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Statistical Analysis of the BitCoin and South African Rand Exchange Rates Risks when the Tails are Somewhat Heavy

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Abstract: In the work of the global financial crisis, in which incorrect and normal distribution-based risk models were used, this paper employs some selected statistical parent distribution models to better quantify and compare the financial riskiness of BitCoin and the Rand; both measured against the United States Dollar. The positive returns (gains) and negative returns (losses) are fitted separately to selected statistical distributions with varying degrees of tail heaviness, hence providing for various levels of risk. Riskiness is measured using the Value at Risk and Expected Shortfall. Relatively heavy-tailed models like: the exponential, Weibull, and the Burr distributions are considered and are non-normal, a feature that is common in financial, and currency data. The Anderson-Darling test is used to confirm the goodness-of-fit of the distributions. Using the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC), BitCoin gains are confirmed to follow the heavy-tailed Burr distribution, while the losses follow the lighter-tailed exponential distribution. In the case of the Rand, both gains and losses follow the Weibull distribution with tails that are even lighter than the Exponential distribution tail. BitCoin has a greater upside and downside risk hence rendering it riskier than the Rand. The derived information helps investors in BitCoin and the South African Rand understand the comparative risk they are exposed to when they convert their investments from the Rand to invest in BitCoin. Risk managers can use the approach and the results to compute risk-adjusted capital requirements.

Keywords: AIC; BIC; Burr; Expected Shortfall; Exponential; Heavy-tailed; Value at Risk; Weibull.

1 Introduction

Emerging countries' currencies, like the South African Rand, are risky. Currency risk is defined as a change in the value of one currency in relation to another currency (most commonly against the United States dollar) [1]. The liberalisation of the Rand posts South African independence in 1994 meant that the Rand value was now determined by market forces, making it a volatile asset [2]. Investors' loss-aversion sentiments imply a greater reaction toward losses than gains [3]. This leads to investors seeking alternative investments to mitigate potential losses [4]. BitCoin can be that alternative investment [5].

Although BitCoin is also a risky investment due to the lack of regulation and backing by a central bank [6], it is still an attractive investment as witnessed by the growth in share price and volume. Recent studies have shown that BitCoin can be considered an alternative safe haven asset under certain circumstances. [7], [8], and [9] concluded that cryptocurrencies can serve as a safe-haven asset for the BRICS countries (Brazil, Russia, India, China, and South Africa) equity markets, before and during market crisis and global crises, like the Covid-19 pandemic. South Africa is a founding member of the BRICS.[10] posits that the mean and variance (risk) of returns are the most important factors to consider when constructing an optimal portfolio. [10] theorem assumes normally distributed returns. Empirical evidence has shown that the returns are not normally distributed.

The idea in [10] theorem is to maximise return for a given level of risk or to minimise risk for a given level of return. Quantifying risk has gained a lot of attraction post the global financial crisis (GFC) of 2008 due to the recognition of deviation of returns from the normal distribution properties [11]; [12].

The Value at Risk (VaR) is a quantile function that measures the riskiness of an asset or portfolio over a specified time horizon [13]. VaR is defined as the maximum monetary amount (say dollars) likely to be lost over a specific time horizon, at a pre-defined confidence level. The Basel Committee on Banking Supervision (BCBS) noted some weaknesses when using VaR for determining risk-adjusted capital requirements, including the incoherent property [14] and the assumed normality of data property. [15] went further and recommended that banks use the expected shortfall (ES) instead of VaR in calculating market, credit, and operational risks. The ES is calculated by averaging all of the returns in the distribution that are worse than the VaR of the portfolio at a given level of confidence.

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This study seeks to quantify and compare the riskiness of two currency assets, namely the South African Rand and BitCoin using the relatively heavy-tailed parent distributions. This is to help BitCoin investors and traders in South Africa, understand aggregate risk more accurately. The two currencies are measured against the US dollar (USD). Identifying the correct statistical distribution for the returns is critical in calculating accurate values of VaR and ES.

2 Literature Review

The parametric methods approach to estimating financial risk measures, like Value at Risk (VaR) and Expected Shortfall (ES), requires one to first fit a suitable statistical distribution model that best describes the financial returns [16]. This distribution must fairly suit the stylised facts of the financial assets returns including the relatively heavy tails associated with financial data [17]. A heavy-tailed distribution is often regarded as a tail that is heavier than a normal distribution or an exponential distribution [18]. This implies that it has more observations further from the mean, which is a common feature in financial time series data. The pool of heavy-tailed distributions is very large [19]. Their applicability ranges from physical phenomena, like rainfall to insurance and financial data [20]; [21].

[22] compared the suitability of eight heavy-tailed models in describing the insurance loss data. They fitted the Kavya-Manoharan power Lomax distribution, the power Lomax (PLo) model, the odd log-logistic modified Weibull, the Kumaraswamy-Weibull, the generalised modified Weibull, extended odd Weibull Lomax, the Weibull-Lomax, the Marshall–Olkin power Lomax, the Marshall–Olkin alpha power Lomax, the exponentiated generalised alpha power exponential distributions, the Kumaraswamy generalised power Lomax model and the Marshall-Olkin alpha power exponentiated Weibull distributions to the data set. They used several information criteria including the Akaike information criterion (AIC), Bayesian information criterion (BIC), the consistent Akaike information criterion (CAIC), the Hannan-Quinn information criterion (HQIC)) and the distance of the Kolmogorov Smirnov and the p-values. They concluded that the Kavya-Manoharan power Lomax distribution was the best-fitting statistical distribution model. The best-fit distribution model was then used to quantify market risk.

[23] observed the presence of heavy tails and excess kurtosis in the returns data of BitCoin. [24] observed a high negative skewness and more volatility in BitCoin returns when compared to other stock returns.

[25] investigated BitCoin, Litecoin, and Ripple's volatility behaviour using statistical analysis (copulas) and extreme value analysis of their returns and concluded that their volatility shows heavy tails and instability over time, resulting in extremely high volatility regularly. BitCoin was the least volatile out of all the cryptocurrencies investigated, with an annual volatility of 62%. This figure suggests that BitCoin could be riskier than fiat currencies from developed economies. Similar findings were recorded by [26] who concluded that the range of volatility appeared to be smallest for BitCoin compared to the other cryptocurrencies.

[27] investigated the risk of an investment in BitCoin from a statistical point of view, using univariate extreme value analysis. After using the Value at Risk and Expected Shortfall as risk measures, they found that BitCoin is extremely volatile compared to traditional G10 currencies, with a loss of more than 10% to be expected within 20 days.

[28] confirmed the adequacy of the heavy-tailed models in estimating the VaR of BitCoin and Ethereum returns using Kupiec's backtesting procedure. They employed a generalised autoregressive score (GAS) combined with heavy-tailed distributions, like generalised lambda distribution, Laplace distribution, the asymmetric Student's t-distribution, and the skewed Student's t-distribution. [29] also compared the performance of extreme value mixture models in quantifying tail related risk of BitCoin and Ethereum. They used mixture models of the normal distribution with the generalised Pareto tails (GPD-Normal-GPD) and kernel distribution estimator with the generalised Pareto tails (GPD-KDE-GPD) models in their study. The GPD-KDE-GPD was found to be superior to the GPD-Normal-GPD.

While the Rand is a fiat currency, the South African Reserve Bank (SARB) adopted a market-driven exchange rate post-independence in 1994 [2]. The Rand is affected by domestic and external disturbances (and is perceived as being risky), like foreign investor sentiment and contagion emanating from association with similar economies, thus elevating the Rand volatility to be above the volatility index (VIX), (US stock price volatility, a commonly used indicator of global risk appetite) [30]. [31] used stable distributions to model and estimate market risk of selected indices in the Johannesburg Stock Exchange and the South African Rand. They employed the Nolan's S_0 parameterisation of stable distribution. The findings showed the robustness of the stable distribution in quantifying market risks in South Africa's stock indices and currency.

3 Materials and Methods

The parametric models' approach adopted in this study to quantify the riskiness of the financial assets is done by following these steps:

1. Using the Hill and the generalised Pareto quantile to quantile (Q-Q)plots to ascertain the domain of attraction in the tails of gains and losses of the currencies understudy through estimating the Extreme Value Index (EVI).
2. Fitting the statistical parent distribution to the gains and losses returns data separately and deducing the tail behaviour of the returns and to infer relative risk. The empirical densities, cumulative distribution function (CDF), quantile to quantile (Q-Q), and probability to probability (P-P) plots and the Anderson-Daring test will be used to ascertain the goodness of fit visually.
3. Use information criterion values to choose the best model fit.
4. Simulate (1000) the returns using the best fitted model.
5. Quantifying VaR and ES for Bitcoin and the Rand gains and losses using the selected statistical parent distribution.
6. Use the results from (4) to compare the riskiness of the two currencies.

Since a heavy-tailed model is by definition, a distribution with tails heavier than an exponential, it therefore suffices that we start by quantifying risk using the exponential distribution. In terms of heavy-tailedness, the statistical distributions can be ranked as follows, from the less heavy-tailedness to the heavier-tailedness: (1) Weibull with parameter $K > 1$ (2) exponential (3) Weibull with parameter $K < 1$, and (4) Burr.

3.1 Plots for detecting heavy tails in data

The Hill Estimator

Let $\{y_1, y_2, \dots, y_m\}$ be the observed returns with the empirical j^{th} quantiles from the observed returns be $\{Y_{j,m}\}$. The Hill estimator by [32], an estimate of the extreme value index (EVI) γ is defined as:

$$H_{k,m} = \hat{\gamma}_{k,m} = \frac{1}{k}\sum_{j=1}^{k} j(\log Y_{m-j+1,m} - \log Y_{m-j,m}), \tag{1}$$

where k is number of tail observations.

The Hill plot measures the resultant estimate of the heaviness of the tails, or of an associated quantile estimate [33].

The generalised Q-Q plot

The generalised Q-Q plot is defined as follows:

$$\left(\log\frac{m+1}{k+1}; \log\left(Y_{m-k,m}H_{k,m}\right)\right). \tag{2}$$

A generalised Hill estimator of the EVI

A generalised Hill estimator of the extreme value index (EVI) γ, also known as the slope at the last k points of the generalised Q-Q plot is given by:

$$\hat{\gamma}_{k,m} = H_{k+1,m} + \frac{1}{k}\sum_{j=1}^{k}\left(\log H_{j,m} - \log H_{k+1,m}\right). \tag{3}$$

A generalised Hill estimator is more versatile to use than the Hill estimator. A positive slope on the plot indicates a heavy tail of the Pareto type (e.g. a Burr has a Pareto type tail). A slope of zero indicates a light tail associated with distributions like the exponential. A negative slope indicates a bounded tail associated with say, a uniform distribution.

3.2 Exponential distribution

The probability distribution function (PDF) and the cumulative distribution (CDF) of the exponential distribution is:

$$f(y, \lambda) = \lambda e^{-\lambda y}, \tag{4}$$

$$F(y, \lambda) = 1 - e^{-\lambda y}, \tag{5}$$

where y represents the currency returns and $\lambda > 0$ is the scale /rate parameter.

The Maximum Likelihood Estimation (MLE) of the rate parameter λ is given in the following theorem:

Theorem 1

If Y is exponentially distributed with the PDF given in equation (1) where $\lambda > 0$ then, the MLE of λ is given as:

$$\hat{\lambda} = \frac{m}{\sum_{i=1}^{m} y_i}, \tag{6}$$

m is the sample size.

Risk Measures for the Exponential Distribution

The formula to compute Value at Risk (VaR) and Expected Shortfall (ES) for a tail probability p, and total sample size m.

$$VaR_p(y) = -\frac{1}{\hat{\lambda}}\log(1-p), \tag{7}$$

$$ES_p(y) = -\frac{1}{p\hat{\lambda}}[\log(1-p)\,p - p - \log(1-p)], \tag{8}$$

for $y > 0$, $0 < p < 1$, and _ $\hat{\lambda} > 0$, the scale/rate parameter.

3.3 Weibull distribution.

The probability distribution function (PDF) and the cumulative distribution (CDF) of the Weibull distribution is:

$$f(y,\beta,K) = \frac{K}{\beta}\left(\frac{y}{\beta}\right)^{K-1} e^{-\left(\frac{y}{\beta}\right)^{K}}, \tag{9}$$

$$F(y,\beta,K) = 1 - e^{-\left(\frac{y}{\beta}\right)^{K}}, \tag{10}$$

where y represents the currency returns, $K > 0$ is the shape parameter, and $\beta = \frac{1}{\lambda} > 0$ is the scale parameter. When $K = 1$, the Weibull distribution reduces to the exponential distribution.

The Maximum Likelihood Estimation (MLE) of the scale/rate and shape parameter of the Weibull distribution is given in the following theorem:

Theorem 2

If Y follows a Weibull distribution with the PDF given in equation (9) where $K > 0$ and $\beta > 0$ then, the MLE estimates of K and β are given as:

$$\hat{\beta} = \frac{m}{\sum_{i=1}^{m} y_i{}^{K}}, \tag{11}$$

where K can be computed by solving numerically the following equation:

$$m + K\sum_{i=1}^{m} \ln y_i = \frac{mK\sum_{i=1}^{m} y_i^K \ln y_i}{\sum_{i=1}^{m} y_i^K}. \tag{12}$$

Risk Measures for the Weibull Distribution

The formula to compute Value at Risk (VaR) and Expected Shortfall (ES) for a tail probability p, and total sample size m.

$$VaR_p = \hat{\beta}[-\log(1-p)]^{\frac{1}{\hat{K}}}, \tag{13}$$

$$ES_p = \frac{\hat{\beta}}{p}\gamma\left[1+\frac{1}{\hat{K}}, -\log(1-p)\right], \tag{14}$$

for $y > 0, 0 < p < 1, \hat{K} > 0$ *is* the shape parameter, and $\hat{\beta} > 0$ is the scale parameter. Where $\gamma[\alpha, y]$ is the upper incomplete gamma function defined by $\gamma[\alpha, y] = \int_0^y t^{x-1}e^{-t}\mathrm{dt}.$

3.4 The Burr Distribution

The PDF and CDF of the Burr distribution as presented by [34] are as follows:

$$f(y; a, c, \beta) = \frac{ac}{\beta}\left(\frac{y}{\beta}\right)^{a-1}\left(1+\left(\frac{y}{\beta}\right)^{a}\right)^{-(c+1)}, \tag{15}$$

$$F(y; a, c, \beta) = 1 - \left(1+\left(\frac{y}{\beta}\right)^{a}\right)^{-c}. \tag{16}$$

The Burr has two shape parameters, a > 0 and c > 0. $\beta = \left(\frac{1}{\lambda}\right)$ > 0 represents the rate parameter. Parameters are obtained using the MLE method.

Risk Measures for the Burr distribution

Using the formula,

$$\mathrm{VaR}_p(y) = F_p^{-1}(y), \tag{17}$$

we obtain the following risk measures at p level of significance

$$\mathrm{VaR}_p = \beta\left[-1-(1-p)^{-\frac{1}{c}}\right]^{\frac{1}{a}}, \tag{18}$$

$$ES_p = \frac{1}{p}\int_0^p \beta\left[-1-(1-t)^{-\frac{1}{c}}\right]^{\frac{1}{a}} dt. \tag{19}$$

3.5 The Anderson-Darling (A-D) Test

The Anderson-Darling test for goodness of fit is used to confirm if the data sample comes from a specific distribution [35]. The null hypothesis claims that the data follow a specified distribution. The A-D test statistic A^2 is defined as:

$$A^2 = -m - \sum_{j=1}^{m}\frac{(2j-1)}{m}\left[\ln F(y_j) + \ln\left(1 - F(y_{m+1-j})\right)\right], \tag{20}$$

where $F(y_j)$ is the specified theoretical CDF and $\{y_1 < \cdots < y_m\}$ is the ordered sample data.

3.6 Information Criterion

The information criterion measures tell us how well the model fits the data. It uses the likelihood function and the number of parameters to strike a balance between model fit and complexity. In this study, two information criteria are used, namely, the Akaike information criterion (AIC) and the Bayesian information criterion (BIC).

The Akaike information criterion (AIC)

Developed by [36], AIC estimates the relative quality of different models while penalising for model complexity. The AIC formula is:

$$AIC = -2 \log L + 2p, \tag{21}$$

where L is the likelihood function and p is the number of parameters. It encourages the selection of models that fit the data well but avoids excessive complexity, preventing overfitting.

The Bayesian Information Criterion (BIC)

Developed by [37], the BIC is similar to AIC though it introduces a stronger penalty for model complexity compared to AIC.

$$BIC = -2 \log L + 2p \times \log m, \tag{22}$$

where the term $\log(m)$ represents the logarithm of the sample size (m).The penalty term scales with the sample size.

4 Results

Exchange rate data used in this research were obtained from the www.investing.com/currencies website, which serves the financial industry. R [38], RStudio [39], VaRES [40], and fitdistrplus [41], DescTools [42] statistical packages were used for the analysis. The statistical parent distribution models were fitted with the daily adjusted closing prices from 1 January 2015 to 30 June 2021. The returns were calculated and used for modelling. The formula for the log returns is $y_t = \log\left[\frac{S_t}{S_{t-1}}\right]$, where S_t and S_{t-1} are the current and the previous day's adjusted exchange rate values, respectively. Gains (positive returns) and losses (negative returns) were separated and fitted independently of each other. Negative returns are transformed to positive losses using the formula; $l_t = -1 \times y_t$ for negative y_t values only.

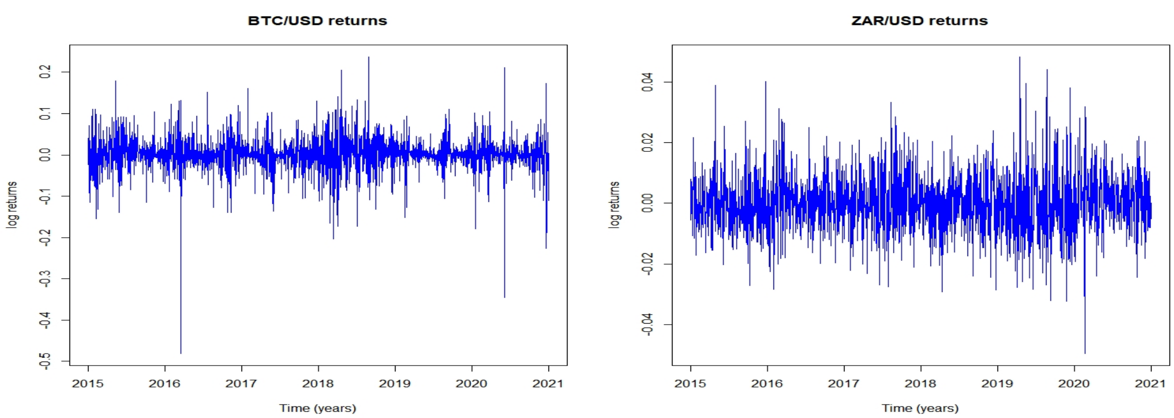

Fig. 1: Time series graph of daily returns series for BitCoin and the Rand.

Table 1: Descriptive Statistics.

	BitCoin Gains	Returns Losses	Rand Gains	Returns Losses
Observations	1080	1275	822	869
Mean	0.0270881	0.002664	0.008325	0.007632
Maximum	0.4809040	0.2372	0.04897	0.04987
Minimum	0.0000506	0.00004494	0.00000713	0.00000742
Skewness	4.241746	2.265626	1.555208	1.462761
Kurtosis	39.53633	7.57467	3.564153	3.818191
J-B Test (p-value)	<0.0001	<0.0001	<0.0001	<0.0001
ADF Test (p-value)	<0.001	<0.001	<0.001	<0.001
PP Test (p-value)	<0.001	<0.001	<0.001	<0.001

Table 1 shows the descriptive statistics, normality, and stationarity tests for the daily returns of the BitCoin and the Rand. The p-values of the normality and stationarity tests are also presented in Table 1.

In both cases, the exchange rates' gains and losses, positive skewness is detected (4.241746 and 2.265626) for Bitcoin's gains and losses respectively, and (1.555208 and 1.462761) for Rand's gains and losses respectively. Both currencies' gains and losses exhibit excess kurtosis (39.53633 and 7.57467) for Bitcoin's gains and losses, and (3.564153 and 3.818191) for Rand's gains and losses respectively.

The Jarque–Bera rejects the null hypothesis of normality at the 5% level of significance, suggesting the use of heavier-tailed distributions in analysing the above-mentioned return series. The preliminary diagnosis suggests that the skewed and heavy-tailed statistical parent distributions are the most relevant to fit for this type of data since they account for skewness, excess kurtosis, and hence the heavier tails (Hakim et al 2018).

The augmented Dickey-Fuller test (ADF) and Phillips-Perron test (PP) show that, at the 1% level of significance, the null hypothesis of a unit root is rejected, and it can be concluded that both exchange rate return series are stationary.

4.1 Fitting distributions to BitCoin/USD

The Hill plot and the generalised Q-Q plot are used to classify the nature of the tails for the BitCoin gains.

4.1.1 Fitting distributions to BitCoin gains

Figure 2 suggests a positive γ value in the Hill plot of the Bitcoin gains, but this estimate is often associated with bias.

Hill Plot and the generalised Q-Q plot

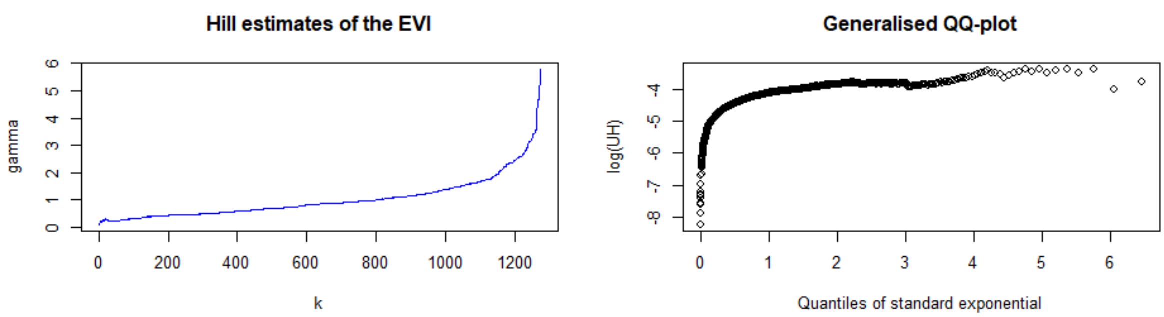

Fig. 2: The Hill plot (left) and the generalised Q-Q plot (right) for BitCoin gains (both plots estimate gamma).

The gradient of γ in the upper tail of the generalised Q-Q plot is approximately 0 implying the distribution of the data lies in the Gumbel domain, but it could also be viewed as slightly positive. Therefore, the distributions that could possibly give a good fit are the exponential, Weibull, and the Pareto type tail if the gradient in the upper tail is deemed positive. The Burr distribution would be ideal in this instance of a positive slope.

Exponential Distribution

The BitCoins gains exponential distribution parameter estimate is $\hat{\lambda} = 36.91659$ with standard error =1.123336.

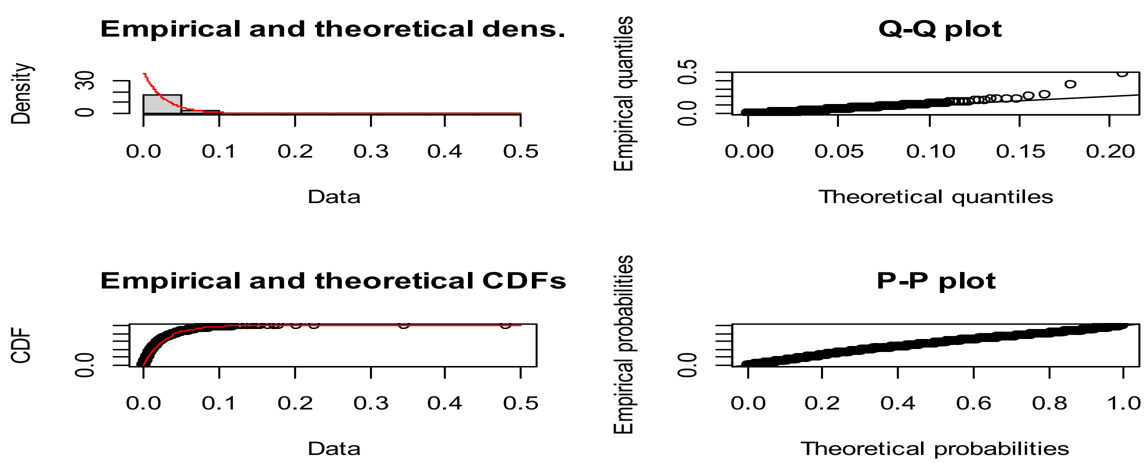

Fig. 3: Diagnostic plots for the BitCoin gains fitted using exponential distribution.

In Figure 3, the Q-Q plot's upper tail is convex implying that the distribution of the right tail of the BitCoin gains is heavier-tailed than any exponential distribution. The density plot does trace well the histogram confirming that the exponential distribution might be a good distribution for the main body of the data, but not in the extreme upper tail of the distribution. The risk needs to be modelled accurately in the tails as well to avoid problems associated with wrong models leading to the GFC [43].

Weibull Distribution

The BitCoin gains Weibull distribution parameter estimates of the shape and the scale are $\widehat{K} = 0.88001364$ and $\hat{\beta} = 0.02530289$ with standard errors of 0.0202735504 and 0.0009200785 respectively. Both errors are very small signifying that the parameters are significant. The $\widehat{K} < 1$ signifying that the right tail of the gains is heavier than implied by any exponential distribution. Figure 4 gives the diagnostic plots.

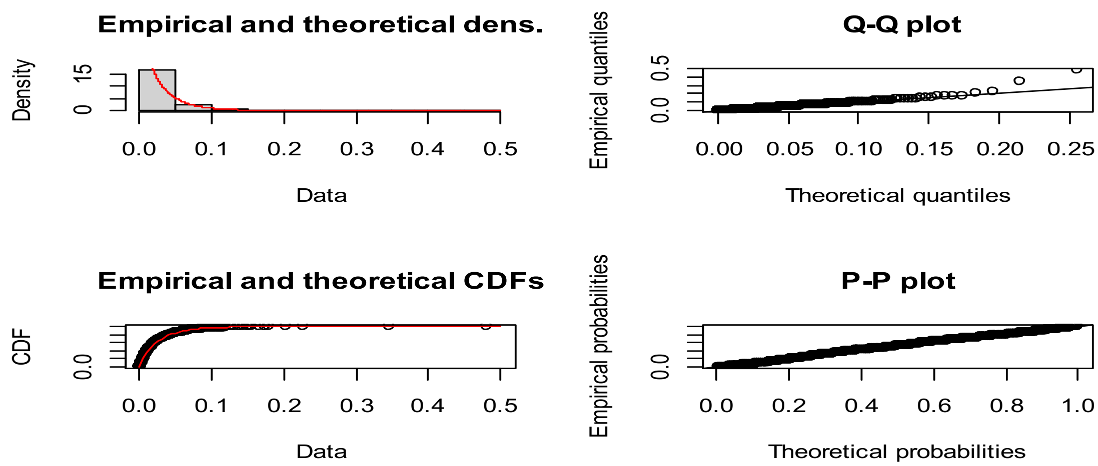

Fig. 4: Diagnostic plots for the BitCoin gains fitted using the Weibull distribution.

The Q-Q plot's upper tail is convex implying that the distribution of the BitCoin gains is heavier-tailed than any Weibull distribution. The density plot does trace well the histogram confirming that the Weibull distribution might be a good distribution, except for the heavier upper tail.

Burr Distribution

The BitCoin gains Burr distribution parameter estimates are: shape1 $\hat{a} = 4.9956152$, shape2 $\hat{c} = 0.9948668$ and the scale is $\hat{\beta} = 9.1793483$ respectively. Figure 5 gives the diagnostic plots.

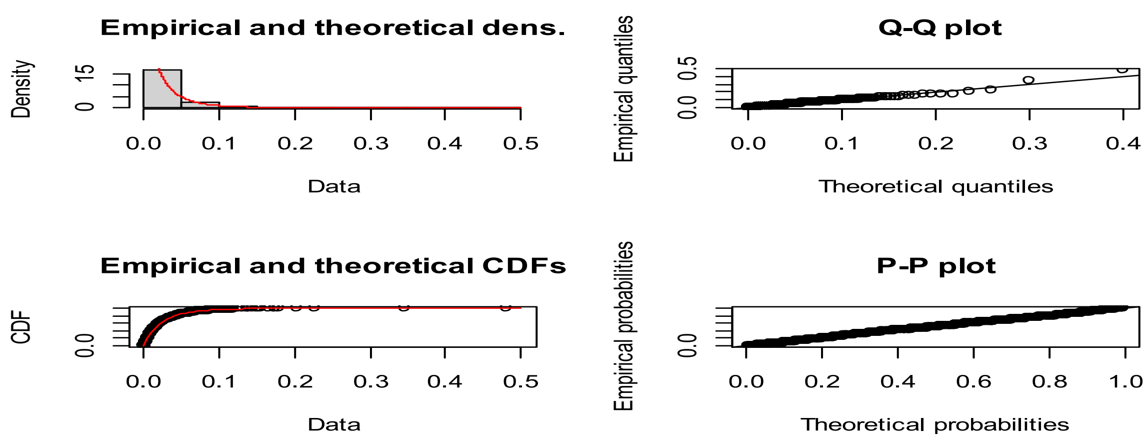

Fig. 5: Diagnostic plots for the BitCoin gains fitted using Burr distribution.

The Burr Q-Q plot's upper tail is approximately linear implying that the Burr distribution is a good fit for the BitCoin gains. The Burr fit to the upper tail is better than any of the distributions considered so far.

4.1.2 Fitting distributions to BitCoin losses

The Hill plot and the generalised Q-Q plot are used to classify the nature of the tails for the BitCoin losses. Figure 6 suggests a positive γ value in the Hill plot, but this estimate is often associated with bias.

The generalised Q-Q plot and Hill plot

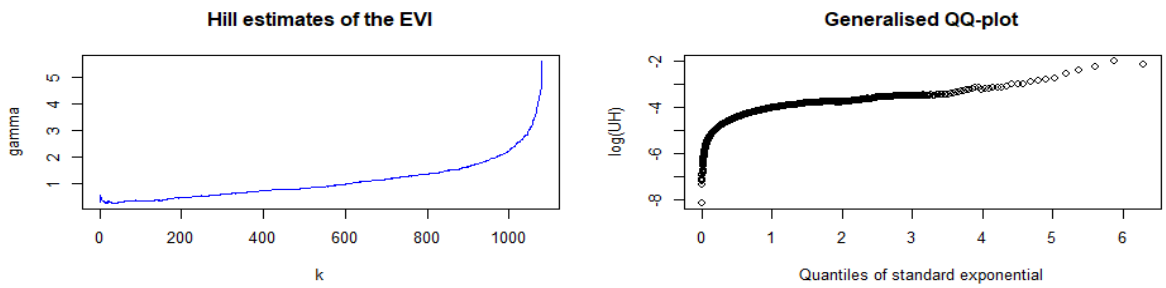

Fig. 6: The Hill Plot (left) and the generalised Q-Q plot (right) for BitCoin losses.

The generalised Q-Q plot gradient γ is more positive than the gains. This implies that the data belongs to a heavy-tailed Fréchet domain. The possible statistical parent distributions in the Fréchet domain are the Pareto type distributions, such as the Burr distribution. The lighter tailed distributions of the exponential and Weibull will still be fitted for comparison.

Exponential distribution

The BitCoin exponential distribution losses parameter estimate is $\hat{\lambda} = 37.53324$ with standard error =1.051141.

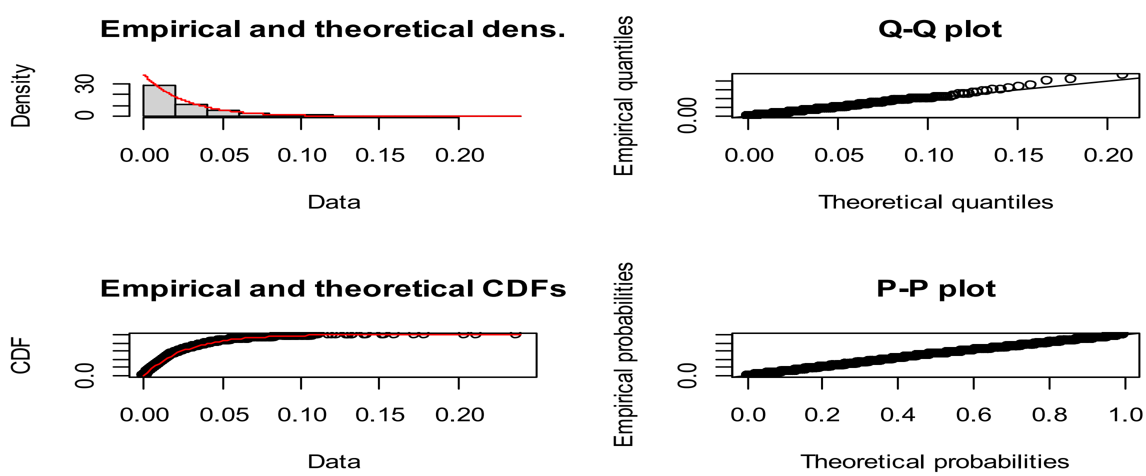

Fig. 7: Diagnostic plots for the BitCoin losses fitted using exponential distribution.

The Q-Q plot's upper tail has few observations deviating from the line implying that the distribution of the BitCoin losses could be slightly heavier-tailed than any exponential distribution. The density plot does trace well the histogram confirming that the exponential distribution might be a good candidate, save for the right or upper tail.

Weibull Distribution

The BitCoin losses Weibull distribution parameter estimates of the shape and the scale are $\hat{K} = 0.96448535$ and $\hat{\beta} = 0.02620907$ with standard errors of 0.0207903254 and 0.0007994854 respectively. Both errors are very small, signifying that the parameters are significant. The $\hat{K} < 1$, signifying that the right tail is heavier than an exponential distribution tail.

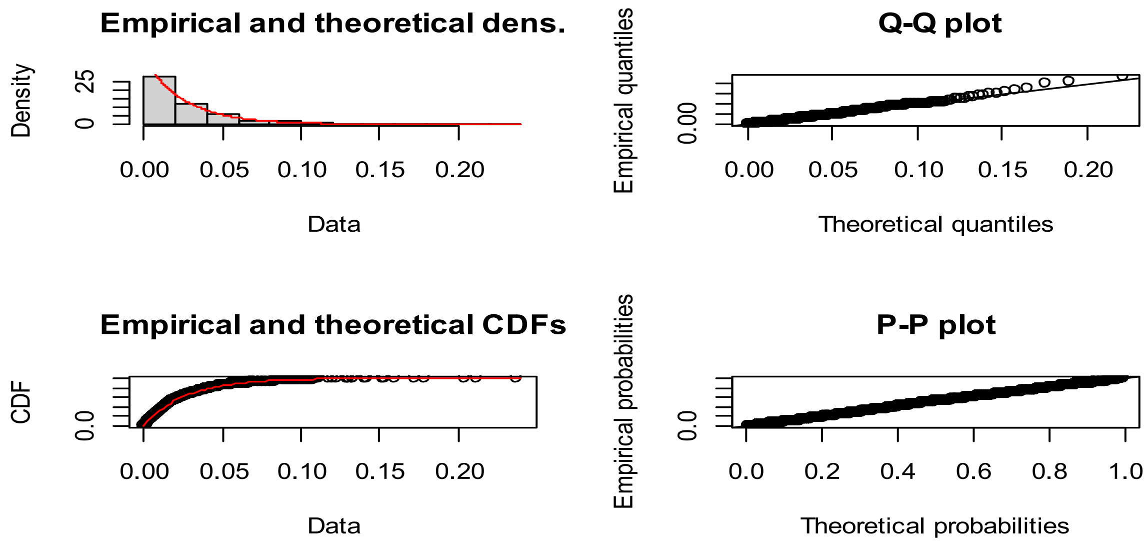

Fig. 8: Diagnostic plots for the BitCoin losses fitted using Weibull distribution.

The Q-Q plot's upper tail has few observations deviating from the line implying that the distribution of the BitCoin losses could be following the Weibull distribution, save for some extreme upper tail returns. The density plot does trace well the histogram confirming that the Weibull distribution might be a good candidate.

Burr distribution

The BitCoin losses Burr distribution parameter estimates are: shape1 parameter, $\hat{\alpha} = 10.287301$, shape2 parameter $\hat{c} = 1.022257$, and the scale parameter is $\hat{\beta} = 4.196602$ respectively.

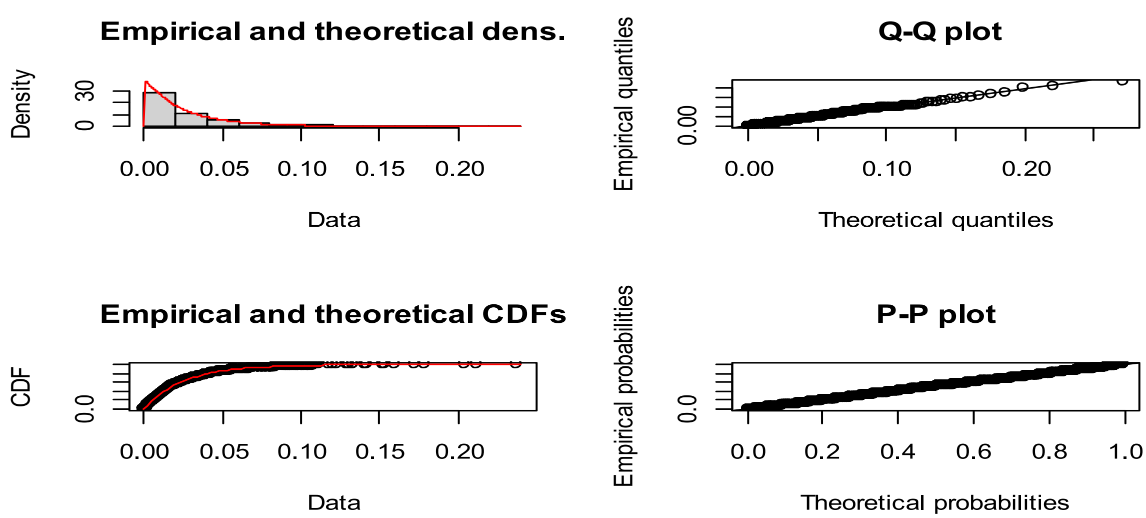

Fig. 9: Diagnostic plots for the BitCoin losses fitted using Burr distribution.

The Burr Q-Q plot's upper tail is approximately linear implying that the Burr distribution is a good fit for the BitCoin losses. The density plot traces well the histogram suggesting that the Burr distribution might be the ideal distribution.

4.2 Fitting distributions to Rand/USD

The Hill plot and the generalised Q-Q plot are used to classify the nature of the tails for the Rand gains. Figure 10 suggests a positive γ value in the Hill plot, but this estimate is often associated with bias.

4.2.1 Fitting distributions to Rand gains

Hill plot and generalised QQ-plot

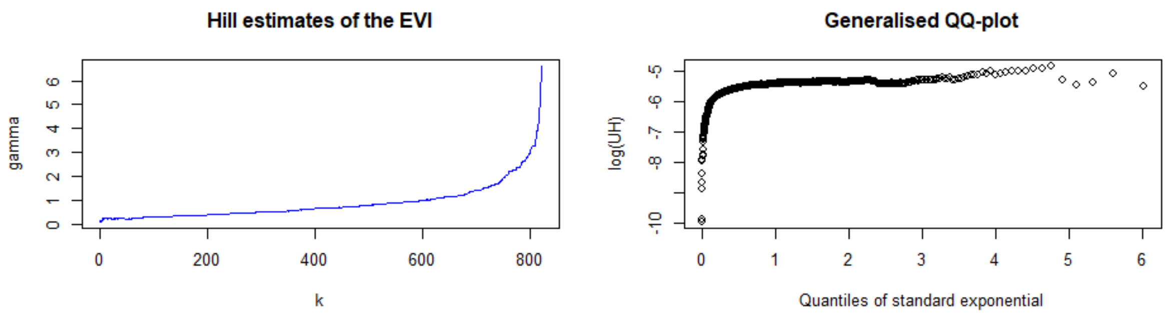

Fig. 10: Hill plot (left) and generalised Q-Q plot (right) of the Rand gains.

The generalised QQ plot's upper tail appears to be constant, indicating that $\gamma=0$. It could also be seen as slightly positive. This implies that the distribution of the Rand gains falls within the Gumbel domain if $\gamma=0$, and distributions like exponential distribution, Weibull distribution etc could be suitable fit for the data. If $\gamma>0$, then the Burr would give a better fit.

Exponential distribution

The Rand gains exponential distribution parameter estimate is $\hat{\lambda} = 120.53$ & standard error =4.189.

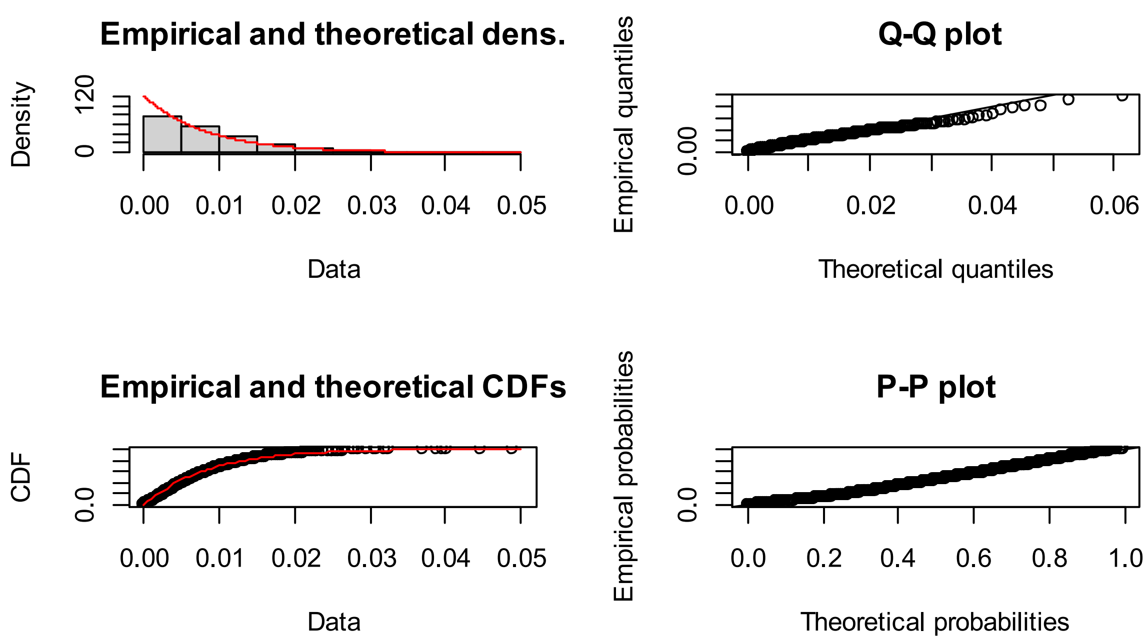

Fig. 11: Diagnostic plots for the Rand gains fitted using exponential distribution.

The density plot appears to be a good estimate of the histogram data for the Rand gains. The probability (P-P plot) and quantile (Q-Q plot) plots are almost linear, confirming that the exponential distribution could be a good fit.

Weibull distribution

The Rand gains Weibull distribution parameter estimates of the shape and the scale are $\hat{K} = 1.162302557$ and $\hat{\beta} = 0.008763113$ with standard errors of 0.0319970548 and 0.0002656102 respectively. Both errors are very small, signifying that the parameters are significant.

The $\hat{K} > 1$, signifying that the right tail is lighter than an exponential distribution tail.

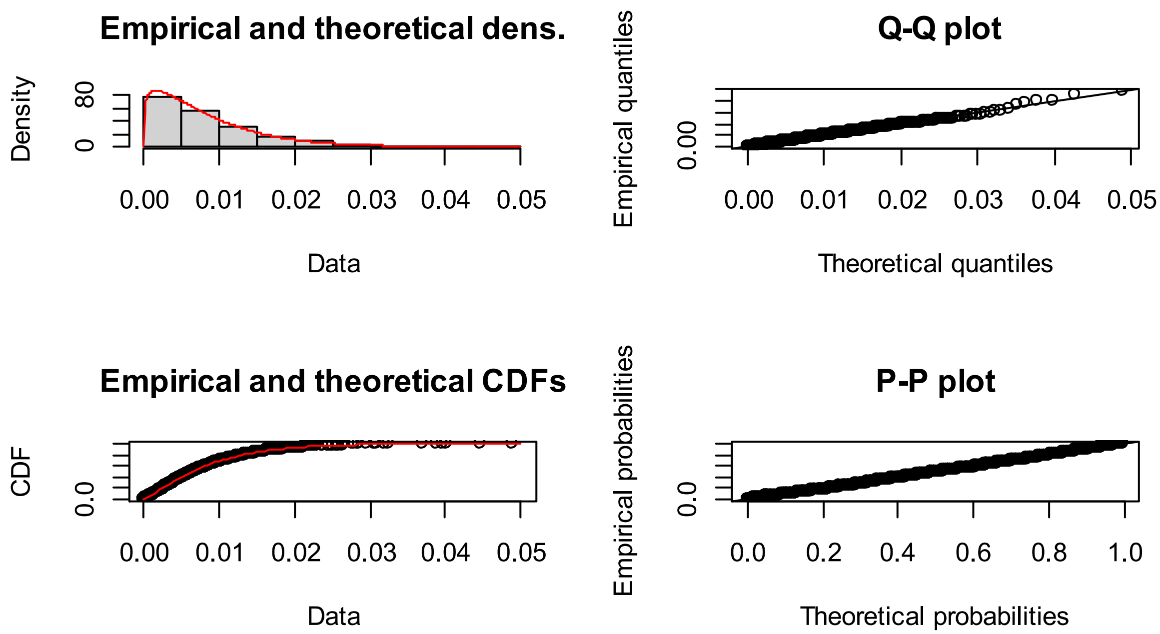

Fig. 12: Diagnostic plots for the Rand gains fitted using Weibull distribution.

The density plot appears to be a good estimate of the histogram data for the Rand gains. The probability (P-P plot) and quantile (Q-Q plot) plots are almost linear, confirming that the Weibull distribution could be a good fit. There are no deviations towards the upper tail in the Q-Q plot.

The Burr distribution

The Rand gains Burr distribution parameter estimates are: shape1 parameter, $\hat{\alpha} = 473.8418958$, shape2 parameter $\hat{c} = 1.1637584$, and the scale parameter is $\hat{\beta} = 0.5738171$.

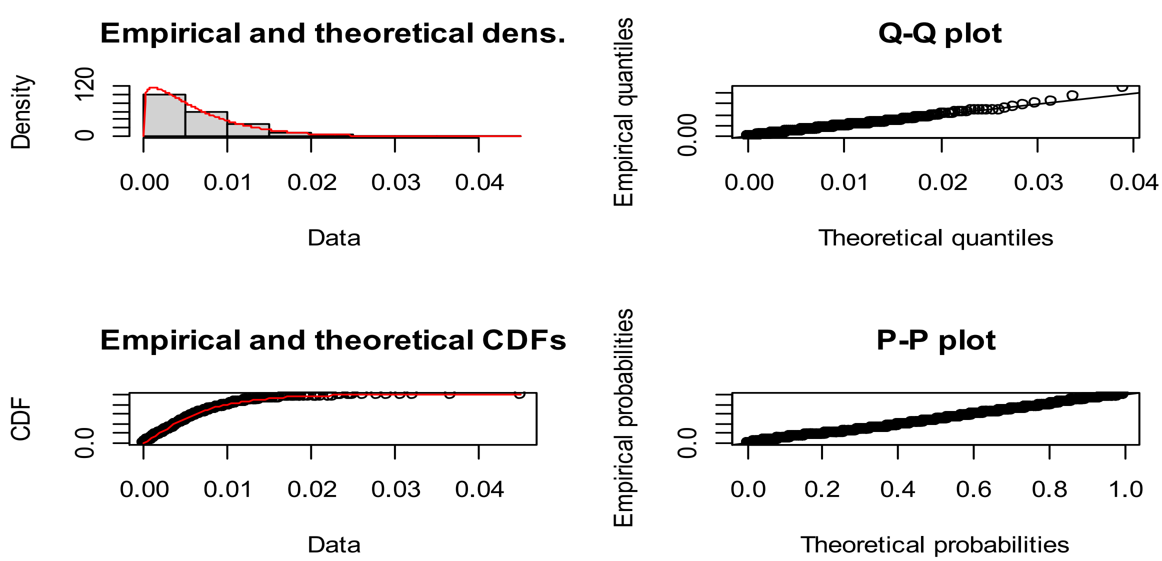

Fig. 13: Diagnostic plots for the Rand gains fitted using Burr distribution.

The Burr Q-Q plot's upper tail is approximately linear implying that the Burr distribution is a good fit for the Rand gains. The density plot appears to be a good estimate of the histogram data for the Rand gains.

4.2.2 Fitting distributions to Rand losses

The Hill plot and the generalised Q-Q plot are used to classify the nature of the tails for the Rand losses. Figure 14 suggests a positive γ value in the Hill plot, but this estimate is often associated with bias.

Hill plot and generalised Q-Q plot

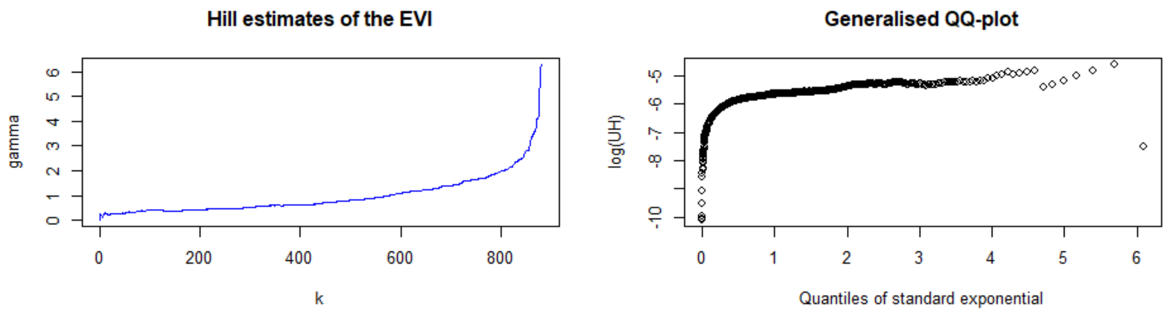

Fig. 14: Hill plot (left) and generalised Q-Q plot (right) for the Rand losses.

The generalised Q-Q plot appears to be almost flat, indicating that $\gamma \approx 0$. This implies that the distribution of Rand losses falls within the Gumbel domain distributions. The candidate distributions would be the exponential and the Weibull.

Exponential distribution

The Rand losses exponential distribution parameter estimate is $\hat{\lambda} = 131.0335$ with standard error $= 4.445005$.

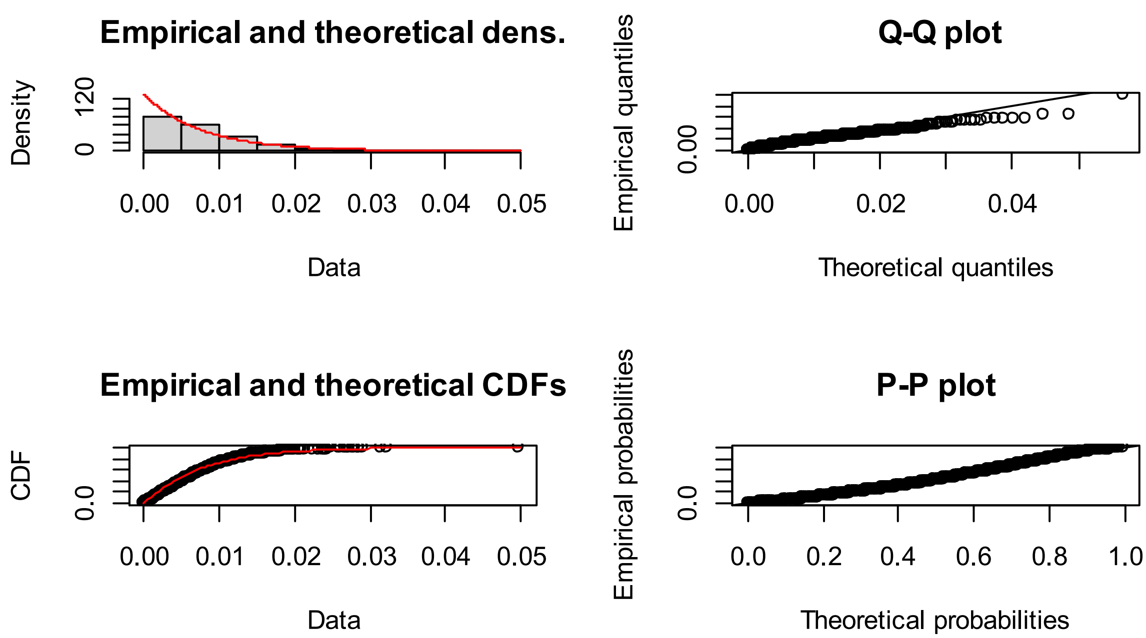

Fig. 15: Diagnostic plots for the Rand losses fitted using exponential distribution.

The Q-Q plot's upper tail deviates and goes below the line, implying that the distribution of the Rand losses is lighter-tailed than any exponential distribution. This suggests a Weibull distribution with $\hat{K} > 1$ would give the best fit. The density plot does trace well the histogram confirming that the exponential distribution might be the ideal distribution, save for the lighter right or upper tail.

Weibull distribution

The Rand losses parameter estimates of the shape and the scale are $\hat{K}=1.249641445$ and $\hat{\beta}=0.008174867$ with standard errors of 0.0335429195 and 0.0002224204 respectively. Both errors are very small, signifying that the parameters are significant. The $\hat{K}>1$, signifying that the right tail is lighter than an exponential distribution tail.

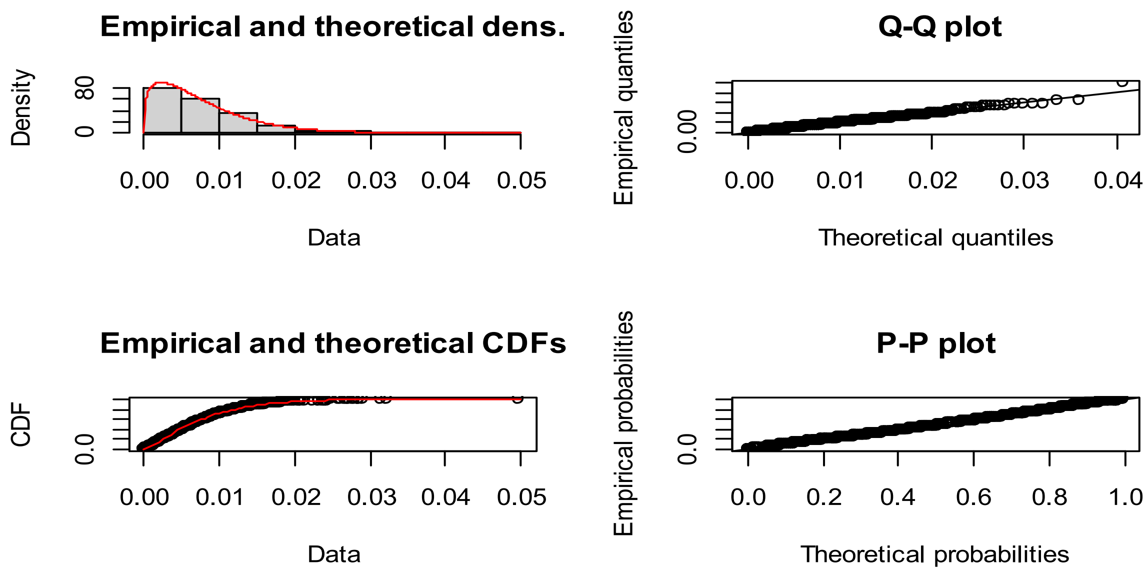

Fig. 16: Diagnostic plots for the Rand losses fitted using Weibull distribution.

The density plot appears to be a good estimate of the histogram data for the Rand losses. The probability (P-P plot) and quantile (Q-Q plot) plots are almost linear, confirming that the Weibull distribution could be a good fit. There are no deviations towards the upper tails in the Q-Q plot.

Burr Distributions

The Rand losses parameter estimates are: shape1 parameter $\hat{a}=579.5304410$, shape2 parameter $\hat{c}=1.2506426$, and the scale parameter is $\hat{\beta}=0.7559712$.

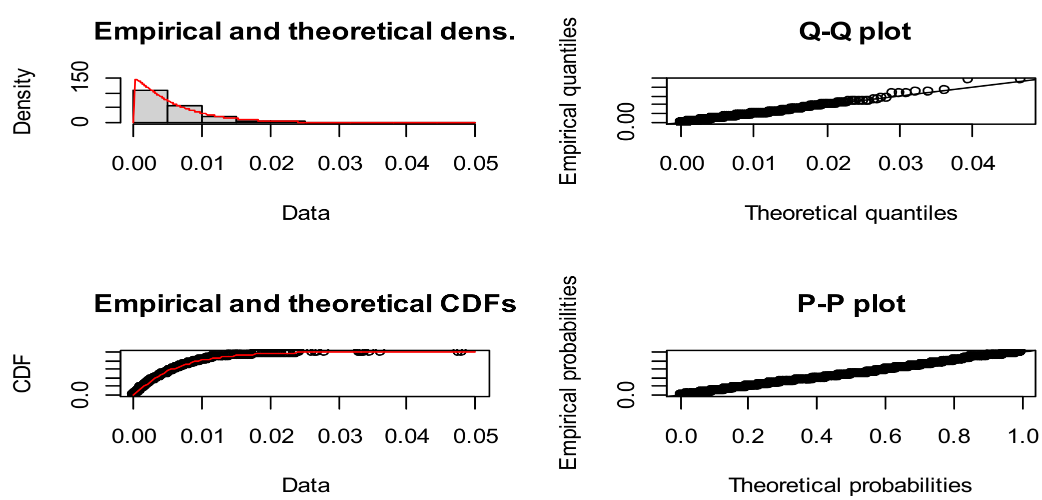

Fig. 17: Diagnostic plots for the Rand losses fitted using Burr distribution.

The density plot appears to be a good estimate of the histogram data for the Rand losses. The probability (P-P plot) and quantile (Q-Q plot) plots are almost linear, confirming that the Burr distribution could be a good fit.

4.3 Model Adequacy

Table 2: Anderson Darling Test.

	A-D Test	BitCoin		Rand	
		Gains	**Losses**	**Gains**	**Losses**
Exponential	**Statistic**	9.97	1.6662	6.9435	14.186
	p-value	<0.00001	0.1413	0.0003527	<0.00001
Weibull	**Statistic**	1.6298	0.8313	0.49653	0.72325
	p-value	0.1483	0.459	0.7503	0.5397
Burr	**Statistic**	0.79483	0.4243	0.50136	0.7279
	p-value	0.4848	0.8243	0.7454	0.5359

Table 2 present the Anderson-Darling statistic and their respective p-values of the selected distribution. All p-values that are greater than 0.05 imply that the models are adequate for the data sets. Based on the results in Table 2, all models are adequate for the data sets using the Anderson-Darling test except for the exponential distribution. The exponential distribution is inadequate in all datasets, except for BitCoin losses.

4.4 Information Criterion

The AIC and BIC will now be used to confirm the best fitting distributions, taking cognisance of the data behaviour in the tails of the distributions.

Table 3: AIC and BIC.

	Information Criterion	BitCoin		Rand	
		Gains	**Losses**	**Gains**	**Losses**
Exponential	**AIC**	-5632.708	**-6692.329**	-6226.218	-6733.538
	BIC	-5627.723	**-6687.178**	-6221.507	-6728.771
Weibull	**AIC**	-5663.818	-6693.207	**-6252.158**	**-6793.803**
	BIC	-5653.849	-6682.906	**-6242.734**	**-6784.268**
Burr	**AIC**	**-5676.795**	-6694.966	-6250.08	-6791.738
	BIC	**-5661.84**	-6679.514	-6235.945	-6777.435

Using the AIC and BIC presented in Table 3, the best fit statistical parent distributions for the BitCoin are Burr for the gains and the exponential for the losses. The Bitcoin gains follow the heavier tailed Burr distribution, but the losses follow the lighter-tailed exponential distribution. Bitcoin gains are therefore more volatile giving the potential for large gains when compared to their losses.

The Rand gains and losses both follow a Weibull distribution. Both gains and losses of the Rand have $\hat{K} > 1$, implying that they possess tails that are less heavy tailed than the exponential tail. The tail behavior characteristic of these gains and losses follows the same distribution.

The BitCoin gains have heavier tails than the Rand gains, and hence BitCoin gains are more volatile than the Rand gains. The BitCoin losses have heavier tails than the Rand losses, and hence BitCoin losses are more volatile than the Rand losses. BitCoin is riskier than the Rand on both upside risk and downside risk.

These selected models were used to simulate 1000 returns and to compute out of sample VaR and ES statistics.

4.5 Risk Measures

Table 4 gives out sample VaR and ES measures.

Table 4: VaR and ES.

Currency		VaR		ES	
		95%	**99%**	**95%**	**99%**
BitCoin	Gains (Burr)	0.0898	0.1701	0.1418	0.2492
	Losses (Exponential)	0.0831	0.1188	0.1086	0.1435
Rand	Gains (Weibull)	0.0226	0.0331	0.0289	0.0398
	Losses (Weibull)	0.0197	0.0282	0.0248	0.0334

The values obtained from the VaR and ES models, as presented in Table 4, reveal that both at 95% and 99% confidence levels, the BitCoin statistic is higher than the Rand statistic i.e. the BitCoin gains VaR of $0.0898 and $0.1701 are greater than the Rand's $0.0226 and $0.0331 per dollar invested. This implies that BitCoin has a greater upside risk and hence rendering it riskier than the Rand. A similar conclusion can be drawn using the ES values for the gains. The BitCoin statistics are higher than the Rand statistics i.e. the BitCoin gains ES of $0.1418 and $0.2492 at 95% and 99%, are greater than the Rand's $0.0289 and $0.0398 respectively per dollar invested.

On the negative returns (losses), the BitCoin VaR statistics for losses of $0.0831 and $0.1188 are greater than Rand's $0.0197 and $0.0282 per dollar invested at both 95% and 99% confidence levels respectively. This implies that BitCoin has a greater downside risk and hence rendering it riskier than the Rand.

Comparing losses to gains for BitCoin, the values presented in Table 4, shows that the upside risk (likelihood of potential gains) is greater than the downside risk (the prospects of potential losses). A similar conclusion can be drawn for the Rand, the upside risk is greater than the downside risk.

5 Discussion

The study applied statistical parent distributions when fitting them to gains and losses of the two currencies' returns: namely the BitCoin and the Rand, both measured against the USD. The Hill and the generalised Q-Q plots confirm the presence of light to heavy tails in the currencies. The approach adopted in this paper of classifying the tails of the data set into 4 groups with increasing risk ((1) Weibull with parameter $K > 1$ (2) exponential (3) Weibull with parameter $K < 1$, and (4) Burr) to carter for varying degrees of risk is a novel approach. This is especially so in the wake of the GFC problems which caused miscalculation of risks when using the normal distribution-based models [43]. The normal distribution grossly underestimates extreme risks.

The Anderson-Darling tests model adequacy of the heavy tailed distribution in BitCoin and Rand sample data sets and confirmed the presence of heavy tails in currency data. A similar conclusion was drawn by [28], [23]; [27].

The best-fit distribution for BitCoin is the Burr and the exponential for gains and losses respectively, while for Rand returns, the Weibull distribution with tails even less heavy than the exponential tail is suggested, for both gains and losses. This implies that the BitCoin has heavier tails than the Rand, confirming that the BitCoin is more volatile than the Rand

The selected models were used to compute the value at risk and expected shortfall, and to further compare the riskiness of the two currencies returns. Table 4 summarises the VaR and ES statistics and it can be concluded that the BitCoin is riskier than the Rand. This is consistent with findings by [25] who noted that the BitCoin is more volatile than the G10 countries or fiat currencies.

This study concentrates on fitting the tails of the data at the expense of the main body of the data. However, the interest and risk are in these tails of the data. The generalised extreme distribution (GEVD) and the generalised Pareto distribution (GPD) do ignore the main body of this data set altogether by only working with extremes in the data set. These models will be considered in another study.

6 Conclusion

In this study, statistical parent distribution-based risk measures (VaR and ES) show that BitCoin is riskier than the Rand. Investors who convert their savings from the Rand to the BitCoin are exposed to greater financial risk. [16] concluded that BitCoin is less volatile when compared to other cryptocurrencies, giving an impression of a safe currency. [17] findings on the presence of heavy tails in the financial data are confirmed in this study using BitCoin and South African Rand.

While the data set used in this study ranges from 2015 to 2021, the findings on the presence of heavy tails can be generalised considering that several authors [28], [23]; [27] also found the presence of heavy tails to be prevalent in cryptocurrency data. This information is useful to local foreign currency traders and investors who need to fully appreciate their risk exposure when they convert their savings or investments to BitCoin from the South African currency, the Rand. This is particularly important when the market enters a turbulent time. The conclusion is BitCoin is riskier than the South African Rand, a developing country's currency.

Conflicts of Interest Statement

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Ethics Statement: *This research did not require ethical approval. Data Availability Statement Data associated with the manuscript is public and has been referenced appropriately.*

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