article, Definitely help mathematicians to work on other problems such as prime numbers, The Riemann hypothesis, The Goldbachs conjecture in math, And some problems in astronomy which can be solved by the Collatz conjecture. These aspects set this article apart from others.

Keywords: Number theory, Powers of the number (2) and number (3), Collatz conjecture, Dangerous problem

1 Introduction

Collatz conjecture is known by different names in the mathematical world, Such as the (3n+1) conjecture, The Ulam conjecture, The Syracuse problem, And the hailstone sequence.
Mathematicians around the world have made significant efforts to prove this problem [4,5,6,7,8,9,10,11], For example, Prof. Terence Tao [2] and Prof. Jeffrey Lagarias [3]. However, No one has ever been able to prove this problem. At this stage, I believe mathematics is now ready to prove this conjecture.
In this paper, I attempt to explore and prove this conjecture from a fresh perspective. I present and validate several points, Each of which is a doorway to prove this problem [1].
First, I categorize numbers in to groups and utilize powers of the number (2) to prove this conjecture. Then, I intrance to details and explain the features of different numbers in their groups, And step by step, I prove that this conjecture is true for all numbers, And each number you select eventually reaches to the number (1).
To prove this conjecture, We need to analyze all numbers, But because all numbers do not have the same features, So I categorize numbers with similar features into groups and I convert infinite numbers into finite groups that have the same sequence. Then, I prove the accuracy of the Collatz conjecture for all numbers.

2 Categorize numbers in groups

Collatz conjecture says that if you select an even number, You must divide it by number two and if you choose an odd number, First you have to multiply it by number (3), Then add to the number one, Then divide it by number (2). This conjecture says that for each number you select if you continue this process, Finally you lead up to number (1).
We know that all odd numbers were an even number which divided by the number (2). If we add number one and then number two to this number, We have three numbers in the arrow [1] (K is an odd number)

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2K,2K+1,2K+2.
2k+1- $\frac{(2k+2)}{4}$= $\frac{(3k+1)}{2}$ In this case, We have the Collatz formula [1].
We categorize all numbers into four groups with these shapes (4k,4k+1,4k+2,4k+3), And ($K \geq 0$) [1]. About even numbers with these shapes (4k,4k+2), If an even number is a power of number two, After several divisions by number two, Finally reaches to the number (1), And the Collatz conjecture is true for these numbers. But if an even number is not a power of number two, After some divisions by number two, Finally we have an odd number. So, If we prove the Collatz conjecture for odd numbers with these shapes (4k+1,4k+3), We prove this conjecture for all numbers.
I want to explain how we can find the next number from an odd number.

Lemma (1.2): If $k_1 = 4k+1$ as we said before, We have 2(4k+1),2(4k+1)+1,2(4k+1)+2 then we have 8k+3-$\frac{(8k+4)}{4}$=6k+2 [1].
And if $k_1 = 4k+3$ as we said before, We have 2(4k+3), 2(4k+3)+1, 2(4k+3)+2 then we have 8k+7-$\frac{8k+8}{4} = 6k+5$ [1].

Lemma (2.2): We assume that (k) belongs to Arithmetic numbers [1].

$$(K \in W)$$

With below table [1] according to Collatz conjecture, We can find all odd numbers and their next numbers [1].

Table 1: Constructing next numbers

k	4k+1	6k+2	4k+3	6k+5
0	1	2	3	5
1	5	8	7	11
2	9	14	11	17
3	13	20	15	23
4	17	26	19	29
5	21	32	23	35
6	25	38	27	41
7	29	44	31	47
8	33	50	35	53
9	37	56	39	59
10	41	62	43	65
11	45	68	47	71
12	49	74	51	77
13	53	80	55	83
14	57	86	59	89
15	61	92	63	95
16	65	98	67	101
17	69	104	71	107
18	73	110	75	113
19	77	116	79	119
20	81	122	83	125
21	85	128	87	131
22	89	134	91	137
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
85	341	512	343	515

*(6k+2) is the next number constructed by (4k+1) and (6k+5) is the next number constructed by (4k+3).

3 Beautiful features in numbers, lemmas, And conjectures proof by lemmas

Lemma (1.3): If k=4k_1 or k=4k_2+2 (k is an even number) then 4k+1= 4(4k_1)+1=16k_1+1 then 6k+2=24k_1+2 , $6k+2 \overset{2}{\equiv} 0$
4k+1=4(4k_2 + 2)+1=16k_2 + 9 then 6k+2=24k_2 + 14 , $6k+2 \overset{2}{\equiv} 0$
And if k=4k_1+1 or k=4k_2+3 (k is an odd number) then, 4k+1=4(4k_1 + 1)+1=16k_1 + 5 then, 6k+2=24k_1 + 8 , $6k+2 \overset{4}{\equiv} 0$
4k+1=4(4k_2 + 3)+1=16k_2 + 13 then, 6k+2=24k_2 + 20 , $6k+2 \overset{4}{\equiv} 0$

Note(1.3): In all Ks in 6k+2 according to Collatz conjecture, We have to divide this number by number (2), And we lead up to 4k_1+1 or 4k_1 +3 in another k such k_1 but in 6k+5, Because this number is odd, This number directly equals to 4k_1+1 or 4k_1+3 in another k such k_1.

Lemma (2.3): In k=2^m we have 4k+1=2^{m+2}+1 so we have 6k+2=2^{m+3}-2^{m+1}+2 and according to Collatz conjecture, We have to divide this number by number (2), So we lead up to 3k+1=2^{m+2}-2^m+1=3(2^m)+1=4(2^{m-2}(3))+1, And 4k+3=2^{m+2} +3, So we have 6k+5=4(2^{m-1}(3)+1)+1

Result (1.3): In all ks with this shape, k=2^m in 6k+2 we lead up from k to k_1=3(2^{m-2}) in 4k_1+1 and in 6k+5 we lead up from k to 4k_2+1 in k_2=3(2^{m-1})+1=6(2^{m-2})+1=(6k_3+2)-1 in k_3=2^{m-2}

Lemma (3.3): In all ks with this shape, k=2^m-1 we have 4k+1=2^{m+2}-3 then, We have 6k+2=4(2^{m+1}-2^{m-1}-1) and according to Collatz conjecture, We lead up to 2^{m+1}-2^{m-1}-1=3(2^{m-1})-1 And we have 4k+3=2^{m+2}-1 then 6k+5=3(2^{m+1})-1=4(3(2^{m-1})-1)+3

Result (2.3): In k=2^m-1 in 6k+2, if m=1 we leadup to number (2) and if m=2 we leadup to number 5=4k_1+1 in k_1=1 and for ($m \geq 3$) we have 6k+2=4(3(2^{m-3})-1)+3 so we leadup from k=2^m-1 in 6k+2 to k_1=3(2^{m-3})-1 in 4k_1+3 and in 6k+5 we leadup from k=2^m-1 to k_1=3(2^{m-1})-1 in 4k_1+3

Lemma(4.3): In all ks with this shape, k=2^m+1 we have 4k+1=2^{m+2}+5 then, We have 6k+2=2^{m+3}-2^{m+1}+2^3 and according to Collatz conjecture, We leadup to 2^{m-2}(3)+1=6(2^{m-3})+1=(6k_1+2)-1 in k_1=2^{m-3} and we have 4k+3=2^{m+2}+7 then, We have 6k+5=2^{m+3}-2^{m+1}+11=4(2^{m+1}-2^{m-1}+2)+3

Result(3.3): In all ks with this shape, k=2^m+1 from 6k+2 we leadup to 6k_1+1=(6k_1+2)-1 in k_1=2^{m-3} and we leadup from 6k+5 to 4k_2+3 in k_2=2^{m-1}(3)+2=6(2^{m-2})+2 in $k_3=2^{m-2}$

Lemma(5.3): In all ks with this shape, k=2^{2m}+2^{2m-2}+.....+2^2+2^0 we have 4k+1=2^{2m+2}+2^{2m}+ ...+2^2+2^0 then 6k+2=2^{2m+3}, So we lead up to the power of the number (2) and according to Collatz conjecture, Eventually we reach to the number (1), So this problem is correct for these numbers [1]. And for 4k+3, We have 6k+5=2^{2m+3}+3=4(2^{2m+1})+3

Result(4.3): In all ks with this shape, k=2^{2m}+2^{2m-2}+...+2^2+2^0 from 6k+2 we lead up to (2^{2m+3}), power of the number (2), And after several divisions by number (2), Eventually we lead up to the number (1), And Collatz conjecture has proven for these numbers. And from 6k+5 we leadup to $4k_1$+3 in $k_1=2^{2m+1}$

Lemma(6.3): In all ks with this shape, k=2^{2m+1}+2^{2m-1}+...+2^1 in 4k+1=2^{2m+3}+2^{2m+1}+...+2^3+1 then, We have 6k+2=2^{2m+4}-2 so we leadup to 3k+1=2^{2m+3}-1, And for 4k+3=(4k+1)+2 we have 6k+5=2^{2m+4}+1=4(2^{2m+2})+1

Result(5.3): In all ks with this shape, k=2^{2m+1}+2^{2m-1}+...+2^1 from 6k+2 we leadup to $4k_1$+3 in $k_1=2^{2m+1}$-1 and from 6k+5 we leadup to $4k_2$+1 in $k_2=2^{2m+2}$

Lemma(7.3): In all ks with this shape, k=10(2^{2m}+2^{2m-2}+...+2^0)+3=2^{2m+3}+2^{2m+2}+2^{2m}+...+2^2+2^0 we have
4k+1=2^{2m+5}+2^{2m+4}+...+2^4+2^2+2^0
then 6k+2=2^{2m+4}(5) [1] and for 4k+3=(4k+1)+2 we have 6k+5=5(2^{2m+4})+3=4(5(2^{2m+2}))+3

Result(6.3): In all ks with this shape, k=10(2^{2m}+2^{2m-2}+...+2^0)+3 we leadup from 6k+2 to 5(2^{2m+4}), So after division by number (2) for (2m+4) times, Finally we reach to the number (5), And we know that according to Collatz conjecture, From number (5) we leadup to number (8=2^3), Power of the number (2), And eventually we lead up to the number (1), And this conjecture has been proved for these numbers too. And from 6k+5, We leadup to $4k_1$+3 in k_1=5(2^{2m+2})

Lemma(8.3): In all ks with this shape, k= 2^{2m+1}-2^{2m-1}-...-2^1 we have 4k+1=2^{2m+3}-2^{2m+1}-...-2^3+1 then
6k+2=2^{2m+4}-2^{2m+3}+6 so we leadup to 3k+1=2^{2m+3}-2^{2m+2}+3=4(2^{2m+1}-2^{2m})+3=4(2^{2m})+3 and for 4k+3=2^{2m+3}-2^{2m+1}-...-2^3+3 then 6k+5=2^{2m+4}-2^{2m+3}+9=4(2^{2m+2}-2^{2m+1}+2)+1=4(2(2^{2m}+1))+1

Result(7.3): In all ks with this shape, k=2^{2m+1}-2^{2m-1}-...-2^1 from 6k+2, We leadup to $4k_1$+3 in $k_1=2^{2m}$ and from 6k+5, We leadup to $4k_2$+1 in k_2=2(2^{2m}+1)

Lemma(9.3): In all ks with this shape, k=2^{2m}-2^{2m-2}-...-2^2-2^0 we have 4k+1=2^{2m+2}-2^{2m}-...-2^4-2^2+1
then 6k+2=4(2^{2m+1}-2^{2m}+1) so according to Collatz conjecture, We lead up to 2^{2m+1}-2^{2m}+1=2^{2m}+1 and for 4k+3=(4k+1)+2, We have 6k+5=4(2^{2m}+1)+3

Result(8.3): In all ks with this shape, k=2^{2m}-2^{2m-2}...-2^2-2^0 we lead up from 6k+2 to $4k_1$+1 in $k_1=2^{2m-2}$ and from 6k+5, We lead up to $4k_2$+3 in $k_2=2^{2m}$+1

Remark(1.3): If we add numbers in 6k+2 or 6k+5 in k, We lead up to another $6k_1$+2 or $6k_1$+5 in k_1, And this fact is true for all numbers.
For example: In k=6 we have 6k+2=38, So 3+8=11, And 11=$6k_1$+5 in k_1=1, And in k=5 we have 6k+5=35, So 5+3=8, And 8=$6k_2$+2 in k_2=1

Lemma(10.3): In all k=$3k_1$+2 in $4k_1$+1 and in all k=$3k_1$ in $4k_1$+3 we have multiple of the number (3). All $4k_1$+1, Which are the multiple of the number (3), Are in k_1= 6k+2 or 6k+5 in k, And all $4k_1$+3, Which are the multiple of the number (3), Are in k_1=(6k+2)+1 or (6k+5)+1 in k.
We have (3^m-1)=6k+2 ($m \geq 1$) in k=0,1,4,13,..., Except k=o other ks are obtained with this formula, k=3^0+3^1+3^2+...

Remark(2.3): In all ks, We know that we can make numbers with (4k,4k+1,4k+2 and 4k+3). For example, In k=1, We have four numbers (4,5,6,7). As we said before, With the 6k+2 and the 6k+5 formula, We can construct the next number from 4k+1 and 4k+3. So we can construct next number from (4k,4k+2) with the 6k+2 and 6k+5 with the following method too.
In (4k+1)-1=4k, Then we have (6k+2)-1=6k+1 so we lead up from k_1=4k in $4k_1$+3 to k_2=6k+1 in $4k_2$+1 and in (4k+3)-1=4k+2, Then we have (6k+5)-1=6k+4 so we lead up from k_1=4k+2 in $4k_1$+3 to k_2=6k+4 in $4k_2$+1.
In k_1=4k in $4k_1$+1 we have $\frac{6k+2}{2}$-1=3k, And in k_1=4k+2 in $4k_1$+1 we have $\frac{6k+2}{2}$=3k+1. According to Table (1) [1], We can find the next number from each number we want, Easily and fast.

Remark(3.3): In all k=$4k_1$ in 6k+2, We lead up to k_2=3k in $4k_2$+1, And in 6k+5 we leadup to k_2=$6k_1$+1 in $4k_2$+1.
In all k=$4k_1$+1 in 6k+2, We have,
6($4k_1$+1)+2=$24k_1$+8=4($6k_1$+2), And according to Collatz conjecture we lead up to $6k_1$+2 in k_1 and in 6k+5, We have 6k+5=6($4k_1$+1)+5=$24k_1$+11=4($6k_1$+2)+3, So we lead up to k_2=$6k_1$+2 in $4k_2$+3 [1].
In all k=$4k_1$+2 in 6k+2, We have,
6k+2=6($4k_1$+2)+2=$24k_1$+14=2($12k_1$+7)=2(4($3k_1$+1)+3),
So we lead up to k_2=$3k_1$+1 in $4k_2$+3 and in 6k+5 we have 6k+5=6($4k_1$+2)+5=$24k_1$+17=4($6k_1$+4)+1, So we lead up

to k_2=6k_1+4 in 4k_2+1.
In all k=4k_1+3 in 6k+2 we have,
6k+2=6(4k_1+3)+2=24k_1+20=4(6k_1+5), So according to Collatz conjecture we lead up to 6k_1+5 in k_1 and in 6k+5 we have 6k+5=6(4k_1+3)+5=24k_1+23=4(6k_1+5)+3, So we lead up to k_2=6k_1+5 in 4k_2+3 [1].

Lemma(11.3): In all ks with each of the four forms mentioned, we have two odd numbers from (4k+1 and 4k+3) formulas, And in 4k+1, We have 6k+2. If we select 4k+1 and construct an odd number with the 4k+1 formula again, Then we have 6k+2, And according to Collatz conjecture, We lead to first 6k+2 in first (k).
k_1=4k+1 so we have 6k_1+2=6(4k+1)+2=24k+8=2^2(6k+2) and if k_2=4(4k+1)+1 then we have 6k_2+2=2^4(6k+2), So if we continue this process, We leadup to this formula, 2^{2m}(6k+2) and according to Collatz conjecture after (2^{2m}) times division by number (2), We leadup to first (6k+2) and (m) is the number of steps that are from (4k+1) in the first (k). For example, in k=7 in 4k+1=29 we have 6k+2=44 and according to Collatz conjecture, $\frac{44}{4} = 11$ so we leadup from k=7 to number (11) and in k=4(29)+1 we have 6k+2=704=16(44) and finally we lead up to $\frac{704}{64} = 11$ so we leadup from k=4(29)+1=117 to number (11) too.
So in general, In stage (m), (k) is,
k=First k(2^{2m})+(2^{2m-2}+...+2^2+2^0) so 4k+1=First (4k+1)(2^{2m})+(2^{2m-2}+...+2^2+2^0) so we have 6k+2=2^{2m}(6k+2)
In k in (4k+3), We have (6k+5), And if we select (4k+3) as a k and we continue the process in 4k+1, We lead up to the first (6k+5) in k. For example, in k=7 in 4k+3=31 and 6k+5=47 and in k=31 6k+2=188 and $\frac{188}{4} = 47$ so we leadup to 6k+5=47 too, so in k_1=4k+3 we have 6k_1+2=6(4k+3)+2=24k+20=2^2(6k+5) and we leadup to (6k+5) in k. So in general in k in (4k+3) we have (6k+5) and in k_1=4k+3 in 6k_1+2=2^{2m}(6k+5) and after (2m) times division by number (2), Finally we leadup to first (6k+5).
In k in 4k+3 we have 6k+5 and if we select k_1=4k+3 in 6k_1+5=6(4k+3)+5=24k+23=2^2(6k+5)+3 so we leadup from k_1=4k+3 to k_2=6k+5.
So in general, k in step (m) is, k=First k(2^{2m})+(2^{2m}-1) then we have 4k+3=First (4k+3)(2^{2m})+(2^{2m}-1) and 6k+5=First (6k+5)(2^{2m})+(2^{2m}-1). For example in k=3 4k+3=15 and 6k+5=23 then in k_1=15=2^2(3)+(2^2-1) we have 4k+3=2^2(15)+(2^2-1)=63 and 6k+5=2^2(23)+(2^2-1)=95.
So, In general, We can say that we have a cycle of numbers which have the same reference. For example, in k=4 we have 4k+3=19 and 6k+5=29 if k_1=19 6k_1+5=119=116+3=4(29)+3 so we lead up to k_2=29 in 4k_2+3 and 6k_2+5=179=4(44)+3 we leadup from k=29 to k=44 in 4k+3, And as you know, we leadup from k=4 in 4k+3=19 and 6k+5=29 to k_1=7 in 4k_1+1=29 in 6k_1+2=44 and if you continue the process we lead up to the same result from the same reference.

Proof(1.3): According to lemma(2.3), we lead up from k=2^m in 6k+2 to k_1=3(2^{m-2}) in 4k_1+1, If m=4k (the multiple of the number (4)), We continue the process and eventually we lead up to k=$3^{\frac{m}{2}}$ and in 6k+5 we have 6k+5=6(2^m)+5=2^2(6(2^{m-2})+1)+1, So we lead up from k=2^m in 6k+5 to k_1=6(2^{m-2})+1 in 4k_1+1, And because m=4k, k_1=6(2^{4k-2})+2-1 in k_2=2^{4k-2}. As we said before, We lead up from k=4k_1 to k= 3k_1, So when we have k=$2^{m=4k}$ eventually we lead up to k_1=3(2^{4k-3})=3(2^{4n+1}), And according to lemma(11.3), We lead up to k_1=4k+2=3(2(4^{2n})=2(3^{2n+1}) in 4k+1, So we have 6k_1+2, And then we have 3k_1+1=4k_2+3 in k_2=3^0+3^1+3^2+..., For example, In k=256=2^8 in 6k+5=1541=4(385)+1 in k_1=385=4(96)+1 in k_2=96=3(2^5)=3(2(4^2))=2(3^3)=4k+2 in 4k_2+1 and according to lemma(11.3), We have 6k_2+2=578, Then 3k_2+1=289=4(72)+1 in k=72 and 6k+2=6(72)+2=434 3k+1=217=4(54)+1 in k=54=2(3^3)=4k+2 in 6k+2=326, Then 3k+1=163=4(40)+3=4k+3 in k=40=3^0+3^1+3^2+3^3 in 4k+3, Then we have 6k+5=61=$\frac{122}{2}$,6(81)+2= 6(3^4)+2=6(k_1)+2 in
k_1=$3^{\frac{8}{2}}$, So 6(81)+2=488 and according to Collatz conjecture we have, $\frac{488}{4} = 122$ So from k=3^0+3^1+... in 4k+3 in 6k+5 we lead up to k_1=$\frac{6k+2}{2}$ in k=$3^{\frac{m}{2}}$ in 4k_1+1.

Lemma(12.3): We know that $3^{2n} \overset{4}{\equiv} 1$, So we have even powers of the number (3) in 4k+1 in k=2(3^0+3^2+...), For example, 3^2=9 is in k=2(3^0)=2 in 4k+1 and 3^4=81 is in k=2(3^0+3^2)=20 in 4k+1.
And as you know, $3^{2n+1} \overset{4}{\equiv} 3$, So we have all odd powers of the number (3) in 4k+3 in k=2(3^1+3^3+...), For example, 3^3=27 is in k= 2(3^1)=6 in 4k+3.

Proof(2.3): As we said in proof (1.3), In k=$2^{m=4k}$ in 6k+2 we lead up to k_1=$3^{\frac{m}{2}}$=$3^{\frac{4k}{2}}$=3^{2k}, So we lead up to even powers of the number(3). In k=$3^{\frac{m}{2}}$ in 6k+2, After division by number (2), According to Collatz conjecture, we have 3k+1=$3^{\frac{m+2}{2}}$ +1 and as we said in lemma(12.3), $3k+1 \overset{4}{\equiv} 0$. And according to remark(3.3), We lead up from k=$3^{\frac{m}{2}}$ in 6k+2 to k_1=2(3^0+3^2+...), And according to lemma(12.3), In odd sentences (1,3,5,...), In k=2,182,...,(4n+2), We lead up to k_1 in 4k_1+3 and in even sentences (2,4,6,...), In k=20,1640,...,(4n), We lead up to k_1=3n in 4k_1+1, And according to lemma(11.3), Remark(2.3) and remark(3.3), If we continue the process eventually we have the same process which we have in k=$2^{\frac{m}{4}}$ and finally we reach to the number (1).
In k=$3^{\frac{m}{2}}$ in 6k+5 we have 6k+5=6($3^{\frac{m}{2}}$)+5, For example, In k=3^2=9 in 6k+5=59=4(14)+3, So we lead up to k=14 in 4k+3 and 6k+5=89=88+1 in k=22 in 4k+1 and if we continue the process, We have k=2^4 in 6k+5 and eventually we lead up to k=2(3^0) in 4k+1=9 and 6k+2=14 then 3k+1=7=4k+3 in k=1. So according to lemma(11.3), We can conclude that From k=$2^{m=4k}$ in 6k+2 we have k_1=$3^{\frac{m}{2}}$, So in 6k_1+5 finally we lead up to

k=2($3^0+3^2+3^4$+...) in 4k+1=$3^{\frac{m}{2}}$, And then 6k+2 and according to proof(1.3), Eventually we reach to the number (1).

Proof(3.3): In k=2^{4k+1} in 6k+2, As we said before, From each 4k, We lead up to 3k, For example, In k=4(8) we lead up to k=3(8), So according to lemma(11.3), And this point, From k=2^{4k+1} in 6k+2, Eventually we lead up to k_1=2($3^{\frac{4k}{2}}$)=4n+2 in 4k_1+1 and from 6k_1+2, Then we lead up to k_2=$3^0+3^1+3^2$+...=6k+1=(6k+2)-1 in k=2(3^0+3^2+...), And as you know, We have 4k+1=13 in k=3=3^1 and 4k+1=121 in k=30=3^1+3^3 so we have k=($3^1+3^3+3^5$+...) in 4k+1=(6k_1+2)-1, And according to lemma(11.3), As we said we have the cycle of numbers with the same reference and eventually we lead up to powers of the number (2), And then we have powers of the number (3), And according to former proofs we reach to the number (1).
And for k=2^{4k+1} in 6k+5, We lead up to k_1=$3^{\frac{4k}{2}}$ in 4k_1+1, And as we said before, We lead up to the same process in k=2($3^0+3^2+3^4$+...) in 4k+1 and 6k+2.

Proof(4.3): In k=2^{4k+2} in 6k+2, We lead up to k_1=$3^{\frac{4k+2}{2}}$=3^{2k+1}, Odd powers of the number (3). And as we said before, We have all odd powers of the number (3), In k=2($3^1+3^3+3^5$+...) in 4k+3. In k=3^{2k+1} in 6k+2, We lead up to k_1=($3^0+3^2+3^4$+...) in 4k_1+1, And then we have 6k_1+2, So according to former proofs, We lead up to powers of the number (2), And as you know, From powers of the number (2), We lead up to powers of the number (3), And finally we reach to the number (1). And in k=3^{2k+1} in 6k+5, According to lemma(11.3), We lead up to k_2=4k_1+1 in k_1=(3^0+3^2+...), And as we said before, We have the same process with the same reference, So with proofs and points that we said before, Easily we reach to the number (1). And for k=2^{4k+2} in 6k+5 we have, 6k+5=4(3(2^{4k+1})+1)+1, So we lead up to k_1=3(2^{4k+1})+1 in 4k_1+1 and k_1=3k_2+1=$\frac{6k_2+2}{2}$ in k_2=2^{4k+1}. So according to proof (3.3), We said how we reach to the number (1).

Proof(5.3): In k=2^{4k+3} in 6k+2, We lead up to k_1=2($3^{\frac{4k+2}{2}}$) in 4k_1+1, And then we reach to k_2=($3^0+3^1+3^2$+...) in 4k_2+3. So, As we said in lemma(11.3), And former proofs, We lead up to the process which we have in k=2(3^0+3^2+...) in 4k+2=$3^{\frac{m}{2}}$, The even powers of the number (3), And then 6k+2 and as we said in proof(1.3), And proof(2.3), We proved that we reach to the number (1). And for k=2^{4k+3} in 6k+5, We have the 6k+5=4(3(2^{4k+2})+1)+1, So as you see, We reach to k_1=3(2^{4k+2})+1 in 4k_1+1 and k_1=3k_2+1=$\frac{6k_2+2}{2}$ and k_2=2^{4k+2}, If we continue the process, We reach to odd powers of the number (3), In k=$3^{\frac{4k+2}{2}}$=3^{2k+1} in 4k+1, And as we said in proof(4.3), We proved that from these numbers we finally reach to the number (1).

Proof(6.3): According to lemma(3.3), In k=2^m-1 we know that we have 4k+3=2^{m+2}-1, And according to remark(3.3), And lemma(11.3), In 6k+2, We lead up from k to k_1=2^{m-2}-1 in 4k_1+3=k, Then we have 6k_1+5=3(2^{m-1})-1=4(3(2^{m-3})-1)+3, So we lead up to k_2=3(2^{m-3})-1=(6(2^{m-4})+2)-3 in 4k_2+3. Except 2^1-1=1=4k+1 in k=0, We have all 2^m-1 in 4k+3 in k=2^{m-2}-1, So according to remark(3.3), Lemma(11.3), Lemma(5.3) and lemma(6.3), We lead up to 2^m+1=6k+5, Then as we said before, We have 6k+5=4k_1+1=2^m+1 in k_1=2^{m-2}, Then we have 6k_1+2, So according to former proofs about k=2^m, We lead up to powers of the number (3), Then as we said before, We reach to the number (1). According to lemma(5.3) and lemma(11.3), We lead to powers of the number (2) in 4k+3, So as we said in former proofs about the powers of the number (2), We have powers of the number (3), Then as we said we reach to the number (1).
For example, In k=7=2^3-1 in 6k+2=44, We have in k=1, 4k+3=7, Then 6k+5=11 and 4k+3=11 in k=2=2^1, So 6k+5=17=2^4+1 and we have 4k+1=17 in k=4=2^2 in 4k+1 and then 6k+2, Then we reach to k=3^1=3 in 4k+1, Then easily we lead up to the number (1).
In k=2^4-1=15 we have 4k+3=15 in k=3 and 6k+5=23 then 23=4(5=2^2+2^0)+3 in k=5=2^2+2^0 and then in 6k+5=35=4(8=2^3)+3, So we leadup to k=2^3 in 4k+3 and then we have 6k+5, So as we said before, We reach to the power of the number (3), Then we lead up to the number(1).
In k=2^m-1 in 6k+5, According to lemma(11.3), Remark(2.3), and remark(3.3), We have the same cycle of numbers, So as we said, In this proof we lead to the number (1). For example, In k=7 in 6k+5=47=4(11)+3, So we lead up from k=7 to k=11 in 4k+3, Then we reach from k=11 to k=17=2^4+1, And if we continue the process, Eventually we have the power of the number (2), And then we have the power of the number (3), And according to proofs that we said, Finally we reach to the number (1).

Proof(7.3): According to lemma(4.3), As we said before, We have all k=2^m+1 in 4k_1+1 in k_1=2^{m-2}, So according to remark(3.3), We have 6k+2 in k_1, So according to proofs about the powers of the number (2), We lead up to the number (1). And in 6k+5, According to lemma(11.3) and remark (3.3), We have the same cycle of numbers, So as we proved before in powers of the number (2), We easily reach the number (1).

Proof(8.3): According to lemma(5.3), In k=$2^{2m}+2^{2m-2}$+...+2^0 in 6k+2=2^{2m+3} we reach to powers of the number (2), So we easily lead up to the number (1) and in 6k+5=4(2^{2m+1})+3, So we lead up to k_1=2^{2m+1} in 4k_1+3 and according to proofs about the number (2), We can reach to the number (1).

Proof(9.3): According to lemma(6.3), In k=$2^{2m+1}+2^{2m-1}$+...+2^1 in 6k+2=2^{2m+4}-2 and according to Collatz conjecture we have 3k+1=2^{2m+3}-1=4(2^{2m+1}-1)+3, So we reach to $k_1=2^{2m+1}$-1 in $4k_1$+3, So from now on, We act according to proof(6.3), And finally we reach to the number (1). In 6k+5=4(2^{2m+2})+1, So we lead up to $k_1=2^{2m+2}$ in $4k_1$+1, So according to proofs about the number (2), We lead up to powers of the number (3), Then as we said before, We reach to the number (1).

Proof(10.3): According to lemma(7.3), In 6k+2=5(2^{2m+4}), After (2m+4) times division by number (2), Finally we reach to the number (5), And as you know, 4k+1=5 in k=1, Then 6k+2=8=2^3, We have power of the number (2), So we reach to the number (1). In 6k+5=4(5(2^{2m+2}))+3, So we lead up to k_1=5(2^{2m+2}) in $4k_1$+3, Then $6k_1$+5=4(15(2^{2m+1}))+1, So we lead up to k_2=15(2^{2m+1}) in $4k_2$+1, If we continue the process and according to lemma(11.3), Remark(2.3) and remark(3.3) and use the recursive relation, We find out that each number in this formula, Comes from a number which is the power of the number (2), And there is a relation between the powers of the number (3), And if we continue the process, Again we reach to another power of the number (2), And then we have the power of the number (3), Then as we said before in proofs for powers of the number (2), Finally we reach to the number (1). For example, In k=53=10(2^2+2^0)+3, We have 6k+5=323=4(80)+3, So we reach k=80 in 4k+3, Then 6k+5=485=4(121)+1, So we lead up to k=121=($3^0+3^1+3^2+3^3+3^4$) in 4k+1, 6k+2=728 and according to Collatz conjecture, We reach to $\frac{728}{4}$=182=2($3^0+3^2+3^4$), And we know that we have, 4(182)+1=729=3^6 and as we said before, We lead up from k=2^{12} in 4k+1 to k=3^6 and even powers of the number (3) is in 4k+1 in k=2(3^0+3^2+...). So we realize that from k=2^{12}, We reach k=182=2(91), And according to lemma(11.3), We have the same cycle of numbers, And we reach the same numbers from k=182 or k=22. In k=182 in 6k+5=1097=1096+1 then according to Collatz conjecture, We reach to k=137=136+1=4(34)+1 and in 4k+3=91, in k=22 then 6k+5=137=136+1=4(34)+1.
If we continue the process with k=182, According to proofs about the powers of the number (3), Finally we reach another power of the number (2), k=512=2^9, Then according to the proofs for powers of the number (2), We lead up to the number (1).

Proof(11.3): According to lemma(8.3), in 6k+2, According to Collatz conjecture, We lead up to 3k+1=4(2^{2m})+3, So we reach $k_1=2^{2m}$ in $4k_1$+3 and according to proofs about the powers of the number (2), As we proved we reach to the number (1). In 6k+5=4(2(2^{2m}+1))+1, So we lead up to k_1=2(2^{2m}+1)=$4k_2$+2 in $4k_1$+1, Then we have, $6k_1$+2 and as we said before, 6($4k_2$+2)+2=$24k_2$+14 then $3k_1$+1=$12k_2$+7=4($3k_2$+1)+3, So we lead up to k=$3k_2$+1 in 4k+3, in $4k_1$+1 we have $6k_1$+2=6(2(2^{2m}+1))+2 and then $3k_1$+1=6(2^{2m}+1)+1=($6k_3$+2)-1 in $k_3=2^{2m}$+1 and as you know all $k_3=2^{2m}$+1 is in 4k+1 in k=2^{2m-2}. So if we continue the process, As we said before, According to remark(2.3), Remark(3.3), And lemma(11.3), We have the cycle of numbers that reach another power of the number (2), And according to proofs about the powers of the number (2), Finally, We lead up to the number (1).

Proof(12.3): According to lemma(9.3), In 6k+2=4(2^{2m+1}-2^{2m}+1)=4(2^{2m}+1), So according to Collatz conjecture, We lead up to (2^{2m}+1)=4(2^{2m-2})+1=$4k_1$+1 in $k_1=2^{2m-2}$, So according to proofs about the powers of the number (2), We reach to the number (1). In 6k+5=4(2^{2m}+1)+3, So we lead up to $k_1=2^{2m}$+1 in $4k_1$+3, So from now on we act as we did in proof(7.3) and finally we have number (1).

Proof(13.3): According to lemma(11.3), Remark(2.3), Remark(3.3), and with the recursive relation between numbers, We find out that all numbers are made of powers of the number (2) and we have the same cycle of numbers that are made from the formula which I said before, So according to proofs about the powers of the number (2), Step by step we easily reach to the number (1) for all numbers.
For example, In k=18 in 4k+1=4(18)+1=73, And we know that 2(73)=146=6(24)+2, So 4k+1=$\frac{6k_1+2}{2}$=73 in k_1=24 and $4k_1$+1=4(24)+1=97, And we know that 2(97)=194=6(32)+2, So $4k_1$+1=$\frac{6k_2+2}{2}$=97 in k_2=32=2^5.
So, As you saw, From k_2=32=2^5 (the power of the number (2)) in $4k_2$+1=129 we have $6k_2$+2=194, Then after performing several operations, We lead up to k=18 in 4k+1. Now, If we continue the process with the methods I said before, We lead up from k=18 in 4k+1, Then 6k+2 to k_1=40=($3^0+3^1+3^2+3^3$) in $4k_1$+1, We reach to a number that has relation between powers of the number (3), Then we have k_2=30=(3^1+3^3) in $4k_2$+1, Finally, We reach to k=512=2^9 in 4k+3, As you saw we lead up to another power of the number (2), So according to proofs about the powers of number (2), Finally, We reach to the number (1).

4 Discussion

This manuscript demonstrates that Collatz conjecture is true for all numbers.
I categorize numbers in groups, To use their specific features to prove this problem easily. The proof of this conjecture is based on powers of numbers (2) and (3). In this paper, It is clearly proven that each number you select, If you proceed according to the instructions mentioned in different parts of this article, Has a history of powers of the number (2) or number (3). So if we reach the powers of the number (3), According to proofs in this paper we finally reach the powers of the number (2). So for any number we start with the power of the number (2) and in a cycle that I have explained in every step of this

paper, We reach another power of the number (2) again. So from the powers of the number (2) we easily reach the number (1) and the accuracy of this problem for all numbers has been indicated clearly by the results and findings in this paper. Because there are no mathematical methods and formulas in this field, I prove this conjecture step by step with all formulas and achievements in this paper which are from my own several years of research and effort in this field.
Findings and results in this article can clearly help future research in different fields of math such as solving problems in prime numbers, The Riemann hypothesis, And the Goldbachs conjecture, Or even in astronomy problems that have now been found to be related to the Collatz conjecture.

5 Conclusion

In this paper, First I categorized numbers in groups, Then I said how we can find the next number with formulas, So I said how we can find the location of the next number then I said some features in numbers each of which was a doorway to prove this conjecture. I proved how we can find the next number at a given (k) according to remark(2.3) and remark(3.3), and with Table(1), We can find the next number easily and fast and also in proofs that I said, “continue the process“, we use this remark(2.3) and Table(1) [1] to find the next number. I said all features of all numbers in their groups, I wrote formulas to make numbers and according to the return relationship that exists between the numbers and according to the same cycle between the numbers, I proved that according to Collatz conjecture, Each number you select finally reaches to the number (1). I said and proved all formulas that we want to prove this problem and as a matter of fact, I proved this conjecture with powers of the numbers (2) and (3). One interesting point is in the Collatz conjecture that helped me to prove this problem, In this conjecture, We start with the power of the number (2), According to proof(13.3), Each number you consider has an origin of the power of the number (2), Then after performing several operations that I proved all of them in this article, Finally we lead up to another power of the number (2), Then as we said in proofs about the powers of the number (2), We easily reach to the number (1). So according to all the points that I said and proved all along this paper, Now I can decisively say that, According to the Collatz conjecture, Each number you select, Eventually reaches the number (1), And this conjecture is true for all numbers.

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7 Conflict of interest

I declare that there is no conflict of interest regarding the publication of this paper. My research is independent and not influenced by external factors such as financial resources, Personal or professional relationships, Or political affiliations. I have acted with integrity and honesty in doing so. All the results and findings of this article are related to my own research, (Fereshteh Fazeli) the author of this article, And I have written this entire article with my own knowledge.

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