Journal of Statistics Applications & Probability
An International Journal

http://dx.doi.org/10.12785/jsap/020313

Semi-Markov Model of a Series-Parallel System Subject to Preventive Maintenance

M. A. El-Damcese*[*], *A. N. Salem*[*], *and N. S. Temraz

Department of Mathematics, Faculty of Science, TantaUniversity, Tanta, Egypt
[*]*Email: meldamcese@yahoo.com, anmsalem45@yahoo.com*

Received: 13 Sep. 2012, Revised: 24 Feb. 2013, Accepted: 10 Mar. 2013
Published online: 1 Nov. 2013

Abstract: The purpose of this paper is to analyze a series-parallel system by using semi-Markov process. There is a preventive maintenance action provided to the system in order to increase the life time of the system. We suppose that the failure, repair, and maintenance times are stochastically independent random variables each having an arbitrary distribution. The kernel matrix associated with this system is constructed. Expressions for mean sojourn times, steady-state availability, availability function, reliability function, mean time to failure, and mean time to repair are presented. Numerical solutions of the system are obtained.

Keywords: Semi-Markov process, series-parallel system, preventive maintenance (PM), availability, reliability, mean time to failure (MTTF), mean time to repair (MTTR).

1. Introduction

Semi-Markov processes and the related Markov renewal processes have been established by Takáces, Lévy, and Smith in the mid of 1950s.Semi-Markov processes is the generalization of Markov processes by allowing the times between transitions to be arbitrary distributed non-negative random variables which may depend on the current state and the next state. These processes are very important where in reality the life time and repair time in most cases are not necessary to be exponentially distributed. Markov renewal processes is the generalization of renewal processes.Semi-Markov processes and Markov renewal processes have many applications (see Asmussen (2003) andJanssen&Limnios (1999)).

Preventive maintenance is defined as the activity undertaken regularly at preselected intervals while the system is satisfactorily operating (see Sim&Endrenyi (1988)), while repair is the activity to bring the system to as good as new status after failure occurred. The purpose of maintenance is to extend the unit life time and avoid costly consequences of failures.

Limnios (1997) presented a dependability analysis (availability, reliability, maintainability, and mean times) for semi-Markov systems with finite state space by using a new method based on algebraic calculus. Chen &Trivedi (2001) have analytically studied the system periodic preventive maintenance problem by using semi-Markov process assuming that the preventive maintenance with generally distributed parameters. Grabski (2010) presented semi-Markov reliability model of cold standby system and this model is the modification of the model considered by Barlow &Proshan. El-Said & El-Sherbeny (2005) discussed the reliability of two units cold standby system with preventive maintenance and random changes in units.Zajac&Budny (2009) presented the reliability model of intermodal transport which described by semi-Markov processes. They assumed that the probabilities of transition between states are exponential and presented a discussion on Gamma and Weibull distributions in semi-Markov calculations. Cocozza-Thivent&Roussignol (1997) studied the evolution of a multi-component system which is modeled by a semi-Markov process and introduced formulas for the availability and reliability of the system. Also, they obtained numerical solutions for a two-unit parallel system with sequential preventive maintenance.

In this paper we analyzed a series-parallel system by using semi-Markov process and this system is some modification of the model used as an example in Alam (1984) and Csenki (1995). In our model, the system is consisted of two subsystems A and B connected in parallel, where subsystem A consists of $n_1$identical units connected in series and subsystem B consists of $n_2$identical units connected in series. Each subsystem fails when one unit of the subsystem is down and the system fails when the two subsystems fail. There is a preventive maintenance provided to each unit before it fails. Hence, each unit in the system has three states: up, maintenance, and down and we assume that the life, repair, and maintenance times are generally distributed. We constructed the kernel matrix associated with the model and presented the mean sojourn times. Also, we introduced some other measures of the model such as availability function, reliability function, steady-state availability, mean time to failure, and mean time to repair. Numerical solutions, using MAPLE program, were obtained when subsystem A consists of three units and subsystem B consists of four units. These calculations were determined assuming two cases of the life, repair, and maintenance times: the case of distribution and the case of exponential distribution.

2. Notations

n_1: Number of units of subsystem A.

n_2: Number of units of subsystem B.

$F_A(t)$: Distribution function for the failure of subsystem A.

$F_B(t)$: Distribution function for the failure of subsystem B.

$F_1(t)$: Distribution function for the failure of one unit of subsystem A.

$F_2(t)$: Distribution function for the failure of one unit of subsystem B.

$G_A(t)$: Distribution function for subsystem A to go to under PM.

$G_B(t)$: Distribution function for subsystem B to go to under PM.

$G_1(t)$: Distribution function for a unit of subsystem A to go to under PM.

$G_2(t)$: Distribution function for a unit of subsystem B to go to under PM.

$H_A(t)$: Distribution function for the duration of repair of one unit of subsystem A.

$H_B(t)$: Distribution function for the duration of repair of one unit of subsystem B.

$K_A(t)$: Distribution function for the duration of PM of one unit of subsystem A.

$K_B(t)$: Distribution function for the duration of PM of one unit of subsystem B.

$\bar{S}$: In general, [1-S(t)] complement of distribution function.

3. Semi-Markov Processes

To define the semi-Markov process, we first have to define the Markov renewal processes.

Definition1.

A bivariate process $(Z_n, T_n)_{n\in\mathbb{N}}$ is a Markov renewal process with countable state space S and kernel $Q(t)_{t\in\mathbb{R}_+}$, where $Q(t) = \{Q_{ij}(t): i, j \in S\}$ is a family of sub-distribution functions such that $\sum_{j\in S} Q_{ij}(t)$ is a distribution function for all $i \in S$, if it is a Markov process on $S \times \mathbb{R}_+$ such that T_0=0 and

$$Q_{ij}(t) = \Pr\{Z_{n+1} = j, T_{n+1} - T_n \leq t | Z_n = i\}$$

for all $n \in \mathbb{N}, i, j \in S, and\ t \in \mathbb{R}_+$.

Definition 2.

A process $X(t)_{t\in\mathbb{R}_+}$ is a semi-Markov process with countable state space S and kernel $Q(t)$ if

$$X(t) = Z_n, \quad T_n \leq t < T_{n+1}$$

for some Markov renewal process (Z_n, T_n) with state space S and kernel $Q(t)$.

4. Model Description

(1) The system consists of two subsystems A and B connected in parallel. Subsystem A consists of $n_1$identical units connected in series and subsystem B consists of $n_2$identical units connected in series (see Figure 1).
(2) At time t = 0, the system is up and it fails when the two subsystems fail at the same time.
(3) Each subsystem fails when one unit of the subsystem is down and each failed unit is repaired.
(4) There is a preventive maintenance provided to each unit in the subsystem before it fails.
(5) There are asingle repair facilityand a single maintenance facility available,(i.e., in the same time one bad unit only belong to any subsystem under repair or maintenance).
(6) All distributions of the time for repair, maintenance, and time to failure are general.
(7) The model has nine states. All possible states and transitions between them are shown in Figure 2.

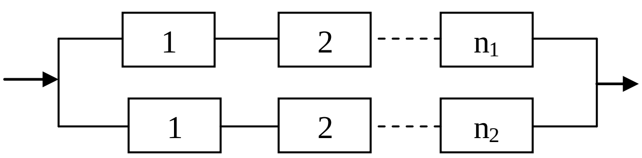

Fig. 1: Diagram of the model

All possible states of the model are given by

Up states : S_1(A Up, B Up), S_2(A PM, B Up), S_3(A Up, B PM), S_4(A Down, B Up),S_5(A Up, B Down), S_6(A PM, B PM),S_7(A PM, B Down), S_8(A Down, B PM)

Down state : S_9(A Down, B Down).

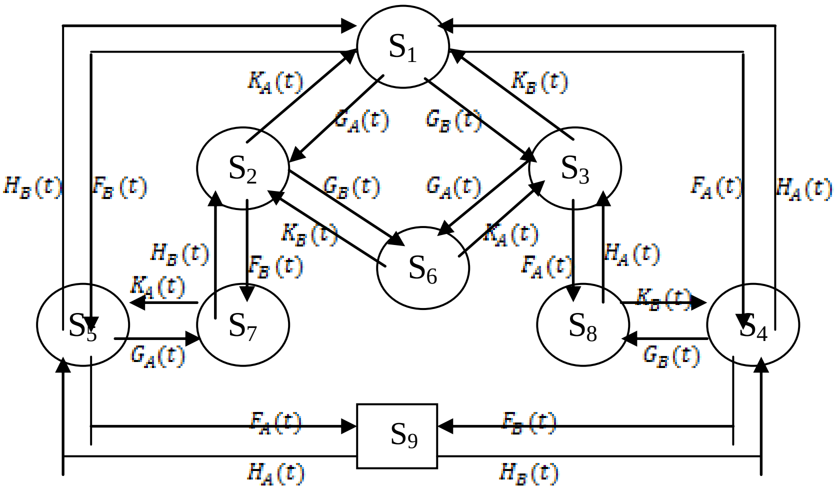

Fig. 2: Transition diagram of the model

Up States Down State

4.1 Kernel Matrix Determination

We construct the kernel matrix associated with the given model and semi-Markov process which has the following structure.

$$\mathbf{Q}(t)=\begin{pmatrix} 0 & Q_{12}(t) & Q_{13}(t) & Q_{14}(t) & Q_{15}(t) & 0 & 0 & 0 & 0\\ Q_{21}(t) & 0 & 0 & 0 & 0 & Q_{26}(t) & Q_{27}(t) & 0 & 0\\ Q_{31}(t) & 0 & 0 & 0 & 0 & Q_{36}(t) & 0 & Q_{38}(t) & 0\\ Q_{41}(t) & 0 & 0 & 0 & 0 & 0 & 0 & Q_{48}(t) & Q_{49}(t)\\ Q_{51}(t) & 0 & 0 & 0 & 0 & 0 & Q_{57}(t) & 0 & Q_{59}(t)\\ 0 & Q_{62}(t) & Q_{63}(t) & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & Q_{72}(t) & 0 & 0 & Q_{75}(t) & 0 & 0 & 0 & 0\\ 0 & 0 & Q_{83}(t) & Q_{84}(t) & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & Q_{94}(t) & Q_{95}(t) & 0 & 0 & 0 & 0 \end{pmatrix}$$

The functions of the kernel matrix are defined as follows

$$Q_{12}(t)=\int_0^t \bar{F}_A(t)\bar{F}_B(t)\bar{G}_B(t)\,dG_A(t)$$

$$Q_{13}(t)=\int_0^t \bar{G}_A(t)\bar{F}_A(t)\bar{F}_B(t)\,dG_B(t)$$

$$Q_{14}(t)=\int_0^t \bar{G}_A(t)\bar{G}_B(t)\bar{F}_B(t)\,dF_A(t)$$

$$Q_{15}(t)=\int_0^t \bar{G}_A(t)\bar{G}_B(t)\bar{F}_A(t)\,dF_B(t)$$

$$Q_{21}(t)=\int_0^t \bar{G}_B(t)\bar{F}_B(t)\,dK_A(t), \qquad Q_{26}(t)=\int_0^t \bar{K}_A(t)\bar{F}_B(t)\,dG_B(t)$$

$$Q_{27}(t)=\int_0^t \bar{K}_A(t)\bar{G}_B(t)\,dF_B(t), \qquad Q_{31}(t)=\int_0^t \bar{G}_A(t)\bar{F}_A(t)\,dK_B(t)$$

$$Q_{36}(t)=\int_0^t \bar{K}_B(t)\bar{F}_A(t)\,dG_A(t), \qquad Q_{38}(t)=\int_0^t \bar{K}_B(t)\bar{G}_A(t)\,dF_A(t)$$

$$Q_{41}(t)=\int_0^t \bar{G}_B(t)\bar{F}_B(t)\,dH_A(t), \qquad Q_{48}(t)=\int_0^t \bar{H}_A(t)\bar{F}_B(t)\,dG_B(t)$$

$$Q_{49}(t)=\int_0^t \bar{H}_A(t)\bar{G}_B(t)\,dF_B(t), \qquad Q_{51}(t)=\int_0^t \bar{G}_A(t)\bar{F}_A(t)\,dH_B(t)$$

$$Q_{57}(t)=\int_0^t \bar{H}_B(t)\bar{F}_A(t)\,dG_A(t), \qquad Q_{59}(t)=\int_0^t \bar{H}_B(t)\bar{G}_A(t)\,dF_A(t)$$

$$Q_{62}(t)=\int_0^t \bar{K}_A(t)\,dK_B(t), \qquad Q_{63}(t)=\int_0^t \bar{K}_B(t)\,dK_A(t)$$

$$Q_{72}(t)=\int_0^t \bar{K}_A(t)\,dH_B(t), \qquad Q_{75}(t)=\int_0^t \bar{H}_B(t)\,dK_A(t)$$

$$Q_{83}(t)=\int_0^t \bar{K}_B(t)\,dH_A(t), \qquad Q_{84}(t)=\int_0^t \bar{H}_A(t)\,dK_B(t)$$

$$Q_{94}(t)=\int_0^t \bar{H}_A(t)\,dH_B(t), \qquad Q_{95}(t)=\int_0^t \bar{H}_B(t)\,dH_A(t)$$

where the $F_A(t)$, $F_B(t)$, $G_A(t)$, and $G_B(t)$ functions are given by

$$F_A(t)- [1-F_1(t)]^{n_1}, F_B(t)=1-[1-F_2(t)]^{n_2} \quad ,$$

$$G_A(t)- [1-G_1(t)]^{n_1},\ G_B(t)=1-[1-G_2(t)]^{n_2}$$

The initial distribution associated with the semi-Markov process is defined as

$$\boldsymbol{\alpha}=[1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$

4.2 Availability and Reliability Analysis

To introduce the availability and reliability functions, we divide the state space and the initial distribution of the semi-Markov process into two states: up (U) and down (D) states. Hence, the kernel matrix and the initial distribution vector can be rewritten as follows (see Limnios (1997))

$$\mathbf{Q}(t)=\begin{pmatrix}\mathbf{Q}_{UU}(t) & \mathbf{Q}_{UD}(t)\\ \mathbf{Q}_{DU}(t) & \mathbf{Q}_{DD}(t)\end{pmatrix}, \qquad \boldsymbol{\alpha}=[\boldsymbol{\alpha}_U\boldsymbol{\alpha}_D]$$

where, as we mentioned, the up and down states are given by
$U = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8\}$ and $D = \{S_9\}$
Thus, the availability function can be obtained from the following relation (see Limnios (1997)), which is written in matrix form, as follows.

$$A(t) = \alpha[\mathbf{I} - \mathbf{Q}(t)]^{-1} \times \mathbf{h}(t)1_{S,U} \tag{1}$$

Where $\mathbf{I}$ is the identity matrix, $\mathbf{h}(t) = \mathbf{I} - diag(\mathbf{Q}(t)1)$, and $1_{S,U} = [1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 0]^T$
The inverse $[\mathbf{I} - \mathbf{Q}(t)]^{-1}$ can be calculated from the following relation

$$[\mathbf{I} - \mathbf{Q}(t)]^{-1} = [\det(\mathbf{I} - \mathbf{Q}(t))]^{-1} \times adj(\mathbf{I} - \mathbf{Q}(t)) \tag{2}$$

Now, to introduce the reliability function, we suppose that the down state is absorbing state. Hence, the reliability function can be obtained from the following relation (see Limnios (1997)), which is written in matrix form, as follows.

$$R(t) = \alpha_U[\mathbf{I} - \mathbf{Q}_{UU}(t)]^{-1} \times \mathbf{h}_1(t)1_U \tag{3}$$

Where $\mathbf{h}_1(t)$ is the restriction of $\mathbf{h}(t)$ on the U set and $1_U = [1\ 1\ 1\ 1\ 1\ 1\ 1\ 1]^T$

4.3 Mean Sojourn Times

The function

$$G_i(t) = \Pr\{T_{n+1} - T_n \le t | X(T_n) = i, X(T_{n+1}) = j\} = \sum_{j \in S} Q_{ij}(t)$$

is the cumulative distribution function of a random variable T_i that is called waiting time in state *i*. The waiting time T_i means the time being spent in state *i* when we do not know the successor state. We give expressions for the cumulative distribution function of a random variable T_i which are given as follows.

$$G_i(t) = \sum_{j=1}^{9} Q_{ij}(t), \qquad i = 1, 2, \dots, 9 \tag{4}$$

Hence,

$$G_1(t) = 1 - \bar{F}_A(t)\bar{F}_B(t)\bar{G}_A(t)\bar{G}_B(t)$$

$$G_2(t) = 1 - \bar{F}_B(t)\bar{K}_A(t)\bar{G}_B(t)$$

$$G_3(t) = 1 - \bar{G}_A(t)\bar{F}_A(t)\bar{K}_B(t)$$

$$G_4(t) = 1 - \bar{H}_A(t)\bar{G}_B(t)\bar{F}_B(t)$$

$$G_5(t) = 1 - \bar{H}_B(t)\bar{G}_A(t)\bar{F}_A(t)$$

$$G_6(t) = 1 - \bar{K}_A(t)\bar{K}_B(t), \qquad G_7(t) = 1 - \bar{K}_A(t)\bar{H}_B(t)$$

$$G_8(t) = 1 - \bar{K}_B(t)\bar{H}_A(t), \qquad G_9(t) = 1 - \bar{H}_A(t)\bar{H}_B(t)$$

Now, the mean sojourn times are obtained as follows

$$E(T_i) = \int_0^\infty \{1 - G_i(t)\}dt, \qquad i = 1, 2, \dots, 9 \qquad (5)$$

$$E(T_1) = \int_0^\infty \bar{F}_A(t)\bar{F}_B(t)\bar{G}_A(t)\bar{G}_B(t)dt$$

$$E(T_2) = \int_0^\infty \bar{F}_B(t)\bar{K}_A(t)\bar{G}_B(t)dt$$

$$E(T_3) = \int_0^\infty \bar{G}_A(t)\bar{F}_A(t)\bar{K}_B(t)dt$$

$$E(T_4) = \int_0^\infty \bar{H}_A(t)\bar{G}_B(t)\bar{F}_B(t)dt$$

$$E(T_5) = \int_0^\infty \bar{H}_B(t)\bar{G}_A(t)\bar{F}_A(t)dt$$

$$E(T_6) = \int_0^\infty \bar{K}_A(t)\bar{K}_B(t)dt, E(T_7) = \int_0^\infty \bar{K}_A(t)\bar{H}_B(t)dt$$

$$E(T_8) = \int_0^\infty \bar{K}_B(t)\bar{H}_A(t)dt, E(T_9) = \int_0^\infty \bar{H}_A(t)\bar{H}_B(t)dt$$

4.4 Transition Probabilities of Embedded Markov Chain

The transition matrix of the embedded Markov chain $\mathbf{P} = [p_{ij}: i, j \in S]$ of the semi-Markov process is obtained as follows.

$$\mathbf{P} = \begin{pmatrix} 0 & p_{12} & p_{13} & p_{14} & p_{15} & 0 & 0 & 0 & 0 \\ p_{21} & 0 & 0 & 0 & 0 & p_{26} & p_{27} & 0 & 0 \\ p_{31} & 0 & 0 & 0 & 0 & p_{36} & 0 & p_{38} & 0 \\ p_{41} & 0 & 0 & 0 & 0 & 0 & 0 & p_{48} & p_{49} \\ p_{51} & 0 & 0 & 0 & 0 & 0 & p_{57} & 0 & p_{59} \\ 0 & p_{62} & p_{63} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & p_{72} & 0 & 0 & p_{75} & 0 & 0 & 0 & 0 \\ 0 & 0 & p_{83} & p_{84} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{94} & p_{95} & 0 & 0 & 0 & 0 \end{pmatrix}$$

Where $p_{ij} = Pr\{X(T_{n+1}) = j | X(T_n) = i\} = \lim_{t\to\infty} Q_{ij}(t)$

and

$$p_{12} + p_{13} + p_{14} + p_{15} = 1, \qquad p_{21} + p_{26} + p_{27} = 1$$

$$p_{31} + p_{36} + p_{38} = 1, \qquad p_{41} + p_{48} + p_{49} = 1$$

$$p_{51} + p_{57} + p_{59} = 1, \qquad p_{62} + p_{63} = 1$$

$$p_{72} + p_{75} = 1, \qquad p_{83} + p_{84} = 1, \qquad p_{94} + p_{95} = 1$$

Now, the embedded Markov chain steady-state equations **v = vP** are given by

$v_1 = v_2 p_{21} + v_3 p_{31} + v_4 p_{41} + v_5 p_{51}$

$v_2 = v_1 p_{12} + v_6 p_{62} + v_7 p_{72}$

$v_3 = v_1 p_{13} + v_6 p_{63} + v_8 p_{83}$

$v_4 = v_1 p_{14} + v_8 p_{84} + v_9 p_{94}$

$v_5 = v_1 p_{15} + v_7 p_{75} + v_9 p_{95}$

$v_6 = v_2 p_{26} + v_3 p_{36}$

$v_7 = v_2 p_{27} + v_5 p_{57}$

$v_8 = v_3 p_{38} + v_4 p_{48}$

$v_9 = v_4 p_{49} + v_5 p_{59}$

And $\sum_{j=1}^{9} v_j = 1$

4.5 Steady-State Availability

The transition probabilities of the semi-Markov process is defined as
$\pi_{ij}(t) = \Pr\{X(t) = j|X(0) = i\}$

Tthe state probabilities of the semi-Markov process is defined as
$\pi_j(t) = \Pr\{X(t) = j\}$

The steady-state availability can be obtained from the following relation

$$A = \sum_{j=1}^{8} \pi_j, \qquad (6)$$

Where$\pi_j = \lim_{t\to\infty} \pi_j(t)$ are the steady-state probabilities of the semi-Markov process and these probabilities can determined from the following relation

$$\pi_j = \frac{v_j\, E(T_j)}{\sum_{j=1}^{9} v_j\, E(T_j)}, \qquad j = 1, 2, \dots, 9 \qquad (7)$$

4.6 Mean Time to Failure and Repair

In order to find mean time to system failure (MTTF), we make a partition on the transition probability matrix of the embedded Markov chain and the mean sojourn times as follows (see Limnios (1997)).

$$\mathbf{P} = \begin{pmatrix} \mathbf{P}_{UU} & \mathbf{P}_{UD} \\ \mathbf{P}_{DU} & \mathbf{P}_{DD} \end{pmatrix}, \qquad \mathbf{E}(T) = [\mathbf{E}_U(T)\mathbf{E}_D(T)]$$

Hence, the mean time to system failure and mean time to system repair (MTTR) can be obtained from the following relations

$$MTTF = \alpha_U[\mathbf{I} - \mathbf{P}_{UU}]^{-1}\mathbf{E}_U(T),$$

$$MTTR = E(T_9)$$

where

$$\mathbf{E}_U(T) = [E(T_1)\ \ E(T_2)\ \ E(T_3)\ \ E(T_4)\ \ E(T_5)\ \ E(T_6)\ \ E(T_7)\ \ E(T_8)]^T$$

5. Numerical Example

We obtain numerical solutions for some measures of the introduced model of semi-Markov process, when $n_1 = 3$ and $n_2 = 4$, in the following two cases.

Case (i)

We suppose that the failure, repair, and maintenance times are distributed as follows.

$$F_1(t) = 1 - e^{-\beta_1 t^2/2}, F_2(t) = 1 - e^{-\beta_2 t^2/2},$$

$$G_1(t) = 1 - e^{-\beta_3 t^2/2}, G_2(t) = 1 - e^{-\beta_4 t^2/2}$$

$$H_A(t) = 1 - e^{-\beta_5 t^2/2}, H_B(t) = 1 - e^{-\beta_6 t^2/2}$$

$$K_A(t) = 1 - e^{-\beta_7 t^2/2}, K_B(t) = 1 - e^{-\beta_8 t^2/2}$$

and given the following data

β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
0.01	0.015	0.02	0.018	0.025	0.03	0.026	0.034

We obtain the availability function by substituting in relation (1) and using MAPLE program. The results for the availability function versus the time and the effect of preventive maintenance are shown in Figure3.

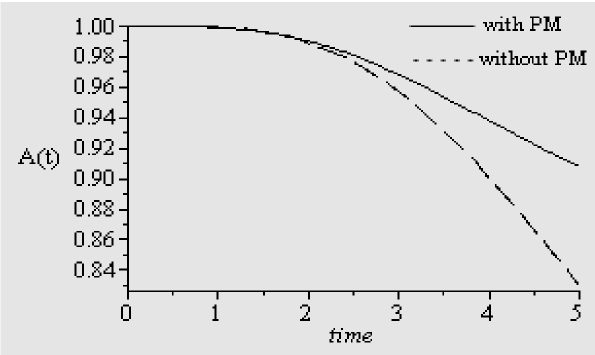

Fig. 3: Comparison of the availability function for the system with and without PM versus the time (case i)

We obtain the reliability function by substituting inrelation (3) and using MAPLE program. The results for the reliability function versus the time and the effect of preventive maintenance are shown in Figure 4.

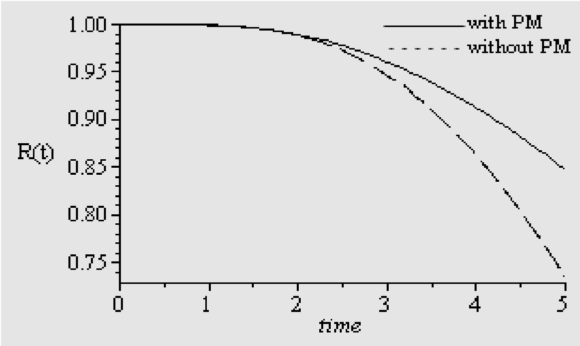

Fig. 4: Comparison of the reliability function for the system with and without PM versus the time (case i)

Now, to find the system steady-state availability, we first calculate the mean sojourn times and the results are given as follows.

$$[E(T_1) \quad E(T_2) \quad E(T_3) \quad E(T_4) \quad E(T_5) \quad E(T_6) \quad E(T_7) \quad E(T_8) \quad E(T_9)] =$$

$$[2.660 \quad 3.153 \quad 3.559 \quad 3.163 \quad 3.618 \quad 5.116 \quad 5.296 \quad 5.159 \quad 5.344]$$

then we solve the embedded Markov chain steady-state equations and obtain the following results.
v_1 = 0.1051531067, v_2 = 0.1727046867, v_3 = 0.1243782374, v_4 = 0.08923804221,
v_5 = 0.1136790340, v_6 = 0.1388838880, v_7 = 0.1224235752, v_8 = 0.07101596133,
v_9 = 0.06252346848

Hence, the steady-state availability probability is obtained by using relation (7) and then substituting in relation (6) and the result is given by.

$$A = 0.91688$$

Case (ii)

As a special case, let us consider that the failure, repair, and maintenance times are exponentially distributed as follows.

$$F_1(t) = 1 - e^{-\gamma_1 t}, F_2(t) = 1 - e^{-\gamma_2 t},$$

$$G_1(t) = 1 - e^{-\gamma_3 t}, G_2(t) = 1 - e^{-\gamma_4 t}$$

$$H_A(t) = 1 - e^{-\gamma_5 t}, H_B(t) = 1 - e^{-\gamma_6 t}$$

$$K_A(t) = 1 - e^{-\gamma_7 t}, K_B(t) = 1 - e^{-\gamma_8 t}$$

and given the following data

γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	γ_8
0.001	0.002	0.003	0.004	0.008	0.009	0.005	0.006

We obtain the availability function by substituting in relation (1) and using MAPLE program. The results for the availability function versus the time and the effect of preventive maintenance are shown in Figure 5.

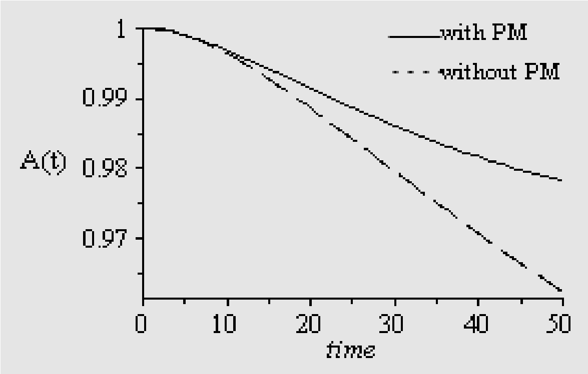

Fig. 5:Comparison of the availability function for the system with and without PM versus the time (case ii)

We obtain the reliability function by substituting in relation (3) and using MAPLE program. The results for the reliability function versus the time and the effect of preventive maintenance are shown in Figure 6.

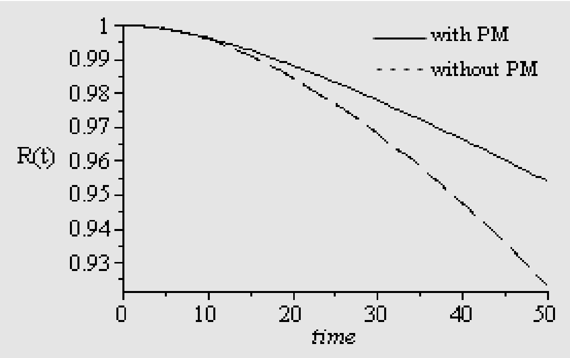

Fig. 6: Comparison of the reliability function for the system with and without PM versus the time (case ii)

Now, to find the system steady-state availability, we first calculate the mean sojourn times and the results are given as follows.

$$[E(T_1) \quad E(T_2) \quad E(T_3) \quad E(T_4) \quad E(T_5) \quad E(T_6) \quad E(T_7) \quad E(T_8) \quad E(T_9)] =$$

$$[27.77 \quad 34.48 \quad 55.55 \quad 31.25 \quad 47.61 \quad 90.90 \quad 71.42 \quad 71.42 \quad 58.82]$$

then we solve the embedded Markov chain steady-state equations and obtain the following results.

v_1 = 0.1375445750, v_2= 0.1994396338, v_3 = 0.1833927667, v_4 = 0.04584819169,

v_5 = 0.07131940929,v_6 = 0.2017320430, v_7 = 0.08558329092, v_8 = 0.05348955679,

v_9 = 0.2165053293

Hence, the steady-state availability probability is obtained by using relation (7) and then substituting in relation (6) and the result is given by.

A = 0.9769

6. Conclusion

In this paper, we analyzed a series-parallel system by using semi-Markov process. The main advantage of semi-Markov process is to permit non-exponential distributions for the transitions between states.Preventive maintenance was provided to each unit of the system in order to increase the life time of the system. The kernel matrix and expressions for mean sojourn times, availability function, reliability function, steady-state availability, MTTF, and MTTR were presented assuming that the life, maintenance, and repair times are generally distributed. Numerical solutions were obtained for some measures of the model when the life, maintenance, and repair times are linearly and exponentially distributed. The effect of preventive maintenance were shown in the numerical example.

References

[1] Alam, M., Unavailability analysis of protective systems subject to sequential testing, *International Journal of Systems Science,* **15**, 1381-1395 (1984).

[2] Asmussen, S., *Applied Probability and Queues,* Springer, New York, (2003).

[3] Chen, D. and Trivedi, K. S., Analysis of periodic preventive maintenance with general system failure distribution, *Proceedings of 2011 Pacific Rim International Symposium on Dependable Computing,* 103-107, (2001).

[4] Cocozza-Thivent, C. and Roussignol, M., Semi-Markov processes for reliability studies, *Journal of ESAIM: Probability and Statistics,* **1**, 207-223 (1997).

[5] Csenki, A., An integral equation approach to the interval reliability of systems modeled by finite semi-Markov processes, *Journal of Reliability Engineering and System Safety,* **47**, 37-45 (1995).

[6] El-Said, K. M. and El-Sherbeny, M. S., Profit analysis of a two unit cold standby system with preventive maintenance and random change in units, *Journal of Mathematics and Statistics,* **1**, 71-77 (2005).

[7] Grabski, F., Semi-Markov reliability model of the cold standby system, *Journal of Applied Quantitative Methods,* **5**, 486-497 (2010).

[8] Janssen, J. and Limnios, N., *Semi-Markov Models and Applications,* Kluwer, Dordrecht, (1999).

[9] Limnios, N., Dependability analysis of semi-Markov systems, *Journal of Reliability Engineering and System Safety,* **55**, 207-207 (1997).

[10] Sim., S. H, and Endrenyi., J., Optimal preventive maintenance with repair, *Journal of IEEE Transactions on Reliability,* **37**, 92-96 (1988).

[11] Zajac, M. and Budny, T., On determination of some characteristics of semi-Markov process for different distributions of transient probabilities, Journal of Reliability & Risk Analysis: Theory & Applications, **2**, 74-84 (2009).