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Variable-Order Hybrid COVID-19 Mathematical Model with Time Delay; Numerical Treatments

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Abstract: In this paper, a variable-order fractional (VOF) hybrid COVID-19 mathematical model with time delay is presented, where its operator can be written as a combination of VOF derivative of Caputo and VOF integral of Riemann-Liouville (RL), where a new parameter ϑ consistent with the physical model problem is introduced. The positivity of the solutions and the local stability of disease-free equilibrium (DFE) of the present model are discussed. Theta nonstandard finite difference method (Θ NFDM) is used to study numerically the model problem. Particular attention is given to investigating the stability of this method. Several numerical experiments are performed with different values of variable-order derivative and time delay.

Keywords: Hybrid variable-order fractional operator, theta nonstandard method, COVID-19 mathematical model with time delay.

1 Introduction

As it is known COVID-19 virus is mainly transmitted through contact with saliva droplets or discharge from an infection person. Symptoms of infection with the emerging coronavirus include shortness of breath, diarrhea, fatigue, fever, and cough. In the most severe cases, COVID-19 leads to pneumonia and death. Due to a large number of susceptible people and more routes of transmission, COVID-19 is more contagious compared to SARS and MERS. Since the end of 2019, The COVID-19 virus broke out around the world at a rapid pace impacting the lives of people and claiming millions of lives. Fighting the COVID-19 is expensive and difficult. In order to prevent and control its spread, the governments have implemented very strict disease control and prevention strategies such as: medical quarantine, self-isolation, social distancing, city closurs, travel restrictions and contact tracing [1,2,3].

Mathematical modeling of infectious diseases is an essential tool to understand and study the mechanism of spread of a disease like COVID-19 in the human population, predict the future course of an outbreak [4]. By including lag factors in the system, the proposed model agrees well with real-world events. The delay factor was included in some models. For details see [5,6,7,8,9,10,11,12,13,14,15]. It is well known that, the VOF derivatives are non-local in nature and they can describe hereditary properties and memory in multiple materials and processes, for more details see [16,17,18,19,20,21, 22].

In this work, VOF hybrid COVID-19 model with time delay is presented and analyzed. The new hybrid VOF derivatives which is more general than the Caputo fractional derivative, it is defined as a linear combination of RLVOF integral and Caputo VOF derivative. Positivity, boundednes and stability of the current model are proved. In addition, Constant proportional-Caputo (CPC) Θ NFDM is constructed to study the proposed model numerically. Depending on $\Theta \in [0,1]$, this method can be an implicit method or an explicit method. Stability analysis is presented.

This work is organized as follows: Some definitions are given in Section 2. A hybrid VOF model is given and positivity, the local stability are discussed, in Section 3. CPC-Θ NFDM and its stability analysis are studied in Section 4. Numerical experiments are introduced in Section 5, and in Section 6 the conclusions are ultimately outlined.

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2 Preliminaries

In the following, let us consider $\alpha := \alpha(t)$.

Definition 1. *[23] Let $\alpha \in \mathbb{R}^+$, $-\infty < a < b < +\infty$, the Caputo VOF derivatives (right and left sides) of order α on $f(t) \in AC^m[a,b]$ are given as follows respectively:*

$$ {}^{C}D^{\alpha}_{b-}f(t) = {}^{C}_{t}D^{\alpha}_{b}f(t) = (-1)^n(\Gamma(n-\alpha))^{-1}\int_t^b (z-t)^{n-\alpha-1}f^{(n)}(z)dz, \quad b>t, \tag{1} $$

and

$$ {}^{C}D^{\alpha}_{a+}f(t) = {}^{C}_{a}D^{\alpha}_{t}f(t) = (\Gamma(n-\alpha))^{-1}\int_a^t (t-z)^{n-\alpha-1}f^{(n)}(z)dz, \quad a<t, \tag{2} $$

where $n = \lfloor \alpha \rfloor + 1, \quad \alpha \notin \mathbb{N}_0, \quad n-1 < \alpha < n.$

Definition 2. *[23] Let $\alpha \in \mathbb{R}^+$, $-\infty < a < b < +\infty$, the right and left sides of RLVOF derivatives of order α on $f(t)$, $f(t) \in L_1[a,b]$ are given as following, respectively:*

$$ {}^{RL}_{t}D^{\alpha}_{b}f(t) = (-1)^n(\Gamma(n-\alpha))^{-1}\frac{d^n}{dt^n}\int_t^b (z-t)^{n-\alpha-1}f(z)dz, \quad b>t, \tag{3} $$

and

$$ {}^{RL}_{a}D^{\alpha}_{t}f(t) = (\Gamma(n-\alpha))^{-1}\frac{d^n}{dt^n}\int_a^t f(z)(t-z)^{n-\alpha-1}dz, \quad t>a, \tag{4} $$

where $n = \lfloor \alpha \rfloor + 1, \quad n-1 < \alpha < n.$

Definition 3. *[23] Let $\alpha \in \mathbb{R}^+$, $-\infty < a < b < +\infty$, the right and left sides of RLVOF integrals of order α on $f(t) \in L_1[a,b]$ are given as following, respectively:*

$$ {}_{t}I^{\alpha}_{b}f(t) = (\Gamma(\alpha))^{-1}\int_t^b (z-t)^{\alpha-1}f(z)dz, \quad b>t, \tag{5} $$

and

$$ {}_{a}I^{\alpha}_{t}f(t) = (\Gamma(\alpha))^{-1}\int_a^t f(z)(t-z)^{\alpha-1}dz, \quad t>a, \tag{6} $$

where $0 < \alpha < 1$. *For* $\alpha = 0$, *we set* ${}_{a}I^{0}_{t} := I$, *the identity operator.*

Definition 4. *[24] The proportional-Caputo (PC) VOF hybrid operator can be defined in two ways:*

–The first way (as a general form):

$$ \begin{aligned} {}^{PC}_{0}D^{\alpha}_{t}f(t) &= (\Gamma(1-\alpha))^{-1}\int_0^t \Big(f'(z)L_0(\alpha,z) + f(z)L_1(\alpha,z) \Big)(t-z)^{-\alpha}dz \\ &= {}^{RL}_{0}I^{1-\alpha}_{t}\Big(f'(t)L_0(\alpha,t) + f(t)L_1(\alpha,t) \Big) \\ &= (\Gamma(1-\alpha)t^{\alpha})^{-1}\Big(f(t)L_1(\alpha,t) + f'(t)L_0(\alpha,t) \Big), \end{aligned} \tag{7} $$

where $L_1(\alpha,t) = t^{\alpha}(1-\alpha)$ *and* $L_0(\alpha,t) = t^{(1-\alpha)}\alpha$ *for* $1 > \alpha > 0$.

–The second way (as a simpler expression form):

The constant proportional-Caputo (CPC) VOF hybrid operator:

$$
\begin{aligned}
{}_0^{CPC}D_t^{\alpha} f(t) &= (\Gamma(1-\alpha)(t-z)^{\alpha})^{-1} \int_0^t \left(L_1(\alpha) f(z) + L_0(\alpha) f'(z) \right) dz \\
&= L_1(\alpha)\, {}_0^{RL}I_t^{1-\alpha} f(t) + L_0(\alpha)\, {}_0^{C}D_t^{\alpha} f(t), \qquad (8)
\end{aligned}
$$

for Q *is a constant,* $Q^{(1-\alpha)}\alpha = L_0(\alpha)$ *and* $Q^{\alpha}(1-\alpha) = L_1(\alpha)$.

Definition 5. *[24] The inverse operator of the CPCVOF derivative is given by:*

$$
{}_0^{CPC}I_t^{\alpha} f(t) = (L_0(\alpha))^{-1} \int_0^t exp\left(\frac{L_1(\alpha)}{L_0(\alpha)} \right) {}_0^{RL}D_t^{1-\alpha}(t-z) f(z) dz. \qquad (9)
$$

3 Mathematical Model

VOF Coronavirus disease model with delay is presented here as an extension of the model given in [25] by using a new hybrid VOF operator. An auxiliary parameter ϑ is added to the VOF operator to satisfy the dimensional matching between the two sides of the resulting VOF equations. In this way, the left side has the dimension of day^{-1} [26].
Below, the updated mathematical model:

$$
\begin{aligned}
\frac{1}{\vartheta^{1-\alpha}}\, {}_0^{CPC}D_t^{\alpha} S &= \pi_s - e^{-\mu\tau}(\beta_1 S(t_1) I(t_1)) - e^{-\mu\tau}(\beta_2 S(t_1) A(t_1)) - \mu S, \\
\frac{1}{\vartheta^{1-\alpha}}\, {}_0^{CPC}D_t^{\alpha} I &= \omega_1 E - (\omega_4 + \mu) I, \\
\frac{1}{\vartheta^{1-\alpha}}\, {}_0^{CPC}D_t^{\alpha} A &= \omega_2 E - (\omega_5 + \mu) A, \\
\frac{1}{\vartheta^{1-\alpha}}\, {}_0^{CPC}D_t^{\alpha} R &= \omega_3 E + \omega_4 I + \omega_5 A - \mu R, \\
\frac{1}{\vartheta^{1-\alpha}}\, {}_0^{CPC}D_t^{\alpha} E &= e^{-\mu\tau}(\beta_1 S(t_1) I(t_1)) + e^{-\mu\tau}(\beta_2 S(t_1) A(t_1)) \\
&\quad - (\omega_1 + \omega_2 + \omega_3 + \mu) E, \qquad (10)
\end{aligned}
$$

where $t_1 = t - \tau$. With initial conditions

$$
E(0) = e_0, \;\; S(0) = s_0, \;\; I(0) = i_0, \;\; A(0) = a_0, \;\; R(0) = r_0, \qquad (11)
$$

where $s_0,\ e_0,\ i_0,\ a_0$ *and* $r_0 \geq 0$.

Table 1: Variables of System (10).

Variable(t)	Interpretation
E	Exposed population
S	Susceptible population
I	Symptomatic population
A	Asymptomatic population
R	Recovered population

Table 2: System (10) parameters definition.

Parameter	Interpretation	Parameter value
π_s	The recruitment rate	0.5
μ	The mortality rate due to disease infection	0.5
β_1, β_2	Symptomatic, and asymptomatic rate of infection	1.05
ω_1	Symptomatic rate infected interaction	0.4787
ω_2	The interaction with the infected asymptomatic rate	1.0004
ω_3	The exposed person recovered from disease	0.0854
ω_4	Symptomatic person recovered after quarantine	0.0987
ω_5	The rate of the quarantine of asymptomatic infected persons	0.1234

3.1 *Positive Solutions*

Lemma 1. *Let $t \geq 0$, all solutions of (10) remain non-negative under conditions (11) [27].*

Proof. Using (11), we have:

$$\begin{aligned}
{}_0^{CPC}D_t^{\alpha} S(t)|_{S=0} &= \vartheta^{1-\alpha}\pi_s \geq 0,\\
{}_0^{CPC}D_t^{\alpha} E(t)|_{E=0} &= \vartheta^{1-\alpha} S(t_1)(\beta_2 A(t_1) + \beta_1 I(t_1))e^{-\mu\tau} \geq 0,\\
{}_0^{CPC}D_t^{\alpha} A(t)|_{A=0} &= \vartheta^{1-\alpha}\omega_2 E \geq 0,\\
{}_0^{CPC}D_t^{\alpha} I(t)|_{I=0} &= \vartheta^{1-\alpha}\omega_1 E \geq 0,\\
{}_0^{CPC}D_t^{\alpha} R(t)|_{R=0} &= \vartheta^{1-\alpha}(\omega_3 E + \omega_4 I + \omega_5 A) \geq 0,
\end{aligned} \tag{12}$$

where $t_1 = t - \tau$.

Thus, all solutions of (10) are non-negative.

3.2 *Local Stability*

The local stability of DFE is discussed in the following [28,29]:
To find the DFE points of system (10), we study $\frac{1}{\vartheta^{1-\alpha}}\,{}_0^{CPC}D_t^{\alpha}(.) = 0$, therefore, we have the following equilibrium points: the first one is DFE which is given as $E_0 = (\frac{\pi_s}{\mu}, 0, 0, 0, 0)$ in case vanishing all state variables, and the second point is called endmeic point and given as

$$E^* = (\bar{S}, \bar{E}, \bar{I}, \bar{A}, \bar{R}),$$

where

$$\begin{aligned}
\bar{S} &= \frac{(\omega_4 + \mu)(\mu + \omega_1 + \omega_2 + \omega_3)(\mu + \omega_5)}{\beta_1 e^{-\mu\tau}\omega_1(\omega_5 + \mu) + \omega_2 e^{-\mu\tau}\beta_1(\mu + \omega_4)},\\
\bar{E} &= \frac{\pi_s - \mu\bar{S}}{\mu + \omega_1 + \omega_2 + \omega_3},\\
\bar{I} &= \frac{\omega_1\bar{E}}{\mu + \omega_4},\\
\bar{A} &= \frac{\omega_2\bar{E}}{\omega_5 - \mu},\\
\bar{R} &= \frac{\omega_5\bar{A} + \omega_3\bar{E} + \omega_4\bar{I}}{\mu + \omega_5}.
\end{aligned} \tag{13}$$

The basic reproduction number (R_0) can be obtained by generation matrix as follows [30]:

$$F = \vartheta^{1-\alpha}\begin{pmatrix} 0 & \frac{\beta_1\pi_s e^{-\mu\tau}}{\mu} & \frac{\beta_2\pi_s e^{-\mu\tau}}{\mu} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$V = \vartheta^{1-\alpha}\begin{pmatrix} \mu+\omega_1+\omega_2+\omega_3 & 0 & 0 & 0 \\ -\omega_1 & \mu+\omega_4 & 0 & 0 \\ -\omega_2 & 0 & \mu+\omega_5 & 0 \\ -\omega_3 & -\omega_4 & -\omega_5 & \mu \end{pmatrix},$$

where F is matrix of a new infectious and V is matrix of the transfer of individuals between compartments.

$$FV^{-1} = \begin{pmatrix} \frac{\pi_s e^{-\mu\tau}(\beta_1\omega_1(\mu+\omega_5)+\beta_2\omega_2(\mu+\omega_4))}{\mu(\omega_1+\omega_2+\omega_3+\mu)(\mu+\omega_4)(\mu+\omega_5)} & \frac{\beta_1\pi_s e^{-\mu\tau}}{\mu(\omega_4+\mu)} & \frac{\beta_2\pi_s e^{-\mu\tau}}{\mu(\omega_5+\mu)} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$R_0 = \rho(FV^{-1}) = \vartheta^{1-\alpha}\left[\frac{\pi_s e^{-\mu\tau}(\beta_1\omega_1(\mu+\omega_5)+\beta_2\omega_2(\mu+\omega_4))}{(\mu+\omega_4)(\mu+\omega_5)\mu(\mu+\omega_3+\omega_2+\omega_1)}\right],$$

where ρ is the convergence radius.

Theorem 1. *DFE (E_0) of system (10) is locally asymptotically stable if $1 > R_0$ and it's unstable if $1 < R_0$.*

Proof. To investigate the local stability of (10), cosider Jacobian matrix of (10) at the DFE.

$$J(E_0) = \vartheta^{1-\alpha}\begin{pmatrix} -\mu & 0 & L & L_1 & 0 \\ 0 & -(\mu+\omega_1+\omega_2+\omega_3) & L & L_1 & 0 \\ 0 & \omega_1 & -\omega_4-\mu & 0 & 0 \\ 0 & \omega_2 & 0 & -\omega_5-\mu & 0 \\ 0 & \omega_3 & \omega_4 & \omega_5 & -\mu \end{pmatrix},$$

where $L = \frac{-\beta_1\pi_s e^{-\mu\tau}}{\mu}$, $L_1 = \frac{\beta_2\pi_s e^{-\mu\tau}}{\mu}$.

The characteristic equation

$$\lambda^3+\lambda^2[d_1+d_4+d_5]+\lambda[d_5(d_1+d_4)+d_1d_4-d_2\omega_1-c_3\omega_2]+(d_1d_2d_5-\omega_2d_3d_4-d_5d_2\omega_1)=0, \tag{14}$$

where

$$\begin{aligned} d_1 &= (\omega_1+\omega_2+\omega_3+\mu)\vartheta^{1-\alpha} > 0; \\ d_2 &= \frac{\beta_1\pi_s e^{-\mu\tau}}{\mu}\vartheta^{1-\alpha} > 0; \\ d_3 &= \vartheta^{1-\alpha}\frac{\beta_2\pi_s e^{-\mu\tau}}{\mu} > 0; \\ d_4 &= (\omega_4+\mu)\vartheta^{1-\alpha} > 0; \\ d_5 &= (\omega_5+\mu)\vartheta^{1-\alpha} > 0. \end{aligned}$$

Applying Routh-Hurwitz critertion, equation (14) has roots with negative real part iff $1 > R_0$. So DFE is locally asymptotidy stable if $1 > R_0$.

4 Numerical Investigation

4.1 CPC-Θ NFDM

Consider following hybrid VOF derivatives equation, $1 \geq \alpha > 0$:

$$ {}_{0}^{CPC}D_t^{\alpha} f(t) = \xi(t, f(t)), \quad f(0) = f_0. \tag{15} $$

$$ \begin{aligned} Using(8): \qquad {}_{0}^{CPC}D_t^{\alpha} f(t) &= \frac{1}{1-\Gamma(\alpha)} \int_0^t \frac{1}{(t-z)^{\alpha}} (f(z)L_1(\alpha) + f'(z)L_0(\alpha))dz \\ &= L_0(\alpha){}_0^{C}D_t^{\alpha} f(t) + L_1(\alpha){}_0^{RL}I_t^{1-\alpha} f(t) \\ &= L_0(\alpha){}_0^{C}D_t^{\alpha} f(t) + L_1(\alpha){}_0^{RL}D_t^{\alpha-1} f(t). \end{aligned} \tag{16} $$

We can discretize (16) using the discretizationnof Grünwald-Letnikov as follos:

$$ \begin{aligned} {}_{0}^{CPC}D_t^{\alpha} f(t)|_{t=t^n} = &\frac{L_1(\alpha(t_n))}{\tau^{\alpha(t_n)-1}} \left(f_{n+1} + \sum_{i=1}^{1+n} \omega_i f_{n+1-i} \right) \\ &+ \frac{L_0(\alpha(t_n))}{\tau^{\alpha_n}} \left(f_{n+1} - \sum_{i=1}^{n+1} \mu_i f_{n+1-i} - q_{n+1} y_0 \right), \end{aligned} \tag{17} $$

$$ \begin{aligned} \frac{L_1(\alpha(t_n))}{\tau^{\alpha(t_n)-1}} \left(f_{n+1} + \sum_{i=1}^{n+1} \omega_i f_{n+1-i} \right) + \frac{L_0(\alpha(t_n))}{\tau^{\alpha(t_n)}} \left(f_{n+1} - \sum_{i=1}^{n+1} \mu_i f_{n+1-i} - q_{n+1} f_0 \right) \\ = (1-\theta)\xi(f(t_{n+1}), t_{n+1}) + (\theta)\xi(f(t_n), t_n), \end{aligned} \tag{18} $$

$0 \leq \theta \leq 1$, $\mu_1 = \alpha(t_n)$, $\mu_i = (-1)^{i-1} \binom{\alpha(t_n)}{i}$, $\omega_0 = 1$, $\omega_i = (1 - \frac{\alpha(t_n)}{i})\omega_{i-1}$, $\tau = \frac{T_f}{N_n}$, $t^n = n\tau$, N_n is a natural number, $q_i = \frac{i^{\alpha(t_n)}}{\Gamma(1-\alpha(t_n))}$ and $i = 1, 2, ..., n+1$.
Assume that [31]:

$$ 0 < q_{i+1} < q_i < ... < q_1 = \frac{1}{\Gamma(-\alpha(t_n)+1)}, $$
$$ 0 < \mu_{i+1} < \mu_i < ... < \mu_1 = \alpha(t_n) < 1. $$

4.2 Stability of CPC-Θ NFDM

To investigate Stability of the implicit method $(\theta < 1)$, consider a model test problem of linear VOF delay differential equation [32]:

$$ \begin{aligned} ({}_{0}^{CPC}D_t^{\alpha} f)(t) &= \rho_0 f(t) + \rho_1 f(t-\tau), \ t \geq 0, \ 1 \geq \alpha > 0, \\ f(t) &= \Psi(t), \ t \in [-\tau, 0], \ f(0) = f_0, \end{aligned} \tag{19} $$

such that $\rho_0 < 0$, $\rho_0 > \rho_1$ and $\Psi(t)$ is bounded and continuous function.
Let $f_n = f(t_n)$ is an approximate solution, by applying CPC-Θ NFDM with (8) and rewrite (19) as following:

$$ \begin{aligned} \frac{L_1(\alpha(t_n))}{\tau^{\alpha(t_n)-1}} \left(f_{n+1} + \sum_{i=1}^{n+1} \omega_i f_{n+1-i} \right) + \frac{L_0(\alpha(t_n))}{\tau^{\alpha(t_n)}} \left(f_{n+1} - \sum_{i=1}^{n+1} \mu_i f_{n+1-i} - q_{n+1} f_0 \right) \\ = \theta(\rho_0 f(t_n) + \rho_1 f(t_{n-q})) + (1-\theta)(\rho_0 f(t_{n+1}) + \rho_1 f(t_{n-q+1})), \end{aligned} \tag{20} $$

we put:

$$ gg = \ L_1(\alpha(t_n))\tau^{1-\alpha(t_n)}, \quad and \quad gg1 = \ L_0(\alpha(t_n))\tau^{-\alpha(t_n)}, $$

we get:

$$f_{n+1} = \frac{1}{(gg+gg1)-(1-\theta)\rho_0}\left(gg1\sum_{i=1}^{1+n}\mu_i f_{n+1-i} - q_{n+1}f_0 - gg\sum_{i=1}^{1+n}\omega_i f_{n+1-i}\theta(\rho_0 f(t_n)+\rho_1 f(t_{n-q})) + (1-\theta)\rho_1 f(t_{n-q+1})\right), \tag{21}$$

we have

$$\frac{1}{(gg+gg1)-(1-\theta)\rho_0} < 1,$$

$$f_1 \leq f_0,$$

$$f_n \leq f_{n-1} \leq f_{n-2} \leq \ldots \leq f_1 \leq f_0.$$

So this scheme is stable.

5 Numerical Results

We have used CPC-Θ NFDM (18) to solve the hybrid VOF systems (10) numerically, with initial conditions: $s_0 = 0.5,\ e_0 = 0.2,\ i_0 = a_0 = r_0 = 0.1$.
Figure 1 (a)-(e) shows the plot of each compartment of the model at fixed delay term ($\tau = 0.2$) and different values of α, where the susceptible population is increasing over time while the exposed population is reduced, and in short time a slight increment in asymptomatic and symptomatic population, then it becomes stable. However, the recovered population is declining over time. All compartments reach their minimum or maximum at a lower VOF. At $\alpha = 1$, the integer model dynamics [5] represents by the red color curves, and at $\alpha = 0.90$, the fractional model dynamics [25] represents by the black color curves. Then we can conclude that the model (10) is generalizes the model in [25].

Let fixed delay term ($\tau = 0.5$) for different values of the VOF α in Figure 2 (a)-(e).
Moreover, for the VOF $\alpha = 0.99 - sin0.0005t$, Figure 3 (a)-(e) simulate each compartment of the proposed model whit different (τ). When τ increases, the susceptible population increases, while the exposed population decreases. According to our results, without a shift in the transmission rate, increased delay strategies lead to a reduction in the infected population. We note that at $\tau = 3$, $\tau = 2$ and $\tau = 0.8$ the number of symptomatically infected individuals decreases, see figure 3(c). Then, we could overcome the pandemic by delay strategies e.g., quarantine, isolation for about 60 days, social distancing, holiday extensions, travel restrictions and hospitalization.

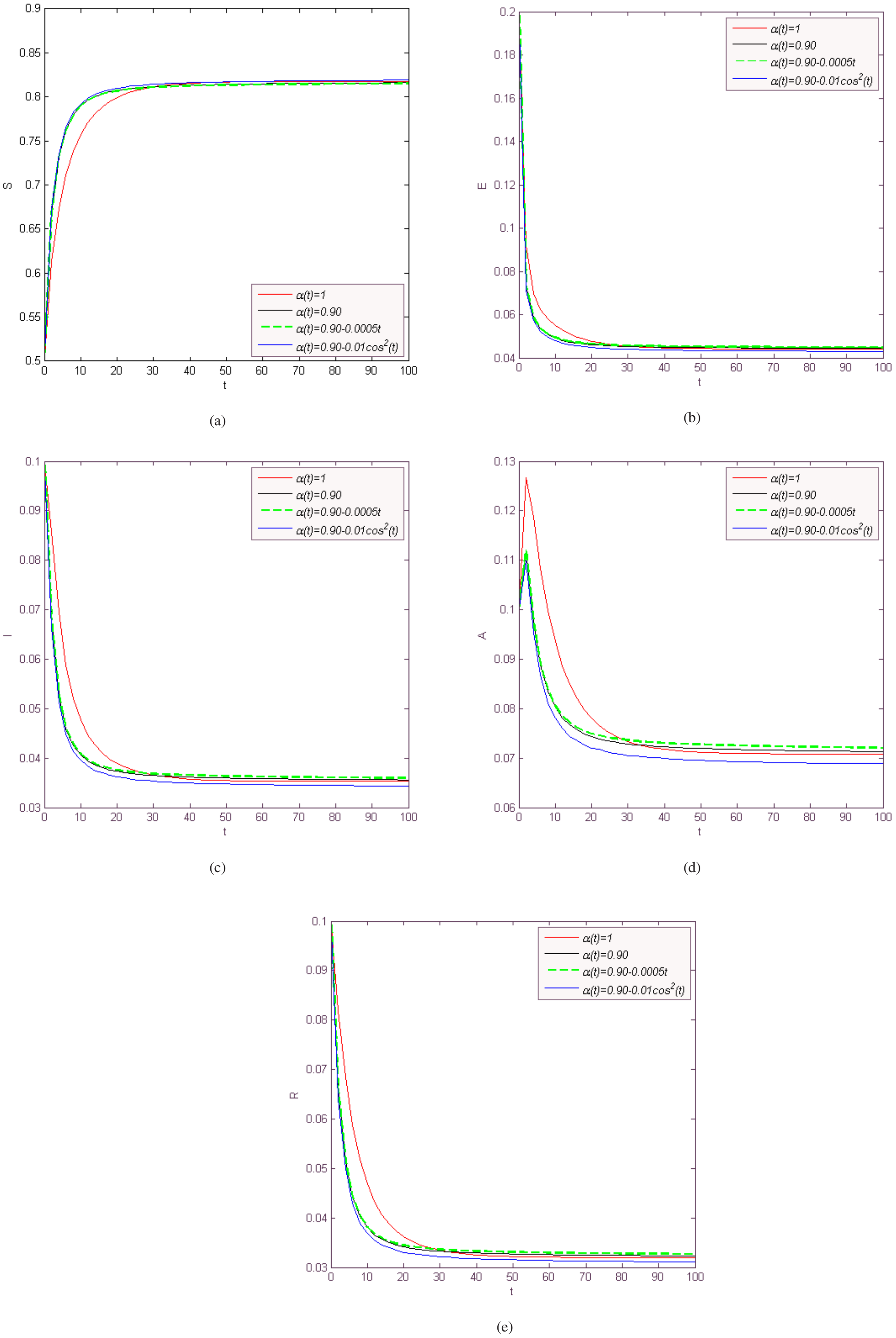


Fig. 1: At $\tau = 0.2$ with different values of α

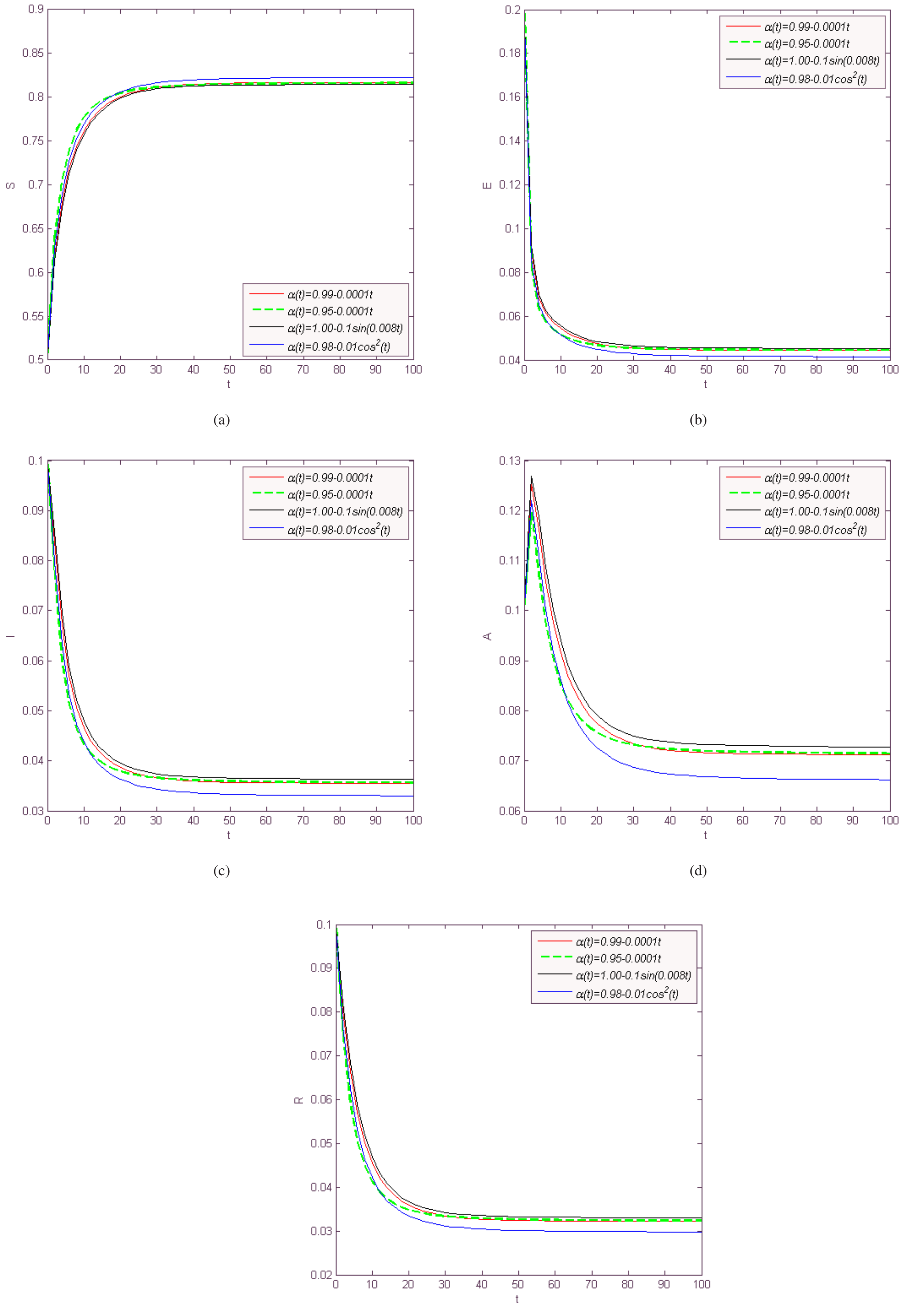
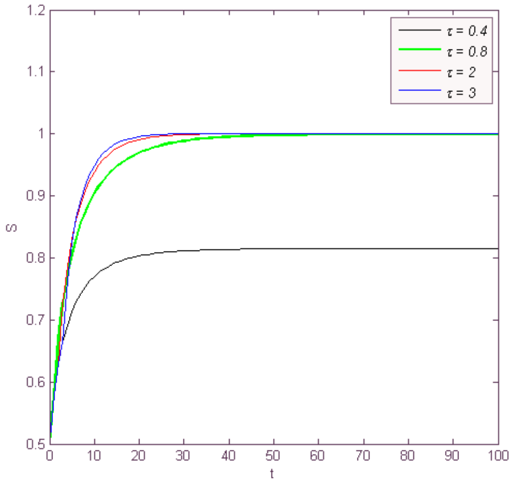
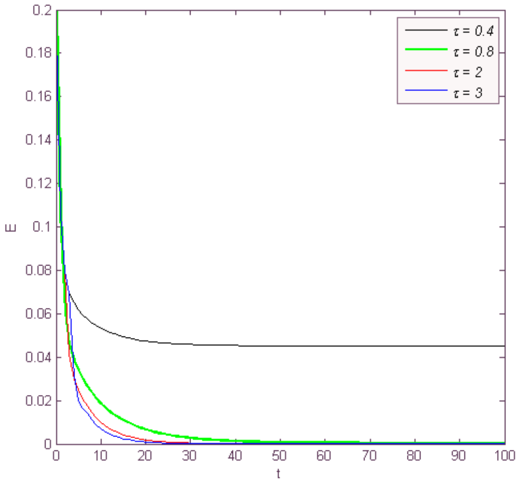


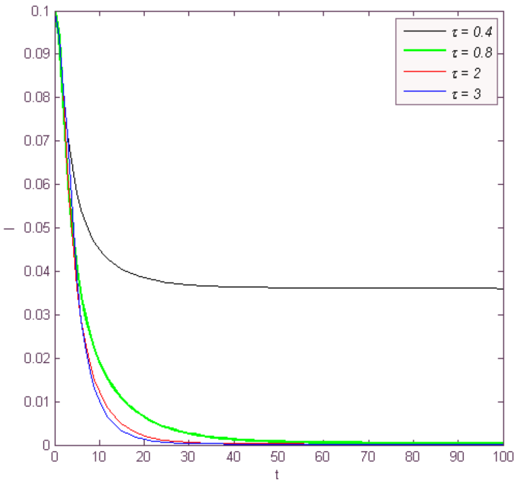
Fig. 2: At $\tau = 0.5$ with different values of α



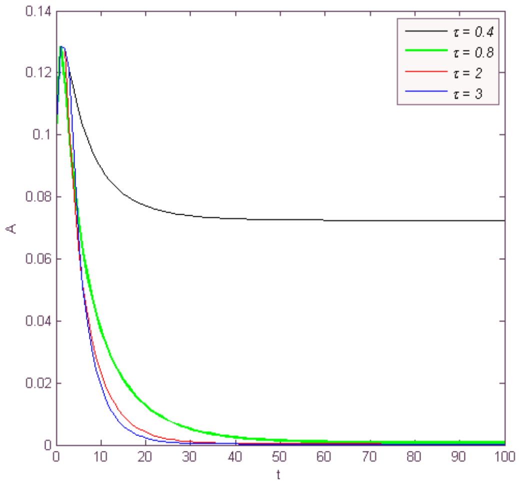
(a)



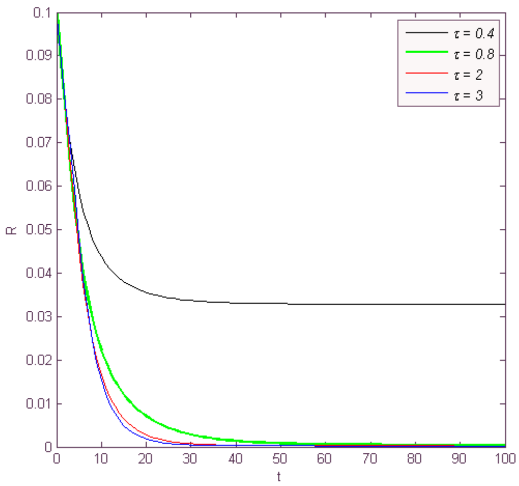
(b)



(c)



(d)



(e)

Fig. 3: At $\alpha = 0.99 - sin0.0005t$ with different values of τ.

6 Conclusion

In this work, we presented and analyzed VOF for COVID-19 mathematical model with time delay. While VOF derivatives with time delay increases its complexity, it improves of the model dynamics. It is also CPC fractional operator here can be obtained as a special case from CPC VOF operator. The proposed model Positivity, boundedness and stability are discussed. In order to make the probosed model physically compatible, we are introduced a new parameter ϑ. Numerically studies using CPC-Θ NFDM for the model proplem. When $\Theta = 1$, $\Theta = 0$ the numerical scheme is called explicit, fully implicit schemes respectively. The results obtained by the CPC-Θ NFDM at $\Theta = 0$ are more stable, some graphs are provided. Accordingly, the approach Taken in this paper opens several avenues for future research.

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