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Investigation on the Real-Time Tracking of Single Magnetic Target under the Geomagnetic Background

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Abstract: This paper presents a method of tracking a single magnetic target in the typical background of smooth geomagnetic field and geomagnetic anomalies. Mathematically, a set of linear equations is firstly derived, which serves as a solution to the real-time tracking of a single magnetic object for small area. And then using the obtained location information, the equivalent magnetic moment for the target is obtained. Finally computer simulation and error analysis are provided. The results show that this mathematical model is relatively satisfying for medium/short-range continuous tracking under geomagnetic background and this simple system also has some reference value for practical applications to some extent.

Keywords: Real-Time Tracking; Single Magnetic Target; Geomagnetic Field

1. Introduction

The tracking technique of a single magnetic target has attracted more and more attention because of its essential role in numerous military and civilian applications. In the past decades, several theories and practical systems have been proposed to realize this goal in an increasing efficient way. However, they often need a large collection of the relative information [1] or require the target to be equipped with complex electronic attachments. In some cases, the complex optimization algorithm suffers an unacceptable time costs for real-time tracking systems while still cannot totally get rid of the accumulated error. Considering the limited time interval between each measurement in medium/ short-range real-time target tracking systems, those drawbacks become even more significant.

In recent years, the latest magnetic sensors have already achieved a very high accuracy [2], for instance, the optically pumped magnetometer and the superconducting quantum interference magnetometer have reached an accuracy level of pT and even fT. This actually provides us a new chance to realize the high-accuracy real-time localization just with a simple attachment, such as an RFID tag, to the target [3, 6].

Some investigations on tracking a single target through the detection of certain characters in the weak magnetic field indicate that this promising approach is of easy access in simple electromagnetic environment [2-6]. However, since in many practical engineering applications, the geomagnetic field and local geomagnetic anomalies can reshape the total weak magnetic field seriously, those methods cannot arrive at the results predicted theoretically. This paper further explored the real-time tracking principle under this circumstance. For two typical geomagnetic backgrounds, a set of linear equations is derived mathematically as the solution to this problem, respectively. Considering its acceptable computational complexity, each calculation can be finished in a sufficient short time interval with a cheap assistant processing circuit. Meanwhile, computer simulation and error analysis of this tracking method are further provided.

2. Localization Principle in the Typical Geomagnetic Field

For an RFID tag fixed on the mobile target with a loop antenna, it is equivalent to a magnetic dipole with an unknown magnetic moment of $\vec{p}=(p_1,p_2,p_3)^T$ [3]; then the total magnetic field intensity $\vec{H}=(H_x,H_y,H_z)^T$ generated at $\vec{r}=(x,y,z)^T$ is expressed as

$$\vec{H}=\frac{1}{4\pi}\frac{3(\vec{p}\cdot\vec{n})\vec{n}-\vec{p}}{r^3}+\overrightarrow{H_B} \quad (1)$$

where $\vec{n}$ is the unit vector of $\vec{r}$.

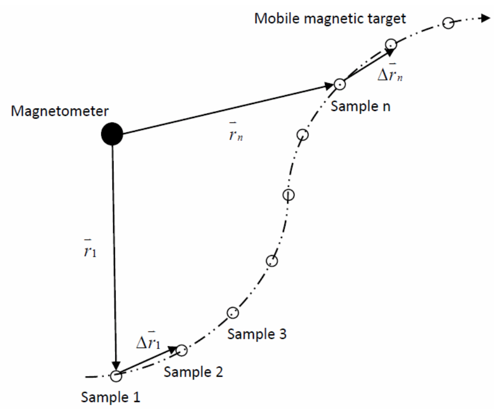

Fig.1 The illustration of the tracking method

By repeatedly measuring the three components of $\vec{H}$ with purposely designed device, which will be described in detail later, the tracking of mobile magnetic target at a successive set of sample points can be achieved, as depicted in Fig.1.

The total magnetic field varies with the space and time. However, compared with the very limited time span of the operation of a medium/short-range tracking system, the geomagnetic field can be considered as being independent of time. Besides, in the near earth space, the background magnetic field performs mainly as the superposition of the geomagnetic field and a series of abnormal magnetic fields caused by natural materials and man-made facilities, such as the magnetic mineral, high-voltage transmission line etc. Based on these considerations, this problem can be abstracted as the following two cases typically.

2.1. $\overrightarrow{H_B}$ *keeps constant*

Usually, the background magnetic field in the near-earth space is not affected by geomagnetic anomalies. Thus for a medium/ short-range localization system, the background magnetic field intensity can be approximately regarded as constant.

Here we denote $\overrightarrow{H_B}$ as $\overrightarrow{H_B}=\begin{pmatrix}H_{0x}\\H_{0y}\\H_{0z}\end{pmatrix}$. In many practical situations, $\vec{p}$ can be taken as constant. By calculating the derivative of $\vec{H}$, we can obtain the partial differential equation of H_x,H_y,H_z:

$$\sum_{k=1}^{3}\frac{\partial H_i}{\partial x_k}\cdot x_k=-3(H_i-H_{0i}) \quad (2)$$

where x_1、x_2、x_3 represent x, y, z, and H_1, H_2, H_3 represent H_x, H_y, H_z.

Therefore the localization equation can be reached as

$$\vec{r}=\begin{pmatrix}x\\y\\z\end{pmatrix}=-3\cdot\begin{pmatrix}\frac{\partial H_x}{\partial x}&\frac{\partial H_x}{\partial y}&\frac{\partial H_x}{\partial z}\\\frac{\partial H_y}{\partial x}&\frac{\partial H_y}{\partial y}&\frac{\partial H_y}{\partial z}\\\frac{\partial H_z}{\partial x}&\frac{\partial H_z}{\partial y}&\frac{\partial H_z}{\partial z}\end{pmatrix}^{-1}\cdot\begin{pmatrix}H_x-H_{0x}\\H_y-H_{0y}\\H_z-H_{0z}\end{pmatrix} \quad (3)$$

In practical engineering application, the three components of the total magnetic field intensity $\vec{H}$ and each of the gradient components required in equation (3) can be measured. Thus the relative coordinates of the magnetic target can be computed directly.

With an acceptable computational complexity, equation (3) can be utilized repeatedly with certain time interval to realize the continuous tracking of the target. Since all the measured information is only related to the current position of the target, no error will be accumulated along the trajectory.

2.2. $\overrightarrow{H_B}$ *changes at a monotonous and steady rate*

For an area in which exist magnetic anomalies, it can be divided into several regions whose background magnetic field changes at a monotonous and steady rate. Considering the limited space span of a medium/short-range localization system, a set of linear functions can be utilized to approximate the actual local background magnetic field intensity.

Without loss of generality, let

$$\overline{H}_B = \begin{pmatrix} k_1 x + h_{0x} \\ k_2 y + h_{0y} \\ k_3 z + h_{0z} \end{pmatrix},$$

then we can arrive at

$$\begin{pmatrix} \frac{\partial H_x}{\partial x} & \frac{\partial H_x}{\partial y} & \frac{\partial H_x}{\partial z} \\ \frac{\partial H_y}{\partial x} & \frac{\partial H_y}{\partial y} & \frac{\partial H_y}{\partial z} \\ \frac{\partial H_z}{\partial x} & \frac{\partial H_z}{\partial y} & \frac{\partial H_z}{\partial z} \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -3 \cdot \begin{pmatrix} H_x - k_1 x - h_{0x} \\ H_y - k_2 y - h_{0y} \\ H_z - k_3 z - h_{0z} \end{pmatrix} + \begin{pmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad (4)$$

Equation (4) leads to the localization of a magnetic target in the background of the linearly changing geomagnetic field, i.e.

$$\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -3 \cdot \begin{pmatrix} \frac{\partial H_x}{\partial x} - k_1 & \frac{\partial H_x}{\partial y} & \frac{\partial H_x}{\partial z} \\ \frac{\partial H_y}{\partial x} & \frac{\partial H_y}{\partial y} - k_2 & \frac{\partial H_y}{\partial z} \\ \frac{\partial H_z}{\partial x} & \frac{\partial H_z}{\partial y} & \frac{\partial H_z}{\partial z} - k_3 \end{pmatrix}^{-1} \cdot \begin{pmatrix} H_x - k_1 x - h_{0x} \\ H_y - k_2 y - h_{0y} \\ H_z - k_3 z - h_{0z} \end{pmatrix} \quad (5)$$

Similarly, all of the required information for localization, i.e. each component of the right side of equation (5), can be obtained through appropriate design of the measuring device. And the limited computational complexity also makes equation (5) an ideal principle to realize the real-time tracing without the accumulated error.

2.3. *The determination of magnetic moment $\overline{p}$*

After having calculated the three-dimensional coordinate of the target through equation (3) or (5), the magnetic moment $\vec{p}$ of the magnetic dipole can further be obtained.

From the original magnetic model described as equation (1), we can arrive at:

$$\overline{H} - \overline{H_B} = \frac{3}{4\pi r^5}(p_1 x + p_2 y + p_3 z) \cdot \vec{r} - \frac{1}{4\pi r^5} \cdot \vec{p} \quad (6)$$

Above equation can be rearranged as:

$$\begin{pmatrix} H_x - H_{Bx} \\ H_y - H_{By} \\ H_z - H_{Bz} \end{pmatrix} = \frac{3}{4\pi r^5} \begin{pmatrix} -y^2 - z^2 & xy & xz \\ xy & -x^2 - z^2 & yz \\ xz & yz & -x^2 - y^2 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} \quad (7)$$

Through calculating the inverse matrix of the coefficient matrix by Cramer Law, the magnetic moment can be finally expressed as:

$$\begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = 2\pi r \cdot \begin{pmatrix} x^2 - 2y^2 - 2z^2 & 3xy & 3xz \\ 3xy & -2x^2 + y^2 - 2z^2 & 3yz \\ 3xz & 3yz & -2x^2 - 2y^2 + z^2 \end{pmatrix} \cdot \begin{pmatrix} H_x - H_{Bx} \\ H_y - H_{By} \\ H_z - H_{Bz} \end{pmatrix} \quad (8)$$

For each computation through equation (3) or (5), the current value of $\vec{r}$ can be acquired, which in turn leads to the determination of the related local magnetic field intensity $\overline{H_B}$. As a result, the three-dimensional value of $\vec{p}$ can be directly calculated from equation (8). Moreover, since we will get a large number of calculated values along the trajectory, the least square method can be used to obtain a more accurate value of $\vec{p}$ if necessary.

2.4. *The structure of the measuring device*

Usually, the design of the measuring device can follow the strategy shown as Fig.2 and Fig.3 [3, 6], where the device consists of seven three-component magnetometers. To obtain the six parameters of $\overrightarrow{H_B}$, no extra device or effort is necessary except a measurement before the signal source (i.e. the magnetic target) begin to work. And for that moment, we have

$$\overline{H}_1 = (h_{0x}, h_{0y}, h_{0z})^T,$$

$$k_1 = \frac{\partial H_x}{\partial x},\ k_2 = \frac{\partial H_y}{\partial y},\ k_3 = \frac{\partial H_z}{\partial z}.$$

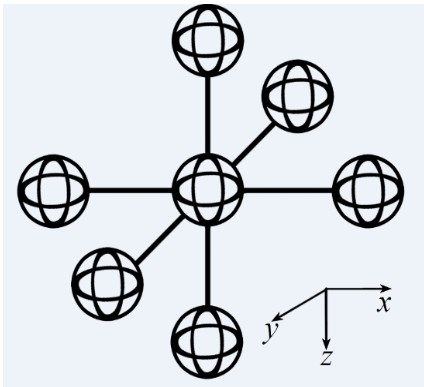

Fig.2. The structure plot of the measuring device

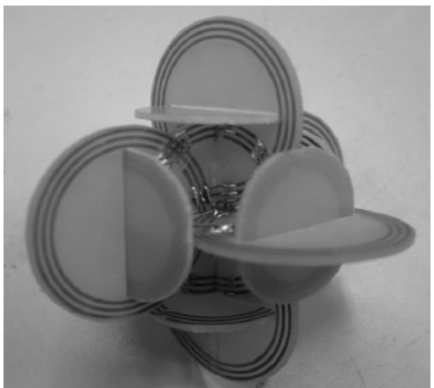
Fig.3. The model of the measuring device [3]

Moreover, according to specific practical background, it is possible to further simplify the device. For instance, in the passive space, the magnetic intensity satisfies the following relationships [7]:

$$\frac{\partial H_x}{\partial x}+\frac{\partial H_y}{\partial y}+\frac{\partial H_z}{\partial z}=0 \tag{9}$$

$$\begin{cases}\dfrac{\partial H_z}{\partial x}=\dfrac{\partial H_x}{\partial z}\\ \dfrac{\partial H_x}{\partial y}=\dfrac{\partial H_y}{\partial x}\\ \dfrac{\partial H_y}{\partial z}=\dfrac{\partial H_z}{\partial y}\end{cases} \tag{10}$$

which means the device can be further simplified through eliminating the nondependent elements.

3. Computer Simulation and Error Analysis

To evaluate the effectiveness of the localization method described as equation (3) and (5), computer simulations are provided as follows. Meanwhile some error analysis is also proposed.

3.1. The computer simulation and error computation of the situation where geomagnetic field keeps steady

Usually, the computer simulation is taken along a one-way path. However, this approach can only investigate the accuracy of the algorithm and cannot demonstrate the accumulation of the tracking error over a long distance efficiently. In this paper, a round-trip path is chosen to further illustrate the stability of the tracking method over the distance travelled.

Assume the magnetic target placed at the coordinate origin has a magnetic moment of $\vec{p}=\begin{pmatrix}4.5\times10^6\\ 3\times10^5\\ 9\times10^6\end{pmatrix}A\cdot m^2$. In addition, at the initial time the mobile target is located at x=0m, y=0m, z=0m and then it moves along the trajectory of $\begin{cases}(x^2+y^2)^2=5000(x^2-y^2)\\ z=x\end{cases}$, $(x\geq 0)$. Without loss of generality, we choose 400 equally spaced points along the line as the inspection points.

Furthermore, the superposition value of a set of white-noise and a constant number is used to simulate the approximately constant geomagnetic field intensity, and here we take the ratio of the fluctuation to the constant number as 1%. The simulation results obtained from equation (3) are depicted in Fig.4-Fig.5. Without loss of generality, only the error for the x-coordinate is provided here as the representative.

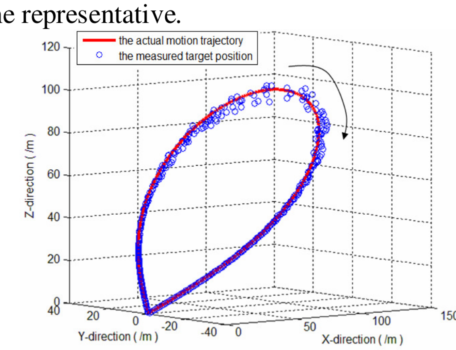

Fig.4 The localization result in three-dimensional constant background geomagnetic field

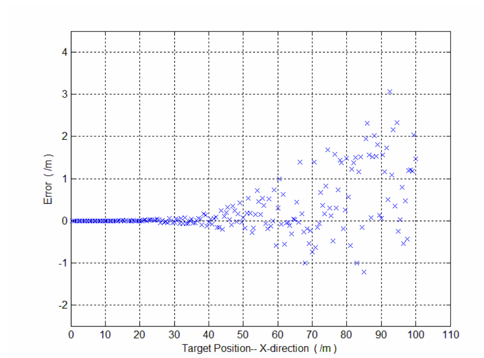

Fig.5 The x-direction absolute localization error in constant background geomagnetic field

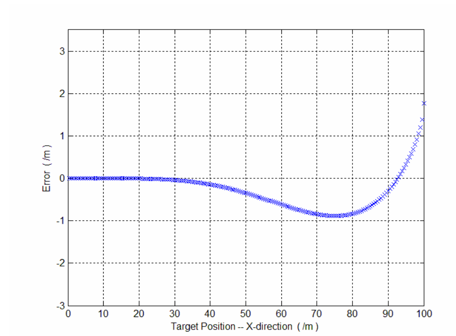

Fig.7 The x-direction absolute localization error in three-dimensional monotonous-and-steady changing background geomagnetic

3.2. *The computer simulation and error computation of the situation where exists magnetic anomalies*

In this case we assume the target has the same location and magnetic moment as mentioned, while a monotonous and steady background magnetic field is generated by a single-point magnetic object located in $x = -10m, y = 300m, z = -35m$. Selecting

$$\vec{p}' = \begin{pmatrix} 9\times10^6 \\ 3\times10^6 \\ 1.5\times10^7 \end{pmatrix} A\cdot m^2$$ as the magnetic moment of the single-point magnetic object, Fig.6-Fig.7 show the simulation and error computation at the same set of inspection points.

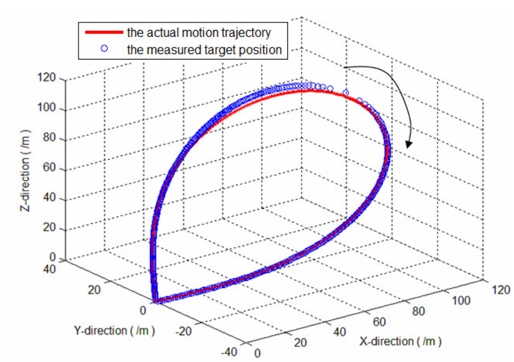

Fig.6 The localization result in three-dimensional monotonous-and-steady changing background geomagnetic field

3.3. *The error analysis of the simulation*

From Fig.4-Fig.7 we can see that the overall tracking result is basically satisfactory, especially when the magnetic target is near the measuring device. For instance, for the range of 0-80m, the x-direction tracking error in the smooth geomagnetic field is smaller than 2m. And for the area with single point magnetic anomaly, the tracking errors in the same direction is smaller than 1m. In the region far from the target, the error between the computed result and the actual distance becomes more significant. This is mainly because the differential signal obtained in that area is relatively weak. In other words, when apart from the target for 80-100m, the differential signal collected by the measuring device is almost equal with or even smaller than the fluctuation of the geomagnetic field. Coupled with the limited accuracy of computation, it finally leads to an increasing error.

Besides, Fig.4 and Fig.6 clearly show that the localization error only relates to the distance between the measuring device and the mobile target, and will not be affected by the total traveled distance. This can be attributed to the fact that all required information in this tracking method does not rely on the previous state of the target. With the increase of the signal intensity, the accuracy achieved will be boosted dramatically.

4. Conclusions

This paper further explored the real-time tracking method under the typical geomagnetic fields. Mathematically, a set of equations is proposed to calculate the location of a single magnetic target under two typical geomagnetic fields respectively. Coupled with a purposely designed measuring device as well as a simple attachment to the target, this algorithm can be realized easily. Moreover, the algebra expression of the equivalent magnetic moment of the target is further derived. Considering the acceptable overall computational complexity as well as the relatively simple structure of the localization system, this algorithm can be utilized as the basic principle of a real-time tracking system operates under the geomagnetic background. This research also has some reference value for the practical tracking of a single target and corresponding design of measuring devices to some extent.

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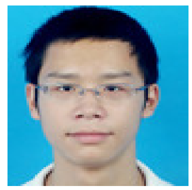

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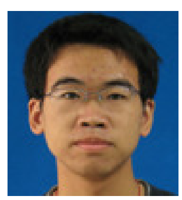

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