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Advancements in Statistical Sampling: Stratified Double Unified Ranked Set Sampling with Perfect Ranking for Symmetric Distributions

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Abstract: In this comprehensive analysis, we unveil enhancements to a refined sample selection technique known as Stratified Double Unified Ranked Set Sampling with Perfect Ranking, specifically designed to accurately determine the population mean. This innovative method's capability to estimate the population means is rigorously evaluated against three established sampling strategies: Stratified Simple Random Sampling (SSRS), Stratified Ranked Set Sampling (SRSS), and Stratified Median Ranked Set Sampling, also encompassing comparisons with Stratified Double Unified Ranked Set Sampling (SMRSS). Our comparative analysis is meticulously conducted across three symmetric probability distributions: the standard normal distribution, the Student-t distribution, and the uniform distribution (0,1). Through our investigations, we ascertain that the estimator emerging from Stratified Random Sampling, utilizing Two Full Cycles of Systematic Selection within a perfectly ordered set, remains unbiased. This holds true across various sample set sizes (m) - specifically, m = 2, 4, 6, 10 - and during two distinct levels of selection rounds (r), namely r = 2 and 5. This study not only underscores the precision and reliability of the enhanced sampling method but also provides a comparative framework that highlights its superior efficiency in population mean estimation under diverse distribution scenarios.

Keywords: Stratified Simple Random Sampling, Stratified Ranked Set Sampling, Stratified Median Ranked Set Sampling, Stratified Double Unified Ranked Set Sampling.

1. Introduction

To enhance the process of selecting sample units from a population, it is imperative to employ advanced statistical sampling methods. These methods are crucial for efficiently identifying representative sample units while maximizing cost-effectiveness and optimizing the overall sampling procedure. Among the various probability-based random sampling methods, Stratified Simple Random Sampling (SSRS) is particularly noteworthy. This method partitions a population of N units into L distinct strata, each sampled using straightforward random techniques [1]. Another vital technique is Stratified Systematic Sampling, where a subset of n units from the total population N is organized into L strata, with each stratum being systematically sampled to ensure thorough representation [2]. Additionally, Stratified Median Ranked Set Sampling is an important strategy where n samples are drawn from a population divided into L strata, employing median ranked set sampling within each stratum to enhance precision [3].

Ranked Set Sampling (RSS) was initially proposed in 1952 by McIntyre as a novel technique aimed at estimating average yields in populations [4]. Since its introduction, RSS has been extensively refined by researchers to improve the accuracy of estimating population parameters. A significant advancement in RSS came about in 1968 when Dell and Clutter provided a mathematical foundation for the method, proving that the sample mean obtained from RSS is an unbiased estimator of the population mean and offers greater variance efficiency than Simple Random Sampling (SRS) of the same sample size [5]. Further validation was given by Takahashi and Wakimoto in 1972, who demonstrated that RSS's variance is equal to or less than that of SRS, regardless of the ranking accuracy [6].

The development of RSS progressed with Al-Saleh's introduction of Stratified Ranked Set Sampling (SRSS) in 1996 [7], followed by Muttlak's formulation of Median Ranked Set Sampling in 1997 [8]. The RSS framework was significantly expanded in 2017 with Bouza's introduction of Unified Ranked Set Sampling (URSS), emphasizing the method's versatility and wide applicability [9]. During this period, significant contributions were made by scholars such as Samawi, Muttlak, Al-Omari, Al-Saleh, and El-Sayyad, who developed nonparametric tests to affirm the perfect ranking assumption in RSS [10] [11] [12] [13] [14]. Another noteworthy development occurred in 2000 when Bouza introduced Double Ranked Set Sampling (DRSS), incorporating two cycles of sampling to enhance efficiency [15].

Building on this extensive legacy, the current study introduces an innovative stratified sampling technique called Stratified

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Double Unified Ranked Set Sampling with Two Full Cycles. This sophisticated method is specifically engineered to provide more accurate and reliable estimates of the population mean, particularly in symmetric distributions [16]. This approach represents the culmination of decades of research and innovation in statistical sampling, offering a more refined, efficient, and precise tool for researchers seeking to analyze and interpret complex population dynamics.

2. Sampling Methods

Simple Random Sampling

Simple Random Sampling (SRS) is a statistical method used for selecting n Units out of N units from a larger population, ensuring that each ${}_NC_n$ possible sample of that number has an equal chance of being chosen. This approach guarantees that every individual unit of the population has an equal opportunity of selection. In practical terms, SRS involves selecting each unit individually and randomly, which upholds the principles of randomness and fairness in the sampling process. This method is fundamental in statistical sampling to avoid bias and to ensure that the sample accurately represents the broader population.

Stratified Sampling Method

In stratified sampling method, the population of N units is divided into L non overlapping subpopulations each of $N_1, N_2, \cdots, N_L$ units, respectively, such that $N_1 + N_2 + \cdots + N_L = N$. these sub populations are called strata. For full benefit from stratification, the size of the h_{th} subpopulation, denoted by N_h for $h = 1, 2, \cdots, L$, must be known. Then the samples are draw independently from each strata, producing samples sizes denoted by $n_1, n_2, \cdots, n_L$, such that the total sample size in $n = \sum_{h=1}^{L} n_h$. If a simple random sample is taken from each stratum, the whole procedure is known as stratified simple random sampling (SSRS)

Ranked Set Sampling (RSS)

The ranked set sampling can be described as follows:

Step 1: Draw a simple random sample of size m^2 units from the target population.

Step 2: Allocate the m^2 selected units as randomly as possible into m sets, each of size m.

Step 3: Without yet knowing any values for the variable of interest, rank the units within each set with respect to variable of interest. This may be based on personal professional judgment or done based on a concomitant variable correlated with the variable of interest.

Step 4: Choose a sample for actual quantification by including the smallest ranked unit in the first set, the second smallest ranked unit in the second set, the process is continued in this way until the largest ranked unit is selected from the last set.

Step 5: Repeat Steps 1 through 5 for r cycles (times) to draw the RSS of size $n = mr$

We denote the suggested method by RSS (m, r) as

Example 1: Let $(m = 6, r = 1)$ be

$$\begin{bmatrix} \boxed{X_{11}}, X_{12}, X_{13}, X_{14}, X_{15}, X_{16} \\ X_{21}, \boxed{X_{22}}, X_{23}, X_{24}, X_{25}, X_{26} \\ X_{31}, X_{32}, \boxed{X_{33}}, X_{34}, X_{35}, X_{36} \\ X_{41}, X_{42}, X_{43}, \boxed{X_{44}}, X_{45}, X_{46} \\ X_{51}, X_{52}, X_{53}, X_{54}, \boxed{X_{55}}, X_{56} \\ X_{61}, X_{62}, X_{63}, X_{64}, X_{65}, \boxed{X_{66}} \end{bmatrix}$$

Then the measured RSS units are $X_{11}, X_{22}, X_{33}, X_{44}, X_{55}, X_{66}$

Considerations:

1. Note: m is set size, r is number of cycles (times), n is sample of size
2. The RSS is used for Population infinite.

Median Ranked Set Sampling (MRSS)

The MRSS procedure as proposed by [5] can be formed by selecting random samples of size n units from the population and rank the units within each sample with respect to a variable of interest. If the sample size n is odd, then from each sample select for the measurement the $\left(\frac{m+1}{2}\right)th$ smallest ranked unit, i.e., the median of the sample. If the sample size n is even, then select for the measurement from the first $\frac{m}{2}$ samples the $\left(\frac{m}{2}\right)th$ smallest ranked unit and from the second $\frac{m}{2}$ samples the $\left(\frac{m+1}{2}\right)th$ smallest ranked. The cycle can be repeated r times if needed to get a sample of size nm units.

Unified Ranked Set Sampling (URSS)

[6] proposed a unified ranked sampling (URSS). The URSS procedure can be executed as follows:

Step 1: Use an SRS method to select m^2 units from the population of interest and rank them with respect to the variable of interest.

Step 2: Select the sample units for measurement as

Divided into 2 cases:

Case: Even Number

From the size of the sample units, divide the sample units into two equal parts:

Part 1: Select sample units from $\left(\frac{m}{2}+(i-1)m\right)$ (let this be denoted by $i=1,\ 2,\cdots,m$).

Part 2: Select sample units from $\left(\left(\frac{m}{2}+1\right)+(i-1)m\right)$ (let this be denoted by $i=1,\ 2,\cdots,m$).

Example 2: Consider the case of. $(m=4, r=1)$. Draw a simple random sample of size $m^2=4^2=16$ units as

$X_{11}, X_{12}, X_{13}, X_{14}, X_{15}, X_{16}, X_{17}, X_{18}, X_{19}, X_{110}, X_{111}, X_{112}, X_{113}, X_{114}, X_{115}, X_{116}$

We select samples each of size

$X_{11}, \boxed{X_{12}}, X_{13}, X_{14}, X_{15}, \boxed{X_{16}}, X_{17}, X_{18} \mid X_{19}, X_{110}, \boxed{X_{111}}, X_{112}, X_{113}, X_{114}, \boxed{X_{115}}, X_{116}$

Let $X_{12}, X_{16}, X_{111}, X_{115}$ is DURSS of size 4

Case: Odd Number

From the size m^2 of the sample units, select sample units from $\left(\frac{m+1}{2}+(i-1)m\right)$ (let this be denoted by $i=1,\ 2,\cdots,m$).

Example 3: Consider the case of $(m=3, r=1)$. Draw a simple random sample of size $m^2=3^2=9$ units as

$X_{11}, X_{12}, X_{13}, X_{14}, X_{15}, X_{16}, X_{17}, X_{18}, X_{19}$

We select samples each of size

$$X_{11}, \boxed{X_{12}}, X_{13}, X_{14}, \boxed{X_{15}}, X_{16}, X_{17}, \boxed{X_{18}}, X_{19}$$

Let X_{12}, X_{15}, X_{18} is URSS of size 3

Step 3: Repeat the process as in the first round of sampling, that is, follow Steps 1 to 2 until the total number of sampling rounds *r* is completed. This will result in *mr* sample units being selected for measuring the characteristics of interest.

The purpose of this research is to propose a modified version of Ranked Set Sampling, known as Stratified Double Unified Ranked Set Sampling (SDURSS) with perfect ranking, to estimate the population mean. Additionally, this study demonstrates the efficiency of the mean estimator based on SDURSS through a simulation conducted under symmetric distributions.

Double Unified Ranked Set Sampling (DURSS)

In this research, DRSS method is adapted to the DURSS approach as proposed by [12]. The procedure is as follows:

Step 1: Utilize the Simple Random Sampling (SRS) method to identify elements from the target population and randomly divide these elements into m sets, each consisting of m elements.

Step 2: Implement the usual Unified Ranked Set Sampling (URSS) procedure on each set to obtain m ranked set samples, each of size m.

Step 3: Apply the URSS procedure again on the results from Step 2 to obtain a DURSS of size m.

This procedure is illustrated for cases of both even and odd numbers in the following example.

Case: even number

Example 4: Consider the case of $(m = 4, r = 1)$. Draw a simple random sample of size $m^3 = 4^3 = 64$ elements

(4 sets of size 16 each). Assume the elements are

$$X_{(1)}^{(1)}, X_{(2)}^{(1)}, X_{(3)}^{(1)}, \cdots, X_{(16)}^{(1)}, X_{(1)}^{(2)}, X_{(2)}^{(2)}, X_{(3)}^{(2)}, \cdots, X_{(16)}^{(2)}, X_{(1)}^{(3)}, X_{(2)}^{(3)}, X_{(3)}^{(3)}, \cdots, X_{(16)}^{(3)}, X_{(1)}^{(4)}, X_{(2)}^{(4)}, X_{(3)}^{(4)}, \cdots, X_{(16)}^{(4)}$$

After ranking the elements of each set obtain 6 ranked set samples of size 4 each($m^2 = 4^2 = 16$).

$$\left[X_{(1)}^{(1)}, X_{(2)}^{(1)}, X_{(3)}^{(1)}, X_{(4)}^{(1)}, X_{(5)}^{(1)}, X_{(6)}^{(1)}, X_{(7)}^{(1)}, X_{(8)}^{(1)}, X_{(9)}^{(1)}, X_{(10)}^{(1)}, X_{(11)}^{(1)}, X_{(12)}^{(1)}, X_{(13)}^{(1)}, X_{(14)}^{(1)}, X_{(15)}^{(1)}, X_{(16)}^{(1)}\right],$$

$$\left[X_{(1)}^{(2)}, X_{(2)}^{(2)}, X_{(3)}^{(2)}, X_{(4)}^{(2)}, X_{(5)}^{(2)}, X_{(6)}^{(2)}, X_{(7)}^{(2)}, X_{(8)}^{(2)}, X_{(9)}^{(2)}, X_{(10)}^{(2)}, X_{(11)}^{(2)}, X_{(12)}^{(2)}, X_{(13)}^{(2)}, X_{(14)}^{(2)}, X_{(15)}^{(2)}, X_{(16)}^{(2)}\right],$$

$$\left[X_{(1)}^{(3)}, X_{(2)}^{(3)}, X_{(3)}^{(3)}, X_{(4)}^{(3)}, X_{(5)}^{(1)}, X_{(6)}^{(3)}, X_{(7)}^{(3)}, X_{(8)}^{(3)}, X_{(9)}^{(3)}, X_{(10)}^{(3)}, X_{(11)}^{(3)}, X_{(12)}^{(3)}, X_{(13)}^{(3)}, X_{(14)}^{(3)}, X_{(15)}^{(3)}, X_{(16)}^{(3)}\right]$$

and

$$\left[X_{(1)}^{(4)}, X_{(2)}^{(4)}, X_{(3)}^{(4)}, X_{(4)}^{(4)}, X_{(5)}^{(4)}, X_{(6)}^{(4)}, X_{(7)}^{(4)}, X_{(8)}^{(4)}, X_{(9)}^{(4)}, X_{(10)}^{(4)}, X_{(11)}^{(4)}, X_{(12)}^{(4)}, X_{(13)}^{(4)}, X_{(14)}^{(4)}, X_{(15)}^{(4)}, X_{(16)}^{(4)}\right]$$

We select the sample unit from the elements of each set obtain 4 ranked set samples of size 4 each as

$$\left[X_{(1)}^{(1)}, \boxed{X_{(2)}^{(1)}}, X_{(3)}^{(1)}, X_{(4)}^{(1)}, X_{(5)}^{(1)}, \boxed{X_{(6)}^{(1)}}, X_{(7)}^{(1)}, X_{(8)}^{(1)}, X_{(9)}^{(1)}, X_{(10)}^{(1)}, \boxed{X_{(11)}^{(1)}}, X_{(12)}^{(1)}, X_{(13)}^{(1)}, X_{(14)}^{(1)}, \boxed{X_{(15)}^{(1)}}, X_{(16)}^{(1)}\right],$$

$$\left[X_{(1)}^{(2)}, \boxed{X_{(2)}^{(2)}}, X_{(3)}^{(2)}, X_{(4)}^{(2)}, X_{(5)}^{(2)}, \boxed{X_{(6)}^{(2)}}, X_{(7)}^{(2)}, X_{(8)}^{(2)}, X_{(9)}^{(2)}, X_{(10)}^{(2)}, \boxed{X_{(11)}^{(2)}}, X_{(12)}^{(2)}, X_{(13)}^{(2)}, X_{(14)}^{(2)}, \boxed{X_{(15)}^{(2)}}, X_{(16)}^{(2)}\right],$$

$$\left[X_{(1)}^{(3)}, \boxed{X_{(2)}^{(3)}}, X_{(3)}^{(3)}, X_{(4)}^{(3)}, X_{(5)}^{(1)}, \boxed{X_{(6)}^{(3)}}, X_{(7)}^{(3)}, X_{(8)}^{(3)}, X_{(9)}^{(3)}, X_{(10)}^{(3)}, \boxed{X_{(11)}^{(3)}}, X_{(12)}^{(3)}, X_{(13)}^{(3)}, X_{(14)}^{(3)}, \boxed{X_{(15)}^{(3)}}, X_{(16)}^{(3)}\right]$$

and

$$\left[X_{(1)}^{(4)}, \boxed{X_{(2)}^{(4)}}, X_{(3)}^{(4)}, X_{(4)}^{(4)}, X_{(5)}^{(4)}, \boxed{X_{(6)}^{(4)}}, X_{(7)}^{(4)}, X_{(8)}^{(4)}, X_{(9)}^{(4)}, X_{(10)}^{(4)}, \boxed{X_{(11)}^{(4)}}, X_{(12)}^{(4)}, X_{(13)}^{(4)}, X_{(14)}^{(4)}, \boxed{X_{(15)}^{(4)}}, X_{(16)}^{(4)}\right]$$

so we have 4 DURSS

$$X^{(1)}_{(2)}, X^{(1)}_{(6)}, X^{(1)}_{(11)}, X^{(1)}_{(15)}, X^{(2)}_{(2)}, X^{(2)}_{(6)}, X^{(2)}_{(11)}, X^{(2)}_{(15)}, X^{(3)}_{(2)}, X^{(3)}_{(6)}, X^{(3)}_{(11)}, X^{(3)}_{(15)}, X^{(4)}_{(2)}, X^{(4)}_{(6)}, X^{(4)}_{(11)}, X^{(4)}_{(15)}$$

We select the sample unit from 4 DURSS

$$X^{(1)}_{(2)}, \boxed{X^{(1)}_{(6)}}, X^{(1)}_{(11)}, X^{(1)}_{(15)}, X^{(2)}_{(2)}, \boxed{X^{(2)}_{(6)}}, X^{(2)}_{(11)}, X^{(2)}_{(15)}, X^{(3)}_{(2)}, X^{(3)}_{(6)}, \boxed{X^{(3)}_{(11)}}, X^{(3)}_{(15)}, X^{(4)}_{(2)}, X^{(4)}_{(6)}, \boxed{X^{(4)}_{(11)}}, X^{(4)}_{(15)}$$

Let $X^{(1)}_{(6)}, X^{(2)}_{(6)}, X^{(3)}_{(11)}, X^{(4)}_{(11)}$ is DURSS of size 4

Case: odd number

Example 5: Consider the case of $(m = 5, r = 1)$. Draw a simple random sample of size $m^3 = 5^3 = 125$ elements (5 sets of size 25 each). Assume the elements are

$$X^{(1)}_{(1)}, X^{(1)}_{(2)}, X^{(1)}_{(3)}, \cdots, X^{(1)}_{(25)}, X^{(2)}_{(1)}, X^{(2)}_{(2)}, X^{(2)}_{(3)}, \cdots, X^{(2)}_{(25)}, X^{(3)}_{(1)}, X^{(3)}_{(2)}, X^{(3)}_{(3)}, \cdots, X^{(3)}_{(25)},$$

$$X^{(4)}_{(1)}, X^{(4)}_{(2)}, X^{(4)}_{(3)}, \cdots, X^{(4)}_{(25)}, X^{(5)}_{(1)}, X^{(5)}_{(2)}, X^{(5)}_{(3)}, \cdots, X^{(5)}_{(25)}$$

After ranking the elements of each set obtain 5 ranked set samples of size 5 each($m^2 = 5^2 = 25$).

$$\begin{bmatrix} X^{(1)}_{(1)}, X^{(1)}_{(2)}, X^{(1)}_{(3)}, X^{(1)}_{(4)}, X^{(1)}_{(5)}, X^{(1)}_{(6)}, X^{(1)}_{(7)}, X^{(1)}_{(8)}, X^{(1)}_{(9)}, X^{(1)}_{(10)}, X^{(1)}_{(11)}, X^{(1)}_{(12)}, X^{(1)}_{(13)}, \\ X^{(1)}_{(14)}, X^{(1)}_{(15)}, X^{(1)}_{(16)}, X^{(1)}_{(17)}, X^{(1)}_{(18)}, X^{(1)}_{(19)}, X^{(1)}_{(20)}, X^{(1)}_{(21)}, X^{(1)}_{(22)}, X^{(1)}_{(23)}, X^{(1)}_{(24)}, X^{(1)}_{(25)} \end{bmatrix},$$

$$\begin{bmatrix} X^{(2)}_{(1)}, X^{(2)}_{(2)}, X^{(2)}_{(3)}, X^{(2)}_{(4)}, X^{(2)}_{(5)}, X^{(2)}_{(6)}, X^{(2)}_{(7)}, X^{(2)}_{(8)}, X^{(2)}_{(9)}, X^{(2)}_{(10)}, X^{(2)}_{(11)}, X^{(2)}_{(12)}, X^{(2)}_{(13)}, \\ X^{(2)}_{(14)}, X^{(2)}_{(15)}, X^{(2)}_{(16)}, X^{(2)}_{(17)}, X^{(2)}_{(18)}, X^{(2)}_{(19)}, X^{(2)}_{(20)}, X^{(2)}_{(21)}, X^{(2)}_{(22)}, X^{(2)}_{(23)}, X^{(2)}_{(24)}, X^{(2)}_{(25)} \end{bmatrix},$$

$$\begin{bmatrix} X^{(3)}_{(1)}, X^{(3)}_{(2)}, X^{(3)}_{(3)}, X^{(3)}_{(4)}, X^{(1)}_{(5)}, X^{(3)}_{(6)}, X^{(3)}_{(7)}, X^{(3)}_{(8)}, X^{(3)}_{(9)}, X^{(3)}_{(10)}, X^{(3)}_{(11)}, X^{(3)}_{(12)}, X^{(3)}_{(13)}, \\ X^{(3)}_{(14)}, X^{(3)}_{(15)}, X^{(3)}_{(16)}, X^{(3)}_{(17)}, X^{(3)}_{(18)}, X^{(3)}_{(19)}, X^{(3)}_{(20)}, X^{(3)}_{(21)}, X^{(3)}_{(22)}, X^{(3)}_{(23)}, X^{(3)}_{(24)}, X^{(3)}_{(25)} \end{bmatrix},$$

$$\begin{bmatrix} X^{(4)}_{(1)}, X^{(4)}_{(2)}, X^{(4)}_{(3)}, X^{(4)}_{(4)}, X^{(4)}_{(5)}, X^{(4)}_{(6)}, X^{(4)}_{(7)}, X^{(4)}_{(8)}, X^{(4)}_{(9)}, X^{(4)}_{(10)}, X^{(4)}_{(11)}, X^{(4)}_{(12)}, X^{(4)}_{(13)}, \\ X^{(4)}_{(14)}, X^{(4)}_{(15)}, X^{(4)}_{(16)}, X^{(4)}_{(17)}, X^{(4)}_{(18)}, X^{(4)}_{(19)}, X^{(4)}_{(20)}, X^{(4)}_{(21)}, X^{(4)}_{(22)}, X^{(4)}_{(23)}, X^{(4)}_{(24)}, X^{(4)}_{(25)} \end{bmatrix}$$

and

$$\begin{bmatrix} X^{(5)}_{(1)}, X^{(5)}_{(2)}, X^{(5)}_{(3)}, X^{(5)}_{(4)}, X^{(5)}_{(5)}, X^{(5)}_{(6)}, X^{(5)}_{(7)}, X^{(5)}_{(8)}, X^{(5)}_{(9)}, X^{(5)}_{(10)}, X^{(5)}_{(11)}, X^{(5)}_{(12)}, X^{(5)}_{(13)}, \\ X^{(5)}_{(14)}, X^{(5)}_{(15)}, X^{(5)}_{(16)}, X^{(5)}_{(17)}, X^{(5)}_{(18)}, X^{(5)}_{(19)}, X^{(5)}_{(20)}, X^{(5)}_{(21)}, X^{(5)}_{(22)}, X^{(5)}_{(23)}, X^{(5)}_{(24)}, X^{(5)}_{(25)} \end{bmatrix}$$

We select the sample unit from the elements of each set obtain 5 ranked set samples of size 5 each as

$$\begin{bmatrix} X^{(1)}_{(1)}, X^{(1)}_{(2)}, \boxed{X^{(1)}_{(3)}}, X^{(1)}_{(4)}, X^{(1)}_{(5)}, X^{(1)}_{(6)}, X^{(1)}_{(7)}, \boxed{X^{(1)}_{(8)}}, X^{(1)}_{(9)}, X^{(1)}_{(10)}, X^{(1)}_{(11)}, X^{(1)}_{(12)}, \boxed{X^{(1)}_{(13)}}, \\ X^{(1)}_{(14)}, X^{(1)}_{(15)}, X^{(1)}_{(16)}, X^{(1)}_{(17)}, \boxed{X^{(1)}_{(18)}}, X^{(1)}_{(19)}, X^{(1)}_{(20)}, X^{(1)}_{(21)}, X^{(1)}_{(22)}, \boxed{X^{(1)}_{(23)}}, X^{(1)}_{(24)}, X^{(1)}_{(25)} \end{bmatrix},$$

$$\begin{bmatrix} X^{(2)}_{(1)}, X^{(2)}_{(2)}, \boxed{X^{(2)}_{(3)}}, X^{(2)}_{(4)}, X^{(2)}_{(5)}, X^{(2)}_{(6)}, X^{(2)}_{(7)}, \boxed{X^{(2)}_{(8)}}, X^{(2)}_{(9)}, X^{(2)}_{(10)}, X^{(2)}_{(11)}, X^{(2)}_{(12)}, \boxed{X^{(2)}_{(13)}}, \\ X^{(2)}_{(14)}, X^{(2)}_{(15)}, X^{(2)}_{(16)}, X^{(2)}_{(17)}, \boxed{X^{(2)}_{(18)}}, X^{(2)}_{(19)}, X^{(2)}_{(20)}, X^{(2)}_{(21)}, X^{(2)}_{(22)}, \boxed{X^{(2)}_{(23)}}, X^{(2)}_{(24)}, X^{(2)}_{(25)} \end{bmatrix},$$

$$\begin{bmatrix} X^{(3)}_{(1)}, X^{(3)}_{(2)}, \boxed{X^{(3)}_{(3)}}, X^{(3)}_{(4)}, X^{(1)}_{(5)}, X^{(3)}_{(6)}, X^{(3)}_{(7)}, \boxed{X^{(3)}_{(8)}}, X^{(3)}_{(9)}, X^{(3)}_{(10)}, X^{(3)}_{(11)}, X^{(3)}_{(12)}, \boxed{X^{(3)}_{(13)}}, \\ X^{(3)}_{(14)}, X^{(3)}_{(15)}, X^{(3)}_{(16)}, X^{(3)}_{(17)}, \boxed{X^{(3)}_{(18)}}, X^{(3)}_{(19)}, X^{(3)}_{(20)}, X^{(3)}_{(21)}, X^{(3)}_{(22)}, \boxed{X^{(3)}_{(23)}}, X^{(3)}_{(24)}, X^{(3)}_{(25)} \end{bmatrix},$$

$$\begin{bmatrix} X^{(4)}_{(1)}, X^{(4)}_{(2)}, \boxed{X^{(4)}_{(3)}}, X^{(4)}_{(4)}, X^{(4)}_{(5)}, X^{(4)}_{(6)}, X^{(4)}_{(7)}, \boxed{X^{(4)}_{(8)}}, X^{(4)}_{(9)}, X^{(4)}_{(10)}, X^{(4)}_{(11)}, X^{(4)}_{(12)}, \boxed{X^{(4)}_{(13)}}, \\ X^{(4)}_{(14)}, X^{(4)}_{(15)}, X^{(4)}_{(16)}, X^{(4)}_{(17)}, \boxed{X^{(4)}_{(18)}}, X^{(4)}_{(19)}, X^{(4)}_{(20)}, X^{(4)}_{(21)}, X^{(4)}_{(22)}, \boxed{X^{(4)}_{(23)}}, X^{(4)}_{(24)}, X^{(4)}_{(25)} \end{bmatrix}$$

and

$$\begin{bmatrix} X^{(5)}_{(1)}, X^{(5)}_{(2)}, \boxed{X^{(5)}_{(3)}}, X^{(5)}_{(4)}, X^{(5)}_{(5)}, X^{(5)}_{(6)}, X^{(5)}_{(7)}, \boxed{X^{(5)}_{(8)}}, X^{(5)}_{(9)}, X^{(5)}_{(10)}, X^{(5)}_{(11)}, X^{(5)}_{(12)}, \boxed{X^{(5)}_{(13)}}, \\ X^{(5)}_{(14)}, X^{(5)}_{(15)}, X^{(5)}_{(16)}, X^{(5)}_{(17)}, \boxed{X^{(5)}_{(18)}}, X^{(5)}_{(19)}, X^{(5)}_{(20)}, X^{(5)}_{(21)}, X^{(5)}_{(22)}, \boxed{X^{(5)}_{(23)}}, X^{(5)}_{(24)}, X^{(5)}_{(25)} \end{bmatrix}$$

so we have 5 DURSS

$$X^{(1)}_{(3)}, X^{(1)}_{(8)}, X^{(1)}_{(13)}, X^{(1)}_{(18)}, X^{(1)}_{(23)}, X^{(2)}_{(3)}, X^{(2)}_{(8)}, X^{(2)}_{(13)}, X^{(2)}_{(18)}, X^{(2)}_{(23)}, X^{(3)}_{(3)}, X^{(3)}_{(8)}, X^{(3)}_{(13)}, X^{(3)}_{(18)}, X^{(3)}_{(23)}, X^{(4)}_{(3)}, X^{(4)}_{(8)}, X^{(4)}_{(13)}, X^{(4)}_{(18)}, X^{(4)}_{(23)}, X^{(5)}_{(3)}, X^{(5)}_{(8)}, X^{(5)}_{(13)}, X^{(5)}_{(18)}, X^{(5)}_{(23)}$$

We select the sample unit from 5 DURSS

$$X^{(1)}_{(3)}, X^{(1)}_{(8)}, \boxed{X^{(1)}_{(13)}}, X^{(1)}_{(18)}, X^{(1)}_{(23)}, X^{(2)}_{(3)}, X^{(2)}_{(8)}, \boxed{X^{(2)}_{(13)}}, X^{(2)}_{(18)}, X^{(2)}_{(23)}, X^{(3)}_{(3)}, X^{(3)}_{(8)}, X^{(3)}_{(13)}, X^{(3)}_{(18)}, X^{(3)}_{(23)}, X^{(4)}_{(3)}, X^{(4)}_{(8)}, \boxed{X^{(4)}_{(13)}}, X^{(4)}_{(18)}, X^{(4)}_{(23)}, X^{(5)}_{(3)}, X^{(5)}_{(8)}, \boxed{X^{(5)}_{(13)}}, X^{(5)}_{(18)}, X^{(5)}_{(23)}$$

Let $X^{(1)}_{(13)}, X^{(2)}_{(13)}, X^{(3)}_{(13)}, X^{(4)}_{(13)}, X^{(5)}_{(13)}$ is DURSS of size 5

Stratified Double Ranked Set Sampling (SDURSS)

If the Double ranked set sampling method is used to select the sample units from each stratum, then the whole procedure is called a Stratified Double Ranked Set Sampling (SDURSS). To illustrate the SDURSS method, let us consider the follows example for sample size.

Example 4: Suppose that we have two strata, i.e. $L = 2$ and $h = 1, 2$. Let (m, r) Assume that from the first stratum we select a sample of size $m \times r = 4 \times 2 = 8$ and from the second stratum we want a sample of size $m \times r = 4 \times 2 = 8$. Then the process as illustrates as follows:

Stratum 1: Now, select 8 samples as follows:

Draw a simple random sample of size $m \times r = 4 \times 2 = 8$ units, 2 times and select of sample units.

Stratum 1	$(r = 1)$	$X^{1(1)}_{(1)1}, X^{1(1)}_{(2)1}, X^{1(1)}_{(3)1}, \cdots, X^{1(1)}_{(16)1}, X^{2(1)}_{(1)1}, X^{2(1)}_{(2)1}, X^{2(1)}_{(3)1}, \cdots, X^{2(1)}_{(16)1},$ $X^{3(1)}_{(1)1}, X^{3(1)}_{(2)1}, X^{3(1)}_{(3)1}, \cdots, X^{3(1)}_{(16)1}, X^{4(1)}_{(1)1}, X^{4(1)}_{(2)1}, X^{4(1)}_{(3)1}, \cdots, X^{4(1)}_{(16)1}$
	$(r = 2)$	$X^{1(1)}_{(1)2}, X^{1(1)}_{(2)2}, X^{1(1)}_{(3)2}, \cdots, X^{1(1)}_{(16)2}, X^{2(1)}_{(1)2}, X^{2(1)}_{(2)2}, X^{2(1)}_{(3)2}, \cdots, X^{2(1)}_{(16)2},$ $X^{3(1)}_{(1)2}, X^{3(1)}_{(2)2}, X^{3(1)}_{(3)2}, \cdots, X^{3(1)}_{(16)2}, X^{4(1)}_{(1)2}, X^{4(1)}_{(2)2}, X^{4(1)}_{(3)2}, \cdots, X^{4(1)}_{(16)2}$

For $h=1$ we have $X^{6(1)}_{(1)1}, X^{6(1)}_{(2)1}, X^{11(1)}_{(3)1}, X^{11(1)}_{(4)1}, X^{6(1)}_{(1)2}, X^{6(1)}_{(2)2}, X^{11(1)}_{(3)2}, X^{11(1)}_{(4)2}$.

Stratum 2: Now, select 8 samples as follows:

Draw a simple random sample of size $m^2 = 4^2 = 16$ units, 2 times and select of sample units.

Stratum 2	$(r=1)$	$X^{1(2)}_{(1)1}, X^{1(2)}_{(2)1}, X^{1(2)}_{(3)1}, \cdots, X^{1(2)}_{(16)1}, X^{2(2)}_{(1)1}, X^{2(2)}_{(2)1}, X^{2(2)}_{(3)1}, \cdots, X^{2(2)}_{(16)1},$ $X^{3(2)}_{(1)1}, X^{3(2)}_{(2)1}, X^{3(2)}_{(3)1}, \cdots, X^{3(2)}_{(16)1}, X^{4(2)}_{(1)1}, X^{4(2)}_{(2)1}, X^{4(2)}_{(3)1}, \cdots, X^{4(2)}_{(16)1}$
	$(r=2)$	$X^{1(2)}_{(1)2}, X^{1(2)}_{(2)2}, X^{1(2)}_{(3)2}, \cdots, X^{1(2)}_{(16)2}, X^{2(2)}_{(1)2}, X^{2(2)}_{(2)2}, X^{2(2)}_{(3)2}, \cdots, X^{2(2)}_{(16)2},$ $X^{3(2)}_{(1)2}, X^{3(2)}_{(2)2}, X^{3(2)}_{(3)2}, \cdots, X^{3(2)}_{(16)2}, X^{4(2)}_{(1)2}, X^{4(2)}_{(2)2}, X^{4(2)}_{(3)2}, \cdots, X^{4(2)}_{(16)2}$

For $h=2$ we have $X^{6(2)}_{(1)1}, X^{6(2)}_{(2)1}, X^{11(2)}_{(3)1}, X^{11(2)}_{(4)1}, X^{6(2)}_{(1)2}, X^{6(2)}_{(2)2}, X^{11(2)}_{(3)2}, X^{11(2)}_{(4)2}$.

(Define: k, $X^{k(h)}_{(i)j}$ is number of ranking the elements of each set, h stratum size, i is number of each set, r is number of cycles (times)

Therefore, the measured SDURSS units are

$X^{6(1)}_{(1)1}, X^{6(1)}_{(2)1}, X^{11(1)}_{(3)1}, X^{11(1)}_{(4)1}, X^{6(1)}_{(1)2}, X^{6(1)}_{(2)2}, X^{11(1)}_{(3)2}, X^{11(1)}_{(4)2}, X^{6(2)}_{(1)1}, X^{6(2)}_{(2)1}, X^{11(2)}_{(3)1}, X^{11(2)}_{(4)1}, X^{6(2)}_{(1)2}, X^{6(2)}_{(2)2}, X^{11(2)}_{(3)2}, X^{11(2)}_{(4)2}$

where their mean of these units is used as an estimator of the population mean.

Estimation of Population Mean

Let $X_1, X_2, \cdots, X_n$ be n independent random variables from a probability density function with mean μ and variance σ^2. follows:

The DURSS estimator of the population mean is given by

$$\bar{x}_{DURSS}(m)r = \frac{1}{mr}\sum_{i=1}^{m}\sum_{j=1}^{r} x_{(l+(i-1)m)j}$$

(Definition: If m is an even number, Part 1 is defined as $l = \frac{m}{2}$ and Part 2 as $l = \frac{m}{2}+1$. If m is an odd number, it is defined as $l = \frac{m}{2}+1$.)

And the variance is

$$\sigma^2_{\bar{x}_{SDURSS}}(m)r = \frac{1}{mr}\left\{\sigma^2 - \frac{1}{m}\sum_{i=1}^{m}\left(\mu_{(l+(i-1)m)} - \mu\right)^2\right\}.$$

The Stratified Double Ranked Set Sampling (SDURSS) estimator of the population mean is given by

Lemma 1: If the distribution is symmetric about μ, then $E\left(\bar{X}_{SDURSS}\right) = \mu$, $\bar{X}_{SDURSS}$ is an unbiased estimator of μ

Proof: the sample size $(m_h r = n_h)$ whining the strata, we have

$$E\left(\bar{X}_{SDURSS}\right) = E\left[\sum_{h=1}^{L} W_h\left(\bar{X}_{DURSS}(m,r)\right)h\right]$$

$$= E\left[\sum_{h=1}^{L}\frac{W_h}{m_h r}\left(\sum_{i=1}^{m_h}\sum_{j=1}^{r} x_{(l+(i-1)m_h)j}\right)\right]$$

$$= \sum_{h=1}^{L}\frac{W_h}{m_h r}\left[\sum_{i=1}^{m_h}\sum_{j=1}^{r} E\left(x_{(l+(i-1)m_h)j}\right)\right]$$

$$= \sum_{h=1}^{L}\frac{W_h}{m_h r}\left[\sum_{i=1}^{m_h}\sum_{j=1}^{r} \mu_{(l+(i-1)m_h)j}\right]$$

Since the distribution is symmetric about μ, then $\mu_{(l+(i-1)m_h)j} = \mu_h$. Therefore, we have

$$E\left(\bar{X}_{SDURSS}\right) = E\left[\sum_{h=1}^{L}\frac{W_h}{m_h r}\left(\sum_{i=1}^{m_h}\sum_{j=1}^{r}\mu_h\right)\right]$$

$$= \sum_{h=1}^{L}\frac{W_h}{n_h}\left(n_h \mu_h\right)$$

$$= \sum_{h=1}^{L} W_h . \mu_h = \mu.$$

Where $W_h = \dfrac{N_h}{N}, N_h$ is the stratum size. The variance of SDURSS is given by

$$Var\left(\bar{X}_{SDURSS}\right) = Var\left[\sum_{h=1}^{L}\frac{W_h}{m_h r}\left(\sum_{i=1}^{m_h}\sum_{j=1}^{r} x_{(i:m_h)j}\right)\right]$$

$$= \sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r^2}\left(\sum_{i=1}^{m_h}\sum_{j=1}^{r} Var\left(x_{(i:m_h)j}\right)\right)$$

$$= \sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r^2}\left(\sum_{i=1}^{m_h}\sum_{j=1}^{r} \sigma^2_{\left[x_{(i:m)j}\right]h}\right)$$

$$= \sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r^2}\sigma^2_{\left[x_{(i:m)j}\right]h}$$

Since $E\left(\bar{X}_{SDURSS}\right) = \mu$, hence $Var\left(\bar{X}_{SDURSS}\right) = MSE\left(\bar{X}_{SDURSS}\right)$.

For the means and variances of SSRS, SRSS and SMRSS estimators are as follows.

The means and variances of SSRS are

$$\bar{X}_{SSRS} = \sum_{h=1}^{L} W_h \bar{X}_{(SRS)h}$$

and

$$MSE\left(\bar{X}_{SSRS}\right)=Var\left(\bar{X}_{SSRS}\right)=\sum_{h=1}^{L}W_h^2\left(\frac{N_h-n_h}{N_h}\right)\frac{S_h^2}{n_h},$$

respectively.

The means and variances of SRSS are

$$\bar{X}_{SRSS}=\sum_{h=1}^{L}W_h\left(\bar{x}_{RSS(m_h,r)}\right)_h$$

and

$$MSE\left(\bar{X}_{SRSS}\right)=Var\left(\bar{X}_{SRSS}\right)=\sum_{h=1}^{L}W_h^2\left(\frac{S^2_{h\left(X_{[i:m]j}\right)_h}}{m_h r}\right)$$

where

$$S^2_{h\left(X_{[i:m]j}\right)_h}=S_h^2+\frac{1}{m_h}\sum_{i=1}^{m}\left(\bar{X}_{\left[(i:m)j\right]_h}-\bar{X}_h\right)^2,$$

respectively.

The means and variances of SMRSS are

$$\bar{X}_{SMRSS1}=\sum_{h=1}^{L}\frac{W_h}{n_h}\left(\sum_{i=1}^{m_h}X_{\left(\frac{m_h+1}{2}\right)}\right)$$

(m = Odd)

$$MSE\left(\bar{X}_{SMRSS1}\right)=Var\left(\bar{X}_{SMRSS1}\right)=\sum_{h=1}^{L}\frac{W_h^2}{n_h}S^2_{h\left(\frac{m_h+1}{2}\right)}$$

(m = Odd)

and

$$\bar{X}_{SMRSS2}=\sum_{h=1}^{L}\frac{W_h}{n_h}\left(\sum_{i=1}^{\frac{m_h}{2}}X_{\left(\frac{m_h}{2}\right)}+\sum_{i=1}^{m_h}X_{\left(\frac{m_h}{2}+1\right)}\right)$$

(m = Even)

$$MSE\left(\bar{X}_{SMRSS2}\right)=Var\left(\bar{X}_{SMRSS2}\right)=\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left(\sum_{i=1}^{\frac{m_h}{2}}S^2_{h\left(\frac{m_h}{2}\right)}+\sum_{i=1}^{m_h}S^2_{h\left(\frac{m_h}{2}+1\right)}\right)$$

(m = Even)

3. Simulation and Results

To evaluate the efficiency of this method relative to SDURSS, SSRS, SRSS and SMRSS, we estimate the population mean.

Let $\bar{x}_{SDURSS} = \sum_{h=1}^{L} W_h \left(\bar{x}_{DURSS}\right)_h$ be the SDURSS estimator, while $\bar{x}_{SSRS} = \sum_{h=1}^{L} W_h . \left(\bar{x}_{SSRS}\right)_h$ is the SSRS estimator, and $\bar{x}_{SRSS} = \sum_{h=1}^{L} W_h . \left(\bar{x}_{SRSS}\right)_h$ is the SRSS estimator, and $\bar{x}_{SMRSS} = \sum_{h=1}^{L} W_h . \left(\bar{x}_{SMRSS}\right)_h$ is the SMRSS estimator.

In this section, a simulation study is designed for symmetric distributions with samples of sizes n. We assume that set, and cycles, to compare the SDURSS with the SSRS, SRSS, and SMRSS methods. We assumed that the population is partitioned into two strata in each Strata divide use proportional allocate. We simulate 5,000 replications by R (Version 4.3.2) for computing of the means and variance. If the underlying distribution is symmetric, the efficiency of SDURSS relative to SSRS, SRSS, and SMRSS, respectively are given by

$$eff\left(\bar{X}_{SDURSS}, \bar{X}_{SSRS}\right) = \frac{MSE\left(\bar{X}_{SSRS}\right)}{MSE\left(\bar{X}_{SDURSS}\right)},$$

$$eff\left(\bar{X}_{SDURSS}, \bar{X}_{SRSS}\right) = \frac{MSE\left(\bar{X}_{SRSS}\right)}{MSE\left(\bar{X}_{SDURSS}\right)},$$

and

$$eff\left(\bar{X}_{SDURSS}, \bar{X}_{SMRSS}\right) = \frac{MSE\left(\bar{X}_{SMRSS}\right)}{MSE\left(\bar{X}_{SDURSS}\right)}.$$

Figure 1-6 presents the mean squared error (MSE) of the population mean estimator under varying set numbers m = 2, 4, 6, 10 and rounds r = 2, 5. Figures 1, 3, and 5 show the number of sets for m = 2, 4, 6, 10 and rounds r = 2 under the standard normal distribution, Student-t distribution, and uniform distribution (0,1) respectively. The MSE of the Stratified Double Unified Ranked Set Sampling (SDURSS) is lower than that of Stratified Simple Random Sampling (SSRS), Stratified Ranked Set Sampling (SRSS), and Stratified Median Ranked Set Sampling (SMRSS). Figures 2, 4, and 6 display the number of sets for m = 2, 4, 6, 10 and rounds r = 5 under the standard normal distribution, Student-t distribution, and uniform distribution (0,1) respectively. The MSE of SDURSS is higher than SSRS, SRSS, and SMRSS in some cases.

Figure 1 illustrates the Mean Squared Error (MSE) based on a normal distribution for different sampling methods with r = 2. The graph compares the performance of four different sampling methods: Stratified Simple Random Sampling (SSRS), Stratified Ranked Set Sampling (SRSS), Stratified Median Ranked Set Sampling (SMRSS), and Stratified Double Unified Ranked Set Sampling (SDURSS). From the figure, it can be observed that the SDURSS method consistently yields lower MSE values compared to SSRS, SRSS, and SMRSS, particularly as the number of units increases. This demonstrates that SDURSS is generally more effective in reducing estimation errors when sampling from a normal distribution under the condition of r = 2. In contrast, SSRS exhibits higher MSE values, especially for larger sets of units, indicating less precision in estimating the population mean.

Figure 2 displays the Mean Squared Error (MSE) based on a normal distribution for different sampling methods under r = 5. The graph compares four sampling methods: Stratified Double Unified Ranked Set Sampling (SDURSS), Stratified Median Ranked Set Sampling (SMRSS), Stratified Ranked Set Sampling (SRSS), and Stratified Simple Random Sampling (SSRS). From the figure, it can be observed that the MSE for SDURSS starts higher compared to other methods but gradually decreases as the number of units increases, converging towards the MSE values of the other methods. By the time the number of units reaches 50, all four methods show similar MSE values, indicating that as the number of units grows, the differences in performance between these sampling methods reduce. However, in the initial stages, SMRSS and SRSS exhibit lower MSEs than SDURSS, suggesting that in smaller unit samples, these methods might be more efficient in minimizing estimation errors.

Figure 3 depicts the Mean Squared Error (MSE) based on the Student's t-distribution for different sampling methods with r = 2. The graph compares the performance of four methods: Stratified Simple Random Sampling (SSRS), Stratified Ranked Set Sampling (SRSS), Stratified Median Ranked Set Sampling (SMRSS), and Stratified Double Unified Ranked Set Sampling (SDURSS). In the early stages, when the number of units is small, SDURSS exhibits the lowest MSE values compared to the other methods, indicating higher efficiency in minimizing errors under the Student's t-distribution. On the other hand, SSRS and SRSS display higher MSE values initially, particularly with fewer units. As the number of units increases, the differences in MSE among the methods diminish, and they converge to similar values, suggesting that the advantage of SDURSS becomes less significant as the sample size grows.

Figure 4 presents the Mean Squared Error (MSE) based on the Student's t-distribution for different sampling methods with r=5. The graph compares four methods: Stratified Double Unified Ranked Set Sampling (SDURSS), Stratified Median Ranked Set Sampling (SMRSS), Stratified Ranked Set Sampling (SRSS), and Stratified Simple Random Sampling (SSRS). At the beginning, when the number of units is small, SDURSS and SMRSS show similar MSE values, both being lower than SSRS and SRSS. As the number of units increases, the MSE values for all methods converge, resulting in nearly identical MSE values by the time the sample size reaches 50 units. This indicates that while SDURSS and SMRSS perform better with smaller sample sizes, the differences between the methods decrease as the number of units grows, with all methods performing similarly when larger numbers of units are used.

Figure 5 displays the Mean Squared Error (MSE) based on a uniform distribution for different sampling methods with r = 2. The graph compares four methods: Stratified Simple Random Sampling (SSRS), Stratified Ranked Set Sampling (SRSS), Stratified Median Ranked Set Sampling (SMRSS), and Stratified Double Unified Ranked Set Sampling (SDURSS). The figure shows that SDURSS consistently yields the lowest MSE values across the range of units, indicating that it performs better in minimizing estimation error under a uniform distribution. In contrast, SSRS exhibits the highest MSE values, followed by SRSS and SMRSS, which show more fluctuating patterns as the number of units increases. Overall, SDURSS outperforms the other methods in terms of MSE reduction under these conditions.

Figure 6 illustrates the Mean Squared Error (MSE) based on a uniform distribution for different sampling methods with r=5. The graph compares four sampling methods: Stratified Simple Random Sampling (SSRS), Stratified Ranked Set Sampling (SRSS), Stratified Median Ranked Set Sampling (SMRSS), and Stratified Double Unified Ranked Set Sampling (SDURSS). In this figure, SDURSS initially shows lower MSE values than the other methods, indicating better performance in reducing error with smaller sample sizes. However, as the number of units increases, the MSE values of all methods converge and become nearly identical. By the time the number of units reaches 50, there is little difference between the methods, suggesting that the advantage of SDURSS diminishes as the sample size grows. SMRSS and SRSS exhibit slightly higher MSE values early on, but they also converge with SDURSS and SSRS as the sample size increases.

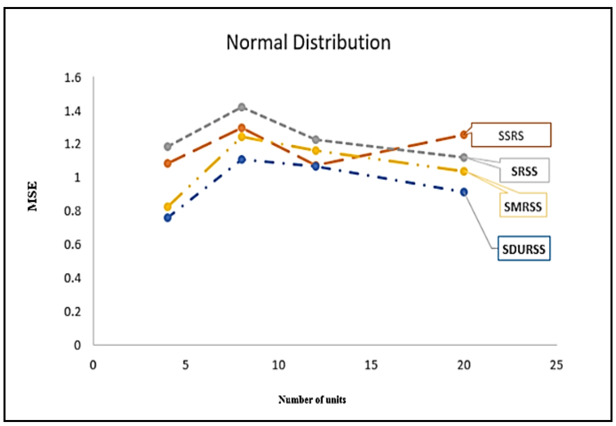

Fig. 1: MSE based on Normal Distribution under r = 2

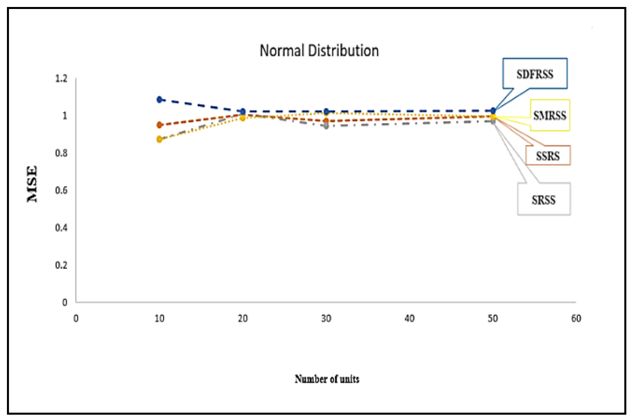

Fig. 2: MSE based on Normal Distribution under r = 5

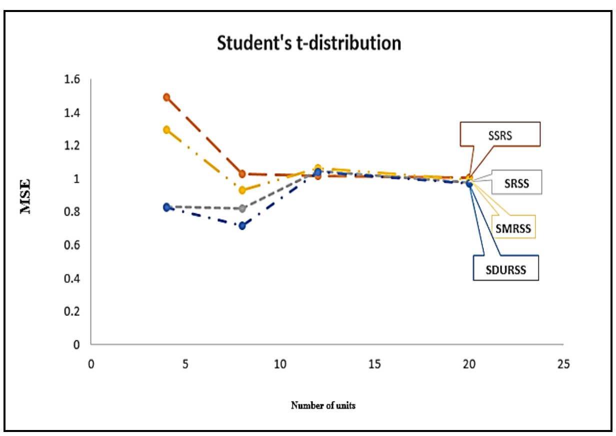

Fig. 3: MSE based on Student-t distribution under r = 2

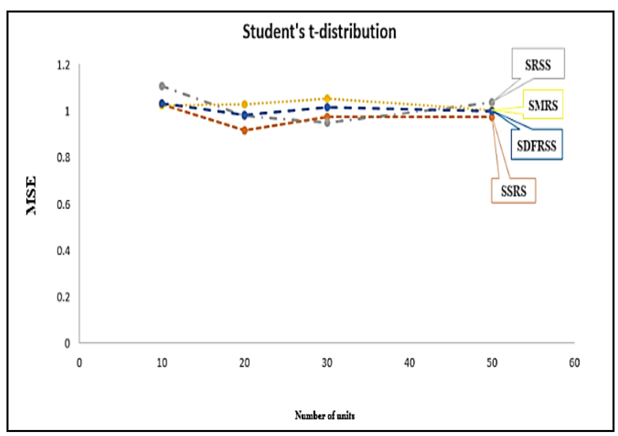

Fig. 4: MSE based on Student-t distribution under r = 5

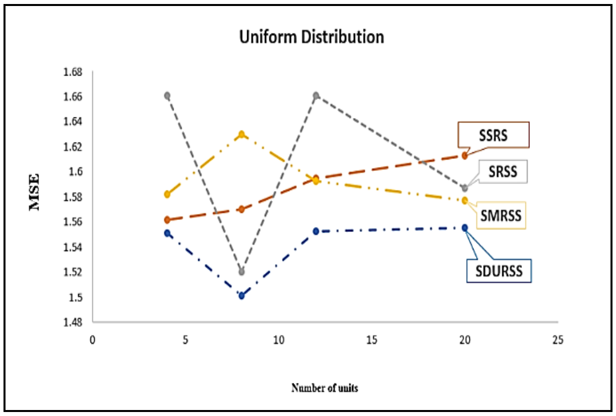

Fig. 5: MSE based on uniform distribution under r = 2

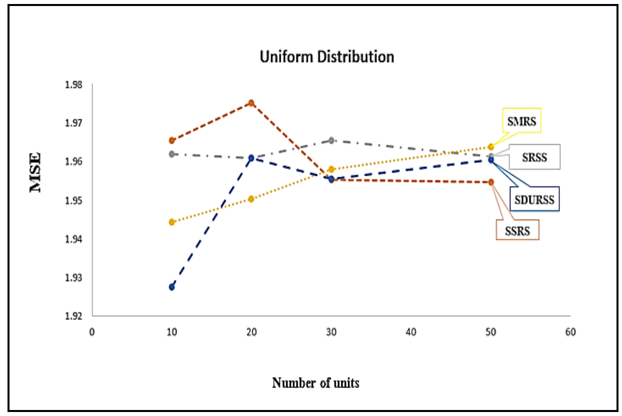

Fig. 6: MSE based on uniform distribution under r = 5

Based on the data presented in Tables 1 to 4. Statistical sampling methods are pivotal in research, providing insights into population parameters through smaller, manageable subsets. This essay explores the efficiency of Stratified Double Unified Ranked Set Sampling (SDURSS) in comparison to three other methods—Stratified Simple Random Sampling (SSRS), Stratified Ranked Set Sampling (SRSS), and Stratified Median Ranked Set Sampling (SMRSS)—for estimating the population mean under various distributions and sample conditions. This analysis draws upon data from four tables which compare the methods across different rounds of sampling and distributions.

The data shows how SDURSS performs under three types of distributions: normal, Student-t, and uniform. Each distribution tests the robustness of SDURSS against traditional methods under both theoretical and practical scenarios. In the case of the normal distribution, SDURSS tends to perform better at lower rounds of sampling (r=2) but shows mixed results as the number of rounds increases (r=5). For example, under a normal distribution with r=2, SDURSS shows higher efficiency ratios (1.4205, 1.5585, 1.0855) compared to when r=5 (0.8773, 0.8045, 0.8060) indicating a decrease in relative efficiency with increased sampling rounds.

The results in Tables 1 to 4 consistently illustrate a trend where SDURSS tends to have higher efficiency at lower rounds across all distributions but does not maintain this edge as the number of rounds increases. This trend is evident in the uniform distribution as well, where initial efficiencies (1.0068, 1.0709, 1.0200 for r=2) slightly decrease or stabilize at higher rounds (1.0197, 1.0179, 1.0088 for r=5). This suggests that while SDURSS is initially more robust, its advantage diminishes with more extensive sampling, which could be due to the increasing complexity and potential biases introduced in multiple rounds of ranked set sampling.

In the Student-t distribution, known for its heavy tails and sensitivity to outliers, SDURSS's performance is more variable. Initially, it outperforms other methods (e.g., 1.8056, 1.0082, 1.5639 for r=2) but then converges closer to the efficiency of other methods with more rounds (0.9936, 1.0698, 0.9888 for r=5). This fluctuation can be attributed to the distribution's characteristics influencing the ranking accuracy and, consequently, the sampling efficiency of ranked set methods.

Table 1: The efficiency of SDURSS relative to SSRS, SRSS, and SMRSS to estimating the population mean with $m = 2$, $r = 2,5$

Distribution	r	$eff\left(\bar{X}_{SDURSS}, \bar{X}_{SSRS}\right)$	$eff\left(\bar{X}_{SDURSS}, \bar{X}_{SRSS}\right)$	$eff\left(\bar{X}_{SDURSS}, \bar{X}_{SMRSS}\right)$
Normal (0,1)	2	1.4205	1.5585	1.0855
	5	0.8773	0.8045	0.8060
Student-t	2	1.8056	1.0082	1.5639
	5	0.9936	1.0698	0.9888
Uniform (0,1)	2	1.0068	1.0709	1.0200
	5	1.0197	1.0179	1.0088

Table 2: The efficiency of SDURSS relative to SSRS, SRSS, and SMRSS to estimating the population mean with $m = 4$, $r = 2,5$

Distribution	r	$eff\left(\bar{X}_{SDURSS}, \bar{X}_{SSRS}\right)$	$eff\left(\bar{X}_{SDURSS}, \bar{X}_{SRSS}\right)$	$eff\left(\bar{X}_{SDURSS}, \bar{X}_{SMRSS}\right)$
Normal (0,1)	2	1.1711	1.2827	1.1240
	5	0.9843	0.9870	0.9678
Student-t	2	1.4386	1.1457	1.2973

Distribution	r	$eff\left(\bar{X}_{SDURSS}, \bar{X}_{SSRS}\right)$	$eff\left(\bar{X}_{SDURSS}, \bar{X}_{SRSS}\right)$	$eff\left(\bar{X}_{SDURSS}, \bar{X}_{SMRSS}\right)$
	5	0.9315	0.9964	1.0469
Uniform (0,1)	2	1.0462	1.0127	1.0858
	5	1.0072	1.0000	0.9945

Table 3: The efficiency of SDURSS relative to SSRS, SRSS, and SMRSS to estimating the population mean with $m = 6$, $r = 2,5$

Distribution	r	$eff\left(\bar{X}_{SDURSS}, \bar{X}_{SSRS}\right)$	$eff\left(\bar{X}_{SDURSS}, \bar{X}_{SRSS}\right)$	$eff\left(\bar{X}_{SDURSS}, \bar{X}_{SMRSS}\right)$
Normal (0,1)	2	1.0053	1.1491	1.0880
	5	0.9501	0.9259	0.9896
Student-t	2	0.9786	1.0037	1.0246
	5	0.9580	0.9350	1.0387
Uniform (0,1)	2	1.0271	1.0696	1.0259
	5	0.9999	1.0051	1.0013

Table 4: The efficiency of SDURSS relative to SSRS, SRSS, and SMRSS to estimating the population mean with $m = 10$, $r = 2,5$

Distribution	r	$eff\left(\bar{X}_{SDURSS}, \bar{X}_{SSRS}\right)$	$eff\left(\bar{X}_{SDURSS}, \bar{X}_{SRSS}\right)$	$eff\left(\bar{X}_{SDURSS}, \bar{X}_{SMRSS}\right)$
Normal (0,1)	2	1.3754	1.2287	1.1350
	5	0.9727	0.9471	0.9712
Student-t	2	1.0385	1.0098	1.0222
	5	0.9732	1.0375	1.0018
Uniform (0,1)	2	1.0371	1.0205	1.0141
	5	0.9970	1.0005	1.0017

4. Conclusions

The analysis of SDURSS compared to SSRS, SRSS, and SMRSS across various distributions and sampling rounds suggests that while SDURSS may offer superior efficiency in initial rounds, its advantages diminish with increased sampling. This pattern underscores the importance of choosing a sampling method that aligns with the specific conditions of the study, including the distribution of the population data and the number of sampling rounds planned. In settings where fewer rounds are practical and distributions are not heavily skewed, SDURSS offers a promising alternative. However, researchers should remain cautious with more extensive sampling and distributions prone to outliers, such as the Student-t. The choice of sampling method, therefore, should be guided not just by efficiency metrics but also by an understanding of the underlying population characteristics and the research objectives.

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